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Paper 60

Optimization of Tendon Geometry of Post-Tensioned Concrete Bridges using Genetic Algorithms

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Abstract

Generally, the technology of post-tensioned concrete is commonly used in construction of medium and long span concrete bridges all around the world. An opportunity for the active modification of the internal force distribution offered by a variable geometry of post-tensioned tendons is one of the main advantages. Unfortunately these kinds of structure are not usually investigated as optimization tasks. This is because of the complexity of the structure and the presence of many design variables which are often discrete. Our contribution presents one of the first examples of the optimum design of a tendon geometry of three span post-tensioned concrete structures investigated to include the necessary design details with the inclusion of discrete variables.

Keywords: post-tensioned concrete, design, bridge, cost, optimization, evolutionary algorithms, genetic algorithms.

1 Introduction

Several studies on the optimization of the post-tensioned concrete structures have been already published in the past. Typically, Marks and Trochymiak [1] presented a work dealing with the optimized design of a tendon geometry in continuous concrete three spans bridge box girders using linear programming. The objective function was focused only on prestressing reinforcement. Design constraints were considered as normal concrete stresses and tendon eccentricities with respect to concrete cover according to Polish code. Similarly, Quiroga and Arroyo [2] published a study with an optimized geometry of prestressing tendons on fixed cross-sectional dimensions. Again, normal stresses of concrete served as constraints. Both these contributions used optimization techniques based on mathematical programming. One of the first examples employing structural requirements leading to discrete formulation of the design problem, Martí and González-Vidosa [3] published a study focused on an optimal design of a prestressed foot bridge using a Simulated Annealing method. The objective function consists of a cost for concrete, nonprestressed and prestressing reinforcement. Constraints were ultimate bending moment capacity, shear capacity and a deflection check according to Spanish EHE code.

However, the mentioned models were solved using linear analysis and did not take into account real behaviour of a structure with construction stages and time effects. Nowadays European standards (Eurocodes) are widely used almost in whole Europe, therefore our optimization is focused on the design and checks according to the design code for concrete bridges (EN1992-2). To the best authors' knowledge, such kind of structural optimization has not been presented in available literature yet.

In this contribution, a post-tensioned concrete three spans bridge is optimized from the prestressing level as well as a geometry point of view. The cross-sectional shape is a three-beam with fixed dimensions of a depth and width. The design and check of the structure is performed according to Eurocodes. The structure is analysed incorporating construction stages with a time dependent analysis of creep and shrinkage behaviour according to annex B from EN 1992-1-1 for a 100 years life time.

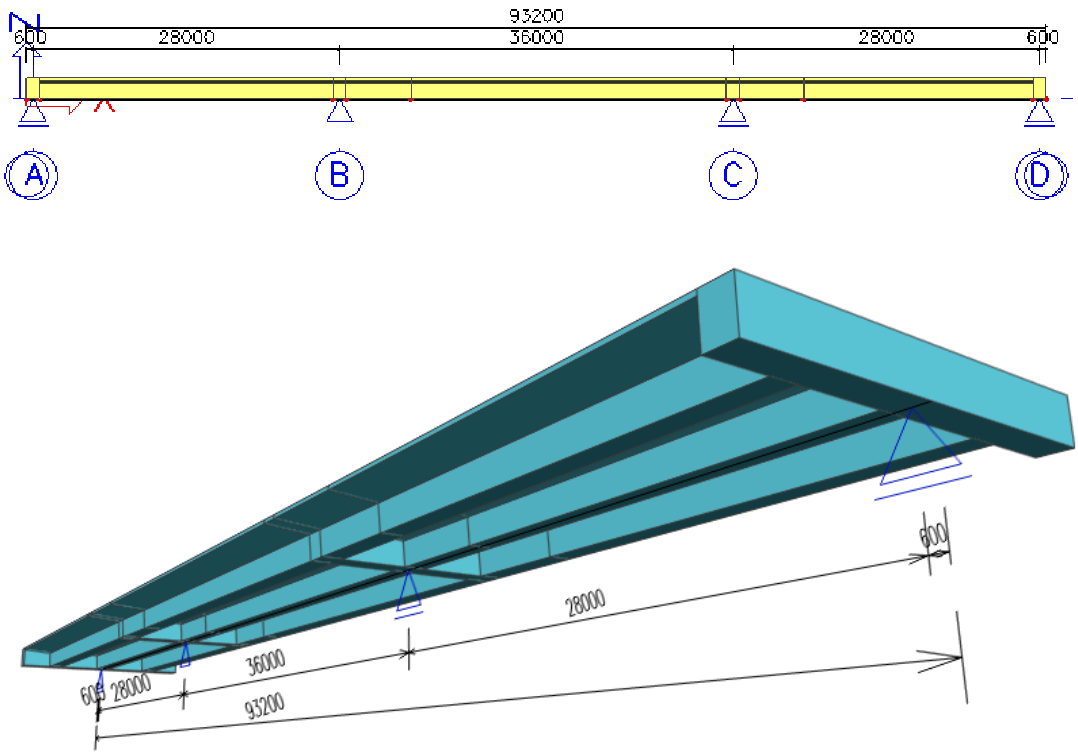

Figure 1: A statical model of the bridge. Dimensions are in millimetres.

2 Three spans post-tensioned bridge

2.1 Description

A post-tensioned concrete three spans (28+36+28 m) bridge is optimized from a prestressing level and a geometry point of view, see Fig. 1. The cross-section shape is three-beams with fixed dimensions of 1.865 m depth and a bottom width of beams equal to 1.2 m. The upper part of a deck is 16.55 m wide. The shape of the cross-section is shown in Fig. 2. A structure is built from concrete C35/45. The design and check of the structure is performed according to Eurocodes with respect to Czech national annex.

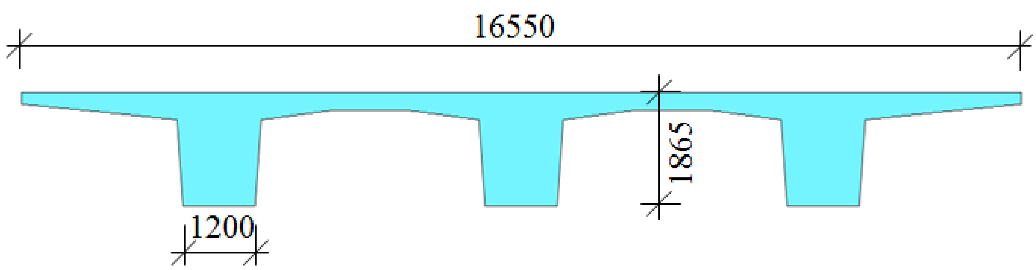

Figure 2: A Bridge cross-section. Dimensions are in millimetres.

The load combinations are considered according to ČSN EN 1990 with respect of A2 attachment. The structure is loaded by a variable traffic load according to ČSN EN1991-2 where a load group `gr1a` (a tandem system, uniform dead load and pedestrians) seems to be dominant. The envelopes of bending moments and shear forces from `gr1a` were evaluated during the analysis. The linear heating and cooling together with initial support displacements were also considered in the calculation. The structure is analysed using construction stages with the time dependent analysis of creep and shrinkage behaviour again according to annex B of ČSN EN 1992-1-1. As usual, the structure is designed for 100 years.

Beam	1	2	3	Sum
Edge load	3624kNm	2290 kNm	831 kNm	6745 kNm
	53.30%	34.00%	12.70%	100%
Load in middle	1823.5kNm	3098 kNm	1823.5 kNm	6745 kNm
	27.25%	45.50%	27.25%	100%

Table 1: Transversal spreading of the load.

2.2 Transversal spreading of the load

Because of the three beams cross-section the transversal spreading load was performed at first. The model for transversal spreading was a 2D model, where the concrete 2D slab was modelled as a deck. Each beam was connected to the slab as a rib. Particularly, the internal beam takes 45.5% and the edge beam 53.3% of the acting variable load, see Tab. 1. Only one cross-section was studied for the next analysis. In the following figure (Fig. 3) a typical studied cross-section is displayed. This is the edge beam which takes 53.3% from the load system of a variable mobile load.

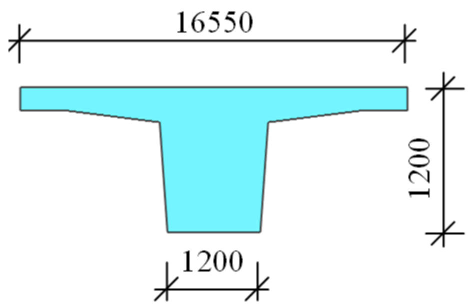

Figure 3: A typical cross-section of one beam.

3 Optimization problem

3.1 Objective function

The prestressing of one beam consists of different tendon geometries A-E as is shown in Fig. 4. The aim of this study was to minimize a necessary area of prestressing reinforcement together with a more effective tendon geometry. The same number of strands was assumed in tendons geometries A, B, C and another in the group of tendons D and E. Hence, the objective function can be expressed as follows

$$min(f_x) = A_{pi} \cdot \sum_{i=1}^{n} n_{t,i} \cdot n_{g,i} \tag{1}$$

where $A_{p,i}$ is the area of one strand Y1770S7-16,0-A; values $n_{t,i}$ and $n_{g,i}$ are a number of strands in a tendon i and a number of tendons in a group for one beam, respectively; $i = 1 \dots n$ where n is a number of a particular tendon geometry.

The length of the tendon varies according to the optimized source geometry. Authors also performed a study where these lengths were included in the objective function. The results gave almost negligible decreasing of the total length of the tendons. There are more than 10 km of strands and a difference just only about 5 meters among optimized solutions was obtained. The area of prestressing reinforcement in case of a continuous tendon source geometry on the whole bridge seems to be the most important factor, and therefore, the effects of the length of the tendon source geometry can be neglected.

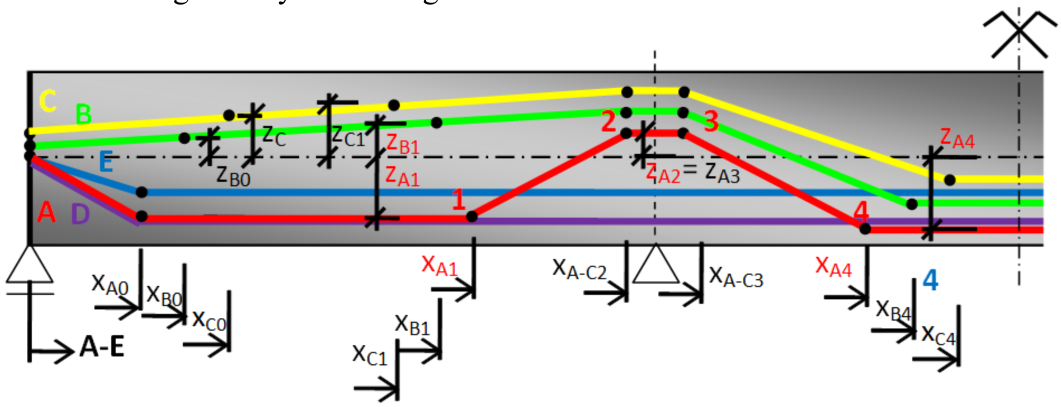

Figure 4: Tendon geometries of post-tensioned tendons. Design variables are marked red.

3.2 Design variables

The number of strands in one tendon and the number of tendons in groups were selected as design variables. There are five different source geometries in one beam (A-E). The number of the same tendons in a group was constant for the source geometries A-C, particularly six tendons in a cross-section were selected, i.e. two tendons in one beam. Next, the number of strands in tendons for the D-E geometries was fixed to 15. Therefore, we have two independent design variables connected to the prestressing area, namely the number of strands for the A-C geometries $n_{t,ABC}$ and the number of groups for the D-E geometries $n_{g,DE}$, see Tab. 2 for initial values and predefined limits.

Parametr	n_{tABC} [-]	n_{gDE} [-]	x_{A1} [m]	z_{A1} [m]	z_{B1} [m]	z_{A2} [m]	x_{A4} [m]
Initial	15	2	19.1	0.15	1.35	1.4	38.1
Minimum	15	0	17.1	0.15	0.45	1.25	36.6
Maximum	19	2	21.1	0.45	1.35	1.55	39.6
Step	2	1	0.5	0.05	0.1	0.05	0.5

Table 2: Initial and limit values for design variables.

Other design variables are geometry coordinates of a source tendon geometry. Each geometry of a tendon has 4 points which positions are optimized (marked red in Fig. 4) and are again listed in Tab. 2. The remaining points of source geometry were calculated based on these design variables. Dependency between all points of a geometry decreases the number of geometry design variables to 7 only. The dependent variables are summarized in the following table. Values x_{step}=2.0 m and x_{step1}=1.5 m and z_{step}=0.15 m are kept constant.

Variable	Formula
z_{A3}	z_{A2}
x_{B4}	$x_{A4}+x_{step}$
z_{B2}	$z_{A2}+z_{step}$
z_{B3}	z_{B2}
z_{B4}	zA4+zstep
x_{C4}	xB4+xstep
z_{C2}	zB2+zstep
z_{C3}	zC2
z_{C4}	zB4+zstep
L	2*L1+L2
x_{A5}	L-xA4
x_{A8}	L-xA1
x_{B5}	L-xB4
x_{C5}	L-xC4
z_{B1pom}	(zB2-1.1)*9.6/(L1-1)+1.1
z_{C1pom}	(zC2-1.5)*11.1/(L1-1)+1.5
x_{B1}	xA1-xstep1
x_{C1}	xB1-xstep1
z_{C1}	zB1+zstep
x_{B8}	L-xB1
x_{C8}	L-xC1
z_{B0}	(zB1<1.1)*zB1+(1.1<=zB1)*zB1pom
z_{C0}	(zC1<1.5)*zC1+(1.5<=zC1)*zC1pom

Table 3: Dependent parameters.

3.3 Constraints

The optimization of prestressing reinforcement was performed based on the serviceability limit state (crack appearance using check of allowable concrete stresses from the characteristic combination) and on the ultimate limit state (a check of capacity calculated using an interaction diagram for acting combination of a normal force and a bending moment). The combinations were evaluated for 100 years of bridge service. Limit values of checks (a ratio of calculated and limit values) was selected in a standard way to 1.0. The used constraints can be divided into three groups.

- Geometrical – these constraints are coming from a geometry of the cross-section and distribution of the individual tendons in the cross-section with respect to a minimal concrete cover and clear distances between tendons.

- The serviceability limit state (Check_calc_max_A) – normal stresses for the characteristic combination during service in 100 years are evaluated as the indication of the longitudinal cracks existence

$$\sigma_{cc} \leq \sigma_{cc,ch} = 0.6 \cdot f_{ck} = 21\ MPa \tag{2}$$

$$\sigma_{ct} \leq f_{ct,eff} = 3.76\ MPa \tag{3}$$

- The ultimate limit state (Check_calc_max_B) – a verification of the beam loaded by the combination of a normal force and a bending moment is performed by a method of the interaction diagram. Fundamental STR/GEO Set B combination was used for verification in this check

$$N_{ed} \leq N_u \tag{4}$$

$$M_{ed} \leq M_u \tag{5}$$

4 Optimization algorithm and results

There exist many optimization algorithms which can be used for optimization. For our case where the number of strands and tendons are discrete variables it is necessary to use methods which are capable to handle discrete variables. Evolutionary algorithms are a group of optimization methods that mimic the evolution of nature in an aim to search optima. Modified simulated annealing (MSA) [4] and Differential Evolution (DE) [5] are well-known examples of these methods. We tested both methods and MSA has been finally selected due to the faster convergence.

The MSA is characterized by the population of candidate solutions, where the population consisted of 15 members. The simulated annealing algorithm is presented in the selection phase, where the acceptance of new solutions is governed by the cooling algorithm. Here, the initial annealing temperature was calculated based on an acceptance of 50% of members from the first population leading to T_{max} = 6686.24. Then the cooling coefficient was set to T_{mult}=0.7356. Note that a special optimization tool was used for the optimization of this kind of structure, see Acknowledgement section. As the results, 9 possible solutions were offered by the program after running 484 iterations, see Tab. 4 for solutions and Fig. 5 for the history of the optimization process. Based on the detailed upcoming analysis we can accept solutions 7, 8 and 9, characterized by 19 strands in a tendon. Tendons with geometry D and E are not necessary at all. When we use tendons with 17 strands we received check values 1.003-1.007 which are slightly out of the limit range. Check values of performed checks of allowable concrete stresses and interaction diagram capacity tend to limit check value of 1.0. Maximal calculated check values are presented in Tab. 5. All obtained results were also verified on other checks for prestressed concrete according to ČSN EN 1992-2. Allowable concrete stresses for characteristic and frequent combinations, allowable stresses in prestressing reinforcement prior and after anchoring and shear verification were performed as well.

Parameter	ntABC [-]	ngDE [-]	xA1 [m]	zA1 [m]	zB1 [m]	zA2 [m]	xA4 [m]	zA4 [m]	Ap,req [mm^2]
Initial	15	2	9.1	0.15	1.35	1.4	38.1	0.15	22500
Sol. 1	17	0	17.1	0.4	1.25	1.4	39.1	0.45	15300
Sol. 2	17	0	17.1	0.4	1.35	1.4	38.6	0.40	15300
Sol. 3	17	0	17.1	0.25	1.25	1.4	38.1	0.45	15300
Sol. 4	17	0	18.1	0.4	1.25	1.4	39.6	0.45	15300
Sol. 5	17	0	17.6	0.25	1.35	1.4	39.1	0.45	15300
Sol. 6	17	0	17.6	0.35	1.25	1.4	39.1	0.40	15300
Sol. 7	19	0	18.6	0.35	1.15	1.4	38.1	0.35	17100
Sol. 8	19	0	17.6	0.45	1.1	1.4	38.6	0.40	17100
Sol. 9	19	0	17.6	0.2	0.55	1.4	37.6	0.35	17100

Table 4: Optimized solutions found by MSA method.

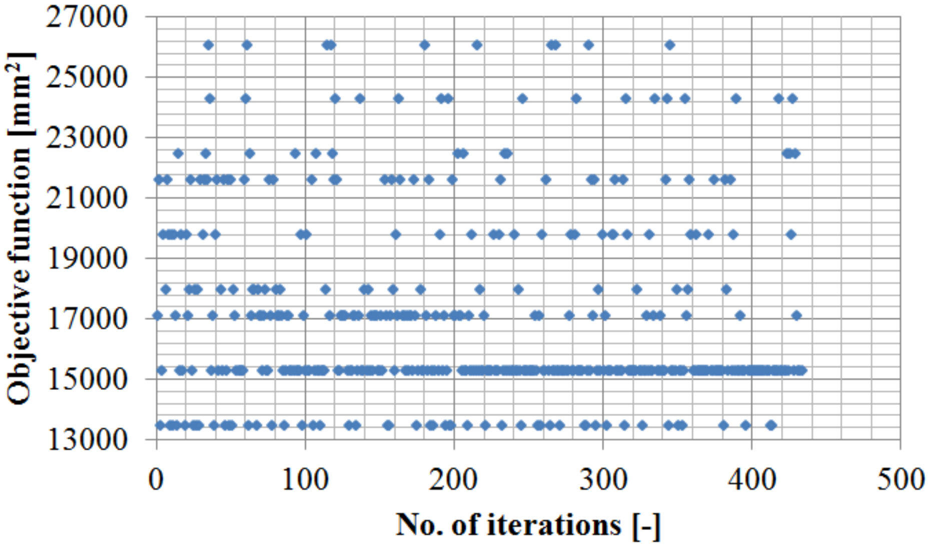

Figure 5: History of the objective function development. Note that all solutions are presented, i.e. including solutions that do not fulfil given constraints.

Original tendon source geometry and a new optimized geometry are compared on the half of the structure in 0. Vertical geometries of the tendons together with the cross-section are five times scaled to highlight the differences. The saving of material is compared for particular solutions together with their check values, see again Tab. 5.

Parameter	Ap,req [mm^2]	Save [%]	Check_calc_max_A [-]	Check_calc_max_B [-]
Initial	22500	-	0.696	0.856
Sol. 1	15300	32	0.710	1.004
Sol. 2	15300	32	0.701	1.005
Sol. 3	15300	32	0.685	1.007
Sol. 4	15300	32	0.711	1.003
Sol. 5	15300	32	0.709	1.003
Sol. 6	15300	32	0.687	1.007
Sol. 7	17100	24	0.680	0.950
Sol. 8	17100	24	0.664	0.945
Sol. 9	17100	24	0.778	0.970

Table 5: Results for offered solutions.

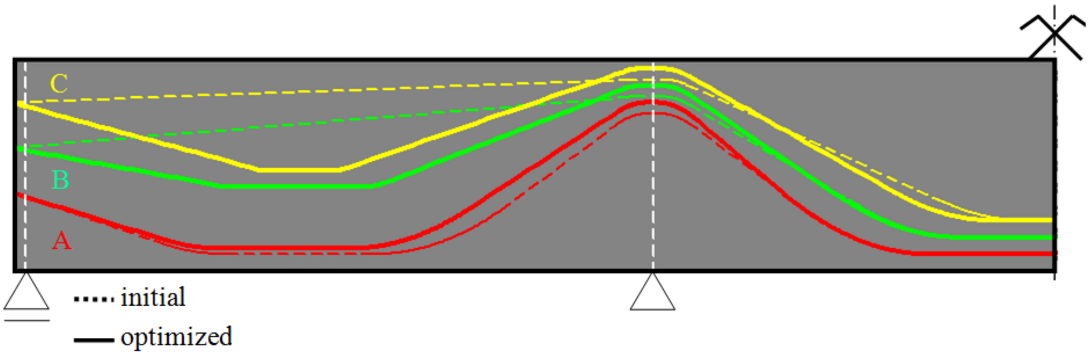

Figure 6: Comparison of original and optimized geometry.

The MSA method decreased amount of prestressing reinforcement by 24% by modifying the tendon geometry. The check values of allowable concrete stresses for characteristic combination and check of capacity using the interaction diagram are very close to the limit check value of 1.0, see Fig. 7.

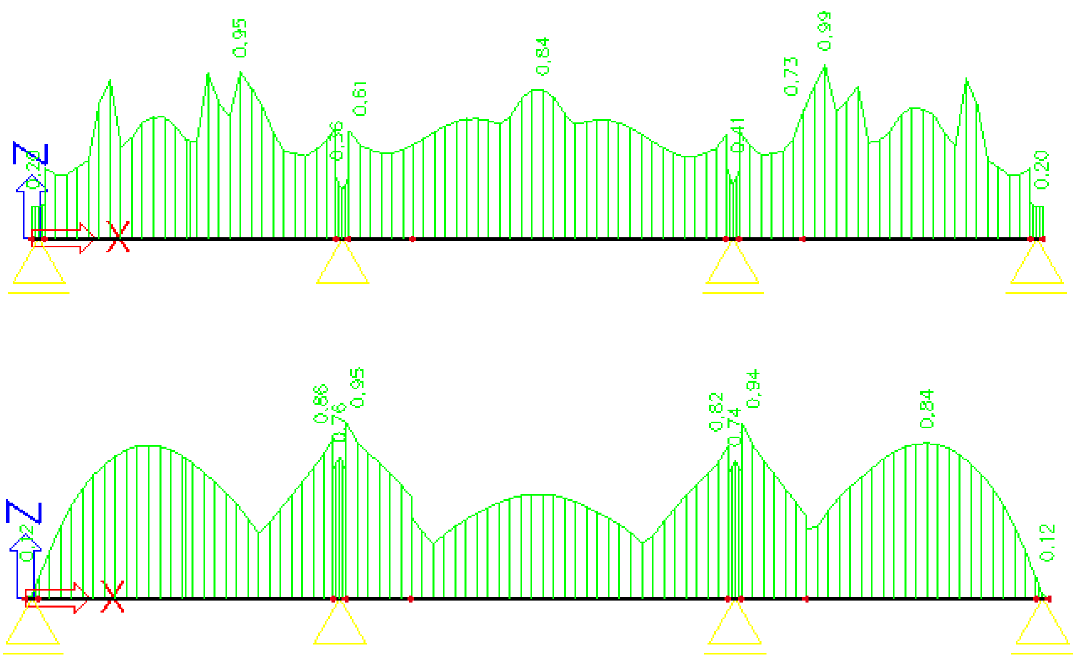

Figure 7: Check value for allowable concrete stresses (top) and capacity using interaction diagram (below) on characteristic combination in 100 years for solution 7.

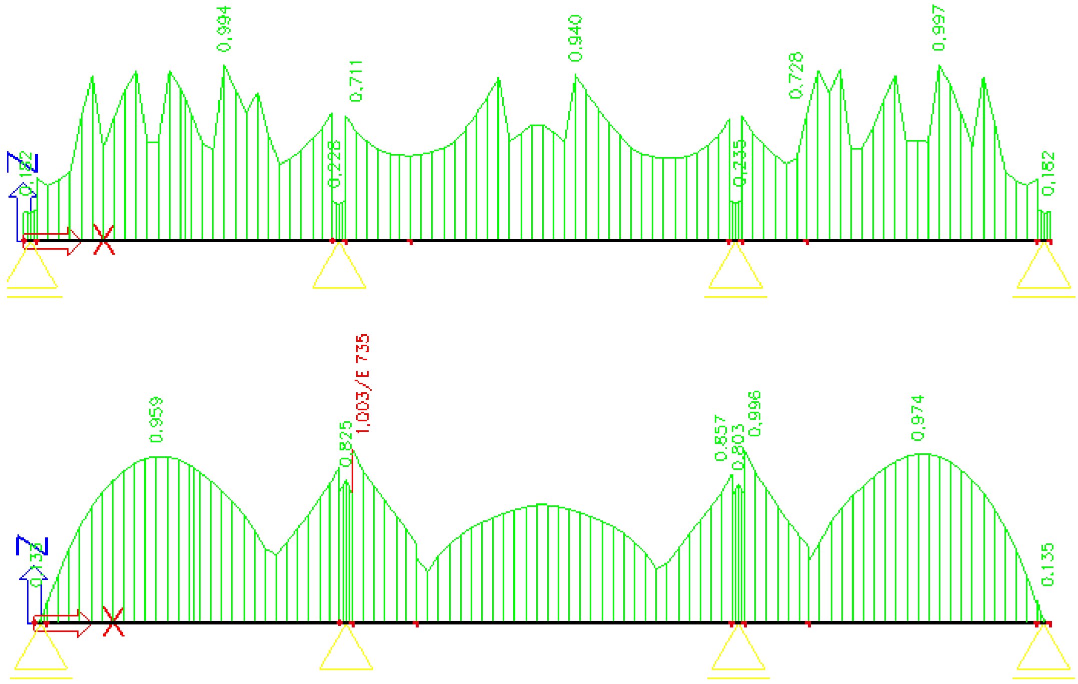

Figure 8: Check value for allowable concrete stresses (top) and capacity using interaction diagram (below) on characteristic combination in 100 years for solution 4.

The results for the solution 4 are mentioned in Fig.8 for illustration purposes. Here, the capacity check using the interaction diagram is not satisfied only with 0.3%.

5 Conclusions

From the obtained results we can conclude that it is possible to used 6 pieces of 19 strands of geometries A-C and that tendons with geometries D and E are not necessary at all. Since the used code includes many safety factors, it is also possible to use the solution 4 violating constraints only by 0.3%. Because the numbers of strands are discrete design variables, it was necessary to apply an optimization method which is able to handle the discrete type of parameters. Here, Modified Simulated Annealing was successfully used. The obtained amount of prestressing reinforcement was decreased by 24% in comparison to the original design.

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References

[1] Marks W., Trochymiak W.: The selection of a system of prestressing tendons in hyperstatic beams as a problem of linear integer programming, Structural and Multidisciplinary Optimization; 1991, Volume 3, Number 1, 59-67

[2] Quiroga S. A., Arroyo U. M. A.: Optimization of prestressed concrete bridge decks, Computers & Structures; 1991, Volume 41, Issue 3, 553-559

[3] Martí J. V., González-Vidosa F.: Design of prestressed concrete precast pedestrian bridges by heuristic optimization, Advances in Engineering Software; July, 2010, Volume 41, Issue 7-8

[4] Lepš, M., Šejnoha, M.: New approach to optimization of reinforced concrete beams. Computers & Structures, 2003. Volume 81, Issues 18–19, 1957–1966

[5] Storn, R., Price, K.: Differential Evolution: A simple and efficient adaptive scheme for global optimization over continuous spaces. Technical Report TR-95-012, University of Berkeley, 1995.