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Multiscale Design and Optimization of Bi-Material Laminated Structures

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Abstract

The focus of this paper is on the compliance minimization of composite structures. It assumes a mixed set of micro and macro independent design variables, to characterize the distribution of two materials to obtain the optimal composite microstructures at the micro design level as well as the optimal fiber orientation at the macro level.

Keywords: multiscale, optimization, composites, homogenization, laminates, topology.

1 Introduction

Two-scale structural design optimization for the simultaneous design of material and structure, has been a topic of research interest during the last decade. This paper is based on the approach presented in [1] and considers two scales, macro and micro, identified with the design domains of the structure and its material (cellular or composite material), respectively. Structure and material evolve for their optimal layouts as a result of updating the density based design variables such that the global compliance is minimized and a global resource volume constraint is satisfied. Here the class of cellular or composite materials, is restricted to single scale periodic materials, with the unit cell topology locally optimized for the given objective function and constraints. Asymptotic homogenization theory is used to compute equivalent elastic properties at the macroscopic level [2]. The design model may include local constraints for the material microstructure depending on the applications. For instance, some constraints may be related with material manufacture requirements as minimum thicknesses as well as mass transport properties and so forth [3].

The structure design domain is usually discretized using a conforming finite element mesh and then each finite element is associated with a cellular material design region matching a proper global volume fraction or density assumed constant in the element. This type of design model leads in general to a very high number of local problems identified with the material microstructure characterization across the whole structure domain. Eventually, the number of local problems is equal to the number of finite elements discretizing the structure domain. Fortunately, parallel processing techniques may be easily applied to this approach ensuring solutions within reasonable computational times [4].

Although the resulting designs obtained with the described methodology are mechanically very efficient, they are hard to manufacture because the changes in material microstructure occur almost from “point-to-point” over the structural domain. Alternatively, the model described so far may be reformulated assuming that the microstructure remains equal in a structural region (design element) defined not based on each finite element but from larger sub-regions consistent with manufacture constraints or structural uniformity. Such a design element may group several finite elements and may be for example one layer in a typical layered composite structure (e.g. laminates or sandwiches panels), see [5,6].

The scope of structural applications in the present work is layered composite structures where typically fiber mats are embedded in a polymeric (resin) matrix covering a large area of the structure domain to form a laminate. This fiber reinforced polymers are usually stacked in a number of layers (laminates) each one having proper fiber orientations. Integer optimization has been applied to determine optimal ply angles and stacking sequences playing with fixed two-material mixing proportions/rules and fiber layouts [5,7,8].

The main contribution given in this work is to investigate and eventually improve the optimality of this type of bi-material composites by applying the reformulated multiscale approach described above. This approach is intended to be lesser restrictive than common design approaches applied to laminated composites. Therefore, macroscopic design sub-regions are here assigned to enclose laminates of a composite structure. A single variable can govern the orientation of the fiber material even though it covers several finite elements in the discretized model (lamina domain). The design space is explored in terms of material directionality (fiber orientation), proper choice of materials and their volume fractions (fiber and resin phases design), stacking sequence, laminate thicknesses, fiber and resin cross-section lay-outs. Optimal composite laminated structures are achieved for maximum stiffness (minimum compliance) with a global material resource constraint that imposes a limit on the total amount of the fiber material. The optimal orientation, volume fraction and cross-section layout of the fiber in each lamina (ply) of the structure is determined by density and angle based design variables. Three-dimensional examples involving laminated plates are shown to demonstrate the relevance of the proposed design methodology and also to recognize the optimality of this type of bi-material composite structures.

2 Multiscale Material Model

The hierarchical material design model considered here is outlined in figure 1. The design domain Ω (linear elastic body) is defined for the macroscale level (global or structural) where the goal is to find an optimal structure layout for given loads and support conditions on $\partial\Omega$, i.e., surface tractions **t** along Γ_t and imposed displacements **u** along Γ_u. Also the design domain Y is defined for the microscale level (local or material) where the aim is to find the optimal design of the material unit cell from which the structure is manufactured (periodic cellular material with periodicity Y is assumed). Furthermore it is assumed that the two scales differ by a proper size ratio, i.e. the material unit cell characteristic size d, is much smaller than the global domain characteristic size D [2].

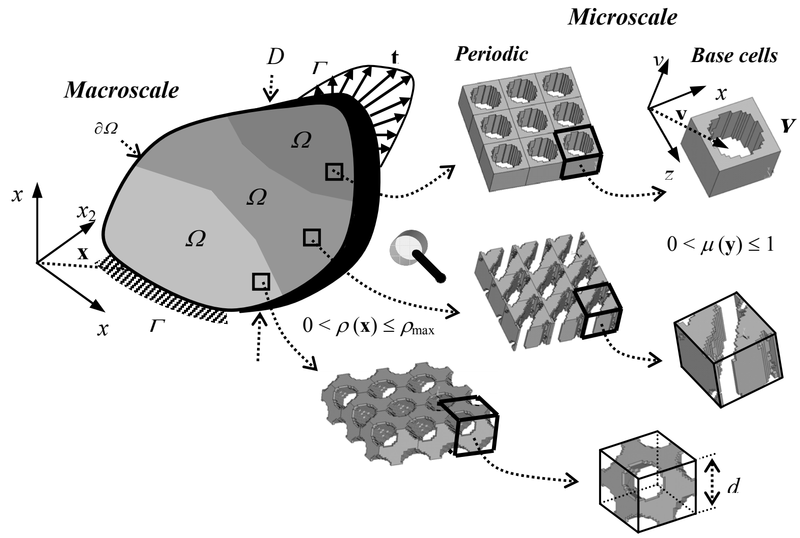

Figure 1: Material model used in two-level topology optimization. Global design domain divided in several design sub-domains with constant microstructure.

In [1] it was considered that each point **x** in Ω is designable, i.e., a relative density design variable ρ is assigned to each **x** and also the material unit cell geometry defined by the "micro" density $\mu(\mathbf{x},\mathbf{y})$ function of **y** in Y is computed at each **x** in Ω. Alternatively, in the present work the designable part of the macroscopic domain is subdivided into "larger" areas (design subdomain or group) Ω_i such that $\Omega = \Sigma\Omega_i$. In each subdomain the designed material is uniform at the macro level. In this work these designable sub-domains are identified with the plies (laminae) of a three-dimensional composite structure. Therefore the optimal design problem is formulated as a two scale material distribution problem (macro and micro) and a "density" field governs each one, $\rho(\mathbf{x})$ and $\mu(\mathbf{x},\mathbf{y})$, respectively. The local problem solution is the topology for a single microstructure (material base or unit cell) that is assumed to be periodically repeated inside each $\Omega_i \in \Omega$. There are as many of these local problems to solve as the number of designable sub-domains Ω_i, or laminates in the particular case treated here. Furthermore, the material model used here for microstructure design is not merely an interpolation between void and solid as presented in [1] but now two materials are mixed to form a fiber/resin compound for each lamina.

3 Optimization Problem Formulation

For compliance based objective function and a constraint on the fiber material total volume, the hierarchical topology optimization problem which minimizes compliance can be formulated as (see e.g. [1,9]):

$$\max_{\substack{\rho(\mathbf{x}) \\ 0<\rho(\mathbf{x})\leq 1 \\ \int_\Omega \rho(\mathbf{x})\mathrm{d}\Omega \leq V^*}} F(\rho) \quad \text{com } F(\rho) = \min_{\substack{\mathbf{u}^r \in U \\ r=1,\ldots,P}} \left[\sum_{i=1}^{n} \int_{\Omega_i} \Phi\left(\rho_i, \mathbf{u}^1, \ldots, \mathbf{u}^P\right) \mathrm{d}\Omega_i - \sum_{r=1}^{P} \alpha^r \left(\int_{\Gamma_t^r} \mathbf{t}^r \cdot \mathbf{u}^r \mathrm{d}\Gamma \right) \right] \tag{1}$$

with the optimal energy density function Φ solution of the local (micro) problem,

$$\Phi\left(\rho_i, \mathbf{u}^1, \ldots, \mathbf{u}^P\right) = \max_{\substack{\mu(\mathbf{y}),\theta_i \\ 0<\mu(\mathbf{y})\leq 1 \\ \int_Y \mu(\mathbf{y})\mathrm{d}Y = \rho_i(\mathbf{x})}} f(\mu, \theta_i) \quad \text{com } f(\mu, \theta_i) = \sum_{r=1}^{P} \alpha^r \left[\frac{1}{2} E^{\mathrm{H}}_{ijkl}(\mu, \theta_i) e_{ij}\left(\mathbf{u}^r\right) e_{kl}\left(\mathbf{u}^r\right) \right] \tag{2}$$

Using a finite element mesh (following each subdomain division) to solve the hierarchical problem (1)-(2), the evaluation of the first integral in (1) becomes simplified as a sum of the strain product terms, $e_{ij}\left(\mathbf{u}^r\right)e_{kl}\left(\mathbf{u}^r\right)$, over all finite elements affected by the density variable ρ_i, i.e., covering all sub-domain of the i^{th} lamina.

The homogenized tensor E^{H}_{ijkl} is assumed constant for each sub-domain Ω_i because the microstructure is constant within each lamina.

The angle design variable, introduced in equation (2), means that the unit cell domain may rotate θ with respect to (w.r.t.) the axis perpendicular to the lamina (*yy* axis) as detailed in next section. This permits the optimization of the fiber orientation angle, while the local material distribution problem identifies the optimal cross-section layout for the fiber.

The homogenized constitutive tensor for the rotated microstructure (3) can be easily computed using the transformation matrix in equation (4).

$$E^{\mathrm{H}}_{ijkl}(\mu, \theta) = R_{im}(\theta) R_{jn}(\theta) R_{kp}(\theta) R_{lq}(\theta) E^{\mathrm{H}}_{mnpq}(\mu) \tag{3}$$

$$R_{ij}(\theta)=\begin{bmatrix}\cos(\theta) & 0 & \mathrm{sen}(\theta)\\ 0 & 1 & 0\\ -\mathrm{sen}(\theta) & 0 & \cos(\theta)\end{bmatrix} \tag{4}$$

The non-rotated homogenized tensor is given by (for further details see [2]):

$$E^{\mathrm{H}}_{ijkl}=\frac{1}{|Y|}\int_Y E_{ijkl}(\mu)\left(\delta_{kp}\delta_{lq}-\frac{\partial\chi_p^{kl}}{\delta y_q}\right)\left(\delta_{ih}\delta_{js}-\frac{\partial\chi_h^{ij}}{\delta y_s}\right)dY \tag{5}$$

The constitutive tensor for the base material is density dependent (6) in order to interpolate between two base materials (see figure 2). This interpolation scheme is based on the generalization of the SIMP model as can been seen in other works addressing multi-phase material optimization [10,11,12,13]. Local (micro) density μ equal 1 means the presence of fiber or stronger material (labeled by superscript 1 in equation 6) whereas the extreme value 0 identifies resin or soft material (labeled by superscript 2).

$$E_{ijkl}(\mu)=\mu^p E^{(1)}_{ijkl}+\left(1-\mu^p\right)E^{(2)}_{ijkl},\quad p\in\mathrm{N} \tag{6}$$

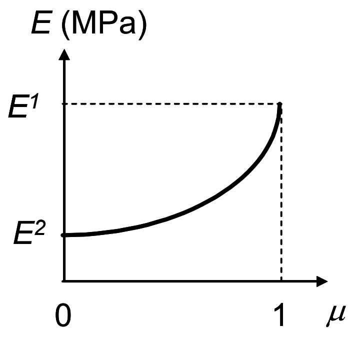

Figure 2: Bi-material interpolation scheme.

The constitutive laws for materials 1 e 2 (assumed solid isotropic linear elastic) are given in terms of the Lamé constants in equations (7) and (8).

$$E^{(m)}_{ijkl}=\ell_1^{(m)}\delta_{ij}\delta_{kl}+\ell_2^{(m)}\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right) \tag{7}$$

$$\ell_1^{(m)}=\frac{E^{(m)}\nu^{(m)}}{\left(1+\nu^{(m)}\right)\left(1-2\nu^{(m)}\right)}\quad;\quad \ell_2^{(m)}=\frac{E^{(m)}}{2\left(1+\nu^{(m)}\right)} \tag{8}$$

4 Structural Examples

In this section preliminary results obtained with the multiscale model applied to layered composites are shown for single loading conditions as simple as traction, pure bending and bending combined with shear. This will allow to identify simple material design solutions and compare them with previous designs reported in the literature.

Figure 3 shows the multiscale material model now applied to laminated plates. The number of layers or laminates considered is 8 and their thicknesses are equal. The layers are assumed to be perfectly bonded together and thus, displacements will be continuous across the thickness. Consequently, interlaminar effects such as delamination are disregarded here. Each laminate corresponds to a patch or designable domain, i.e., associated to a single macro density design variable ρ. Usually in structural applications laminates consist of strong fibers (glass, graphite and boron, for example) bonded together by a relatively compliant resin/polymer matrices.

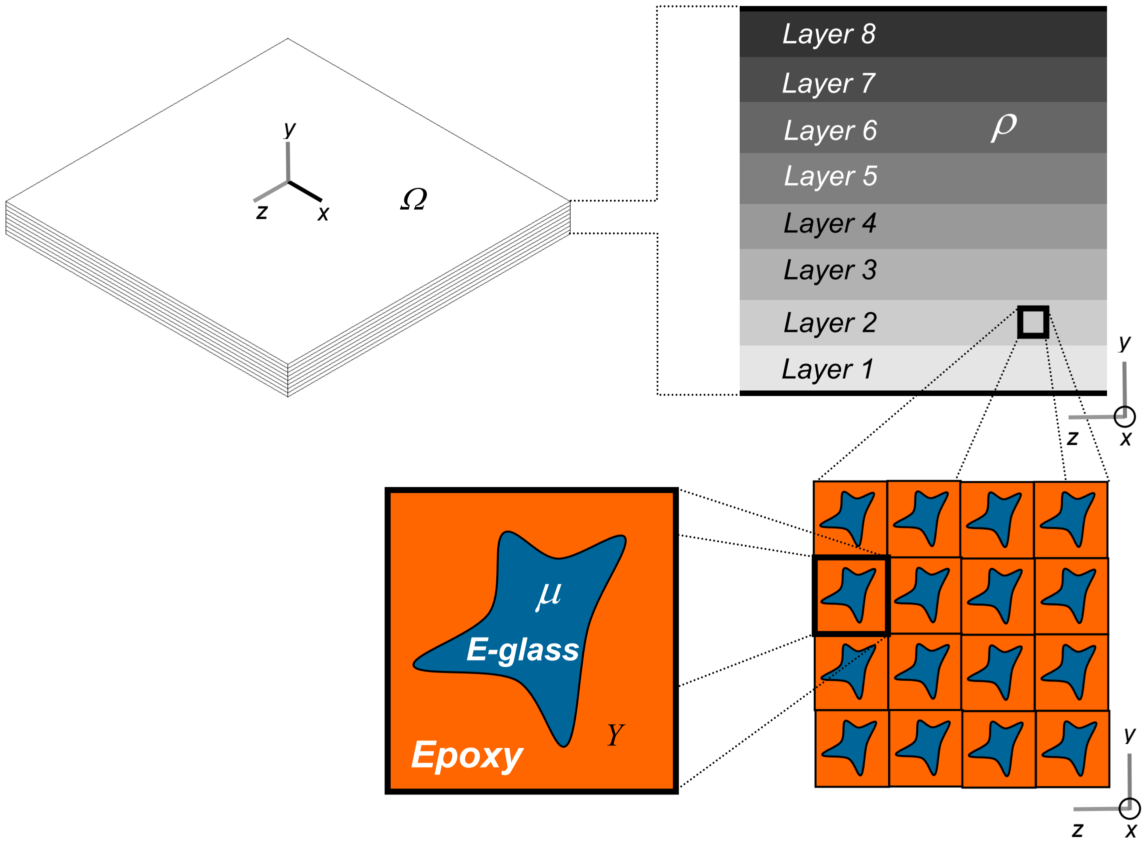

Figure 3: Multiscale material model applied to laminated composites.

Macroscopic density ρ defines the volume fraction of the strong material (fiber). The volume fraction for the resin phase can be determined by setting the difference $1-\rho$. A maximum global amount of fiber material used in the structure design is set to 50%. In this work one considers a mixing of two materials E-Glass (fiber) and Epoxy (resin). The material properties of each constituent are presented in Table 1, see [14]. The Young's modulus ratio between the strong and weak materials is around 20. Microscopic density μ varies between 0 and 1. The value 0 here does not represent void phase but the weak material phase instead while the value 1 identifies the strong material phase. The elastic properties of the material microstructure at each point of the local design domain are dependent on μ by means of the interpolation scheme (6).

Constituents\Material Properties	E(GPa)	ν	ℓ_1 (GPa)	ℓ_2 (GPa)
E-Glass (material 1)	73	0.22	23.51	29.92
Epoxy resin (material 2)	3.45	0.35	2.98	1.28

Table 1: Material properties for Glass filament – Epoxy resin composites.

4.1 Finite Element Model

The finite element model applied to discretize structure and material design domains is shown in figure 4. Hexahedral solid isoparametric finite elements with 8 nodes are used.

The global mesh that discretizes the structure domain Ω is regular, i.e., 8×80×80. This means that each laminate is meshed by 1×80×80 finite elements and the material microstructure remains constant in all these elements unlike the model presented in [1].

Regarding the design of the material microstructure one uses the local mesh shown in figure 4 (right side) to discretize Y. The goal here is to find basically an optimal topology for the cross-section. An optimal material distribution of the fibre or strong material phase is sought and the weak material phase will occupy the remaining part of that local domain Y. Notice that the resulting unit-cell in Y is assumed repeated in the three-dimensional space. Periodicity boundary conditions are imposed and homogenization is used to compute the equivalent macroscopic elastic properties. Moreover, the resulting fiber or material microstructure is oriented in the horizontal or laminate plane by means of an angular variable θ. The optimization algorithm searches for an optimal direction θ of the material microstructure. Figure 5 shows the initial material distribution considered the in material design domain as well as the initial angular orientations in each layer.

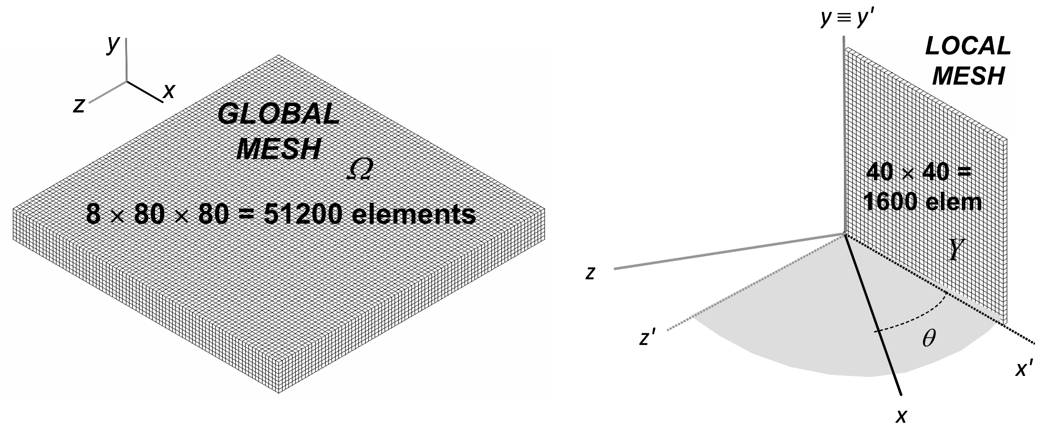

Figure 4: Finite element model for the structure and material domains.

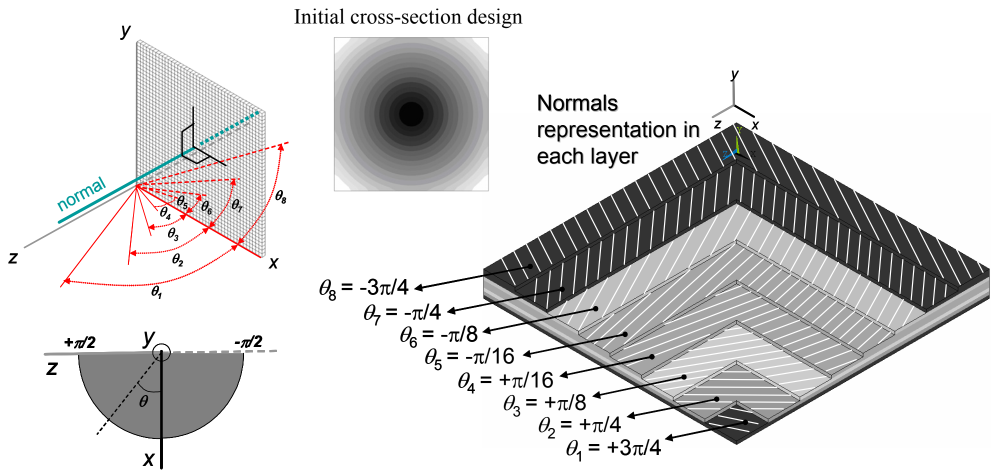

Figure 5: Initial material distribution in Y and orientations θ.

4.2 Plate under uniaxial stress

The plate is subjected to a uniform traction along the *xx* direction (see figure 6). Figure 7 shows the optimum material design obtained at one layer because in the remaining layers the design is the same taking into account that the stress field is uniform.

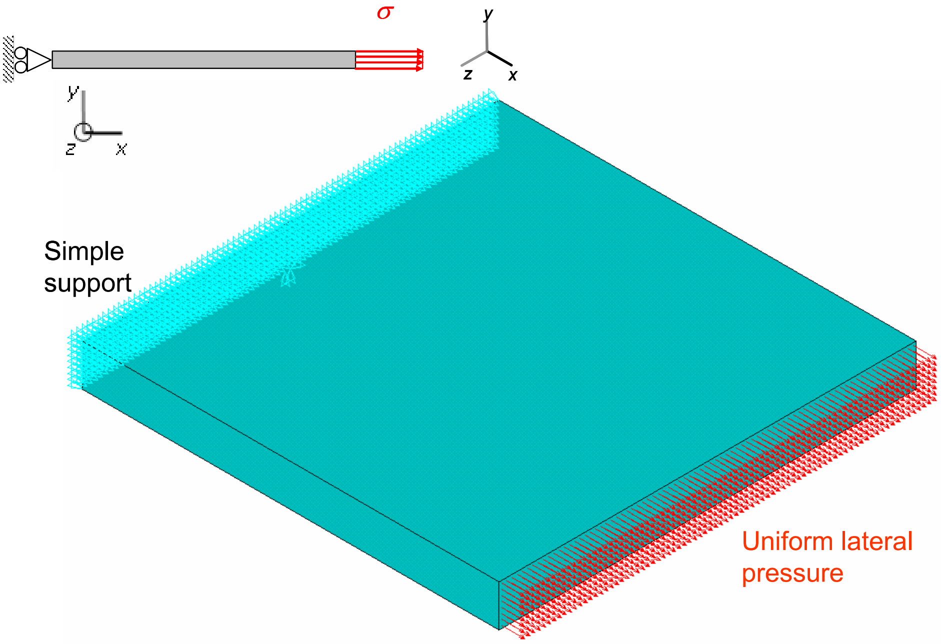

Figure 6: Plate under axial traction.

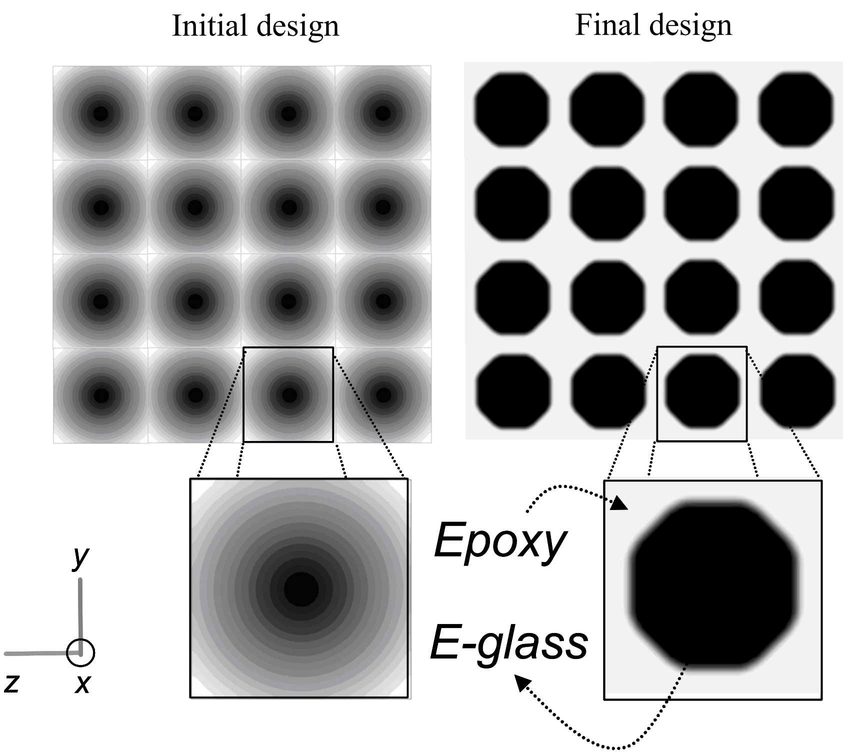

Figure 7: Material microstructure.

The two-material interpolation scheme (6) penalized intermediate densities and led to well defined microstructures. The two phases or one of them are clearly recognized while gray or mixtures are absent as desired. Black is E-Glass and white is Epoxy resin. Material microstructure is presented in terms of the unit cell and a 4×4 array showing the respective periodic pattern. The fibers orientations are coincident with the loading axis as expected. The volume fraction in each layer is 50% of the strong material.

4.3 Plate under pure bending

In the previous load case the stress was uniform in the cross-section of the composite plate which led to the same material solution in each layer. Now a pure bending is applied to the same plate (see figure 8). The normal stresses change between the upper and lower layers due to the application of a bending moment. Pure bending is considered here in order to guarantee that only normal stresses are present (no shear stress). In this particular design problem is obvious that a laminate symmetry w.r.t. the midsurface would figure in the optimal solution (see figure 9). One could save computational cost enforcing a priori such symmetry by assigning the same density design variable to opposite laminae. However this was not done here for the sake of testing the algorithm. The resulting symmetrical distribution of density came up naturally as expected. The extreme layers present the strong material dominating aligned with *xx* axis which is the direction of the normal bending stresses.

In the case of an upper bound equal to 1 for the variation of ρ one obtains extreme layers full with the strong material phase. However, the results obtained correspond to an upper bound of 0.8 for ρ. Setting this limit is in practice consistent with the maximum achievable density of cylindrical shape fibers firmly bounded together and embedded in epoxy resin. In the middle layers, where the normal stresses are low, the soft material dominates and the presence of weak material alone at the layers adjacent to the neutral surface is even allowed because the lower bound of ρ is set to 0.

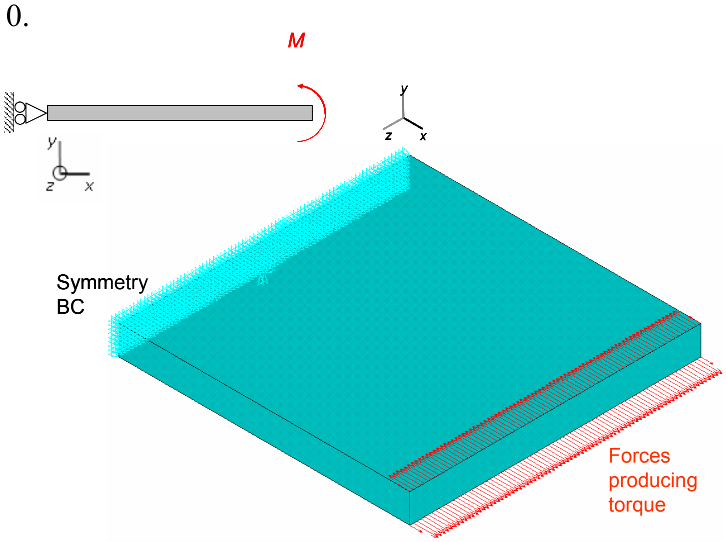

Figure 8: Plate under pure bending.

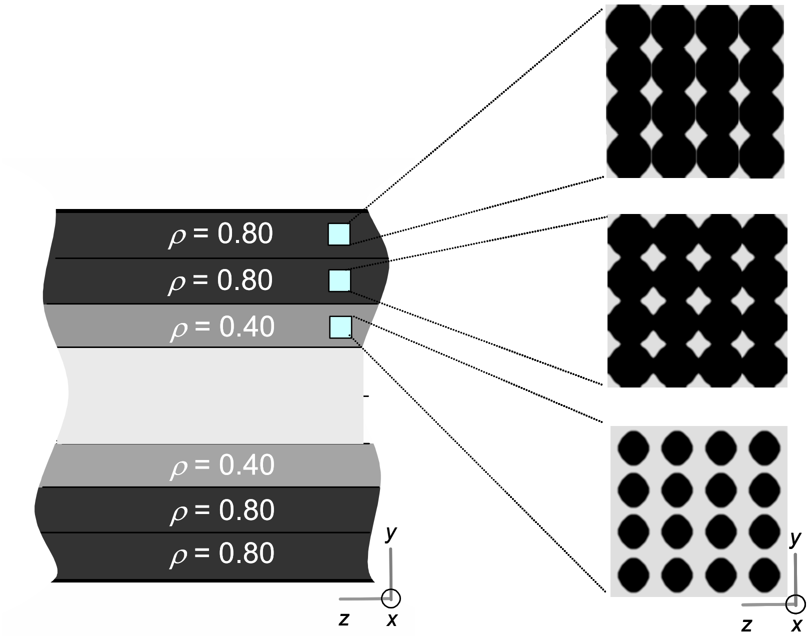

Figure 9: Material microstructure changing among layers.

4.4 Plate under bending and shear

This last example is used here to show the influence of shear in the optimal shape of fibers. Applying an uniform pressure on the top of the simply supported plate leads to normal stresses combined with shear stresses (see figure 10). Maximum normal stresses are in the extreme layers whereas maximum shear forces appear at the plate middle surface. Basically this explains why some intermediate layers in figure 11 present now vertical laminated solutions (parallel to *xy* plane). Vertical laminates improve performance resisting the shear field.

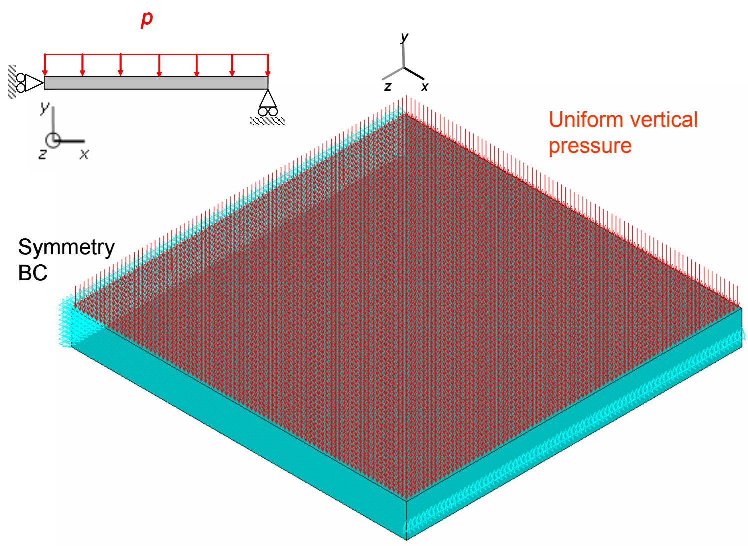

Figure 10: Simply supported plate subjected to an uniform pressure.

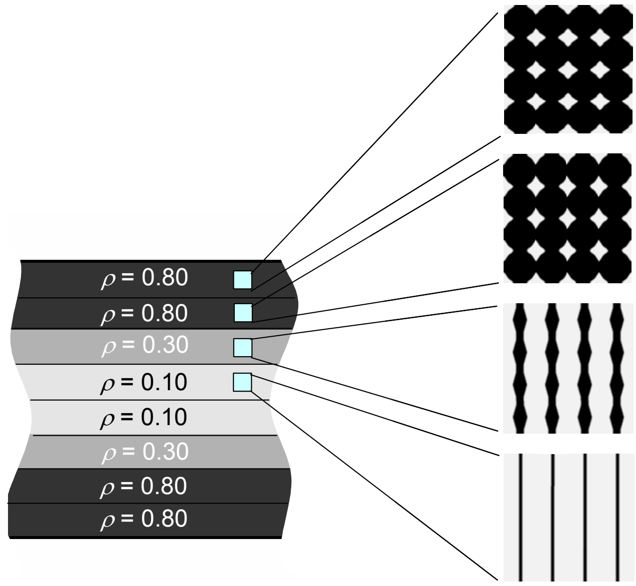

Figure 11: Material microstructures within each layer.

5 Conclusion

The two-scale topology optimization model presented in this paper can contribute for a deeper mechanical perception of the fine structure of laminated composite structures and improve their performance even further. The model is a derivation, or a particular case, of the generalized model presented in [1]. Here the structure design elements are not necessarily each finite element in the mesh. Rather, the designable regions of the structure domain may group several finite elements into a design a subdomain (or group), in our case a layer, where the design is assumed uniform (or constant). The advantages of this approach are apparent from a manufacturability perspective and it fits particularly well in the design of laminated composites structures. This type of structures is usually designed assuming as design variables fiber orientations, staking sequence of layers and their thicknesses. With this model improved design solutions based on the concurrent design of structure and material can now be investigated systematically. Fiber/resin layouts are not fixed a priori but they themselves are optimized as well as fiber orientation and fiber/resin volume fractions. The results obtained in this work, considering simple plate structures, show a good potential of the proposed design approach for future design developments and applications.

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