

Numerical Modelling of Reinforced Masonry Arches

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Abstract

This paper discusses results of numerical analysis of masonry arches which have been reinforced by fibre–glass fabrics. Laboratory tests on reinforced and unreinforced masonry arches have been conducted by Witzany et al [9]. The numerical simulations discussed in this paper supplement this experimental programme. The main aim of these computations is to identify effective and ineffective configurations of reinforcement. The computations are divided into two parts: a linear elastic parametric study and non-linear analyses of selected cases. An elastic–plastic material model is used for non-linear computations.

Keywords: masonry, arches, experimental testing, numerical modelling.

1 Introduction

The team of Professor Witzany at the Czech Technical University in Prague has been executing a long term experimental testing programme [9]. The main tested objects are reinforced and non-reinforced masonry arcs. The reinforcements are usually fibre-glass or carbon fabrics.

These arches are reinforced by various material on their top and bottom faces. There are 3 variants of the arcs with different heights (0.75 m, 1.0 m and 1.5 m). Other parameters are identical for all three types and they are shown in Figure 1. The thickness of the arc (a dimension in the out of plane direction) is 0.75 m.

The main aim of the mentioned experimental programme is an evaluation of different types of reinforcements. These reinforcements can be placed on the bottom face or on the top face on the arc. The reinforcement can cover only a part of the surface. The top and the bottom ones can be combined. Thus there are many possible setups of these reinforcements. The already finished experiments covered several possible

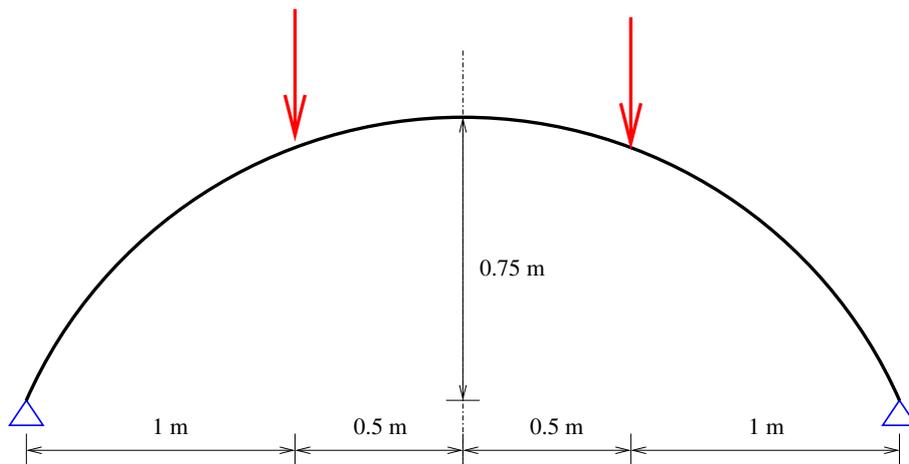


Figure 1: Scheme of experiments.

configurations but they also shown that improper reinforcement use may even limit the bearing capacity of the arc.

It is obvious that addition of a stiffer material would change stress distribution in a masonry arch. The purpose is to lower tensile stresses in masonry. But certain configurations of reinforcing elements can cause enlargement on these stresses in masonry.

The original experimental programme haven't include a numerical counterpart. For a guidance of future experiments it is necessary to carry out some preliminary computational analyses of planned experimental structure configurations.

This article describes an ongoing works in this area. The first part of article discusses a parametric study of reinforcement configurations. This study uses linear elastic material models. It's purpose is to identify the cases with stress concentrations which may lead to earlier start of damage processes.

The second part of this paper discusses a non-linear analysis of one selected case.

2 Parametric study

2.1 Finite element modelling

The parametric study has been carried out with material properties listed in the Table 1. Three problem types have been studied:

- reinforcement on the bottom face,
- reinforcement on the top face,
- reinforcement on the both faces.

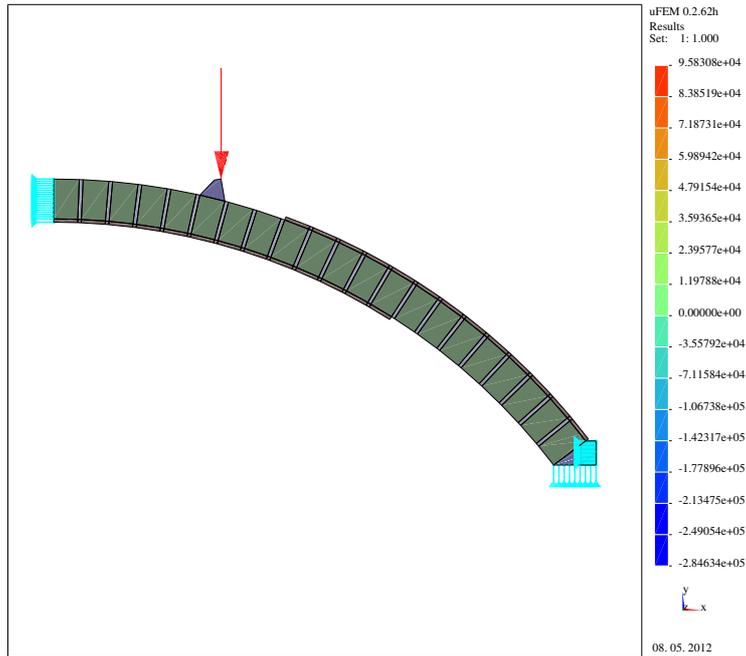


Figure 2: Computational model with partial top and bottom reinforcements.

The variable parameter has been the area of arc face which has been covered by reinforcement. There is more possible combinations of ratio between covered areas of bottom and top surfaces but only one type is studied here. The reinforcement on both side has been proportionally enlarged between computations. One of the cases is shown in Figure 2.

Material	E [MPa]	ν
Bricks	2.50	0.2
Mortar	0.50	0.2
Reinforcement	72.0	0.2
Concrete	40.0	0.2

Table 1: Material properties.

Only a symmetric half of the arc has been modelled. The finite element model has been created from 2D four-node isoparametric elements for plane stress problem. The uFEM [6] software has been used. Reinforcements have been modelled by additional finite elements. Figure 2 shows an example of the computational model. A typical model has included about 4100 finite elements and about 8800 degrees of freedom.

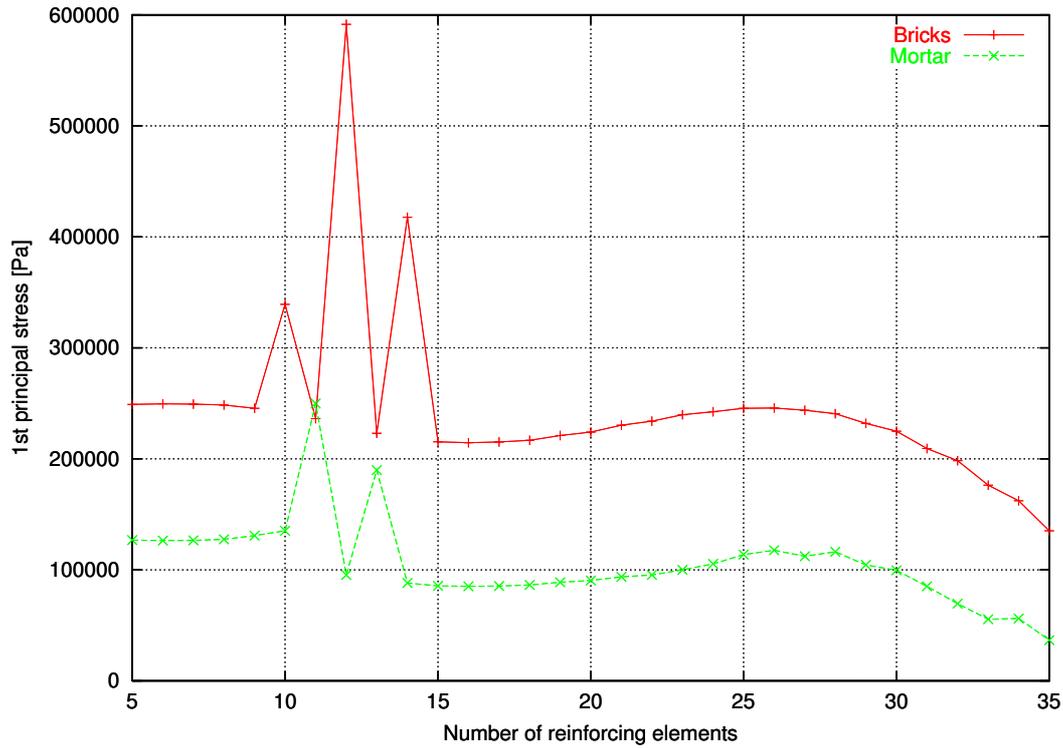


Figure 3: Maximum σ_1 for bottom reinforcement.

2.2 Results

The maximal tensile stress σ_1 has been studied. Figure 3 shows these stresses for the case when only the bottom reinforcement is used. Figure 4 shows the same results for the top reinforcement. Figure 6 shows stresses in the case of reinforcement on both faces.

Every point in these graphs represent a result of one computation. This form of results representation can give an useful information about recommended configurations of reinforcements. But there are some problematic points. The strong peaks (the most visible in Figure 3) represents situation when reinforcements ends near the position of load. It is questionable if the results are correct in this case but such configurations of reinforcements are visibly invalid so results in these points don't need to be studied in detail.

It is interesting that graphs in Figure 4 (that represents cases with reinforcements only on the top face of arc) are much more smooth.

Figure 6 shows results of combination of reinforcements on both sides. This graph has different numbering on its x axis because the approach that was used for previous figures can be confusing here. This figure represents only one strategy of combined reinforcements configurations so any conclusions are limited.

The results show that the maximum tensile stresses are lower then in the case when

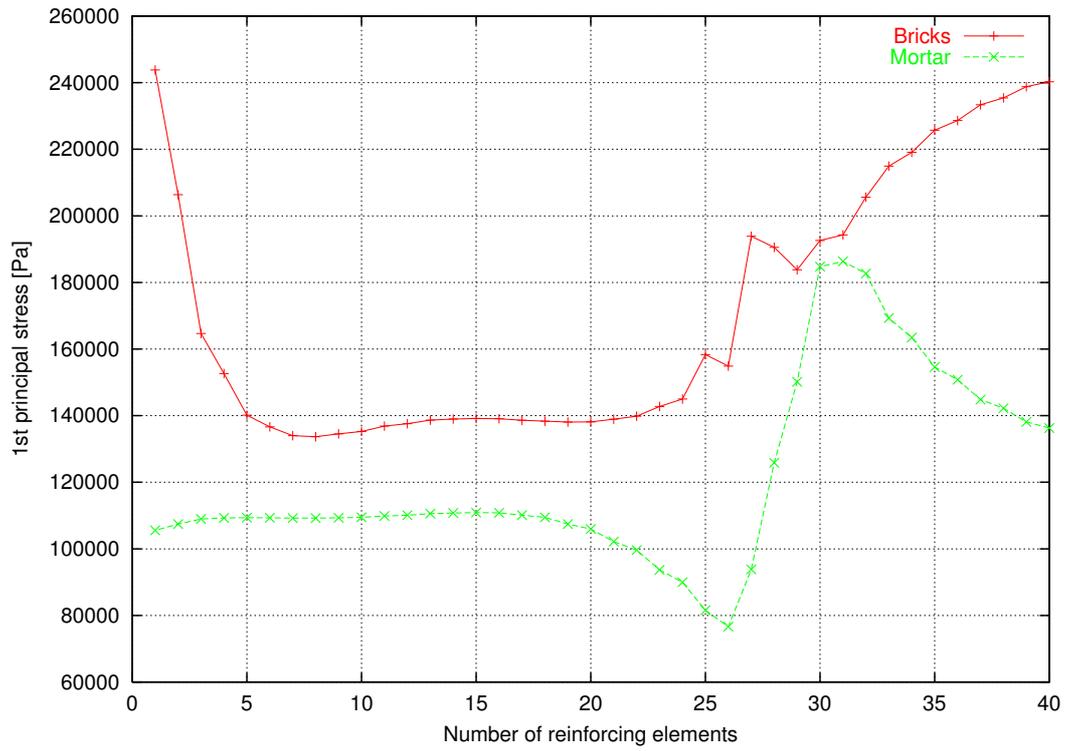


Figure 4: Maximum σ_1 for top reinforcement.

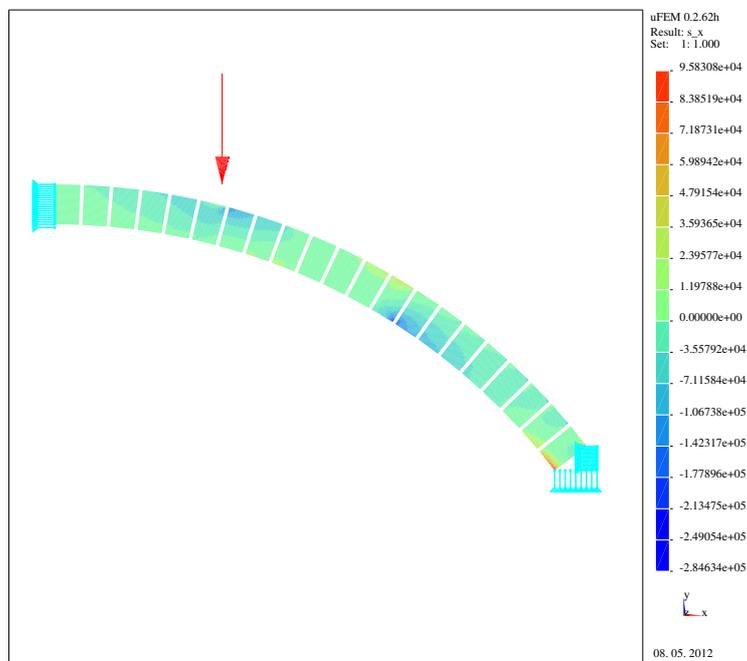


Figure 5: Maximum σ_1 for top reinforcement.

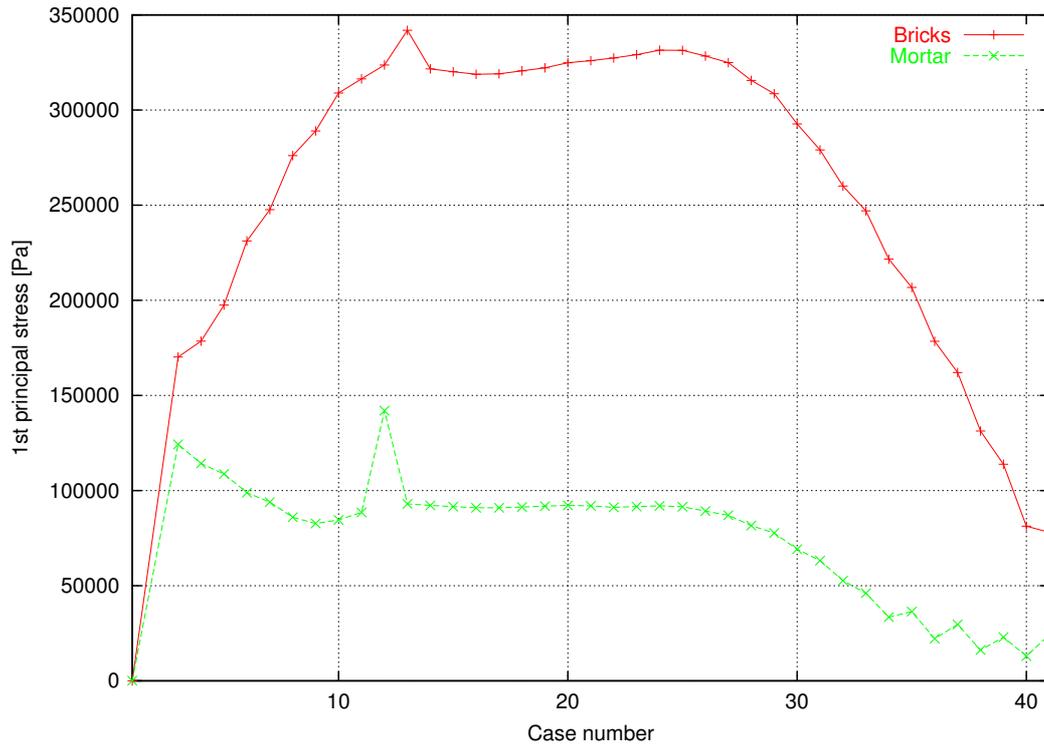


Figure 6: Maximum σ_1 for combined reinforcement.

only the lower face is covered by reinforcement. It can be concluded that use of reinforcement on both sides can effectively reduce maximum stresses in the arc. Figure 5 represents the case numbered 25 which represents the right smooth top of the red line in Figure 6. The computational model for this case is shown in Figure 2.

3 Non-linear analysis

3.1 Finite element model

The non-linear analysis has been done with use of the uFEM [6] and the ANSYS [1] software. However, only results from the ANSYS are discussed here. The finite element model has been created an extension of previously discussed plane model to the third dimension. The original model has been transferred to the ANSYS software and a control analysis has been done. The maximum stresses shown in Figure 8 are not exactly identical to the previously presented data due to different sizes of finite elements.

Because of size of the computational model the 3D version has been re-meshed in order to have an acceptable number of finite elements for the non-linear analysis. The model configuration which has been selected is identical to Figure 2.

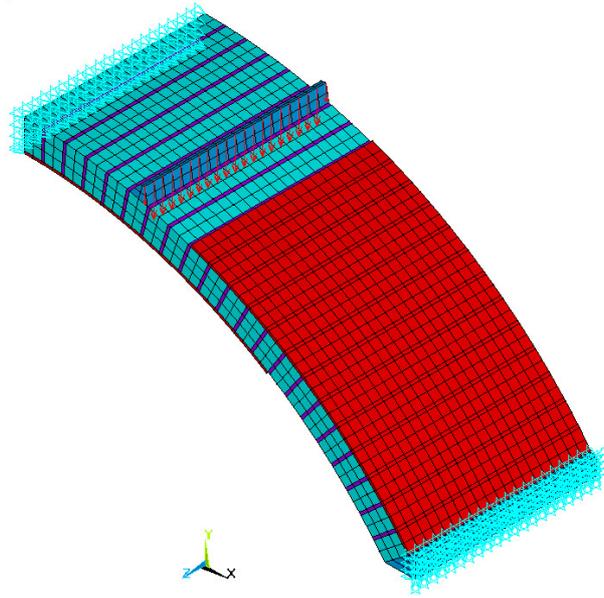


Figure 7: 3D model for non-linear analysis.

The SOLID65 finite element type and the CONCRETE material type have been used. The selected material model was originally developed for modelling of concrete by Willam and Warnke [7, 8]. Material data for this model are listed in Table 2. Only mortar has been modelled with this non-linear model.

This material model [7, 4] has very similar behaviour to the Chen model [2, 3, 5]. There are differences in modelling of hardening. One of the reason to use of the Willam-Warnke model for the presented study was the fact that the obtaining and verification of some parameters for the uFEM material model (which is based of the Chen theory) require additional laboratory works which are not yet finished.

Property	Size	Unit
Open Shear Transfer Coefficient	0.40	-
Closed Shear Transfer Coefficient	0.40	-
Uniaxial Cracking Stress	0.64	MPa
Uniaxial Crushing Stress	3.20	MPa
Biaxial Crushing Stress	3.84	MPa
Hydrostatic Pressure	5.54	MPa
Hydrostatic Uniaxial Crushing Pressure	5.56	MPa
Hydrostatic Biaxial Crushing Pressure	4.64	MPa
Tensile Crack Factor	0.60	-

Table 2: Non-linear material properties.

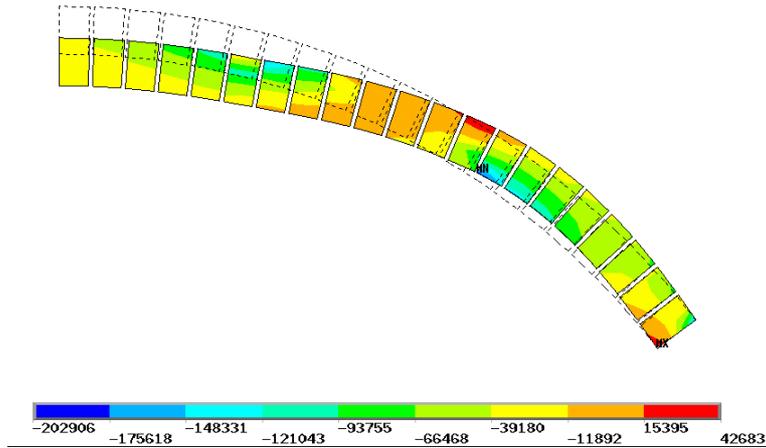


Figure 8: Finite element solution in 2D.

3.2 Results

First a 2D model in the ANSYS software was prepared. This model was based on the data exported from the uFEM solution. An example of obtained results is shown in Figure 8. The 3D finite element model has been derived from this 2D model.

The load–displacement curve for the non-linear analysis is shown in Figure 9. The highly plastic part of the curve is caused by non-linear behaviour of mortar. The position of the plastic zone is shown in 10. It correlates quite well with experimental data (the real arc failed here). It should be noticed that the computed relatively long plastic part of the load–displacement diagram is very optimistic. The real structure failed without so noticeable plastic behaviour. There was a secondary failure on the arc which was not reachable by the used numerical model. Thus there will be further works on material parameters.

4 Conclusion

The paper discussed an ongoing effort on a numerical counterpart of the experimental testing of masonry structural elements [9]. A numerical study has been completed which can serve as a guide for configurations of reinforcements configurations on arches or as a supporting tool for finding configurations for further experimental tests. Other configurations of reinforcement are possible so continuation of this computational works is planned. The main limitation of these numerical studies has been the fact that the linear elastic material was assumed. It means that these studies can only provide information of stress concentrations that may cause non-linear behaviour.

The non-linear part of the computations is performed for cases that have been selected during the parametric study. Only one example of these computations is discussed in this paper.

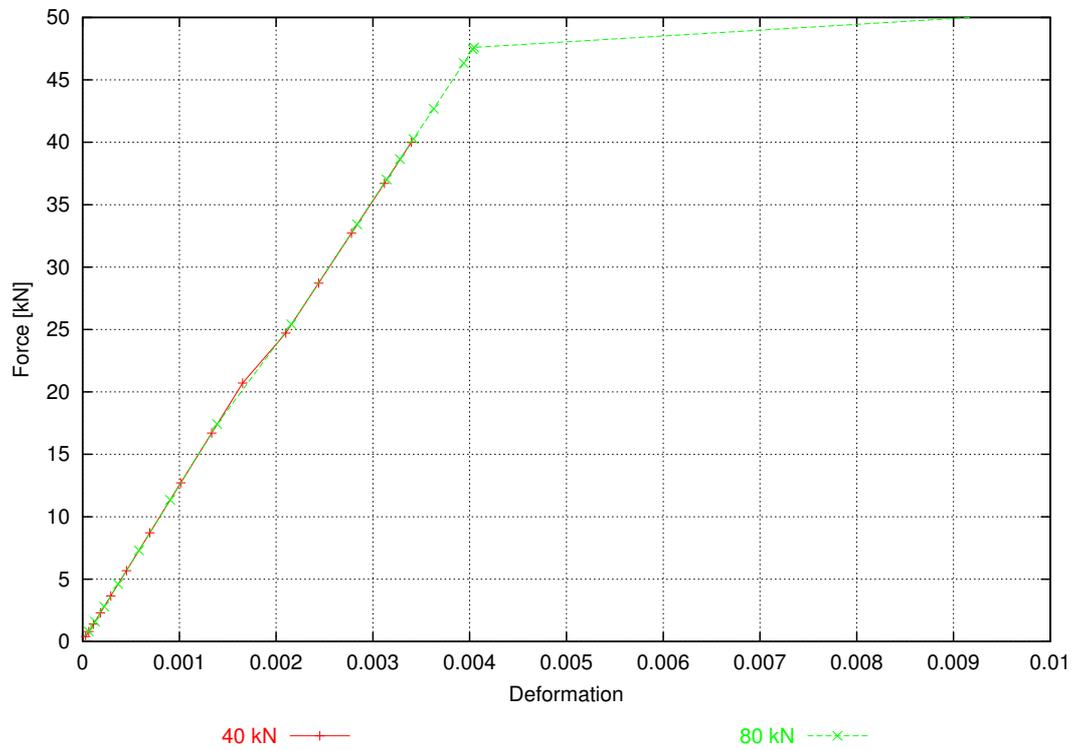


Figure 9: Load–displacement relation for the non-linear model.

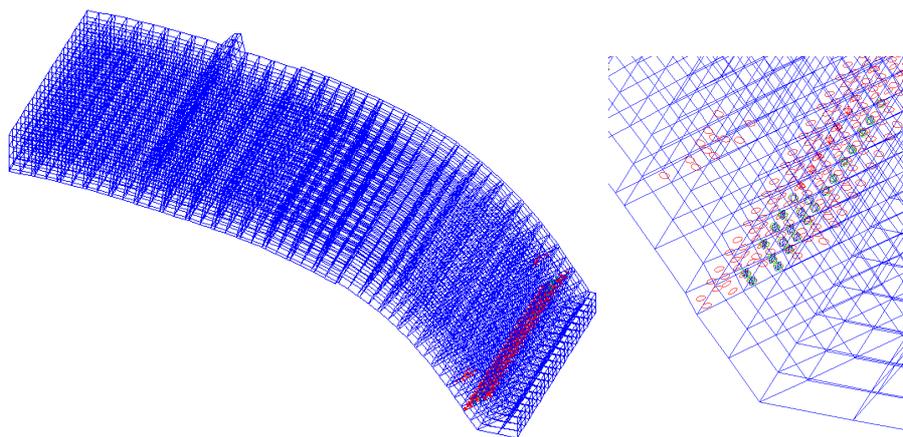


Figure 10: Cracks in mortar.

The selected approach has some disadvantages but it can be successfully used for this particular task. A use of more advanced numerical models or use of homogenisation approaches [11] can improve the precision (of an efficiency) of numerical computations but it may require additional works and it needs additional input data which may not be available.

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