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A One-Dimensional Evolutionary Masonry Model with Low Tensile Strength

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Abstract

In the paper a low tension (LT) elastic-brittle mechanical model for use in the study of structures made of masonry material is discussed. With respect to the NT hypothesis, the model is able to reproduce results in major agreement with the real behaviour of the masonry, since it is able to embed even a low tensile resistance capacity of the material. The LT solution is investigated in order explore its relationship with the other models that are usually adopted in common practice.

Keywords: masonry, material modeling, no-tension model, elastic-brittle model, energy functional, constrained and unconstrained minimum.

1 Introduction

Usually masonry structures lend themselves to some modelling of the basic material that neglects the resistance skill of the masonry to resist tensile stresses [1]-[25].

This is mainly ought to the circumstance that, even if some low resistance to tractions does exist in masonry, it is not reliable and exactly predictable since it usually decays with time.

Besides the typical NT (No Tension) hypothesis, which is often coupled to a indefinite resistance to compressive stresses under purely elastic behaviour in compression, or, otherwise to some ductility in compression, a more realistic mechanical model could be introduced able to take into account such small capacity of resisting tensile stresses, and thus improving the performance of the NT models when applied to the analysis of masonry structures.

In the following the LT (Low Tension) model is introduced, founded on the basic assumption of tensile non-null resistance and elastic-brittle behaviour in tension coupled to indefinite elastic behaviour in compression.

Extensions can be provided for including some ductility in compression.

Some interesting analytical investigation is developed for exploring the properties of the relevant solution, mainly under the energetic profile, and performing a comparison with the solutions of other material models, such as the NT solution.

Actually the reliable forecast of the behaviour of masonry structures is of primary interest also because of possible applications for the protection of monumental heritage, by means, for example, of dynamic control strategies [26]-[30], especially useful in the absence of the environmental forecast [31]-[32].

2 The masonry arch under the NT assumption

2.1 The problem set up for NT arches, portal arches and vaults

In structural patterns like masonry arches, portal arches and vaults, as shown for example for the case of a single span portal arch in Figure 1, when adopting the NT (No Tension) assumption, one may search for the NT solution by a stress approach based by the constrained minimization of a suitably defined complementary energy functional.

In such cases, the set of stress fields equilibrating the applied loads can be built up by superposition while the stress field can be inferred from the internal forces on every cross section by a bi-linear distribution pattern as shown in Figure 2.

The system has only three static redundancies, so that the number of static unknowns is much smaller than the number of kinematic ones.

The solution of the structural problem, in this case, can be best approached by the Minimum principle of Complementary Energy, and the procedure is aimed at identifying the redundant reactions allowing constraint compatibility while minimizing the Complementary Energy functional, whose expression is inferred by the stress distribution in Figure 2.

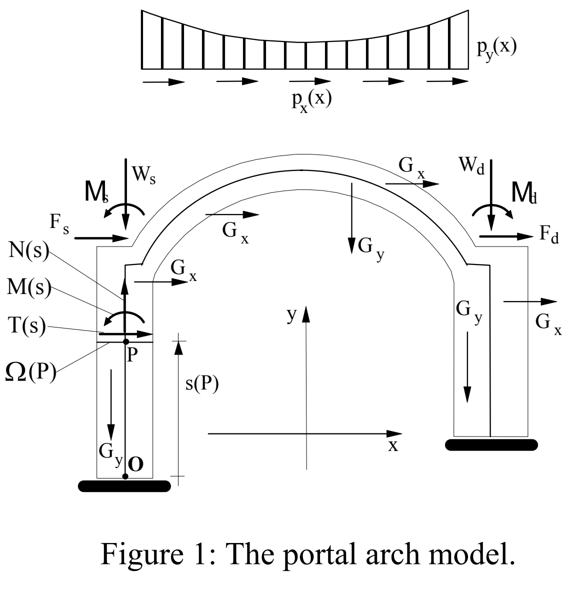

Figure 1: The portal arch model.

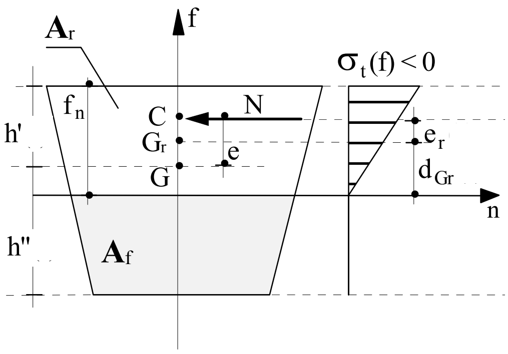

Figure 2: The bi-linear stress pattern at the generic cross section.

In details, with reference to the single span portal arch model subject to a load pattern shown in Figure 1, where **p** is the surface load on the arch and **G** are the body forces, both due to gravity acceleration g and to an eventual field of horizontal base accelerations

$$\mathbf{p} = \begin{bmatrix} p_x(x) \\ p_y(x) \end{bmatrix} = \begin{bmatrix} cp(x) \\ p(x) \end{bmatrix} = p(x)\begin{bmatrix} c \\ 1 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} -a_{max} \\ -g \end{bmatrix} = -\rho g\begin{bmatrix} c \\ 1 \end{bmatrix} \tag{1}$$

with ρ the (constant) material density, and a_{max} the maximum horizontal ground acceleration, set equal to a fraction ″c″ of the gravity acceleration. Equilibrium fields can be built up from stress resultants fields.

At the generic curvilinear abscissa ″s″ on the model mid-line (with origin in O), the stress resultants on the relevant cross-section in terms of shear force, normal force and bending moment are denoted by T(s), N(s) and M(s), respectively.

Since the structure is characterised by three static redundancies, the set of stress fields equilibrating the applied loads can be built up by a superposition scheme, where three redundant stress components are recognized in the thrust force X_1, the support force X_2 and the bending moment X_3 at the section where the arch is supported by the abutment on the left.

Once collected the stress resultants at the generic abscissa in a vector **S**(s) =[T(s) N(s) M(s)]^T one can build up by superposition the stress field in equilibrium with the applied loads for any value of the unknown static variables X_i as follows

$$\mathbf{S}(s) = \mathbf{S}(s|X_1, X_2, X_3) = \mathbf{S}_0(s) + \sum_{i=1}^{3} X_i \mathbf{S}_i(s) \tag{2}$$

where $\mathbf{S}_0$(s) represents the stress resultants vector at the considered curvilinear abscissa corresponding to the applied loads, whilst $\mathbf{S}_i$(s) denote those relevant to the other schemes where the *i*-th static unknown X_i is assumed equal to unit.

On the other side, the condition for static admissibility requires that the same fields do not violate the condition for the resistance of the material. One should consider that, for static admissibility the thrust line must be internal to the profile of the arch, so that for any cross-section the stress resultant forms a small angle with respect to the mid-line, and the shear force results to be small with respect to the normal force.

If one neglects the influence of the shear stress on the stress admissibility at any point, one can assume that the stress state is mono-axial at any point (i.e. the resultant force on any cross-section is orthogonal to the section itself) and the resistance condition may be written in the form

$$\sigma_\theta(Q) \le 0 \quad \forall\, Q \in V \tag{3}$$

where "θ" denotes the direction tangent to the barycentre line at the point where the cross-section containing Q intersects the barycentre line.

The admissibility of the stress field is, then, guaranteed by the condition that the force resultant line is everywhere in the interior of the arch profile; this condition implies that the eccentricity e(s) (which is given by e(s) = M(s) / N(s)) of the stress resultant N(s), at the generic curvilinear abscissa, is required to be bounded by the distances h′(s) and h″(s) of the upper and lower profiles of the arch from the cross-section barycentre (Figure 2).

The NT admissibility conditions may be written in the form

$$\begin{cases} N(s) \le 0 \\ -h'(s) \le \dfrac{M(s)}{N(s)} \le h''(s) \end{cases} \qquad \forall s \in (0, \ell) \tag{4}$$

where ℓ is the length of the model mid-line.

2.2 The MCE approach for NT arches, portal arches and vaults

By the complementary energy approach one should search for the NT solution of the stress problem in the class of equilibrated and NT admissible solutions, as follows

$$\begin{cases} \text{Find} \ \min\limits_{X_1,X_2,X_3} \mathcal{C}(X_1,X_2,X_3) = \dfrac{1}{2}\displaystyle\int_0^\ell \frac{N^2(s)}{E_c A_r(s)}\left(1+\frac{e_r(s)}{d_{Gr}(s)}\right) ds - N_d u_d - T_d v_d - M_d \phi_d \\ \text{Sub} \begin{cases} \text{equilibrium} \begin{cases} N(s) = N_0(s) + \sum\limits_{i=1}^{3} X_i N_i(s) \\ T(s) = T_0(s) + \sum\limits_{i=1}^{3} X_i T_i(s) \\ M(s) = M_0(s) + \sum\limits_{i=1}^{3} X_i M_i(s) \end{cases} \qquad \forall s \in (0,\ell) \\ \text{admissibility} \begin{cases} N(s) \le 0 \\ -h'(s) \le \dfrac{M(s)}{N(s)} \le h''(s) \end{cases} \end{cases} \end{cases} \tag{5}$$

where E_c is the elastic modulus in compression of the masonry, e_r is the distance of the solicitation centre C(s) from the barycentre $G_r(s)$ of the resistant part $A_r(s)$ of the cross-section, and $d_{Gr}(s)$ denotes the distance of $G_r(s)$ from the neutral axis n(s) (as shown in Figure 2); moreover, N_d =N(0), T_d =T(0), M_d =M(0) denote the values assumed by the static redundancies at the basis of the leftward abutment, and u_d, v_d, ϕ_d the settlements of the foundation basis of the leftward abutment; moreover $N_0(s)$, $T_0(s)$, $M_0(s)$ represent the stress resultants referred to the isostatic scheme under the applied loads, $N_i(s)$, $T_i(s)$, $M_i(s)$ represent the stress resultants referred to the isostatic scheme under the *i*-th static redundancy X_i assumed equal to the unit.

The constraint conditions set in Eqs (5) can be observed to be of linear type.

The minimum of the convex functional $\mathcal{C}$ (X_1, X_2, X_3) over the convex set **X** defined by the linear inequalities given in Eqs (5), is a problem of convex optimisation.

Because of the convexity of $\mathcal{C}$ (X_1, X_2, X_3), if the admissible set $\mathbb{D}_{NT}$ is not empty, the NT solution exists and is unique [3], [8], [9].

If, on the contrary, no solution exists for the inequalities in Eqs (5), the set $\mathbb{D}_{NT}$ is empty, in that no purely compressive stress distribution on the cross-section can equilibrate a force applied at a point exterior to the section. Therefore, in this case, despite the inequalities in Eqs (5) have been written with reference to a particular stress pattern, no other stress pattern can hold.

By the principles of Masonry Limit Analysis if no solution exists to Eqs (5), the structure is over the failure condition in the sense of the Static Theorem. It is worthwhile to note, however, that Limit Analysis in the sense of the Kinematical Theorem can be viewed at more as a test for existence of solutions than as a tool for safety assessment, thus adding some more reasons to the question asked in [33]. Collapse mechanisms associated to unilateral hinges activation, in fact, can hardly be associated to admissible values of stress, and/or -if plasticity is assumed in compression- ductility of masonry is a hope rather than a fact.

3 Relationships of the stress solutions for the masonry arch relevant to different material assumptions

3.1 Selection of purely elastic, elastic-plastic, NT and LT mechanical models

As shown in the previous section, in structures of the type of arches, portal arches, and vaults under investigation, one may neglect the influence of the shear stress on the stress admissibility at any point, and one assumes that the stress state is mono-axial at any point.

The stress solution is thus desirable since it represents the fastest way to pursue the solution with the lowest computational effort, by performing a complementary energy minimization under the admissibility conditions relevant to the adopted material model.

In the following, one considers four different mechanical models of the material, i.e. i) the NT model, ii) the purely elastic model, iii) the LT elastic-brittle model (Low Tensile resistance) model, and iv) the elastic-plastic model, in order to analyse the relationships existing between the relevant stress solutions.

The stress-strain diagrams for the four models are reported in Figures 3 and 4.

One should emphasize that some of the mentioned mechanical models are unreliable when applied to the masonry material (like in the case of the elastic and, even more, elastic-plastic assumptions) and, therefore, their investigation should be considered of purely academic interest.

Anyway available commercial software based on such assumptions is often forced to comply with the masonry behaviour, and, thus, some practical interest arises in the deepening of the matter.

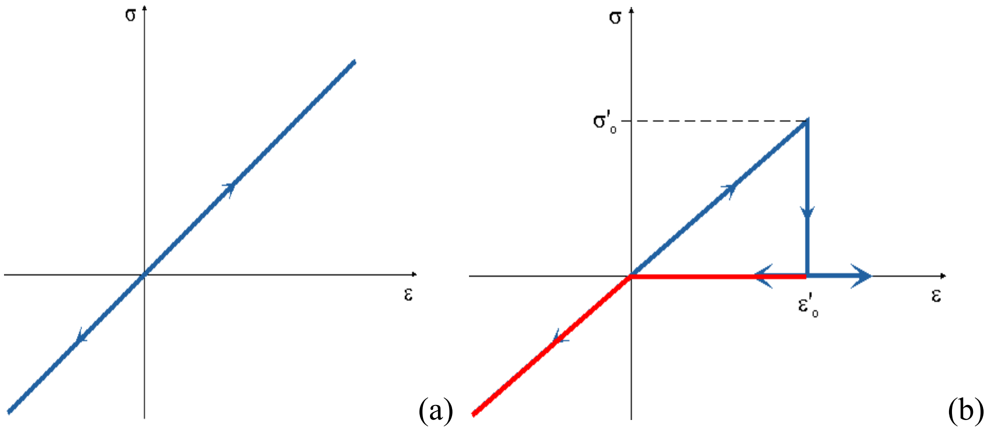

Figure 3: (a) Purely elastic behaviour; (b) Elastic-Brittle LT behaviour.

3.2 The NT material

Under the NT hypothesis, the material obeys a number of conditions regarding the strain ε, composed by an elastic and fracture component, ε_e and ε_f, and the stress σ, which express the conditions that allow or not the development of the fractures.

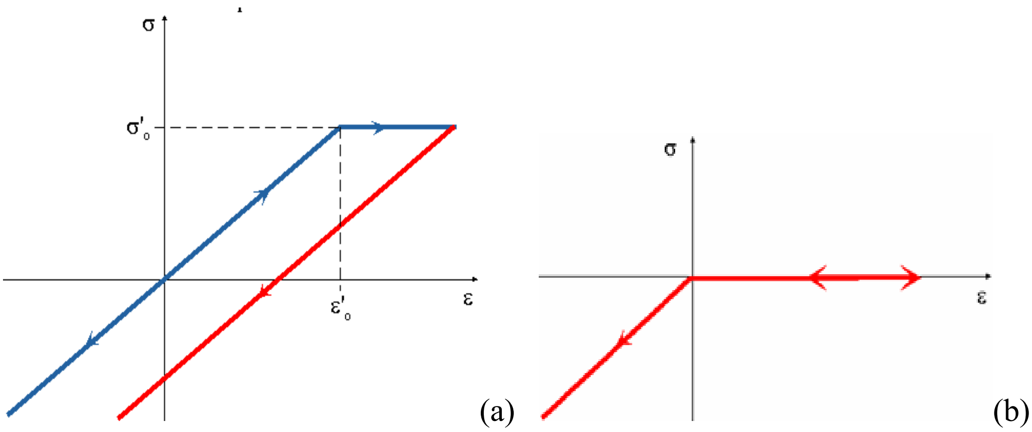

Figure 4: (a) Elastic-Plastic behaviour; (b) NT behaviour.

For NT admissibility the fracture strain is required to be non negative, which means that only detachment are allowed, and the stress to be non positive, that is to say purely compressive.

Thus, first of all, one has

$$\varepsilon = \varepsilon_e + \varepsilon_f,\ \ \varepsilon_e = \frac{\sigma}{E_c},\qquad \left.\begin{array}{l} \varepsilon_f \geq 0 \\ \sigma \leq 0 \end{array}\right\} \Rightarrow \sigma \cdot \varepsilon_f \leq 0 \tag{6}$$

The development of fractures and local detachment along a plane surface at a point can only occur when no stress acts on that surface. Therefore, marking by σ_{NT} the solution stress, the following conditions apply in solution

$$\begin{array}{l} \sigma_{NT} = 0 \ \Rightarrow \varepsilon_e = 0, \quad \varepsilon_e = \dfrac{\sigma_{NT}}{E_c} \\ \left.\begin{array}{l} \varepsilon_f > 0 \Rightarrow \sigma_{NT} = 0 \\ \sigma_{NT} < 0 \Rightarrow \varepsilon_f = 0 \end{array}\right\} \Rightarrow \sigma_{NT} \cdot \varepsilon_f = 0 \end{array} \tag{7}$$

After introducing the reactions $\mathbf{T}_{NT}$ and $\mathbf{T}$ equilibrated respectively by the stress solution σ_{NT} and by any NT statically admissible stress σ, and writing down the expressions of the CE (Complementary Energy) functional for the two cases $\mathcal{C}_{NT}$ and $\mathcal{C}$ (dependence on the stress field is omitted)

$$\mathcal{C}_{NT} = \frac{1}{2E_c}\int_V \sigma_{NT}^2 dV - \int_{S_1} \mathbf{T}_{NT} \cdot \mathbf{u}\, dS, \qquad \mathcal{C} = \frac{1}{2E_c}\int_V \sigma^2 dV - \int_{S_1} \mathbf{T} \cdot \mathbf{u}\, dS \tag{8}$$

where $\mathbf{u}$ denotes the field of imposed displacement on the constrained part S_1 of the surface of the body, one may show that their difference $\Delta\mathcal{C} = \mathcal{C} - \mathcal{C}_{NT}$ is non-negative for any statically admissible stress field and the CE functional attains its minimum in solution.

Starting from the application of the PVW, after some developments, one gets

$$\begin{array}{l} \displaystyle\int_V (\sigma - \sigma_{NT}) \cdot \varepsilon dV = \int_{S_1} (\mathbf{T} - \mathbf{T}_{NT}) \cdot \mathbf{u} dS \\ \displaystyle\rightarrow \frac{1}{E_c}\int_V (\sigma - \sigma_{NT}) \cdot \sigma_{NT} dV + \int_V \sigma \cdot \varepsilon_f\, dV = \int_{S_1} (\mathbf{T} - \mathbf{T}_{NT}) \cdot \mathbf{u} dS \\ \displaystyle\rightarrow \Delta\mathcal{C} = \mathcal{C} - \mathcal{C}_{NT} = \frac{1}{2E_c}\int_V (\sigma^2 - \sigma_{NT}^2) dV - \int_{S_1} (\mathbf{T} - \mathbf{T}_{NT}) \cdot \mathbf{u} dS = \frac{1}{2E_c}\int_V (\sigma_{NT} - \sigma)^2\, dV - \int_V \sigma \cdot \varepsilon_f\, dV \geq 0 \end{array} \tag{9}$$

whence $\mathcal{C} \geq \mathcal{C}_{NT}$.

Therefore, since $\mathcal{C} \geq \mathcal{C}_{NT}$, the NT complementary energy functional in solution attains a value that is always upper-bounded by the one relevant to any other NT statically admissible stress field, i.e. any stress filed which equilibrates the applied loads and satisfies the NT material admissibility conditions, that identify the NT solution domain $\mathbb{D}_{NT}$.

Therefore the NT stress solution is characterized by the constraint minimum of the Complementary Energy functional

$$\sigma_{NT} : \mathcal{C}_{NT} = \mathcal{C}(\sigma_{NT}) = \min_{\sigma \in \mathbb{D}_e \cap \mathbb{D}_{NT}} \mathcal{C}(\sigma) \tag{10}$$

where $\mathbb{D}_e$ represent the purely elastic solution domain where no constraints apply.

3.3 The linear elastic material

Since the stress solution σ_e in case of linear-elastic behaviour of the continuum would be characterized, in turn, by the minimum of the complementary energy under the only constraint imposed by the equilibrium with the applied loads

$$\sigma_e : \mathcal{C}_e = \mathcal{C}(\sigma_e) = \min_{\sigma \in \mathbb{D}_e} \mathcal{C}(\sigma) \tag{11}$$

one may infer that the NT solution is lower-bounded by the linear-elastic one in its definition domain

$$\mathcal{C}_e \leq \mathcal{C}_{NT} \tag{12}$$

3.4 The LT elastic-brittle material

Altough it is temporary because of the decay with time and brittle, some low tensile resistance is exhibited by the masonry and it could be embedded in a more realistic mechanical model.

Whilst in general the ductility of the masonry is poorly reliable, a ductile behaviour is exibithed by masonry whenm once attined the tensile yield treshold, it experiences indefinite deformations under constant stress, which is to say without any loss of resistance capacity.

The formulation of a LT (Low-Tension) behaviour should represent an improvement of the NT modelling, and it is of some interest to investigate the relationships between the two solutions.

One adopts a brittle behaviour in tension. After denoting by "τ" the parameter governing the loading process (for example the time variable), and, for any value "t" of "τ", one gets

$$\sigma(t)=\begin{cases} E\varepsilon(t) & \text{if} \quad \begin{cases}\varepsilon(t)\le 0\\ \text{or}\\ \max\limits_{0\le\tau\le t}\varepsilon(\tau)\le\varepsilon'_o\end{cases}\\ 0 & \text{if} \quad \begin{cases}\varepsilon(t)\ge 0\\ \text{and}\\ \max\limits_{0\le\tau\le t}\varepsilon(\tau)>\varepsilon'_o\end{cases}\end{cases} \tag{13}$$

One should notice that, even in this case, one has that the work of fracture is null

$$\sigma(t)\cdot\varepsilon_f(t)=0 \quad \forall t \tag{14}$$

Denoting by σ_{NT} and $\mathcal{C}_{NT}$ the stress and the complementary energy values attained in the NT solution, and by σ_{LT} and $\mathcal{C}_{LT}$ the same quantities relevant to the LT elastic-brittle solution, after developing some algebraic operations, one may demonstrate that

$$\mathcal{C}_{NT} \ge \mathcal{C}_{LT} \tag{15}$$

whence

$$\mathcal{C}_{NT} \ge \mathcal{C}_{LT} \ge \mathcal{C}_{e} \tag{16}$$

Therefore the elastic-brittle solution is closer as regards to the complementary energy to the elastic one than the NT solution.

3.5 The elastic-plastic material

As mentioned in the above the elastic-plastic hypothesis should be considered purely academic for masonry, since it is usually scarcely credible except for few particular cases.

After denoting by σ_{LT} and $\mathcal{C}_{LT}$ the stress and the complementary energy values attained in the LT elastic-brittle solution, and by σ_p and $\mathcal{C}_p$ the same quantities relevant to the elastic-plastic solution, since the $\sigma_{LT} \le \sigma'_o$ and the elastic-plastic stress solution attains the minimum value of the complementary energy functional between all the possible stress fields equilibrating the loads and with stresses lower than the tensile yield stress σ'_o, one infers that

$$\mathcal{C}_{LT} \ge \mathcal{C}_{p} \tag{17}$$

Therefore the elastic-plastic solution is closer as regards to the complementary energy to the elastic one than the LT solution.

3.6 Relationaships between the stress solutions for different mechanical models

By coupling the inequalities inferred with reference to the four considered mechanical models (NT, elastic-brittle LT, elastic-plastic and purely elastic) one gets the following relationships between the relevant stress solutions

$$\mathcal{C}_{NT} \geq \mathcal{C}_{LT} \geq \mathcal{C}_{p} \geq \mathcal{C}_{e} \tag{18}$$

In Figure 5, the complementary energy functional is represented through its isostatic contour lines where the functional is constant with decreasing values as one moves towards th central minimum point, togheter with the constraints imposed by the admissibility conditions required by the different materials.

One can observe that the elastic-plastic admissibility allows to identify the elastic-plastic domain $\mathbb{D}_p$ where the relevant solution σ_p should be searched for, whilst the NT solution σ_{NT} should be searched for in the relevant statically admissible stress domain $\mathbb{D}_{NT}$

Therefore:

i) the elastic solution σ_e is attained at the unconstrained minimum point of the CE functional;

ii) the plastic solution σ_p, since it should attain the minimum admissible value, is placed at the tangent point of the CE isostatic line tangent to the contour of the plastic domain $\mathbb{D}_p$;

iii) the NT solution σ_{NT}, since it should attain the minimum admissible value, is placed at the tangent point of the CE isostatic line tangent to the contour of the NT domain $\mathbb{D}_{NT}$;

iv) finally the LT elastic-brittle solution σ_{LT}, is placed in the area bounded by the contour of the elastic-plastic domain $\mathbb{D}_p$ and the CE isostatic line relevant to the NT solution.

Based on the above considerations, one may infer that:

- The LT solution belongs to a very narrow area bounded between the plastic and the NT solutions, as shown in Figure 5, so it is very close to the NT solution.
- That narrow area, which is a part of the region comprised between the two definition domains of the plastic and the NT solutions, gets even smaller as the tensile resistance of the material is smaller, or decreases because of decay in time.
- When the yield tensile resistance in attained in the arch, the stress level is substantially the same as in the NT case.
- Definitely, even if the NT case works with a safety advantage, the stress regimes relevant to the LT and the NT solution are approximately the same.

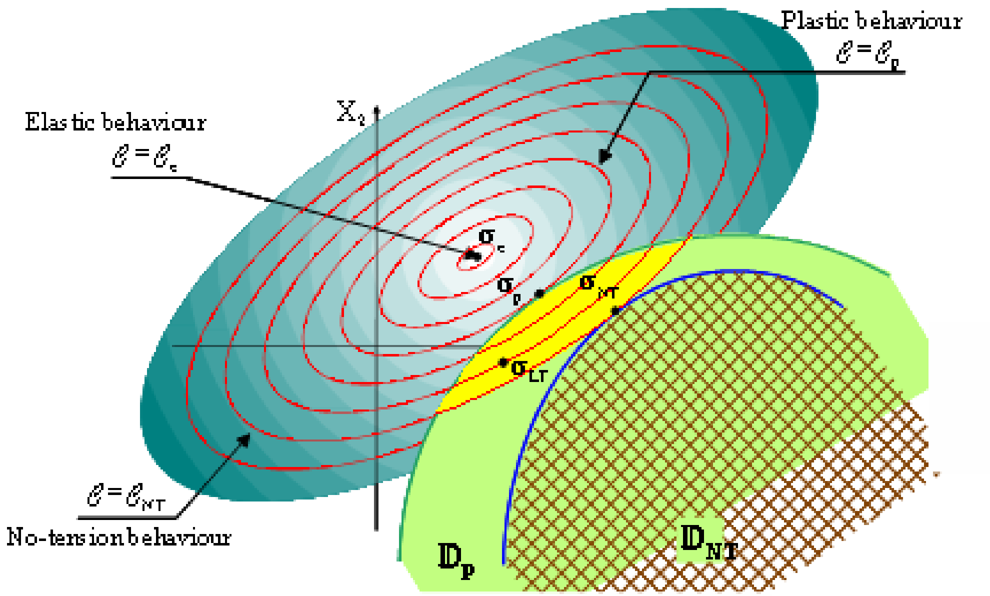

Figure 5: The Complementary- Energy optimum properties of the stress solutions.

4 Conclusions

In the paper the LT elastic-brittle model is developed for modelling the masonry material with some improvement with respect to the NT models. The presented analytical approach is aimed at investigating some properties of the LT solution, which, in conclusion, is shown to be, in energetic terms, upper bounded by the NT solution and lower bounded by the elastic-plastic solution. From this result one may infer that the NT solution works with some safety advantage because it cannot rely upon any tensile resistance, but, definitely, the LT and NT solutions are very close to each other and are characterized by approximately equal stress regimes in the two cases.

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