

# **Local and Global Pareto Dominance in Multi-Populations Evolution applied to Optimization of Composites**

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## **Abstract**

The goal of the proposed multi-objective optimization algorithm is the search of the Pareto-optimal front keeping good population diversity using a hierarchical genetic algorithm with co-evolution of multi-populations. A self-adaptive genetic search incorporating Pareto dominance and elitism is presented. Two concepts of dominance are used: the first one denoted by local non-dominance is performed at the isolation stage of populations and the second one called global non-dominance is performed on the age structured population. The age structured population is used to store the ranked solutions with the aim of obtaining the Pareto front after the evolutionary process. The application to composite structures with the compromise between minimum strain energy and minimum weight is presented.

**Keywords:** multi-objective, co-evolution, non-domination, self-adaptive, age control, composites.

## **1 Introduction**

Structural applications of composite materials have been growing due to their excellent specific stiffness, low weight, and reduced energy consumption. One approach for decreasing costs in composite structures is to adopt a hybrid construction where expensive and high-stiffness materials are used together with inexpensive and low-stiffness material. The optimization problem of topology associated with material/stacking sequence design of hybrid composites is very complex when sizing variables, as ply angle and layer thickness are simultaneously considered. Furthermore, since the balance between weight/cost and stiffness is important in hybrid laminates construction the use of multi-objective design procedures are necessary. Only a few researchers have presented multi-objective design approaches for hybrid composites. In particular, Rahul et al. [1] minimize the

cost/weight of hybrid laminates while maximizing the strength under impact loads. Baier et al. [2] combine strength and manufacturing requirements to optimise structures using metal parts and carbon composites. Ghiasi et al. [3] proposed a model for structural and manufacturing simultaneous design of composites. Also, Pelletier and Vel [4] proposed two models. In the first model the objectives are to maximize the loading capacity and minimize the mass, and in the second one the objectives are to maximize the axial and hoop rigidities and minimize the mass of a graphite/epoxy pressure vessel. Recently Irisarri et al. [5] presents a multi-objective optimization strategy of composite stiffened panels. The global optimization is addressed as: approximation of objective functions to limit calculation costs and decomposition of design domain using iteratively the variables associated with the skin and the stiffeners.

In general, a multi-objective optimization algorithm leads to a set of optimal solutions known as Pareto-optimal solutions. The reason for this is that no solution can be considered better than other relatively to the objective functions. The principal goal of a multi-objective optimization algorithm is the search of the global Pareto-optimal front keeping population diversity in the Pareto-optimal solutions. In this paper such a challenge is performed using a multi-objective hierarchical genetic algorithm (MOHGA) with co-evolution of multi-populations [6-8].

## 2 Optimization of hybrid composites

The multi-objective optimization of composite structures is defined here as a bi-criterion problem. The hybrid construction used in composite structures deals with the compromise between minimum strain energy and minimum weight/cost, exploring the alternative optimal design solutions.

Cost reduction and use of alternative optimal design solutions are important challenges for designers concerned with sustainable energy consumption. The optimization problem is formulated attempting to minimize the weight/cost  $W(\mathbf{u}, \boldsymbol{\pi})$  and the strain energy  $U(\mathbf{u}, \boldsymbol{\pi})$  of the structure, subject to constraints related to structural integrity [6-8]. The design variables  $\mathbf{u}$  and  $\boldsymbol{\pi}$  are associated with the sizing and material distribution, respectively. The constraints are imposed on the critical load factor,  $\lambda_{crit}$ , and on the critical displacement,  $d_{crit}$ , both of which are associated with buckling and first ply failure [6]. The first ply failure is determined based on the Huber-Mises law [6]. Thus, the multi-objective optimization of plates and shells of hybrid composite structures reinforced with beams under static loading can be formulated as follows:

$$\text{Minimise } OBJ(\mathbf{u}, \boldsymbol{\pi}) = (W(\mathbf{u}, \boldsymbol{\pi}), U(\mathbf{u}, \boldsymbol{\pi})) \quad (1)$$

subject to

$$\varphi_1(\mathbf{u}, \boldsymbol{\pi}) = 1 - \frac{\lambda_{crit}(\mathbf{u}, \boldsymbol{\pi})}{\lambda_a} \leq 0 \quad (2)$$

$$\varphi_2(\mathbf{u}, \boldsymbol{\pi}) = \frac{d_{crit}(\mathbf{u}, \boldsymbol{\pi})}{\bar{d}_a} - 1 \leq 0 \quad (3)$$

with  $\bar{\lambda}_a$  and  $\bar{d}_a$  the allowable values for critical load factor and critical displacement, and the size constraints:

$$u_j^l \leq u_j \leq u_j^u, \quad j=1, \dots, \bar{N}_u \quad (4)$$

satisfying the equilibrium equation set:

$$\boldsymbol{\Psi}({}^t \mathbf{d}, {}^t \lambda, \mathbf{u}, \boldsymbol{\pi}) = \mathbf{r}({}^t \mathbf{d}, \mathbf{u}, \boldsymbol{\pi}) - {}^t \lambda \bar{\mathbf{F}} = 0 \quad (5)$$

and the additional arc-length method equation:

$$Q(\mathbf{d}, \lambda, \mathbf{x}, \boldsymbol{\pi}) = 0 \quad (6)$$

In equilibrium Equation (5),  $\mathbf{r}({}^t \mathbf{d}, \mathbf{u}, \boldsymbol{\pi})$  denotes the internal forces in the structural system,  $\mathbf{d}$  is the displacement vector,  ${}^t \lambda$  is the load factor, and  $\bar{\mathbf{F}}$  is the vector of the applied external forces. Since the nonlinear geometric behaviour is considered, the equilibrium is reached through an iterative and incremental loading process based on the arc-length method in Equation (6) for a load level  $t$ .

The composite structures considered in this approach are composed of plate, shell, or beam laminates. The design variables represented by vector  $\mathbf{u}$  are defined as follows: the angle  $\theta_{i,j}$  and the thickness  $\bar{t}_{i,j}$  of the  $i$ -th ply of the  $j$ -th plate or shell laminate, grouped in the vectors  $\boldsymbol{\theta}$  and  $\bar{\mathbf{t}}$ , respectively; the height  $h_j$  and the width  $w_j$  of the rectangular cross section of the  $j$ -th beam laminate, grouped in the vectors  $\mathbf{h}$  and  $\mathbf{w}$ , respectively. The macro-mechanical properties of each laminate depend on the design variables previously defined and also on the ply material and on the laminates distribution on the structure. The material distribution at laminate level and structure level is defined through the variable  $\pi_j$ , associated with the  $j$ -th plate or shell laminate. These variables related to the hybridisation are important in the optimal design of laminated structures with multi-materials [6-8]. In the approach considered in this work, they are grouped in vector  $\boldsymbol{\pi}$ .

### 3 Multi-objective genetic algorithm

#### 3.1 Co-evolution of populations

An approach for structural robust design that simultaneously considers minimum weight/cost and maximum performance/minimum strain energy is proposed in this

paper. The trade-off between the performance target, depending on given stress, displacement and buckling constraints imposed on composite structures, against minimum weight/cost, is searched. The Pareto front is built and such a challenge is performed here using a modified version of previously proposed hierarchical genetic algorithm with co-evolution of multi-populations [6-8] and is denoted by MOHGA. A self-adaptive genetic search incorporating Pareto dominance and an elitist strategy storing the non-dominated solutions found during the evolutionary process is considered [7]. The MOHGA with age structure used in this work to solve the multi-objective optimization problem is a mixed model applying multiple crossover and mutation operators aimed at exploring the synergy and adaptive properties of that algorithmic topology. The algorithm considers a sequential hierarchical relationship between subpopulations evolving in separated isolation stages followed by migration. Improvements based on the *Species Conservation* paradigm are performed to avoid genetic tendencies due to elitist strategies used in hierarchical subpopulations [6].

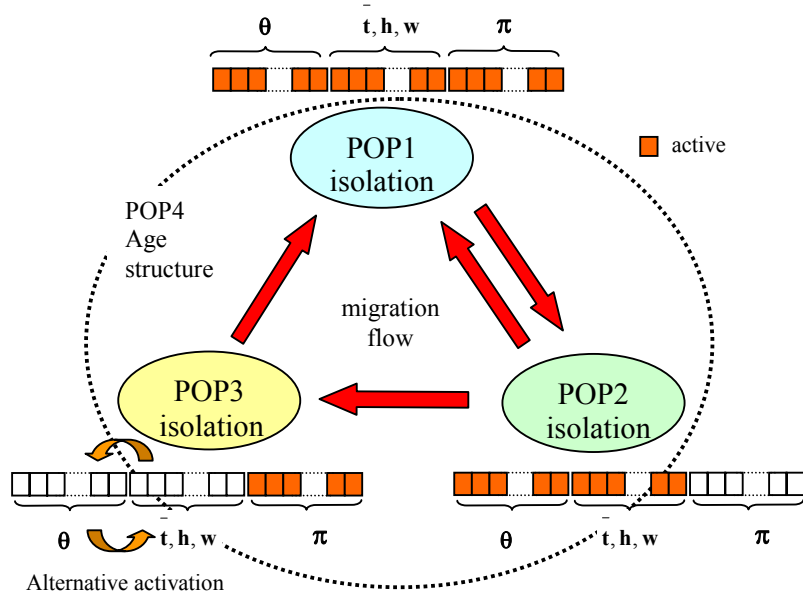


Figure 1: Hierarchical relationship of sub-populations of the MOHGA and chromosome segmentation.

In the proposed MOHGA model, three subpopulations are arranged in a ring and have a hierarchical relationship going from the upper level subpopulation POP1 to the lower level subpopulation POP3. Each MOHGA subpopulation has an independent evolution during a time period denoted by *Isolation Stage*, where the crossover and mutation operators are applied in a sequence of operations. After isolation, a *Migration Stage* occurs with individuals moving towards the subsequent subpopulations in the ring net as shown in Figure 1. The initialization of the subsequent populations is performed with migrated individuals together with new randomly generated individuals. A life cycle from POP1 to POP3 is called *epoch*.

The use of elitist strategies in the subpopulations of the MOHGA seems contradictory to the *Species Conservation* paradigm, and to overcome this difficulty

an enlarged population POP4 is considered [6-8]. Then, all individuals generated as “new” during the evolutionary process are stored into POP4 with an age structure.

Depending on the evolving subpopulation, different design variables are considered in the optimization model corresponding to active and non-active segments of each chromosome as shown in Figure 1. The use of different active segments of the chromosome corresponds to a decomposition of the design space. Chromosome activation deals with segmentation of the population in niches and is driven together with the use of species concept. Individuals with identical material distribution topology belong to the same species having equal code value for the 3<sup>rd</sup> segment of the chromosome associated with that material distribution on laminate and on structure. To induce niche behaviour, rules for species conservation and species dominance are adopted, and the number of individuals belonging to a species is controlled [6]. More specifically, the implementation of the *Species Conservation* paradigm is considered at the *Isolation* and *Migration* stages based on the following rules at each evolution stage: in *Isolation stage* the number of individuals belonging to the same species is limited; and in *Migration stage* all candidates to migration belong to different species [6,8].

### 3.2 Self-adaptive Procedures

It is recognized that the efficiency of genetic algorithms improves if some adaptive rules are included. Adaptive rules perform using additional information related to the behaviour of state and design variables of the structural problem. At each generation, the self-adaptation of the genetic parameters to evolutionary conditions attempts to improve the efficiency of the genetic search. The introduction of adaptive rules occurs at selection, mutation, crossover and migration operators [6-8]. Self-adaptation has proved to be highly beneficial in automatically and dynamically adjusting evolutionary parameters. In order to preserve the diversity of Pareto optimality a population entropy control is proposed. The introduction of adaptive rules occurs at three levels: (i) when defining the search domain in each generation; (ii) considering a crossover operator based on commonality and local improvements; and (iii) by controlling mutation, including behavioural data. Self-adaptation has proved to be highly beneficial in automatically and dynamically adjusting evolutionary parameters.

Crossover	Mechanism of recombination			Taxonomy analysis
	MSM	OGM	OSM	
EHCgi	Elitist based on fitness	Hybrid crossover with genetic improvement	Elite group transferring	NBCO
EpUC	Elitist based on fitness	Uniform parameterised crossover	Elite group transferring	DCO
ApUC	Non-elitist based on age and dominance	Uniform parameterised crossover	Lethal age control + dominating sorting	DCO

Table 1: Crossover operators for self-adaptive procedure

Table 1 shows the mechanisms of recombination of the proposed crossover operators, grouping them according to taxonomy analysis [6-8]. The crossover operators based on its application to both parents are grouped as: Discrete Crossover Operators (DCOs), Aggregation Based Crossover Operators (ABCOs), and Neighbourhood-Based Crossover Operators (NBCOs). The mechanisms of recombination are the Mating Selection Mechanism (MSM), the Offspring Generation Mechanism (OGM), and the Offspring Selection Mechanism (OSM).

The selection of crossover operators depends on the quality of the offspring generated by recombination. During the evolutionary process, the best offspring solution obtained from the crossover is compared with the worst solution of the elite group. This last solution is the best candidate to be eliminated in the next generation. A success event happens when the best-fitted offspring is better than the worst-fitted individual of the elite group of the population. The quality of the solutions obtained from a crossover is then associated with the number of success events divided by the number of generations and is denoted by the success rate. A self-adaptive probability to select a crossover operator scheme can be established depending on the cumulative number of success events. In this work, two crossover schemes can be selected in each subpopulation of MOHGA as described by Conceição António [8].

The incorporation of data related to the behaviour of state variables of the structural system is the main objective of the *Controlled Mutation* [6-8]. The establishment of a relationship between the stress field in the composite structure and the chosen genes to mutate can control the mutation process. This is also a self-adaptive procedure adopted for MOHGA

### 3.3 Non-dominated sorting at isolation stage

At isolation stage of each MOHGA sub-population defined here as set  $\mathbf{Q} \subseteq \mathcal{R}^n$  the individuals are sorted and ranked according to local non-constrain-dominance. Following the definition by Deb [9], an individual  $\mathbf{x}_i \in \mathbf{Q}$  is said to constrain-dominate an individual  $\mathbf{x}_j \in \mathbf{Q}$ , if any of the following conditions are verified:

- (1)  $\mathbf{x}_i$  and  $\mathbf{x}_j$  are feasible, with
  - (i)  $\mathbf{x}_i$  is no worse than  $\mathbf{x}_j$  for all objectives, and
  - (ii)  $\mathbf{x}_i$  is strictly better than  $\mathbf{x}_j$  in at least one objective,
- (2)  $\mathbf{x}_i$  is feasible while individual  $\mathbf{x}_j$  is not,
- (3)  $\mathbf{x}_i$  and  $\mathbf{x}_j$  are both infeasible, but  $\mathbf{x}_i$  has smaller constraint violation.

The constraint violation of an individual  $\mathbf{x}$  is defined to be equal to the sum of the violated constraint function values in the multi-objective optimization problem formulated from (1) to (6):

$$\xi(\mathbf{u}, \boldsymbol{\pi}) = \sum_{i=1}^2 \Gamma_i(\mathbf{u}, \boldsymbol{\pi}) \quad (7)$$

with

$$\Gamma_i[\varphi_i(\mathbf{u}, \boldsymbol{\pi})] = \begin{cases} 0 & \text{if } \varphi_i(\mathbf{u}, \boldsymbol{\pi}) \leq 0 \\ \varphi_i(\mathbf{u}, \boldsymbol{\pi}) & \text{if } \varphi_i(\mathbf{u}, \boldsymbol{\pi}) > 0 \end{cases} \quad (8)$$

The concept of constrain-domination enables to compare two individuals in problems having multiple objectives and constraints, since if  $\mathbf{x}_i$  constrain-dominates  $\mathbf{x}_j$ , then  $\mathbf{x}_i$  is better than  $\mathbf{x}_j$ . If none of the three conditions referred above are verified, then  $\mathbf{x}_i$  does not constrain-dominate  $\mathbf{x}_j$ .

The problem of stacking sequence design of composite structures is well known for having many local optima, and so, dominated solutions are expected. The approach proposed in this work uses a mixture of developed techniques and new techniques in order to find multiple Pareto-optimal solutions in parallel. The principal aspects are: the storage of the Pareto-optimal solutions found, the use of the concept of Pareto dominance in order to assign scalar fitness values to individuals, and the clustering through the co-evolution of sub-populations to reduce the number of non-dominated solutions stored without destroying the characteristics of the Pareto-optimal front.

### 3.4 Fitness assignment procedure at isolation stage

To build the Pareto front it is necessary to rank the population according to non-dominance definitions. The proposal of Fonseca and Fleming [10] is adopted in this paper, a scheme in which the rank of a certain individual corresponds to the number of chromosomes in the current population by which it is dominated. So, let's consider an individual/solution  $\mathbf{x}_i$ , which is dominated by  $p_i$  individuals in the current generation. Its current position in the individual's ranking can be given by:

$$r_i = 1 + p_i \quad (9)$$

All non-dominate individuals are assigned rank 1, denoted by  $\mathbf{r}^1$ . Rank 1 is temporarily disregarded from the population and the non-constrain-dominated solutions of the remaining population are founded and designated as non-constrain-dominated set of rank 2. The procedure continues until all the individuals are ranked. A short analysis reveals that any population, must have at least one solution with rank 1 and that the maximum rank of any population individual cannot be larger than the population size,  $N$ .

After the ranking is performed in ascending order of magnitude, a raw fitness is assigned to each solution based on its rank. The raw fitness is obtained using a linear mapping function taking values between  $N$  for the best ranked solution and 1 for the worst ranked solution. The raw fitness is averaged considering at a time the solutions in each rank. So, for any solution  $\mathbf{x}_i$ , the following average fitness assignment is obtained as:

$$F_i^{aver} = \begin{cases} N - 0.5 (\eta(1) - 1) & \text{if } r_i = 1 \\ N - \sum_{k=1}^{r_i-1} \eta(k) - 0.5(\eta(r_i) - 1) & \text{if } r_i \neq 1 \end{cases} \quad (10)$$

being  $r_i$  the rank of  $\mathbf{x}_i$  as established in equation (9) and  $\eta(r_i)$  the number of solutions in rank  $r_i$ . As follows  $F_i^{aver}$  is the average fitness of all solutions having the same rank  $r_i$ . The mapping and averaging procedures ensures that better ranked solutions have a higher fitness and non-dominated solutions play the most important role in a population.

In order to guarantee the diversity of non-dominated solutions the concept of niching among solutions belonging to each rank is adopted as proposed by Fonseca and Fleming [10]. In the proposed approach a solution located in a less-crowded region will have a better shared fitness. The shared fitness  $F_i^{shar}$  of a solution  $\mathbf{x}_i$  is obtained dividing the corresponding average fitness calculated using equation (10) by the niche count  $nc(\mathbf{x}_i)$ ,

$$F_i^{shar} = \frac{F_i^{aver}}{nc(\mathbf{x}_i)} \quad (11)$$

The niche count of a solution  $\mathbf{x}_i$  is calculated as

$$nc(\mathbf{x}_i) = \sum_{j=1}^{\eta(r_i)} Shar(\delta_{ij}) \quad (12)$$

being  $Shar(\delta_{ij})$  the sharing function defined as

$$Shar(\delta_{ij}) = \begin{cases} 1 - \left( \frac{\delta_{ij}}{\sigma_{share}} \right)^\alpha & \text{if } \delta_{ij} \leq \sigma_{share} \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

where  $\delta_{ij}$  is the normalized distance between any two solutions  $\mathbf{x}_i$  and  $\mathbf{x}_j$  (including itself) in rank  $r_i$ ,  $\alpha$  is the shape parameter of the sharing function and  $\sigma_{share}$  is a reference distance associated with the sharing effect. It is possible that several solutions have the same rank position and to distinguish their quality the metric distance is calculated using the objective values:

$$\delta_{ij} = \sqrt{\sum_{k=1}^M \left( \frac{f_k^{(i)} - f_k^{(j)}}{f_k^{max} - f_k^{min}} \right)^2} \quad (14)$$

where  $f_k^{max}$  and  $f_k^{min}$  are the maximum and the minimum objective function values of the k-th objective. In proposed MOHGA approach those objective function values are evaluated in age structured population POP4. The shape parameter in equation (13) is  $\alpha = .5$ .

In a rank, the average shared fitness value of solutions should remain the same average assigned fitness value before sharing [9]. In order to preserve the referred property a scaling is performed as follows



$$\bar{F}_i^{shar} = \frac{F_i^{aver} \eta(r_i)}{\sum_{k=1} \eta(r_k)} F_i^{shar} \quad (15)$$

The described procedure will continue for all ranks and the fitness assignment process can be described as follows:

1<sup>st</sup> Step: Initialize  $\eta(j) = 0$  for all possible ranks  $j=1, \dots, N$ . Do solution counter  $i=1$ .

2<sup>nd</sup> Step: Calculate the number of solutions  $p_i$  that dominates solution  $\mathbf{x}_i$ . The rank of  $\mathbf{x}_i$  is computed as  $r_i = 1 + p_i$ . Increment the number of solutions in rank as  $\eta(r_i) \leftarrow \eta(r_i) + 1$ .

3<sup>rd</sup> Step: If  $i < N$  do  $i \leftarrow i + 1$  and go to Step 1, else go to Step 4.

4<sup>th</sup> Step: Identify the maximum rank  $\bar{r}$  verifying  $\eta(r_i) > 0$ . Sort based on rank and calculate the average fitness  $F_i^{aver}$  for any solution  $\mathbf{x}_i$  and for  $i=1, \dots, N$ , according to equation (10). Set a rank counter  $r = 1$ .

5<sup>th</sup> Step: For each solution  $\mathbf{x}_i$  in rank  $r$ , calculate the niche count  $nc(\mathbf{x}_i)$  with the other solutions in the same rank by using equation (12) and the shared fitness  $F_i^{shar}$  by using equation (11). Scale the shared fitness using equation (15).

6<sup>th</sup> Step: If  $r < \bar{r}$ , do  $r \leftarrow r + 1$  and go to Step 5, else stop.

In the proposed approach a dynamically updated procedure is adopted for  $\sigma_{share}$  in sharing function defined by equation (13). For two objectives [9] the updating expression is:

$$\sigma_{share} = \frac{f_1^{max} - f_1^{min} + f_2^{max} - f_2^{min}}{N - 1} \quad (16)$$

where  $f_k^{max}$  and  $f_k^{min}$  are the maximum and minimum values of each objective function evaluated in age structured population POP4.

### 3.5 Building Pareto front based on age-structured population

A continuous model of generation of individuals was adopted for age-structured population. An enlarged population with age structure POP4 and performing in parallel with the hierarchical topology of MOHGA is considered in this model [6]. Each individual belonging to population POP4 is characterized by two parameters: *individual age* and *lethal age*. The individual age increases one unit after each generation. Any individual removed from MOHGA sub-populations either by elitist strategy or by finishing of *Isolation stage* of evolution and not selected for migration, will survive in the population with age structure POP4. Furthermore, its *individual age* will continue increasing until removed definitively due to *lethal age*.

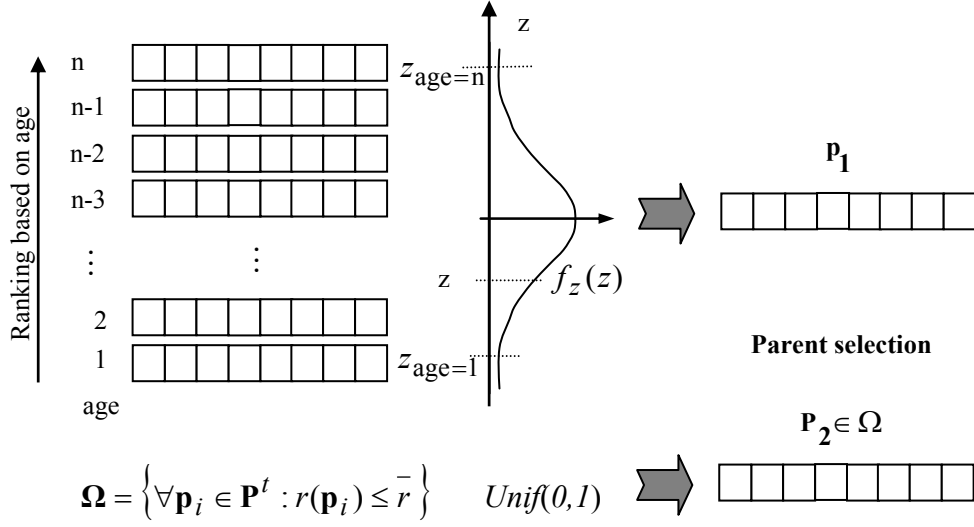


Figure 2: Non-elitist parent selection based on age and dominance (MSM).

The population with age structure is ranked according to non-dominance concepts defined in Section 3.3. This is denoted by global dominance and the corresponding global Pareto front is built.

In the population with age structure (POP4) the mating selection mechanism (MSM) is conditioned by two rules: (i) the candidate age, and (ii) the rank position in non-dominating ranking. One parent comes from enlarged population according to its age and the second parent is selected based on the global dominance.

Assuming that population maturity and potentiality follow a Normal distribution the parent selection is probability dependent and this is applied to first parent choice. Figure 2 presents the Normal probability density function,  $f_z(z)$ , applied to first parent selection in this crossover process. Individuals with ages located at the tails of the Normal density function are the youngest and the oldest of the scale, and they have a very low probability to be selected as parents. Then the reproduction rate by crossover depends on the maturity and degrades as the life cycle goes on till the end.

The selection of second parent is based on its dominance. Solutions which rank  $r(\mathbf{p}_i)$  is lower than  $\bar{r}$  is a mating candidate. The second parent is select from the set group  $\Omega$  with a Uniform probability distribution function. This guarantees the improvement of global Pareto front during the evolutionary process.

## 4 Numerical Example

Aiming to discuss the capabilities of the MOHGA to deal with multi-objective optimization of composite structures a numerical example is presented. A cylindrical shell with rib stiffeners both made of laminated composite materials is considered as shown in Figure 3.

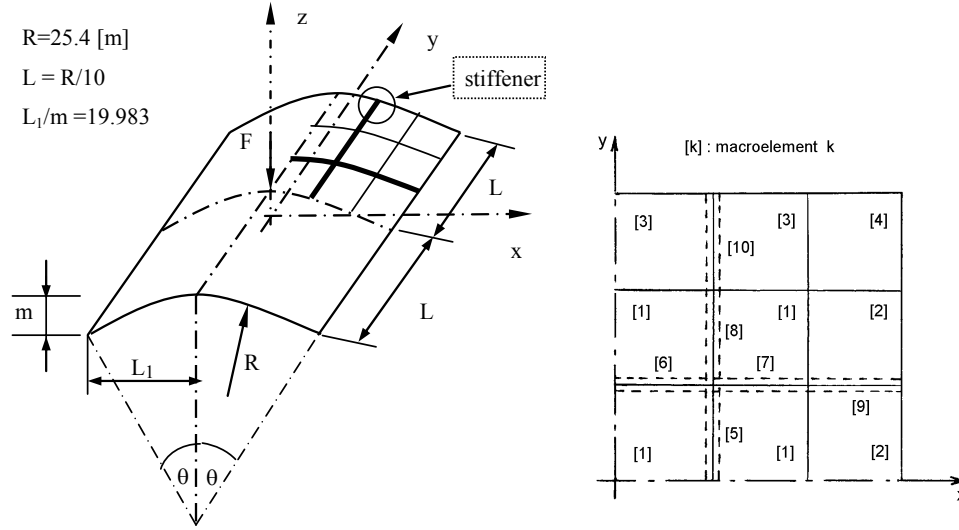


Figure 3: Geometry and laminates of the cylindrical shell reinforced with beams.

The shell is hinged on straight sides and free at its curved boundaries. A central point load  $F_{max}/4 = 1 \text{ kN}$  is applied. Ten laminates were taken into account for the structure, four laminates grouping the shell elements (from 1 to 4) and the others (from 5 to 10) grouping the beam elements of rib stiffeners. All laminates are symmetric and composed by six plies. The rib stiffeners are connected below the shell elements. The mechanical properties of the materials for ply laminate are taken from Tsai [11] and presented in Table 2 with longitudinal strength,  $X$ , transversal strength,  $Y$ , and shear strength,  $S$ , longitudinal Young modulus,  $E_1$ , transversal elastic modulus,  $E_2$ , shear modulus,  $G_{12}$ , Poisson's ratio,  $\nu$ , and specific weight of the material,  $\rho$ .

Material		$E_1$ [GPa]	$E_2$ [GPa]	$G_{12}$ [GPa]	$\nu$
Type	Code				
CFRP: T300/N5208	1	181.00	10.3	7.17	0.28
GFRP: Scotchply 1002	2	38.60	8.27	4.14	0.26
GFRP: E-glass/epoxy	3	43.00	8.90	4.50	0.27
KFRP: Kev 49/epoxy	4	76.00	5.50	2.30	0.34
		$X$ [MPa]	$Y$ [MPa]	$S$ [MPa]	$\rho$ [kg/m <sup>3</sup> ]
CFRP: T300/N5208	1	1500	40	68	1600
GFRP: Scotchply 1002	2	1062	31	72	1800
GFRP: E-glass/epoxy	3	1280	49	69	2000
KFRP: Kev 49/epoxy	4	1400	12	34	1460

Table 2: Mechanical properties of the materials used in the composite laminate

One material from Table 2 is selected for each ply of each laminate. Four composite systems are considered for ply laminates in this study: one carbon/epoxy composite, two glass/epoxy composites and one Kevlar/epoxy composite. The Kevlar/epoxy is considered as a possible material choice only for the inner ply of the symmetric laminates. The remaining materials are free selection and at least two

materials must be considered for hybrid composite laminate construction. Then there are 33 possible combinations of these four materials for the stacking sequence  $\pi_j$  considering the defined rules and six plies in the symmetric  $j$ -th composite shell laminate construction. The beam laminates have six plies made by material number 2 from Table 2 and this kind of material does not change during the optimization process. The maximum allowed value for critical displacement in buckling or FPF failure modes is  $\bar{d}_a = 1.3 \times 10^{-1}$  m. The lower bound for the critical load factor defined in Equation (2) is  $\bar{\lambda}_a = 0.45$ . The size constraints on the design variables are established as:

$$\begin{aligned}
-90^\circ &\leq \theta_{i,j} \leq 90^\circ \\
1.2 \times 10^{-3} \text{ m} &\leq \bar{t}_{i,j} \leq 2.4 \times 10^{-3} \text{ m} \\
2.0 \times 10^{-2} \text{ m} &\leq h_j \leq 4.0 \times 10^{-2} \text{ m} \\
5.0 \times 10^{-3} \text{ m} &\leq w_j \leq 1.5 \times 10^{-2} \text{ m}
\end{aligned} \tag{17}$$

Subpopulation	POP1	POP2	POP3
Population size	30	27	27
Elite group size	10	9	9
Mutation group size	10	9	9
Nr. digits/binary or integer code	4/3/4	4/3/4	4/3/4
Generations/isolation time	6	6	6

Table 3: Genetic Algorithm parameter definitions

The genetic parameters of the MOHGA are presented in Table 3. Five individuals belonging to different species participate in each migration flow between the three MOHGA sub-populations as shown in Figure 1. The number of digits in code format refers the binary coding of the first two segments and the last number refers the integer code used in third segment of the chromosome as shown in Figure 1. In the age-structured population, the *lethal age* is equal to 25 generations. In the self-adaptive crossover procedure the parameters are according to [6]. The mutation operators *Implicit Mutation* and *Controlled Mutation* used in this work have the same probability to be selected as defined previously in author previous research [8].

The MOHGA performs during thirty epochs. Figure 3 shows the Pareto fronts (rank 1) from the 10<sup>th</sup>, 25<sup>th</sup> and 30<sup>th</sup> epochs of the evolutionary process. The results are obtained at the end of each epoch after isolation stage of age structured population POP4 using the associated Pareto front. It can be observed the performance of the proposed approach in search considering a multi-objective optimization problem.

Some Pareto-optimal solutions belonging to rank 1 for the proposed structural problem are presented in Table 4 and Table 5 captured at the 30<sup>th</sup> epoch of age structured population POP4. There are similarities among ply angle solutions for different composite laminates. Also it can be noticed that for all solutions of the Pareto-optimal front most of thickness design variables associated with shell

laminates take the minimum values of the size constraints in Equation (17). This means that most of differences in optimal values for weight objective function depend on variables associated to stiffeners considered in composite structure.

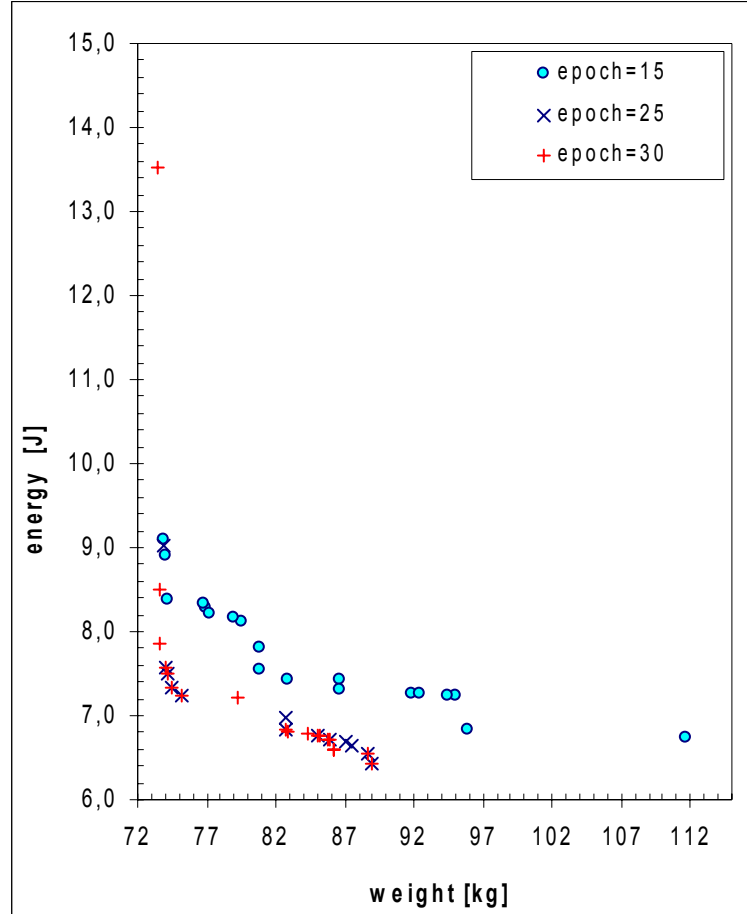


Figure 3: Non-dominated solutions in Pareto fronts during MOHGA evolution.

Table 5 presents the description of the third segment of the chromosome for the seven Pareto-optimal front solutions (rank 1). The optimal material distributions of the structure are based on several kinds of laminates. The positions of the solutions in the Pareto-optimal front are given by the numbering for the particular case of the eighth epoch described in Figure 3 and Table 4.

It is observed that all materials are used in the Pareto-optimal stacking sequence. However the composite system based on Carbon/Epoxy (CFRP: T300/N5208) and the composite system based on Kevlar/Epoxy (Kev 49/epoxy) are the most frequently used in Pareto front. Most of the solutions use the KFRP composite system at the inner ply of the laminate due to its light weight. Also the CFRP composite system is preferred for outer plies due to its higher strength and lower weight. This example shows the efficiency of MOHGA supported by adaptive rules and non-dominated sorting for the construction of the Pareto-optimal front.

Laminate		Design variables	Pareto optimal solutions				
Shell	beam		1st	2nd	3rd	4th	5th
1		$\bar{t}_{1,1} / \theta_{1,1}$	1.20/ 18	1.20/ 18	1.20/ 18	2.23/ 42	2.23/ 42
1		$\bar{t}_{2,1} / \theta_{2,1}$	1.20/ 54	1.20/ 54	1.20/ 78	1.20/ 78	1.20/ 78
1		$\bar{t}_{3,1} / \theta_{3,1}$	1.20/-18	1.20/-54	1.20/-54	1.20/ -54	1.20/ -54
2		$\bar{t}_{1,2} / \theta_{1,2}$	1.20/ -6	1.20/ -6	1.20/ -6	1. 20/-6	1. 20/-6
2		$\bar{t}_{2,2} / \theta_{2,2}$	1.20/ 6	1.20/ 6	1.20/ 6	1. 37/ 6	1. 20/ 6
2		$\bar{t}_{3,2} / \theta_{3,2}$	1.20/-6	1. 20/-6	1.37/-6	1.20/-6	1.20/-6
3		$\bar{t}_{1,3} / \theta_{1,3}$	1 20/ 6	1 20/ 54	1 20/-66	1.20/ 90	1.20/ 90
3		$\bar{t}_{2,3} / \theta_{2,3}$	1. 20/ 66	1. 20/ 66	1. 20/ 66	1.20/ 66	1.20/ 66
3		$\bar{t}_{3,3} / \theta_{3,3}$	1.20/ 90	1.20/ 90	1.20/-66	1. 20/ 90	1. 20/-66
4		$\bar{t}_{1,4} / \theta_{1,4}$	1. 20/-78	1. 20/ 6	1. 20/ 90	1. 20/ 90	1. 20/ 90
4		$\bar{t}_{2,4} / \theta_{2,4}$	1.20/ 54	1.20/ 78	1.20/ 78	1. 20/ 78	1. 20/ 78
4		$\bar{t}_{3,4} / \theta_{3,4}$	1. 20/ 66	1. 20/-18	1. 20/-66	1.20/-66	1.20/-66
	5	$h_1 / w_1$	20.0/ 6.4	20.0/ 6.4	20.0/ 6.4	25.7/ 6.4	25.7/ 6.4
	6	$h_2 / w_2$	31.4/ 5.0	31.4/ 5.0	31.4/ 5.0	31.4/ 5.0	31.4/ 5.0
	7	$h_3 / w_3$	31.4/ 6.4	31.4/12.1	37.1/ 7.9	37.1/15.0	40.0/15.0
	8	$h_4 / w_4$	31.4/ 5.0	20.0/ 5.0	31.4/ 6.4	31.4/ 6.4	31.4/ 6.4
	9	$h_5 / w_5$	25.7/ 5.0	20.0/ 5.0	40.0/12.1	40.0/15.0	40.0/15.0
	10	$h_6 / w_6$	25.7/ 6.4	25.7/ 6.4	28.6/ 6.4	28.6/ 6.4	28.6/ 6.4
Objectives:		Weight [kg]	73.465	73.607	79.303	85.122	86.240
		Energy [J]	13.514	8.508	7.223	6.758	6.592

Table 4: Decoding results of the Pareto-optimal front solutions for the first two segments of chromosome ( $\bar{t}_{i,j}$ ,  $h_i$ ,  $w_i$  [mm] and  $\theta_{i,j}$  [degrees])

Pareto-optimal solutions	Optimal stacking sequence			
	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$
1 <sup>st</sup>	[1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub>			
2 <sup>nd</sup>	[1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub>			
3 <sup>rd</sup>	[1/1/4] <sub>s</sub> ; [1/2/1] <sub>s</sub> ; [1/3/1] <sub>s</sub> ; [3/2/1] <sub>s</sub>			
4 <sup>th</sup>	[1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub>			
5 <sup>th</sup>	[1/1/4] <sub>s</sub> ; [1/1/4] <sub>s</sub> ; [1/3/1] <sub>s</sub> ; [1/1/4] <sub>s</sub>			

Table 5: Decoding results of the Pareto-optimal front solutions corresponding to the third segment of each chromosome (ply material properties defined in Table 2)

To illustrate the local dominance at isolation stage in different populations of the MOHGA approach some results of the fitness assignment procedure are presented.

Table 6 shows the results of MOHGA fitness assignment procedure described in section 3.4 for local dominance. The results correspond to the last generation of 30<sup>th</sup> epoch in isolation stage at population POP3. The non-dominated sorting definition of section 3.3 is previously applied to all solutions in each population and ranked according to its local dominance. In rank column it can be seen that some ranks have no solutions. The use of an elitist strategy guarantees that rank 1 solution drives the evolutionary process.

Solution	Weight [kg]	Energy [J]	Rank	Average fitness	Niche count	Shared fitness	Scaled fitness
1	90.9294	6.4635	1	23.0000	3.8240	6.0146	29.3626
2	73.4646	13.5140	1	23.0000	3.9725	5.7898	28.2651
3	86.3833	6.7735	1	23.0000	4.5243	5.0836	24.8178
4	73.6098	11.0610	1	23.0000	4.8209	4.7708	23.2909
5	79.3032	7.2223	1	23.0000	5.4413	4.2269	20.6354
6	73.8434	8.5453	1	23.0000	5.4549	4.2164	20.5839
7	75.5763	7.6391	1	23.0000	5.5872	4.1165	20.0966
8	78.1174	7.5994	1	23.0000	5.6068	4.1021	20.0263
9	75.1121	8.3739	1	23.0000	5.6363	4.0807	19.9214
10	73.4646	16.6300	2	18.0000	1.0000	18.0000	18.0000
11	79.2498	8.4520	4	17.0000	1.0000	17.0000	17.0000
12	93.1767	8.4585	8	16.0000	1.0000	16.0000	16.0000
13	93.1941	9.0855	10	14.5000	1.5574	9.3103	14.5000
14	92.7368	12.1240	10	14.5000	1.5574	9.3103	14.5000
15	91.7117	15.3230	11	12.0000	1.7551	6.8374	12.8927
16	103.7629	10.7770	11	12.0000	1.9118	6.2769	11.8359
17	97.6002	10.8930	11	12.0000	2.0075	5.9775	11.2714
18	82.3977	9.3893	18	10.0000	1.0000	10.0000	10.0000
19	90.2516	7.0680	19	9.0000	1.0000	9.0000	9.0000
20	101.7097	6.4146	20	8.0000	1.0000	8.0000	8.0000
21	78.0299	6.8309	21	7.0000	1.0000	7.0000	7.0000
22	81.7220	6.0311	22	6.0000	1.0000	6.0000	6.0000
23	86.6537	5.2993	23	5.0000	1.0000	5.0000	5.0000
24	84.7914	4.1186	24	4.0000	1.0000	4.0000	4.0000
25	85.4951	4.2498	25	3.0000	1.0000	3.0000	3.0000
26	83.7449	3.2970	26	2.0000	1.0000	2.0000	2.0000
27	88.8415	3.4336	27	1.0000	1.0000	1.0000	1.0000

Table 6: Fitness assignment procedure at last generation of isolation stage of POP3 corresponding to the end of 30<sup>th</sup> epoch

## 5 Conclusions

A multi-objective hierarchical genetic algorithm (MOHGA) with age structure and based on local and global dominance concepts to deal with multi-objective optimization of composite structures is proposed. The approach based on multi-populations evolution uses the species conservation technique to address the optimal

stacking sequence and material topology of composite structures in multi-objective optimization problems. Thus, the individuals corresponding to the same material selection and topology of the hybrid composite structure belong to the same species. The material distribution of the hybrid composite structures is performed at two levels: the laminate level and the structural topology level.

A structural robust design problem that simultaneously considers minimum weight or cost and minimum strain energy related with maximum performance is presented. The trade-off between the performance target, depending on given stress, displacement and buckling constraints imposed on composite structures, against robustness, is searched. The global Pareto-optimal front is built at age structures population using the concept of Pareto dominance. The concept of local dominance and a sharing function in order to assign scalar fitness values to individuals is used at isolation stage of populations. Such a challenge calls for a multi-objective optimization and is performed here using the proposed hierarchical genetic algorithm with co-evolution of multi-populations. Self-adaptive rules are incorporated in the Pareto front design based on genetic search. The search method adopts an elitist strategy storing non-dominated solutions found during the evolutionary process. The results show that MOHGA is promising in multi-objective optimization of composite structures.

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