Paper 289

Proceedings of the Eleventh International Conference
on Computational Structures Technology,
B.H.V. Topping, (Editor),
Civil-Comp Press, Stirlingshire, Scotland

Accounting for Uncertainty in Bidirectional Ground Motion

C.S. Belsham
Professional Engineer
Southport, United Kingdom

Abstract

The determination of the resultant of the combination of bidirectional horizontal seismic input motion has recently attracted renewed attention in the USA as a result of a change in the National Earthquake Hazards Reduction Program 2009 provisions and has resulted in a number of papers on the topic. This paper proposes a new approach based to random vibration theory and the algebra of normal distributions to determine the probable magnitude of the resultant. The approach is compatible with stochastic based source to site models which are becoming the method of choice for defining modern ground motion prediction equations. The probable magnitudes of the resultant are evaluated and the effects of the resultant are considered in the context of different plan shapes of structures and their vulnerability are explored.

Keywords: seismic, bidirectional ground motion, random vibration theory, structures, algebra of normal distributions, stochastic method.

1 Introduction

A number of articles concerned with the appropriate representation of bidirectional ground motions were presented during the late 1970s to early 1990s; amongst these were [1, 2, 3]. Since then there have been occasional articles, for example [4]. Then since 2006 a significant number of articles have been published, noting in particular [5, 6, 7, 8, 9, 10, 11]. Paper [11] is of particular interest as it is in response to a recommendation by 2009 NEHRP (National Earthquake Hazards Reduction Program) Recommended Seismic Provisions for New Buildings and Other Structures: Part 2, Commentary to ASCE/SEI 7-05, which in 2009 revised the approach to the application of bidirectional horizontal earthquakes to the seismic design of structures. This changed the definition of horizontal ground motion to be used in design from the geometric mean of spectral accelerations for two components, which is defined by equation (1)

$$S_{a-gm} = \sqrt{S_{a-x} \times S_{a-y}} \qquad (1)$$

to the peak acceleration response of a single lumped mass oscillator irrespective of direction, which is defined by equation (2)

$$S_{a-max} = \max\left(S_{a-x}, S_{a-y}\right) \qquad (2)$$

where in equations (1) and (2) S_{a-x} and S_{a-y} are the spectral accelerations for the components of the horizontal ground motions, S_{a-gm} is the geometric mean of spectral accelerations and S_{a-max} is the maximum of those components. Concerns have been raised about this revision [11], amongst which is the increased societal cost, particularly for non-symmetric structures. It also appears that inherent with the approach of using S_{a-gm} is an implicit accounting for the probability that the principal axis of structural response will not align with the direction of maximum spectral input. Paper [11] does identify that for some structural systems, such as flagpoles and circular tanks, which they class as having azimuth-independent properties, the single lumped mass oscillator model that is the basis for maximum-direction ground motions is a good analogue to the real system.

The NEHRP 2009 provision also addresses the combination of the bidirectional components and in the commentary the expectation is quite clear in that the average of the SRSS spectra to the design spectrum is intended to be a factor of 1.3 greater to compensate for the increase associated with taking the SRSS of the two components. In order to clarify matters it is noted that if the two components were perfectly correlated then the SRSS of the average spectra would be larger than the average by the √2. However, the provision also allows the results to be low by as much as 10% resulting in an effective factor of 1.18.

A study [12] draws the conclusion that the reference to the 1.3 factor discussed above when called for in [13] is implicitly referring to design spectra based on average horizontal response.

The development of an alternative approach is given in this paper as to how bidirectional ground motions can be accounted for and takes into consideration the overall problem by extending the investigation to consider the source, the instrumentation and the structure and treating the problem using a probabilistic approach. The results show that using equation (2) can still underestimate the probable input motion particularly in the case of structures with symmetric plans.

The method presented in this paper directly takes this latter consideration into account.

2 The horizontal components of ground motion

With the exception of well-defined faults that break the surface or are near to the surface, the orientation of the fault plain is, in the main, unknown and may be regarded as random. For deep buried faults (> 5km) it is generally accepted that a seismic rupture event can be defined by a double couple acting at the centre of the fault. These couples generate wave motions in the rock which are resolved into primary compression waves called p-waves and shear waves, which are known as secondary waves or s-waves. For the analysis of structures and plant it is primarily s-waves that are of interest and this paper therefore discusses effects in terms of only s-waves.

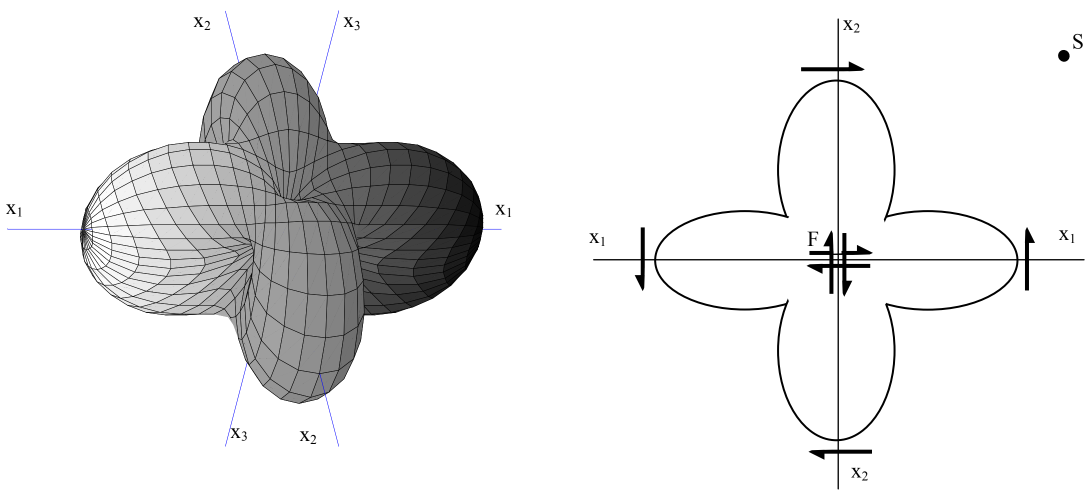

Figure 1: Plots showing the profile of shear wave components from a double couple shear source term that is expanding and a section through on the $x_1 - x_2$ plane showing the direction of the shear at the tips of the lobes where position S is the site

Figure 1 explains why there are two horizontal shear components from an earthquake; as can be seen from figure on the right it is because of the shear forces acting on each of the lobes. In fact the shear forces act across the whole of the surfaces of the lobes which helps to explain why the ground motions at the surface are so complex. Considering the point S in figure 1, which represents the site of interest, it can be seen that it will experience motion from both the directions x_1-x_1 and x_2-x_2 and this is the reason for the two orthogonal horizontal input components.

Now assume that site S has been selected for the installation of a tri-axial accelerometer but that the earthquake focus F in figure 1 is random relative to the site S. The horizontal axes of the tri-axial accelerometer are therefore arbitrarily chosen. As a consequence they are unlikely to align with the maximum ground acceleration when the fault at F in figure 1 ruptures.

The situation at S is shown in figure 2, which also shows the planar area swept out by the locus of the components of the actual ground acceleration such that the perimeter represents the peak ground acceleration in the polar direction.

The creation of the swept area is often described as the orbit of the response of the ground since a path is swept out over time *T*, the duration of the event. It follows that at any time *t* during the period *T* a rigid block at site S is subject to the resultant of the two horizontal components of ground acceleration.

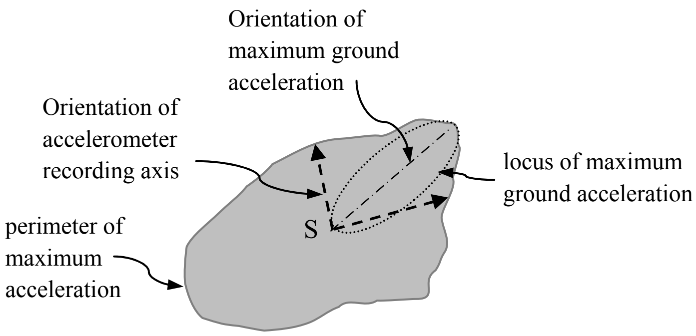

Figure 2: Sketch showing the orientation of the horizontal axes of the accelerometer relative to the maximum ground accelerations and the co-existing orthogonal horizontal component it also shows the planar area swept out by the maximum ground accelerations and a typical locus.

3 Actual and measured ground accelerations

A limitation of the current method of recording ground motions is that the accelerometers are a tri-axial arrangement with two orthogonal horizontal recorders, this means that it can only measure in those two direction. With such an arrangement it is not possible to know if the maximum acceleration has been measured as indicated in figure 3. Even taking the resultant as the maximum could still underestimate the actual value as shown in figure 3, which shows the orientation that the tri-axis would need to be in to record the maximum acceleration.

Figure 3: The resultant from the measurement of the horizontal ground acceleration and to the right the resultant from the actual horizontal ground acceleration

Even by taking the resultant of the measured accelerations it may still be less than the actual maximum even if only one of the components is considered. Considering just the maximum measured value as stated in the 2009 NEHRP provision is therefore likely to be an underestimate of the actual maximum unidirectional ground motion.

This raises the question as to what is the appropriate way of defining the ground motion when measured as two orthogonal components. That, to an extent, depends on the basis by which the ground spectra have been developed.

3.1 Spectra derived from empirical data

If spectra have been built from an extensive database then it could be argued that at least some of the contributing data comes from recordings that did align with the direction of maximum acceleration. This would provide a reasonable approximation to the maximum acceleration but only in one direction. It is then necessary to decide on the value for the orthogonal direction. Having arranged all the records to give the maximum measured value in one direction, it follows that the other direction will have a smaller maximum acceleration. However, with the exception of a few cases, it is probable that neither recording axis has measured the maximum acceleration. Therefore a balanced approach should consider each component as being equal, however that will almost certainly result in a lower maximum acceleration than is currently estimated, the difference being a function of the data set.

3.2 Spectra generated from mathematical models

The Stochastic Method, [14], is a well known example of a mathematical model for generating the responses of earthquakes and these approaches assume equal horizontal components which both achieve the same maximum acceleration. As with the empirical spectra each of the components is consider to be statistically independent of the other. It also follows that since there is no direct physical measurement involved, the components are considered to align with the maximum input directions.

3.3 Resultant of the component motions

As shown in figure 1, there are two ground input components and it follows that there will be a resultant motion which will be larger than either of the two components. If the input motions were the same period and in phase then the calculation of the resultant would be straightforward, resulting in 1.414 times a component. However, the input motions are random and uncorrelated and it is necessary to explore by how much the magnitude of the resultant will exceed the magnitudes of the components. To consider this problem further use is made of the algebra of normal distributions the details of which can be found in [15].

4 The probabilistic magnitude of the resultant of two orthogonal random time histories

The algebra of normal distributions can be applied to the analysis of earthquakes since in most case they can be reasonably approximated by random vibrations with a normal distribution.

When dealing with random processes the peak factor k is defined as the number of standard deviations that the maximum value is from the mean. Therefore the ratio of the standard deviation to the peak can be determined as follows. Let m be the random variable under consideration with a mean value of μ_m and a standard deviation σ_m. Let M be the maximum value, namely the peak acceleration that m reaches during the event, and then k is determined by considering the number of standard deviations M is away from the mean μ_m. Thus

$$k = \frac{M - \mu_m}{\sigma_m} \tag{3}$$

If μ_m is 0, which is typical for earthquakes, then equation (3) reduces to equation (4)

$$k = \frac{M}{\sigma_m} \tag{4}$$

This approach can be applied to the resultant of two orthogonal time histories labelled p and q, which have mean values μ_p and μ_q respectively (taken to be 0) and standard deviations σ_p and σ_q. Since the time histories are considered as being of equal magnitude, they will have approximately the same peak values and although they are statistically independent they will have nearly the same peak factor k. Thus

$$P = k\sigma_p \quad \text{and} \quad Q = k\sigma_q \tag{5}$$

Where P and Q are the peak acceleration reached by the two components. The objective is to determine the resultant r for the two components p and q for the same peak factor k as applies to p and q.

Firstly, although both p and q have 0 mean values, from the algebra of the normal distribution the resultant r does have a non zero mean value given by

$$\mu_r = \frac{\left(\sqrt{2}PQ\right)^{\frac{1}{2}}}{k} \tag{6}$$

Also from the algebra of the normal distribution it can be shown that

$$\sigma_r{}^2 = \sigma_p{}^2 + \sigma_q{}^2 - \sqrt{2}\sigma_p\sigma_q \tag{7}$$

now

$$\sigma_p = \frac{P}{k} \quad , \quad \sigma_q = \frac{Q}{k} \tag{8}$$

substituting equations (8) into equation (7) gives

$$\sigma_r{}^2 = \frac{P^2}{k^2} + \frac{Q^2}{k^2} - \sqrt{2}\frac{PQ}{k^2} \tag{9}$$

or

$$k^2\sigma_r{}^2 = P^2 + Q^2 - \sqrt{2}PQ \tag{10}$$

From equation (3) the maximum value R for the resultant r for the same peak factor k is given by equation (11)

$$R = k\sigma_r + \mu_r \tag{11}$$

substituting from equations (6) and (10) into equation (11) gives equation (12)

$$R = \frac{1}{k}\left(\sqrt{2}PQ\right)^{1/2} + \left(P^2 + Q^2 - \sqrt{2}PQ\right)^{1/2} \tag{12}$$

Which for $P = Q$ gives equation (13)

$$R = \left[\frac{1.1892}{k} + 0.7654\right]P \tag{13}$$

5 A probabilistic representation of the response spectra

Although the above approach may be applied to an empirically derived Ground Response Spectra based on a Ground Motion Prediction Equation it is preferable to use a probabilistic representation that is capable of directly accounting for uncertainties. An equation of this form developed in [16] is shown in equation (14)

$$RS_a\left(\omega,\delta,r,M_W,q,\Delta\sigma\right)=\left[\frac{\pi\omega k\left(r,M_W,q,\delta,\Delta\sigma\right)^2 S_a\left(\omega,r,M_W,\Delta\sigma\right)}{4\delta}\right]^{1/2} \quad (14)$$

where $RS_a(\omega, \delta, r, M_w, q, \Delta\sigma)$ is the ground acceleration response spectra at frequency ω with damping ratio δ located at distance r from the epicentre of a moment magnitude M_w earthquake such that it has a confidence level q and $\Delta\sigma$ is the average stress drop across the fault. $S_a(\omega, \delta, r, M_w, \Delta\sigma)$ is the magnitude of the Fourier spectra at the surface and $k(r, M_w, q, \delta, \Delta\sigma)$ is the peak factor which is discussed in section 6.

6 Determination of the peak factor for combining component responses

Sections 4 and 5 have shown the importance of the peak factor k in determining the resultant. There are a number of expressions which give an estimate of k, the method used in the Stochastic Method, [14] is by Cartwright and Longuet-Higgins [17]. Other well known expressions are those by Davenport [18], Vanmarcke [19] and a simplified form by Vanmarcke and Lai [20]. An expression for k for the source model in equation (14), was developed in [16], which is a function of the percentile of the crossing levels that will exceed q, and which is given by equation (15)

$$k\left(r,M_W,q,\delta,\Delta\sigma\right)=\left[2\ln\left[\frac{2v_0\left(r,M_W,\delta,\Delta\sigma\right)T}{\ln\left(q^{-1}\right)}\right]\right]^{1/2} \quad (15)$$

where v_0 is the estimate of the zero-crossing rate and depends on the properties of the power spectral density function which is a function of the epicentral distance, the moment magnitude and the stress drop, T is the duration of the strong motion. Since this definition of peak factor relates directly to the spectra it will also be appropriate for k in the bidirectional equation (13).

7 The probabilistic resultant of two horizontal components of ground motion

It follows the resultant of two horizontal components of ground motion when they are equal is given by substituting equation (14) into equation (13) leading to the result in equation (16)

$$R=\left[\frac{1.1892}{k\left(r,M_W,q,\delta,\Delta\sigma\right)}+0.7654\right]\left[\frac{\pi\omega k\left(r,M_W,q,\delta,\Delta\sigma\right)^2 S_a\left(\omega,r,M_W,\Delta\sigma\right)}{4\delta}\right]^{1/2} \quad (16)$$

Quantitative results are now given for $Q = P$ and for $Q = 0.8P$ where P is determined from equation (2) and k is assumed to be 3.0, the factor of 0.8 is suggested in [3]. For the case of $Q = P$, equation (16) gives $1.162P$, equation (1) gives $0.894P$ and equation (2) gives P. For the case of $Q = 0.8P$, using equation (12) in equation (16) in place of equation (13) gives $0.987P$, equation (1) gives $0.894P$ as before and equation (2) gives P as before. The case of equation (16) giving $1.162P$ when $Q = P$ is similar to the assumption made by the 2009 NEHRP Provisions. However, if in equation (1) each component was only 0.8 of the maximum values then geometric mean would only be $0.8P$ and the two components would be $0.8P$ and $0.8Q$, which when SRSS gives an effective resultant of $1.13P$, which again differs only slightly from the other results.

The results for high values of q are consistent with what is expected, for example with a value of q of 0.95 k is approximately 4.0, giving a resultant of 1.06. This occurs because of the lower probability of the higher peaks in each component time history occurring together.

8 Effects of horizontal components on different structural arrangements

The structures to which the ground motion generally applies can be divided into three categories by reference to their plan cross-sections; those that are axisymmetric, typically circular; those that are symmetric, typically square and those that are asymmetric, typically rectangular. Each category of structure is discussed separately as it will be shown that the bidirectional input affects each differently.

8.1 Axisymmetric circular plan structures

The axisymmetric circular plan structure shown in figure 4 is the simplest to discuss because its response is the same regardless of the direction of the input motion. It therefore follows that for a spectral analysis the response will be with respect to the resultant ground input motion given by equation (16). Except for low circular structures, consideration needs to be given to the effect of the changing input angle, which results from the random horizontal inputs. This requires the two horizontal inputs to be applied simultaneously as independent time histories. The effect is complex as the response of the structure is forced by the input motion at its various natural frequencies which differ from those of the input motion. As a consequence the resultant response does not follow the same locus as the resultant input motion. This effect will vary from structure to structure and with the specific input motions.

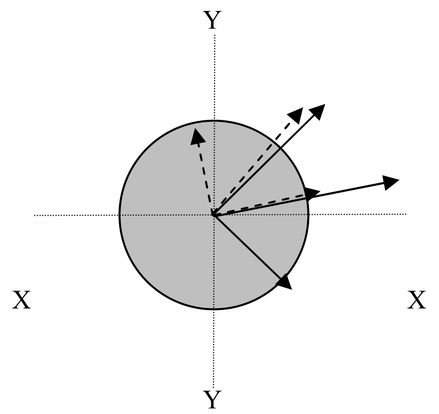

Figure 4: Shows a structure that is circular on plan. The dashed arrows represent the directions of the measured ground accelerations and the resultant with the solid arrows showing the directions of the actual ground accelerations and the resultant.

8.2 Symmetric square plan structures

A square or almost square structure presents an example where the arguments that are normally used do not apply. Consider as an example a simple multi-storey structure with a square plan cross-section with square sections columns with sides *b* in each corner, as shown in figure 5. The stiffness of the structure is proportional to the inertia of the columns, which about the X-X and Y-Y axis of the structure are 4Ixx and 4Iyy where Ixx = Iyy = $b^4/12$. Now consider the stiffness of the columns about the diagonal axes U-U and V-V, here the inertia is across the diagonal of the columns and is given by 4Iuu and 4Ivv where Iuu = Ivv = $b^4/12$.

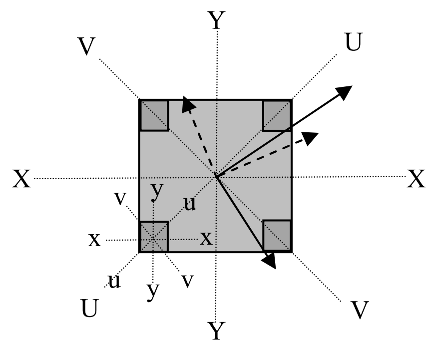

Figure 5: A structure that is square on plan with square columns in each corner. The dashed arrows represent the directions of the measured ground accelerations with the solid arrows showing the directions of the actual ground accelerations. The upper case lettering is for the axes of the structure and the lower case are for the axis of individual members.

The frequencies and modes are therefore the same. It hence follows that such a structure is just as likely to respond diagonally as it is to parallel to a side. This has significant safety implications because of the higher stresses when the structure responds diagonally. This arises firstly because the bending stress in the columns is greater when the structure responds diagonally due to $Zuu = Zvv = b^3/6\sqrt{2}$ whilst $Zxx = Zyy = b^3/6$ and, in addition, the compressive overturning force is applied to one column rather than two in the case of parallel inputs, although this is partly offset by the increased lever-arm in the ratio of 1.414 to 1.0.

In this case the increase in input accelerations due to the combination of the horizontal components will act to further reduce the safety about the diagonal axis. The diagonal can be subject to earthquake input from any one of four directions and at these directions the input can be at least 10° off diagonal with the effect being significantly reduced. This means there is at least probability of 0.22 of the structure suffering diagonal input from an earthquake.

This form of structure is prone to the same issues as have been outlined for the circular plan section.

8.3 Asymmetric rectangular plan structures

For asymmetric structures, which as shown in figure 6 are generally structures with one side appreciably longer than the adjoining sides, in general it is unlikely that the principal directions of the input motion will align with the principal axis of the structure.

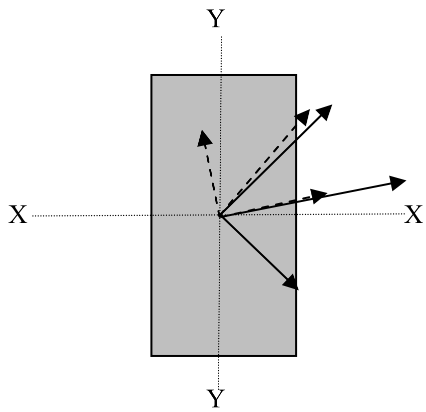

Figure 6: A structure that is rectangular on plan. The dashed arrows represent the directions of the measured ground accelerations and the resultant with the solid arrows showing the directions of the actual ground accelerations and the resultant.

The usual approach when analysing these types of structures is to assume that the response occurs in the directions of the principle axis X-X and Y-Y with the resulting forces combined by a method such as SRSS or the 100-40-40 rule. Prior to 2009 in the US it would mean each direction would be subject to an acceleration ground spectra based on the geometric mean of the data set used in equation (1). Now it will be based on each direction being subject to an acceleration ground spectra based on the maximum component values from the data set used, equation (2). While this revised approach makes a difference to each direction, the spatial combination rules reduce the combined effect.

Although the structure may not align exactly with the resultant of the maximum ground accelerations, in the same manner as discussed for the square structure, there is a probability of 0.11 that the resultant will be in the direction of one of the principle axes of a rectangular building.

9 Discussion and Conclusions

This paper has discussed how two horizontal seismic ground components may be combined into a resultant seismic ground input using a new probabilistic approach. The approach is particularly suited to cases where the fault distance and orientation are unknown; this suggests it applies to Stable Continental Regions which make up most of the global landmass.

The approach has been considered in the context of the effect that it will have on structures of different plan shape. The consequences have been compared with the approach used in the US both before and after the changes introduced by the 2009 National Earthquake Hazards Reduction Program provisions. The results show that the new rule represented by equation (2) is appropriate for the design of rectangular buildings when combined with spatial SRSS.

Even with the increase from the geometric mean to the maximum component there is still a potential underestimate of the resultant force on a symmetric structure which is particularly significant for buildings that are square on plan due to the weaker resistance across the diagonal. It is worth noting that many high-rise building tend to have square or approximately square plan sections.

The analysis has shown that in cases where one of the horizontal components is greater than 0.8 of the other, then the resultant will be larger than the greater of the components for the case of a peak factor k of 3. As the peak factor varies with the magnitude and distance of the earthquake, the ratio of the components that will cause the resultant to exceed the maximum component will also vary. It is however found that even if each of the components are measured as only 0.8 of the maximum, the resultant from the SRSS is close to the probabilistic resultant.

An important observation is that the magnitude of the resultant decreases with increasing peak factor k. An increase in the peak factor is associated with longer return periods. This suggests that for high hazard facilities with return periods of 1 in 10000 years the resultant may only be slightly larger than the maximum components. In contrast facilities being designed for shorter return periods may be subject to proportionally higher resultants.

The approach developed in this paper is also applicable to seismic ground response studies and to seismic soil structure interaction. Both of these dynamic responses have a filtering effect on the seismic input and act to modify the input spectra and possibly introducing correlation which would tend to increase the resultant.

The analysis introduced in this paper is based on an assumption that the seismic ground accelerations are normally distributed. If that is not the case and a different distribution is appropriate then the analysis needs to be carried out numerically using Monte Carlo methods.

References

[1] J. Penzien, M. Watabe, "Characteristics of 3-Dimensional Earthquake Ground Motion", Earthquake Engineering and Structural Dynamics, 3, 365-37, 1975.

[2] E. L. Wilson, M. R. Button, "Three-Dimensional Dynamic Analysis for Multi-component Earthquake Spectra", Earthquake Engineering and Structural Dynamics, 10, 471-476, 1982.

[3] R. A. Clough, J. Penzien, "Dynamics of Structures", McGraw-Hill Kogakusha, Ltd., 1993.

[4] O. A. Lopez, R. Torres, "The Critical Angle of Seismic Incidence and the Maximum Structural Response", Earthquake Engineering and Structural Dynamics, 26, 881 – 894, 1997.

[5] J. W. Baker, C. A. Cornell, "Which Spectral Acceleration Are You Using?", Earthquake Spectra, 22, 293-312, 2006.

[6] K. Beyer, J. J. Bommer, "Relationship between Median Values and between Aleatory Variabilities for different definitions of the Horizontal Component of Motion", Bulletin of Seismological Society America, 96, 1512-1522, 2006.

[7] J. A. Watson-Lamprey, D. M. Boore, "Beyond SaGMRotI: Conversion to Sa_{Arb}, Sa_{SN}, and Sa_{MaxRot}", Bulletin of Seismological Society America, 97, 1511-1524, 2007.

[8] D. M. Boore, J. Watson-Lamprey, N. A. Abrahamson, "Orientation-independent measures of ground motion", Bulletin of Seismological Society America, 96, 1502-1511, 2006.

[9] Y. Huang, A. S. Whittaker, N. Luco, "Maximum Spectral Demands in the Near-fault Region", Earthquake Spectra, 24, 319-341, 2008.

[10] D. M. Boore, "Orientation-independent, non geometric-mean measures of seismic intensity from two horizontal components of motion", Bulletin of Seismological Society America, 100, 1830-1835, 2010.

[11] J. P. Stewart, N. A. Abrahamson, G. M. Atkinson, J. W. Baker, D. M. Boore, Y. Bozorgnia, K. W. Campbell, C. D. Comartin, I. M. Idriss, M. Lew, M.

Mehrain, J. P. Moehle, F. Naeim, T. A. Sabol, “Representation of bidirectional ground motions for design spectra in building codes”, Earthquake Spectra, 27, 927-937, 2011.

[12] NEHRP Consultants Joint Venture, “Selecting and Scaling Earthquake Ground Motions for Performing Response-History Analyses”, Report NIST GCR 11-917-15, 2011.

[13] American Society of Civil Engineers, ASCE/SEI 7-05, “Minimum Design Loads for Buildings and Other Structures”, 2005.

[14] D. M. Boore, “Simulation of ground motion using the stochastic method”, Pure and Applied Geophysics, 160, 635-675, 2003.

[15] E. B. Haugen, “Probabilistic Mechanical Design”, John Wiley & Sons, 1980.

[16] C. S. Belsham, “Exploring Uncertainties in Probabilistic Seismic Hazard Analysis through a Stochastic Source- to-Site Seismic Model”, COMPDYN 2009, ECCOMAS thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, Rhodes, Greece, 22-24 June 2009, 2009.

[17] D. E. Cartwright, M. S. Longuet-Higgins, “The Statistical Distribution of the Maxima of a Random Function”, Proceedings Royal Society London, 237, 212-232, 1956.

[18] A. G. Davenport , “Note on the Distribution of the Largest Value of a Random Function with Application to Gust Loading”, Proceedings of Institution of Civil Engineers, 28, 187-196, 1964.

[19] E. H. Vanmarcke, “On the Distribution of First Passage Time for Normal Stationary Random Processes”, Journal of Applied Mechanics, March 1975, 215-220, 1975.

[20] E. H. Vanmarcke, S. P. Lai, “Strong-motion duration and rms amplitudes of earthquake records”, Bulletin of Seismological Society America, 70, 1293-1307, 1980.