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Seismic Wave Motion over a Geographical Area by a Random Field Model

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Abstract

A statistical method was developed to simulate the propagation of seismic wave over the territory. The seismic intensity at the site is represented by a probability density function whose solution is obtained by the statistical processing of recorded data during some earthquakes that occurred in southern Italy, for which are known the epicentre location and intensity, the intensity at the site, the magnitude, the duration, the occurrence time, the peak acceleration, etc. By means of another statistical processing of the macroseismic parameters the seismic signal propagation over the territory is simulated for high-intensity earthquakes and the simulations are compared to some recorded macroseismic maps.

Keywords: seismic macro-zoning, seismic propagation, seismic simulation, Gaussian function, statistical data elaboration, historical earthquakes.

1 Introduction

The most reliable models for simulating the propagation of the seismic waves over the territory are those of probabilistic type where some macroscopic parameters of an earthquake are considered as random variables. In details, by observing the process by which the seismic signal propagates over the territory it seems that it can be reproduced satisfactorily by a stochastic field. If one has at one's disposal for an earthquake the values of magnitude M, duration time T_o , epicentre E, etc., these parameters can be arranged, for example, in a probabilistic model for the propagation and attenuation of seismic signal, finally getting the macroseismic maps.

From macroseismic maps it is possible to read the distribution of the global seismic hazard of the territory and the seismic hazard specifically referred to a site or an area of interest (Fig. 1). Starting from the data obtained from the seismic hazard maps, other macroseismic parameters can be then identified such as the seismic intensity at the epicentre I_o and the intensity at the site, the peak acceleration a_p , the occurrence time T, etc..

Moreover other local seismic parameters can be known, such as the local response factor, the seismic attenuation coefficient, and the seismic response function, which, coupled with a high degree of vulnerability and with investigation and structural type-specific controls, provide seismic risk. Alternatively a different path may be chosen leading to accelerograms, synthetic and compatible with the conditions of the site, which are the basis for nonlinear analyses of structural type. The synthetic accelerograms can be combined with the analyses and checks carried out on site by getting the local seismic risk (Fig. 1).

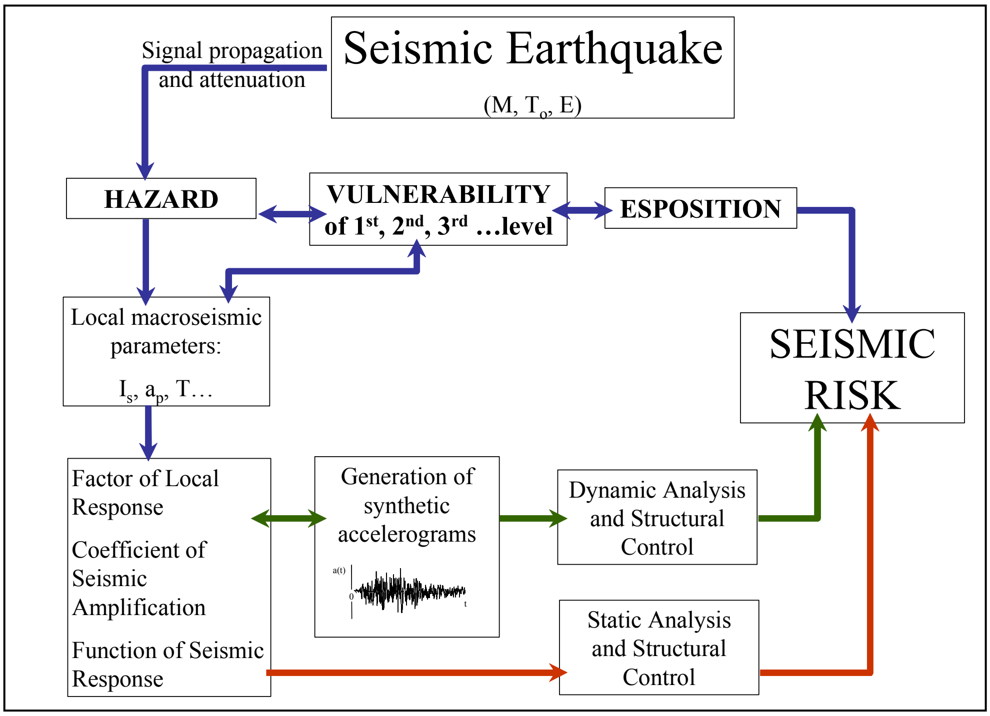

Figure 1: Flow chart for the evaluation of the seismic risk.

The model that is developed in the present paper is a simplification of the classic models such as the probabilistic analysis of macroseismic parameters of Cornell [1], the law of seismicity of Gutenberg-Richter [2], the occurrence time process of earthquakes of Poisson, the law of seismic attenuation.

It should be noted that the simplification of existing models refers only to the type and number of basic parameters needed for the procedure. On the contrary the procedure for the elaboration of the macroseismic parameters uses the basic rules of probability theory. Finally, a stochastic field of simulation about the propagation of the seismic wave over the territory is built with reference to an area of Southern Italy.

2 Epicentral distribution over the territory

In order to test the validity of the procedure built ad hoc for this study the historical data have been processed relevant to earthquakes occurred in an area of southern Italy. In particular, one refers to the main macroseismic parameters, such as epicentre intensity, intensity at site, magnitude, distance from the epicentre of the registration sites, of 542 historical earthquakes with different intensity, magnitude greater than 4 and occurred in the period 1000-1997 within the area of interest.
This selection takes into account the data reported in the main Italian seismic catalogues - “Catalogue of large earthquakes in Italy” [3] until 1980 and “Macroseismic Bulletin of Italy” [4] for the period 1981-1997 - because it ensures the reliability of the used data (Fig. 2).

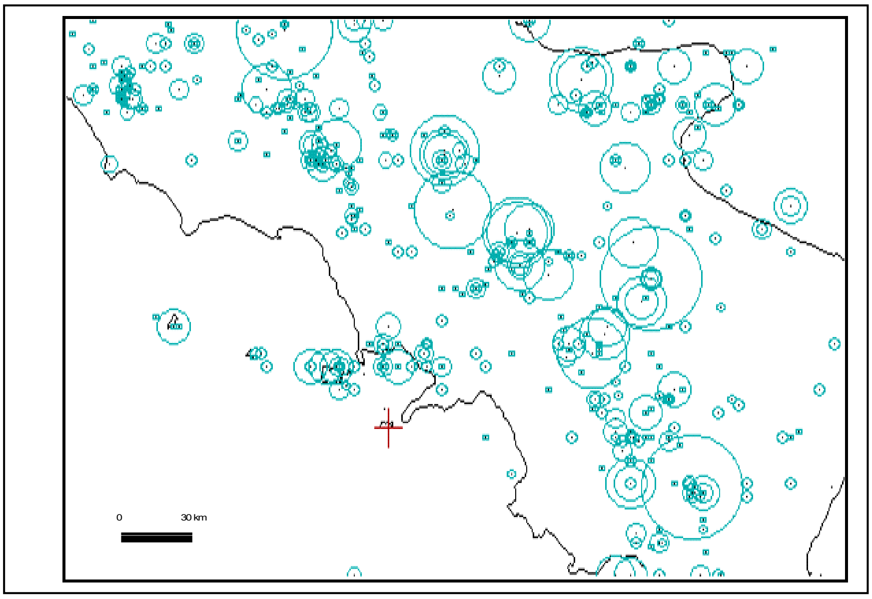

Figure 2: Distribution of epicentres of 542 historical earthquakes (the size of the circle is proportional to the magnitude).

If it is already occurred an earthquake of magnitude M and epicentre E, the location of the epicentre of an expected earthquake can be viewed as a two-dimensional vector expressed by a joint probability density function (JPDF) of the epicentre E as follows

$$P(E|M) = \Pr ob\{\tilde{E} = E\} = \sum_{j=1}^{n} k_j p_j(x_E, y_E, \bar{x}_{Ej}, \bar{y}_{Ej}, \sigma_{xj}, \sigma_{yj}, \rho_j) \quad (1)$$

where k_j are combining coefficients that depend on the magnitude, p_j are probability functions that depend on the geographical location of possible epicentres in the area (with x_E and y_E coordinates) and on other probabilistic parameters (σ_{xj} , σ_{yj} , ρ_j)

$$p_j\left(x_E, y_E, \bar{x}_{Ej}, \bar{y}_{Ej}, \sigma_{xj}, \sigma_{yj}, \rho_j\right) = \frac{e^{\left\{\frac{-1}{2\left(1-\rho_j^2\right)}\cdot\left[\frac{\left(x_{Ej}-\bar{x}_{Ej}\right)^2}{\sigma_{xj}^2}+\frac{\left(y_{Ej}-\bar{y}_{Ej}\right)^2}{\sigma_{yj}^2}-2\rho_j\frac{\left(x_{Ej}-\bar{x}_{Ej}\right)\left(y_{Ej}-\bar{y}_{Ej}\right)}{\sigma_{xj}\sigma_{yj}}\right]\right\}}}{2\pi\sigma_{xj}\sigma_{yj}\sqrt{1-\rho_j^2}} \quad (2)$$

All of these parameters (medium values $\bar{x}_{Ej}, \bar{y}_{Ej}$, standard deviation σ_{xj}, σ_{yj}, correlation coefficients ρ_j and combination coefficients k_j) are essential to resolve the function of propagation of epicentres and can be directly inferred from the data processing of earthquakes already occurred in the area.

After fixing three or more ranges of magnitude (e.g. 4÷5, 5÷6, >6) and selecting the earthquakes relevant to each class, the density function relevant to the epicentre position of an expected earthquake is modelled by a convex combination of a number of bivariate Gaussian distributions, with the geographical coordinates of the epicentre as primary variables.

It should be noted that one of the novelties introduced by this method consists precisely of not considering the single Gaussian function in Eq.(2), but a combination of Gaussian functions, whose final characteristics are obviously different from those of the single function but whose solution is much better because the process thus corrects by itself its inaccuracies.

The parameters that identify the Gaussian distributions, such as medium values, standard deviations and correlation coefficients and their combination coefficients, are calibrated according to two different procedures.

The procedure "A" splits the area using a fixed-pitch grid and resolves various Epicentral probability functions as joint probability density functions (JPDFs) for each quadrant of the grid; the final function is given by the sum of the functions calculated in different quadrants (Fig. 3.a).

The procedure "B" addresses, instead, the Gaussian distribution parameters by minimizing the difference between the simulated moments and the moments derived from data recorded during historical earthquakes occurred in the area. Fig. 3.b shows the distribution of the Epicentral density function for a combination of 3 Gaussians.

The observation of maps included in Fig. 3 shows that the results from the procedure "A", developed for some fixed sized quadrants, are a little more articulate than those obtained with the procedure "B", in which the function is searched for the whole area through the minimization of moments.

Both procedures have shown quite robust compared to the stability tests carried out looking for the variation of the results of the epicentral function for several time intervals. This also demonstrates the reliability of data used for the procedure.

The advantage of both procedures in comparison with other statistical methodologies, is represented by the simplicity and the relatively easy availability of the used basic data, which, as well-known, often represents the real problem for statistical procedures. The disadvantage is typical of all statistics, which require a considerable quantity of data. On the other hand both procedures are structured in a way that there nothing is left to the subjectivity of the operator, which represents another advantage of statistic procedure. In order to deepen the development of the procedures other references of the authors can be consulted (from [5] to [10]). As regards models developed for NT materials such soils, masonry structures and so on and relevant applications one may refer to [11]-[21].

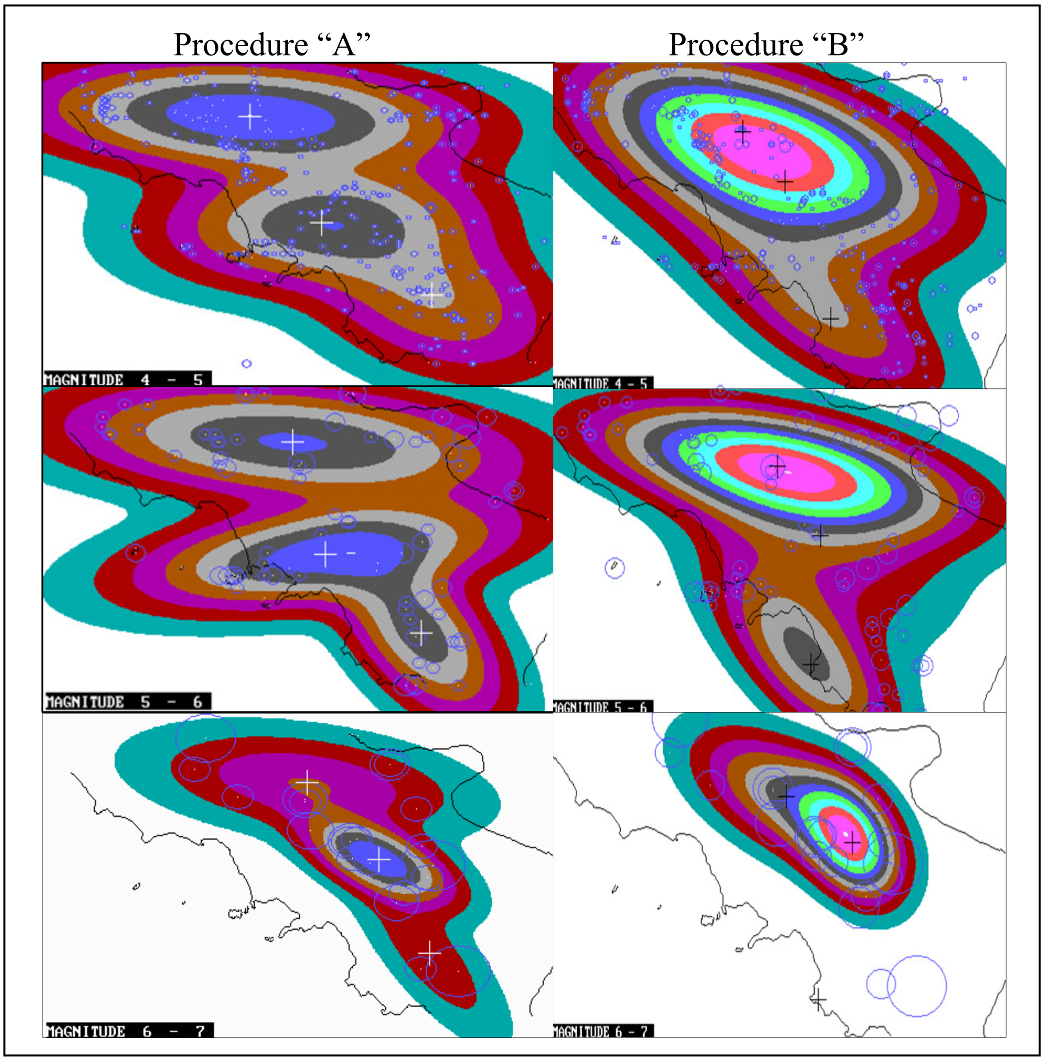

Figure 3: Distribution of Epicentral density function by procedures "A" and "B".

3 Seismic wave propagation over the territory

The identification of the distribution function of epicentres allows a series of developments, the first of which is a model for the propagation of seismic signal over the territory.

During an earthquake, the energy is dissipated in the form of seismic waves which propagate along radial direction from the seismic source in the surrounding territory and within the Earth's crust. It is possible to think that the propagation of a seismic signal $\tilde{s}(r,\alpha)$ from the epicentre E to the recording station S may be modelled as a probabilistic function derived from the sum of a deterministic function of the distance r of the site S from the epicentre E. The random character of the propagation is completely described by the a random process $\tilde{\theta}(\alpha)$, and by a random function that is dependent on both the distance r and the polar angle

$$\tilde{s}(r,\alpha) = s_o \cdot e^{-\tilde{\theta}(\alpha) r}\left[1 + \tilde{d}(r)\right] \tag{3}$$

with s_o the value of the signal at the epicentre, $\tilde{\theta}(\alpha)$ a random function of the polar angle α, $\tilde{d}(r)$ a stationary random function of distance r. The character of both $\tilde{\theta}(\alpha)$ and $\tilde{d}(r)$ can be known using data processing of seismic data relevant to historical earthquakes which are occurred in an area.

If, for example as shown in Fig. 4, the development of a generic seismic wave propagates from the seismic epicentre E to the registration site S with increasing distance r, the propagation function $\tilde{s}(r,\alpha)$ can be regarded as the sum of two distinct effects: on one side, the deterministic portion $s_o\, e^{-\tilde{\theta}(\alpha) r}$ that carries the signal to exponentially decrease as r increases, and, on the other side, a random portion $s_o\, e^{-\tilde{\theta}(\alpha) r}\tilde{d}(\alpha)$ of the signal noise, which goes to overlap the previous one.

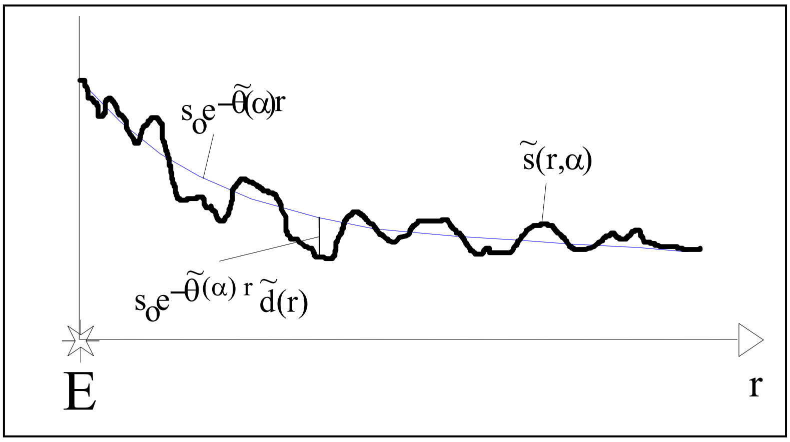

Figure 4: Propagation of a seismic signal from the epicentre along the general direction r.

3.1 Model for radial attenuation

In the first approximation, based on the solution macroseismic parameters, it is possible to get the density function I_s of the intensity at the site, which is conditioned by the prefixed value of the magnitude and by the epicentre location, by means of the epicentre distance r and the angle α with respect to the x-axis, and the epicentre intensity I_o, as follows

$$I_s(r,\alpha) = I_o e^{-\tilde{\theta}(\alpha)\, r} \tag{4}$$

where the process $\tilde{\theta}(\alpha)$ gives the directionality of the signal propagation on the territory

$$\tilde{\theta}(r,\alpha) = -\frac{ln[I_s(r,\alpha)/I_o]}{r} \quad (5)$$

After introducing the characteristic distance $\tilde{R}(\alpha, I_o)$ as the distance at which the intensity reduces by one degree, in the form

$$\tilde{R}(\alpha, I_o) = \frac{-\ln(I_o - 1/I_o)}{\tilde{\theta}(\alpha)} \quad (6)$$

it is easy to understand that the characteristic distance represents the attenuation of a seismic signal over the territory along all the directions that cover radially the surrounding territory starting from the epicentre during an earthquake.

After selecting between the 542 historic earthquakes occurred in the Italian area of interest those with a very high intensity (for a total number N_T = 12) and solving the attenuation law for them, it is possible to notice that:

1) the radial diagrams of the $\tilde{\theta}(\alpha)$ process have a marked random character which depend on the angle α and then on the orientation of the distance epicentre-site with respect to the x axis;

2) the mean quadratic value in all cases has an approximately circular shape and the mean quadratic error is always very small, which is an index of a good grip of the assumed function to the real situation.

In Fig. 5 the example of distributions of the angle α, of the mean quadratic value and the mean quadratic error relevant to the 7/6/1910 Irpinia earthquake in Basilicata (Italy) are shown along all radial directions from the epicentre.

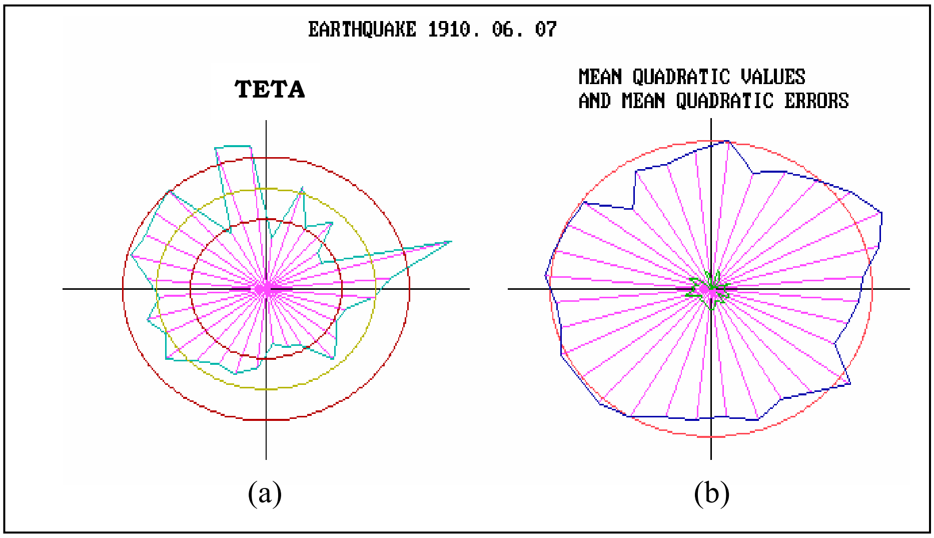

Figure 5: Example of: (a) radial distribution of the angle α and (b) distribution of quadratic mean value (blue line) and the mean quadratic error (green line) for the 7/6/1910 Irpinia earthquake in Basilicata (Italy).

Therefore, if the exponent $\tilde{\theta}(\alpha)$ is highly random the exponential law that expresses the propagation of seismic waves is practically deterministic and seismic signal propagation in the territory can be expressed by Eq.(4) instead of Eq.(3).

Moreover from the radial representation of the inverse function of coefficient $\tilde{\theta}(\alpha)$ plotted on the Italian area of interest, an example of propagation of local intensity function over the territory can be solved for a selected number of earthquakes (Fig. 6).

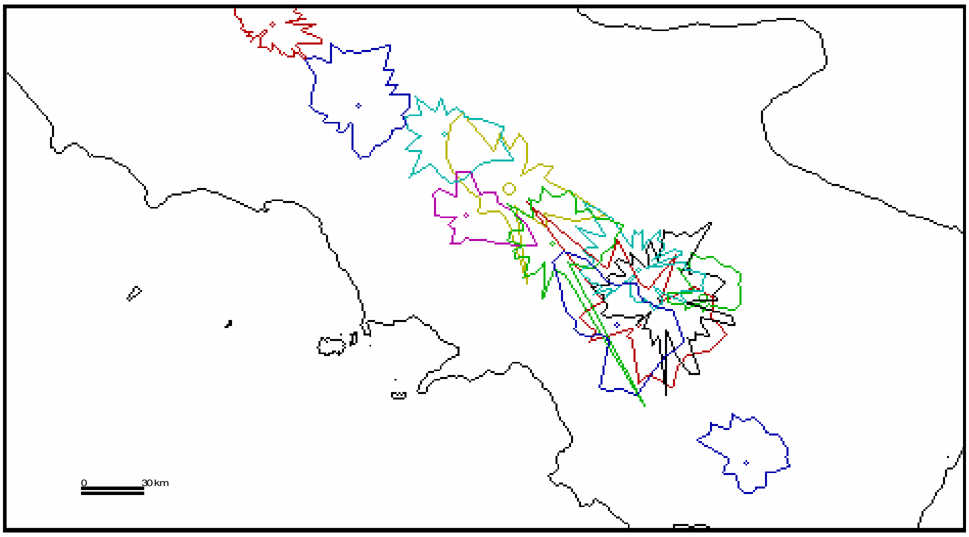

Figure 6: Example of seismic signal attenuation in the territory produced along the radial direction surrounding the epicentre for some historical large earthquakes.

3.2 Attenuation law

Based on the results presented in the previous paragraph, it can be observed that the random function $\tilde{\theta}(\alpha)$ is a periodic process that then can be also expressed as a sum of n-harmonic functions using the development in series of Fourier truncated to a finite number of terms, by means of some random coefficients $\tilde{c}_k$ as

$$\tilde{\theta}(\alpha) = \sum_{k=1}^{n} \left(\tilde{c}_k \cos k\alpha + \tilde{c}_{k+n} \operatorname{sen} k\alpha\right) \tag{7}$$

where the period T of k-harmonic function is given by

$$T_k = \frac{1}{f_k} = \frac{2\pi}{\omega_k} = \frac{1}{k} \tag{8}$$

In order to calculate the statistical values of the coefficients $\tilde{c}_k$, each coefficient can be given some values c_{kj} with j=1,…,N_T (where N_T = 12 is the number of selected historical earthquakes with higher intensity), developing into Fourier series the radial propagation for each individual epicentre of the historic earthquake

$$\theta_j(\alpha)=\sum_{k=1}^{n}\left(c_{kj}\cos k\alpha+c_{k+n,j}\,\mathrm{sen}\,k\alpha\right) \tag{9}$$

Through some other coefficients $c_{\ell j}$ with ℓ =1,…,2n, it can be inferred the relevant probabilistic coefficients, such as the vector of mean values $\bar{c}_i$ and the correlation matrix C_{rs} of the coefficients c_{ij}

$$\bar{c}_i = E[\tilde{c}_i]=\frac{1}{N_T}\sum_{j=1}^{N_T} c_{ij} \tag{10}$$

$$C_{rs} = \mathrm{Cor}[\tilde{c}_r,\tilde{c}_s]=\frac{1}{N_T}\sum_{j=1}^{N_T} c_{rj}c_{sj} \qquad \text{con} \quad r,s=1,\ldots.,2n \tag{11}$$

where j the is referred to historic epicentre, and r and s to the harmonic function order.

The mean value function of $\tilde{\theta}(\alpha)$ and the autocorrelation function are given by

$$\bar{\theta}(\alpha)=E\left[\tilde{\theta}(\alpha)\right]=\sum_{k=1}^{n}\left(\bar{c}_{kj}\cos k\alpha+\bar{c}_{k+n,j}\,\mathrm{sen}\,k\alpha\right) \tag{12}$$

$$\begin{aligned} R_\beta(\alpha,\gamma)&=\mathrm{Cor}\left[\tilde{\theta}(\alpha),\tilde{\theta}(\alpha+\gamma)\right]= \\ &=\sum_{k=1}^{n}\sum_{h=1}^{n}\begin{pmatrix} C_{kh}\cos k\alpha\cos h(\alpha+\gamma)+C_{k+n,h+n}\mathrm{sen}\,k\alpha\,\mathrm{sen}\,h(\alpha+\gamma)+ \\ +C_{k,h+n}\cos k\alpha\,\mathrm{sen}h(\alpha+\gamma)+C_{k+n,h}\mathrm{sen}\,k\alpha\cos h(\alpha+\gamma)\end{pmatrix}\end{aligned} \tag{13}$$

The good approximation between the curves of the function $\tilde{\theta}(\alpha)$ solved from seismic recordings and those solved by means of Fourier series shows a good control of the methodology (Fig. 7).

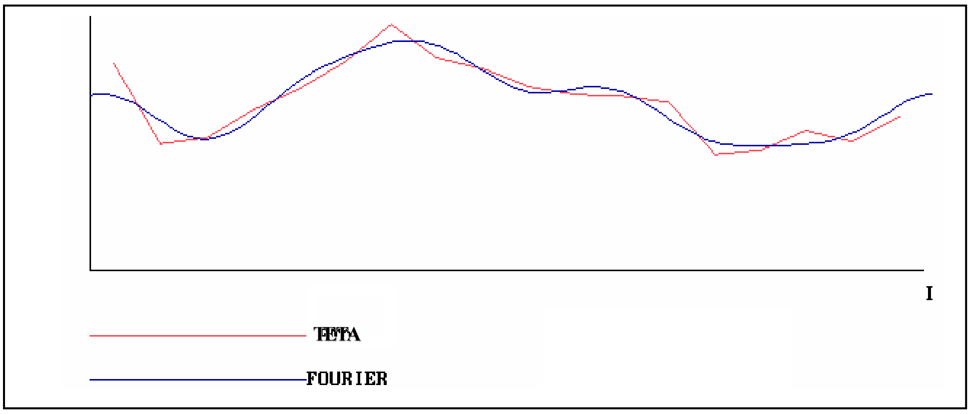

Figure 7: Comparison between the function $\tilde{\theta}(\alpha)$ solved by data of recorded sites (blue line) and simulated function using the Fourier series (red line) for the 7/6/1910-Irpinia earthquake in Basilicata (Italy).

3.3 Simulation of directional propagation process

If the coefficients $\tilde{c}_i$ in Eq.(7) are assumed as Gaussian, the process $\tilde{\theta}(\alpha)$ is Gaussian too. If one discretize in *n*-intervals the time period (0-2π) having $\alpha_1 = 0,\ \alpha_2 ....,\ \alpha_n = 2\pi$, the process $\tilde{\theta}(\alpha)$ itself is discretized into variables $\tilde{\theta}_1 = \tilde{\theta}(\alpha_1), ... ,\ \tilde{\theta}_n = \tilde{\theta}(\alpha_n)$. So it is possible to simulate a number of *n*-dimensional vectors using the mean value and covariance of $\tilde{\theta}_i$

$$\bar{\theta}_i = E(\tilde{\theta}_i) \tag{14}$$

$$C_{\beta ij} = E\left[(\tilde{\theta}_i - \bar{\theta}_i)(\tilde{\theta}_j - \bar{\theta}_j)\right] \tag{15}$$

Considering some random variables relevant to the *n-th* transformation

$$\tilde{\mathbf{y}} = \mathbf{U}\tilde{\boldsymbol{\theta}} \tag{16}$$

they are independent if the variable **U** represents the matrix of eigenvectors of $\mathbf{C}_\beta$.

After simulating *n*-normal values independent y_i, the simulated values of $\tilde{\theta}_i$ corresponding to them are

$$\hat{\theta}_i = \mathbf{U}^{\mathbf{T}}\mathbf{y} \tag{17}$$

which allow the simulation of propagation of seismic intensity over the territory generated by independent random vectors **y** which are transformed by the process of Eq.(17).

The development of the process $\tilde{\theta}(\alpha)$ simulates the propagation of the seismic intensity and then seismic signal.

For each of the earthquakes studied a number of simulations of seismic signal propagation over the territory are produced. These simulations, as the development of a random process, can have very different forms from each other, even if some of these are very similar to the real form of the signal propagation during past earthquakes. To this regard, it should be noted that even real earthquakes, with the same epicentre and magnitude roughly equal, show a seismic intensity distribution in the territory very different from each other, as it is shown below.

Some examples are presented that allow to compare the propagation of the seismic signal recorded on the territory during several earthquakes with some simulations of the propagation obtained by adopting the procedure developed in the above (Figs. 8, 9).

The macroseismic maps in Figs. 8.a) and 9.a) are modelled on the basis of those included in the "Atlas of Isoseismic Maps of Italian Earthquakes" [22].

Macroseismic maps obtained from recordings of two earthquakes occurred in the same area with the same location and magnitude are very different from each other, pointing out clearly the heavily random character of seismic signal propagation.

In particular, by looking at the map of the 9/8/1694-Irpinia earthquake in Basilicata (Italy) with epicentre intensity I_o = 10,5 (Fig. 8.a) one can note that the propagation is preferentially along the direction SE-NW.

This trend is very well represented and captured by both simulations as shown on the right in the figure (Fig. 8.b and c); the simulation also reproduces the secondary seismic signal direction toward SW.

With reference to the 11/23/1980-Irpinia earthquake with epicentre intensity I_o=10 (Fig. 9.a) one may notice that both simulations (Fig. 9.b and c) shown a main direction of spread from SE to NW, with an inverted triangle shape that has two preferential directions of propagation to NE and SW.

In general, one can then conclude that the comparison of the current and simulated macroseismic maps shows that results are reproduce with a good approximation the fundamental propagation characters of the seismic signal.

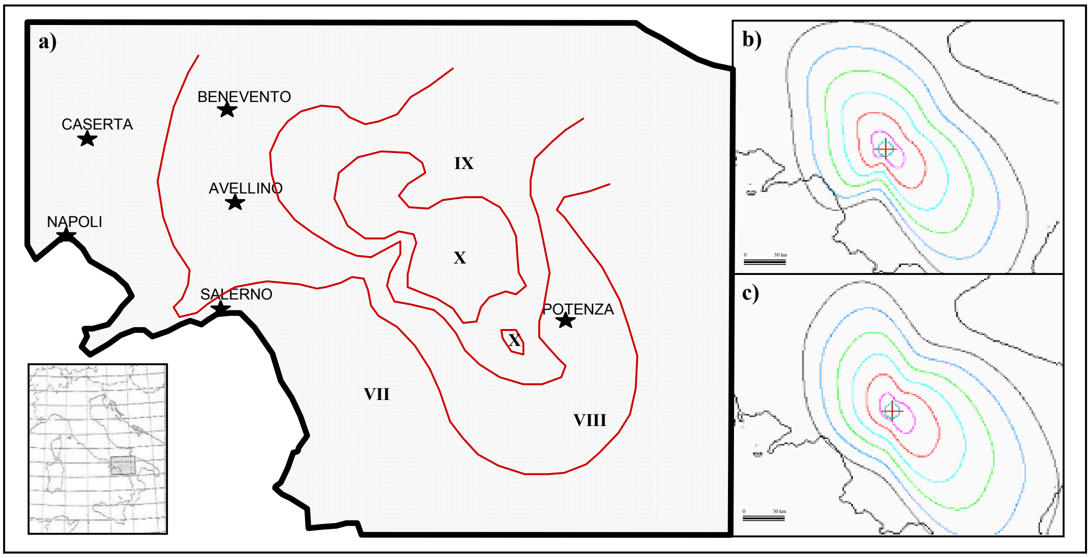

Figure 8: Wave propagation during the 9/8/1694-Irpinia earthquake in Basilicata (I_o= 10,5): on the left the recorded distribution of seismic intensity and, on the right are two examples of simulation).

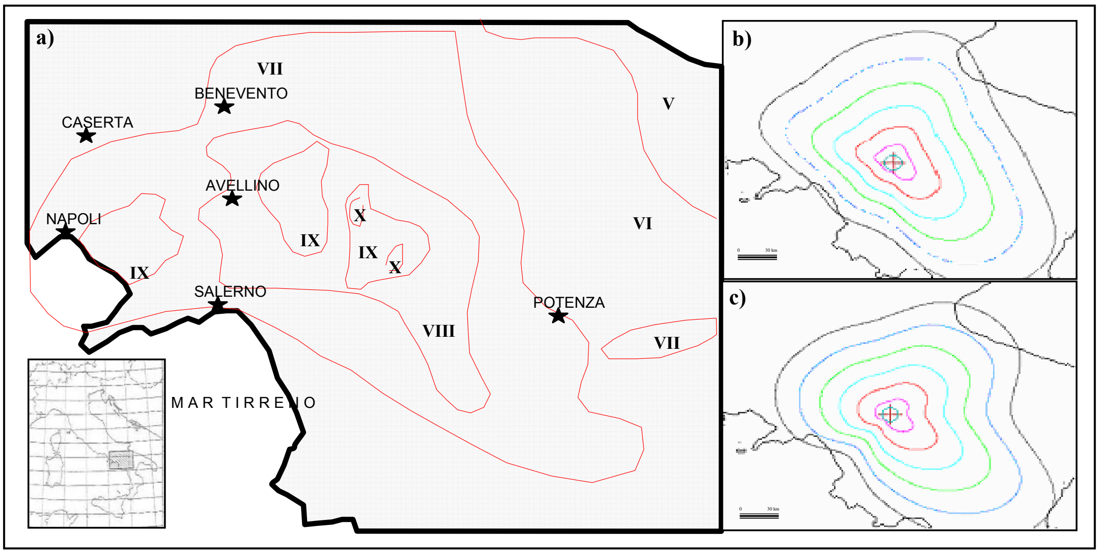

Figure 9: Wave propagation during the 11/23/1980-Irpinia earthquake ($I_o = 10$): on the left the recorded distribution of seismic intensity and, on the right, two examples of simulation.

4 Conclusions

In the paper some results relevant to a research aimed at the development of a simplified methodology for simulation of seismic signal are summarized. First studies developed by the author refers to known literature (the classic Cornell models for the analysis of seismic risk at site, the law of Gutenberg-Richter earthquake, the earthquake occurrence process of Poisson).

Thereafter, the observation of the high randomness in the seismic signal propagation over the territory during some earthquakes really occurred in southern Italy, with the same magnitude and location area, suggested the hypothesis of a probabilistic-type distribution.

To this purpose, the first step consists of the elaboration of macroseismic maps of the seismogenetic areas by building the Epicentral density function. The Epicentral density function expressed by the combination of *n*-Gaussian bivariate distributions solved by means of statistical analysis of data recorded during historic earthquakes really occurred in the area of interest.

Then, assuming that the law of propagation can be expressed by adopting a function of the intensity with an exponential coefficient that completely absorbs the random nature of the function, a stochastic process simulation of signal propagation in the territory is built. The parameters of the process so constructed are obtained through statistical processing of some fundamental macroseismic parameters, such as geographical location of epicentre and distance of the sites from the epicentre, magnitude, epicentre intensity and site intensity, etc., of historical earthquakes already occurred in the area.

The advantage of this procedure is that it represents a generalization of the method of Cornell since it produces the macroseismic maps using automatic statistical processing of macroseismic parameters of historic earthquakes, without any "interpretation" by the operator.

Finally the comparison between the distributions of the seismic signal produced by the development of the built up stochastic process, and the macroseismic maps relevant to occurred historic earthquakes, show a good agreement between the simulated and recorded data, validating the procedure.

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