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Paper 81

A Subspace Fitting Method based on Finite Elements for Identification and Localisation of Damage in Mechanical Systems

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Abstract

In this paper, a subspace fitting method based on finite elements for identification of modal parameters of a mechanical system is proposed. The technique uses prior knowledge resulting from a coarse finite element model (FEM) of the structure. The proposed technique is applied to identify the parameters of several mechanical systems under deterministic and stochastic excitations. Numerical experiments highlight the relevance of the technique compared to the conventional identification techniques. Identification, localization and estimation of the severity of the damage are carried out.

Keywords: subspace identification, subspace fitting, modal analysis, damage localization, finite elements.

1 Introduction

The process of implementing a damage detection strategy is referred to as structural health monitoring (SHM). In recent decades, SHM has attracted worldwide attention due to its significance in the safety evaluation of mechanical structures. The vibration-based SHM process basically involves the observation of a structure over time using dynamic response measurements from an array of sensors, the extraction of damage-sensitive features and the statistical analysis of these features to determine the current state of structural health. SHM can be defined as a four step process [1]: i) determination that damage is present in the structure, ii) determination of the geometric location of the damage, iii) quantification of the severity of the damage and iv) prediction of the remaining service life of the structure.

Vibration-based damage detection [2] is often implemented by identifying changes in the structural dynamic properties before and after damage. So far numerous damage detection techniques have been proposed where the damage features used include natural frequencies, frequency response function, mode shape, mode shape curvature, modal strain energy, modal flexibilities, etc.

Most vibration-based damage detection methods require the modal properties that are obtained from measured signals through the system identification techniques. They normally need intact structural states (undamaged state) or baseline FEM so that structural damage can be identified.

For a linear dynamical system, the subspace model is well suited for capturing the system eigenstructure under operational conditions. The subspace methods can treat system under deterministic or stochastic excitation. The key idea behind subspace identification algorithms is to consider a block Hankel matrix (i.e. constructed from the output and input measurements of a structure) and to project this matrix onto a subspace which is well suited for identifying the structure modal parameters. The procedure consists in formulating an observability matrix from a singular value decomposition of the block Hankel matrix (i.e. once projected onto the considered subspace). Different algorithms, which the best knowns are grouped under the acronyms N4SID, MOESP and CVA [3], allow access to the observability matrix and performing a shift invariance procedure of this observability matrix to recovered modal informations. Unfortunately, if the observability matrix is contaminated by noise, it would be introduces errors.
It is therefore preferable to use the subspace fitting method [4], in which the observability matrix is minimized through a theoretical observability matrix. Although iterative, this method takes the advantage of incorporating prior information about the system.

In this work, we propose a method, in which prior information, from modal data of a coarse FEM of the healthy structure, is introduced in a subspace fitting procedure. The subspace fitting improves the identification of modal frequencies compared to the shift invariance. Incorporating the prior information from the eigenvectors reduces the CPU costs. The method is used for fault identification. For a simulated beam, the method is also able to localize and estimate the severity of damage. Figure 1 summarizes the different steps of this work.

2 Finite element formulation and state space representation

The linear, time invariant, N degrees of freedom of a FEM of a mechanical system considered in this work is one for which the equation of motion and the output equation can be written in matrix form as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \boldsymbol{\gamma}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{u}(t), \tag{1}$$

where $\mathbf{M} \in \mathbb{R}^{N \times N}$, $\boldsymbol{\gamma} \in \mathbb{R}^{N \times N}$ and $\mathbf{K} \in \mathbb{R}^{N \times N}$ are the mass, damping and stiffness matrices respectively, $\mathbf{u}(t)$ is the vector with nodal forces and $\mathbf{q}(t)$ is the vector of

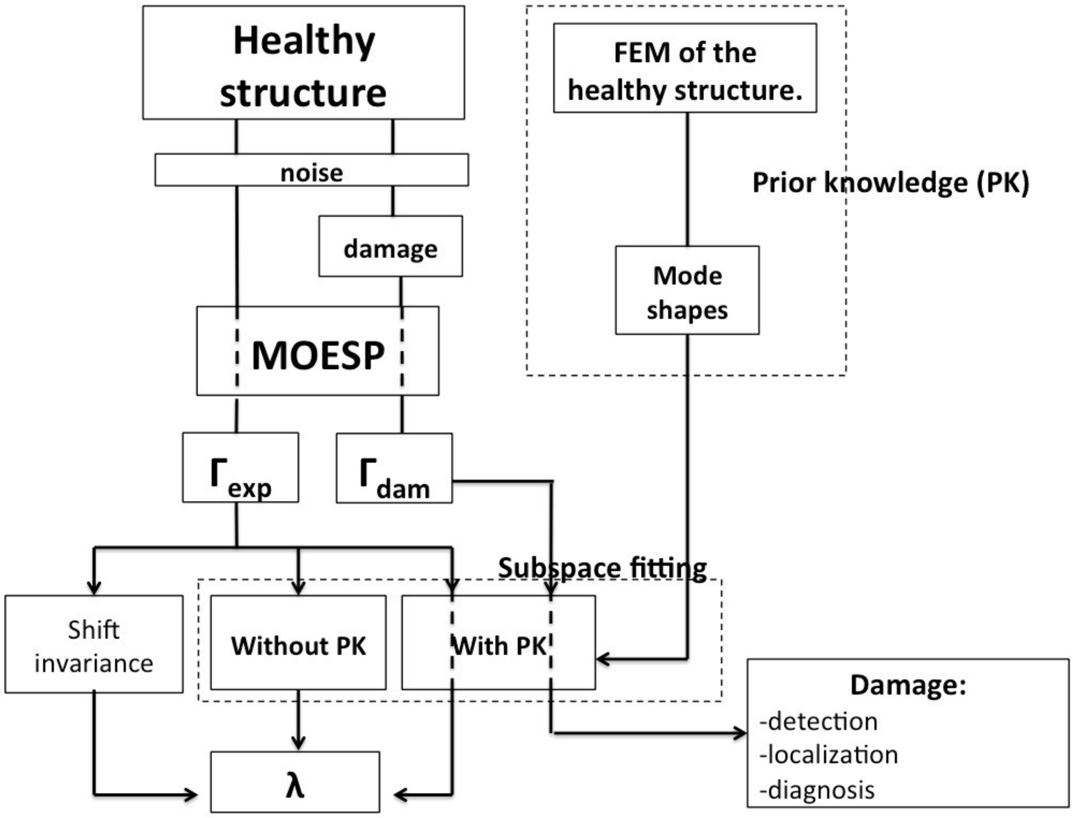


Figure 1: Flow chart of the proposed method.

nodal displacements with ($\dot{\bullet}$) and ($\ddot{\bullet}$) denoting the first and second order derivatives with respect to time t. The free vibrating solutions of equation (1) are usually sought in the following form

$$\mathbf{y}(t) = e^{\mathbf{\Lambda}t}\mathbf{\Phi}. \tag{2}$$

Substituting equation (2) into equation (1) gives

$$(\mathbf{\Lambda}^2\mathbf{M} + \mathbf{\Lambda}\boldsymbol{\gamma} + \mathbf{K})\mathbf{\Phi} = 0, \tag{3}$$

where $\mathbf{\Lambda}$ and $\mathbf{\Phi}$ refer to the complex eigenvalues and corresponding mode shapes of the structure, respectively. The dynamic behavior of the structure can be assessed by mean of the following continuous-time state-space model

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A_c}\mathbf{x}(t) + \mathbf{B_c}\mathbf{u}(t), \\ \mathbf{x}(t) = \begin{bmatrix}\mathbf{q}^T & \dot{\mathbf{q}}^T\end{bmatrix}^T, \end{cases} \tag{4}$$

where

$$\mathbf{A_c} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\boldsymbol{\gamma} \end{bmatrix}, \tag{5}$$

and

$$\mathbf{B_c} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}. \tag{6}$$

$\mathbf{A_c} \in \mathbb{C}^{2N\times 2N}$ is the state transition matrix characterizing the dynamics of the system, while $\mathbf{B_c}$ is an input matrix. After discretization in time, the state-space model of the mechanical structure is to be expressed as

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{f}_k, \\ \mathbf{y}_k = \mathbf{C}\mathbf{x}_k, \end{cases} \tag{7}$$

where

$$\mathbf{A} = e^{\mathbf{A_c}T_s} \quad \text{and} \quad \mathbf{B} = (\mathbf{A}-\mathbf{I})\mathbf{A_c}^{-1}\mathbf{B_c} \tag{8}$$

where T_s is the discrete-time step and $\mathbf{C}$ is an output matrix $\in \mathbb{R}^{l\times 2N}$, with l the number of outputs .

If the system is corrupted by some measurement noise and unknown inputs, Equations (7) are expressed as

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{f}_k + \mathbf{w}_k, \\ \mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k. \end{cases} \tag{9}$$

The stochastic terms $\mathbf{w}_k$ and $\mathbf{v}_k$ are unknowns noise process and noise output. If it is assumed that they have a discrete white noise nature with an expected value equal to zero and that they have covariance matrices equal to

$$\mathbb{E}\left[\begin{bmatrix}\mathbf{w}_p \\ \mathbf{v}_p\end{bmatrix}\begin{bmatrix}\mathbf{w}_q^T & \mathbf{v}_q^T\end{bmatrix}\right] = \begin{bmatrix}\mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{R}\end{bmatrix}\delta_{pq}, \tag{10}$$

where δ denotes the Kronecker symbol.

In modal analysis applications, the modal parameters are extracted from the state space model. An eigenvalue decomposition is applied to the dynamical system matrix $\mathbf{A}$

$$\mathbf{A} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^{-1}, \tag{11}$$

where $\mathbf{\Psi} = \begin{bmatrix}\mathbf{\Phi} \\ \mathbf{\Lambda\Phi}\end{bmatrix} \in \mathbb{C}^{2N\times 2N}$ is the eigenvector matrix (in the present case, it is assumed that the matrix $\mathbf{\Psi}$ is invertible) and $\mathbf{\Lambda} \in \mathbb{C}^{2N\times 2N}$ is the diagonal eigenvalue matrix. The matrix $\mathbf{\Lambda}$ contains the $2N$ discrete-time eigenvalues μ_i, of which the complex conjugated pairs contribute to the vibration modes. They are related to the continuous-time eigenvalues λ_i as

$$\mu_i = e^{\lambda_i T_s}. \tag{12}$$

The resonance frequencies f_i and the damping ratios ξ_i can then be found from

$$\lambda_i, \lambda_i^* = -\xi_i f_i \pm j\sqrt{(1-\xi_i^2)}f_i. \tag{13}$$

3 Subspace Identification method

Among the algorithms that identify experimentally the modal parameters of a mechanical structure, subspace identification algorithms have gained increasing attention due to their inherent robustness and their ability to deal with a large numbers of inputs and outputs. Subspace identification techniques derive models for linear systems solely by applying well-conditioned operations.

Many algorithms have been developed [3] to estimate the matrices of the state-space model. We are interested in those grouped under the acronym MOESP [5], since they simply provide, the matrices of modal parameters of the system.

Block Hankel matrices with input and output data are the basic starting point for subspace identification algorithms. An input block Hankel matrices is defined as follows

$$\mathbf{U} = \left(\begin{array}{ccccc} u_1 & u_2 & u_3 & \dots & u_j \\ u_2 & u_3 & u_4 & \dots & u_{j+1} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ u_i & u_{i+1} & u_{i+2} & \dots & u_{i+j-1} \\ \hline u_{i+1} & u_{i+2} & u_{i+3} & \dots & u_{i+j} \\ u_{i+2} & u_{i+3} & u_{i+4} & \dots & u_{i+j+1} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ u_{2i} & u_{2i+1} & u_{2i+2} & \dots & u_{2i+j-1} \end{array}\right) = \left(\frac{\mathbf{U_p}}{\mathbf{U_f}}\right), \tag{14}$$

where the number of block rows i in $\mathbf{U_p}$ and $\mathbf{U_f}$ is a user-defined index, which is large enough, i.e. $il \geq n$, the number of columns j is typically equal to $s - 2i + 1$, where s is the number of available data samples. The subscript 'p' stands for 'past' and the subscript 'f' for 'future'. The output block Hankel matrices $\mathbf{Y}$, $\mathbf{Y_p}$ and $\mathbf{Y_f}$ are defined in a similar way.

The MOESP algorithms, start from the so called past and future data equations constructed from Equations (9)

$$\mathbf{Y_p} = \mathbf{\Gamma_i X_p} + \mathbf{H_i U_p} + \mathbf{N_p}, \tag{15}$$

$$\mathbf{Y_f} = \mathbf{\Gamma_i X_f} + \mathbf{H_i U_f} + \mathbf{N_f}, \tag{16}$$

where $\mathbf{\Gamma_i} = \begin{pmatrix} \mathbf{C}^T & [\mathbf{CA}]^T & \dots & [\mathbf{CA}^{i-1}]^T \end{pmatrix}^T \in \mathbb{R}^{li \times n}$ is the extended observability matrix, $\mathbf{X_p}$ (respectively $\mathbf{X_f}$) is a past (respectively future) state sequence, $\mathbf{H_i}$ is a block Toeplitz matrix of the (unknown) impulse response from $\mathbf{u}$ to $\mathbf{y}$ and $\mathbf{N_p}$ (respectively $\mathbf{N_f}$) a particular combinaison of the past (respectively future) block Hankel matrices of the perturbations $\mathbf{v}$ and $\mathbf{w}$. For simultaneously removing the term $\mathbf{H_i U_f}$ from $\mathbf{Y_f}$ and decorrelating the noise, it is proposed to consider the following quantity $\mathbf{Y_f \Pi_{U_f^\perp}}$, where $\mathbf{\Pi_{U_f^\perp}} = \mathbf{I} - \mathbf{U_f}^T(\mathbf{U_f U_f}^T)^{-1}\mathbf{U_f}$ is an orthogonal projection for Ordinary-MOESP. In practice, this projection can be obtained by performing a LQ-factorization of the input and output data, which is numerically much more efficient than evaluating the large projection matrix

$$\begin{bmatrix} \mathbf{U_f} \\ \mathbf{Y_f} \end{bmatrix} = \begin{bmatrix} \mathbf{L_{11}} & \mathbf{0} \\ \mathbf{L_{21}} & \mathbf{L_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{Q_1} \\ \mathbf{Q_2} \end{bmatrix}. \tag{17}$$

Now, it can be equivalently written

$$\mathbf{Y_f}\mathbf{\Pi}_{\mathbf{U_f^{\perp}}} = \mathbf{\Gamma_i}\mathbf{X_f}\mathbf{\Pi}_{\mathbf{U_f^{\perp}}} = \mathbf{L_{22}}\mathbf{Q_2}. \tag{18}$$

As a result, it turns out that

$$\text{range}(\mathbf{\Gamma_i}) = \text{range}(\mathbf{L_{22}}). \tag{19}$$

Thus, the column-space of $\mathbf{L_{22}}$ serves as a basis for the column space of the extended observability matrix $\mathbf{\Gamma_i}$. Performing a singular value decomposition of $\mathbf{L_{22}}$ gives

$$\mathbf{L_{22}} = \mathbf{U}_n\mathbf{\Sigma}_n\mathbf{V}_n^T, \tag{20}$$

where n is the number of dominant singular values and also the order of underlying system including the noise model.

The columns of $\mathbf{U}_n$ provide a basis for $\mathbf{\Gamma_i}$. A gap between successive singular values will often indicate the system order.

4 Identification of the parameters of the structure

4.1 Shift invariance

An estimate for the matrices $\mathbf{A}$ and $\mathbf{C}$, up to a similarity transformation, can then be obtained by enforcing the shift invariance structure of the observability matrix [3] as follows: $\hat{\mathbf{C}}$ is equal to the first l rows of $\hat{\mathbf{\Gamma}}$ and $\hat{\mathbf{A}}$ is equal to $\underline{\hat{\mathbf{\Gamma}}}^{\dagger}\overline{\hat{\mathbf{\Gamma}}}$, with $\underline{\hat{\mathbf{\Gamma}}}$ and $\overline{\hat{\mathbf{\Gamma}}}$ denote notations for $\hat{\mathbf{\Gamma}}$ with its last, respectively first l lines removed and where $\hat{\mathbf{\Gamma}}^{\dagger}$ is the pseudo-inverse of $\hat{\mathbf{\Gamma}}$.

The shift invariance property is not satisfied exactly for finite data when stochastic disturbances are present and hence it has to be solved approximately.

4.2 Subspace fitting

Subspace fitting method [4, 6] exploit the full structure of the extended observability matrix. The problem is reformulated in a separable non linear least square fitting minimization problem [7] of the form

$$\min_{\mathbf{A},\mathbf{C},\mathbf{T}} ||\hat{\mathbf{\Gamma}} - \mathbf{\Gamma_{th}}\mathbf{T}||_F^2, \tag{21}$$

where $\mathbf{\Gamma_{th}} = \begin{pmatrix}\mathbf{C}^T & [\mathbf{CA}]^T & \ldots & [\mathbf{CA}^{i-1}]^T\end{pmatrix}^T$ is a function of the elements of $\mathbf{A}$ and $\mathbf{C}$. Just as there are many realizations or coordinate systems that can be used to describe the state space, there are many identifiable $\mathbf{A}$ and $\mathbf{C}$ matrices that can be chosen, each yielding a different $\mathbf{T} \in \mathbb{R}^{n\times n}$ matrix that satisfies equation (21).

Subspace fitting uses the fact that $\mathbf{T}$ that appears linearly in the model function $\mathbf{\Gamma_{th}}\mathbf{T}$ can be optimally expressed as a linear least squares (LS) solution depending on the matrices $\{\mathbf{A}, \mathbf{C}\}$: $\mathbf{T}^{LS} = \mathbf{\Gamma_{th}}^{\dagger}\hat{\mathbf{\Gamma}}$. Therefore, this closed formula of $\mathbf{T}$ can be plugged into the original minimization problem, yielding the equivalent problem only in $\{\mathbf{A}, \mathbf{C}\}$:

$$\min_{\mathbf{A},\mathbf{C}} ||\mathbf{P}_{\perp}(\mathbf{A}, \mathbf{C})\hat{\mathbf{\Gamma}}||_F^2 = \min_{\mathbf{A},\mathbf{C}} \left[\mathrm{trace}(\mathbf{P}_{\perp}(\mathbf{A}, \mathbf{C})\hat{\mathbf{\Gamma}}\hat{\mathbf{\Gamma}}^H)\right], \tag{22}$$

where $\mathbf{P}_{\perp} = \left(\mathbf{I} - \mathbf{\Gamma_{th}}\mathbf{\Gamma_{th}}^{\dagger}\right)$ can be obtained from the QR decomposition of $\mathbf{\Gamma_{th}}$ and $\hat{\mathbf{\Gamma}}^H$ is the conjugate transpose of $\hat{\mathbf{\Gamma}}$.

$$\mathbf{\Gamma_{th}} = \mathbf{QR} = \begin{bmatrix}\mathbf{Q_1} & \mathbf{Q_2}\end{bmatrix}\begin{bmatrix}\mathbf{R_1} \\ \mathbf{0}\end{bmatrix}, \tag{23}$$

and

$$\mathbf{P}_{\perp} = \mathbf{Q_2}\mathbf{Q_2}^H, \tag{24}$$

where $\mathbf{R_1} \in \mathbb{C}^{n\times n}$, $\mathbf{Q_1} \in \mathbb{C}^{j\times n}$ and $\mathbf{Q_2} \in \mathbb{C}^{j\times(j-n)}$. This problem is solved with an iterative procedure like a form of Newton's method.

One of the advantages of the problem formulation considered herein is that any prior knowledge about the structure of $\mathbf{A}$ and $\mathbf{C}$ can be directly incorporated into the problem.

5 Prior knowledge from FEM

By using the eigenvalue decomposition of $\mathbf{A}$ and by constructing $\mathbf{C}$ in agreement with the experimental output sensors placement (see Figure 2 for construction examples), the theoretical observabilty matrix $\mathbf{\Gamma_{th}}$ can be expressed in the modal basis as

$$\mathbf{\Gamma_{th}} = \begin{bmatrix}\mathbf{C_{mod}} \\ \mathbf{C_{mod}}\mathbf{\Lambda} \\ \vdots \\ \mathbf{C_{mod}}\mathbf{\Lambda}^{i-1}\end{bmatrix}, \tag{25}$$

where $\mathbf{C_{mod}} = \mathbf{C}\mathbf{\Psi}$. If the eigenvectors are known, the subspace fitting is reduced to the identification of modal frequencies

$$\hat{\mathbf{\Lambda}} = \arg\min_{\mathbf{\Lambda}} \left[\mathrm{trace}(\mathbf{P}_{\perp}(\mathbf{\Lambda})\hat{\mathbf{\Gamma}}\hat{\mathbf{\Gamma}}^H)\right]. \tag{26}$$

Furthermore $\mathbf{A}$ and $\mathbf{C}$ can be retrieved in the modal basis, which constitutes an interesting feature of the proposed approach.

To verify the subspace fitting method and its possibilities, a numerical example is presented. An Euler Bernoulli cantilever beam made of four elements where only bending in a single plane is considered. Each node has two degrees of freedom,

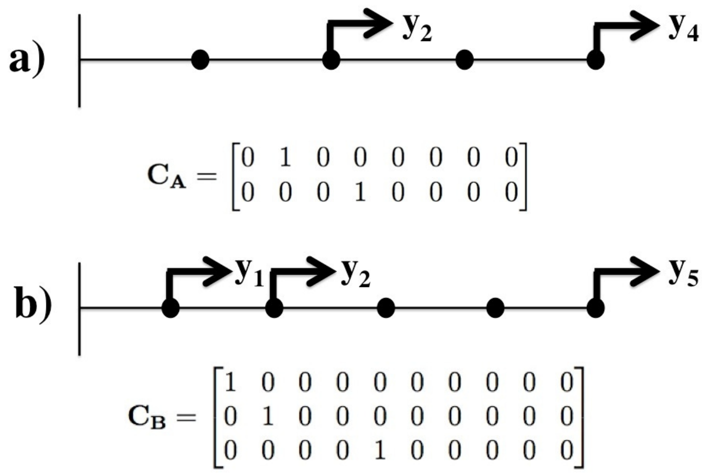


Figure 2: Examples for output matrix construction.

namely the translational displacement and bending rotation. The eigenvectors are extracted from the healthy FEM where Young's modulus (E) is 78×10^9 N/m^2 and the mass density (ρ) 7.85×10^3 kg/m^3. The length of the beam (L) is 1 m, width 0.01 m and thickness 0.01 m. The beam is clamped at one of its end. The beam is excited randomly on its free end and the displacements for each node is recorded with sampling frequency of 5 kHz. Different levels of signal to noise ratio ($\mathrm{SNR_{dB}}$), i.e amplitude ratio between signal and noise expressed using the logarithmic decibel scale, are added to the signal and the algorithm Ordinary-MOESP is applied as shown in the flow chart. First, from the identified observability matrix ($\hat{\mathbf{\Gamma}}$), the modal frequencies are identified either by shift invariance or subspace fitting. Results are summarized in Table 1.

$\mathrm{SNR_{dB}}$	**theoretical frequencies**	6.8	42.4	119.5	235.8	438.6	704.4	1116.8	1832.4
50	**shift invariance**	11.6	43.4	120.5	236.3	438.7	704.4	1116.8	1832.3
	subspace fitting	18	97.1	121.2	232.5	438.7	704.3	1116.8	1832.3
40	**shift invariance**	X	56.6	126.8	273.7	439.0	706.0	1116.8	1832.5
	subspace fitting	X	67.3	125.5	252.1	438.9	705.6	1116.8	1832.4
30	**shift invariance**	X	X	119.4	X	439.8	705.2	1116.4	1825.1
	subspace fitting	X	X	130.1	253.5	439.7	705.5	1116.5	1837.7
20	**shift invariance**	X	X	X	X	590.7	722.1	1118.3	1861.4
	subspace fitting	X	X	X	296.8	405.7	706.1	1116.3	1868.4
15	**shift invariance**	X	X	X	X	530.7	650.7	1266.6	1950.1
	subspace fitting	X	X	X	235.2	415.5	658.5	1249.5	1823.3

Table 1: Comparison between shift invariance and subspace fitting (best result in yellow, X: unidentified frequency).

The $\mathrm{SNR_{dB}}$, shown that when the noise increases, the shift invariance is less efficient. The subspace fitting method with and without prior knowledge were performed, and the results were not dissociated in this case because they produce results equals in term of accuracy due to the fact as same error threshold is used.

The fact that low frequencies are less estimated as higher frequencies is understandable by the fact that the subspace identification methods are based on statistical concepts for which the length of signals are assumed to be sufficiently large compared to the dynamics of the mechanical system. For a finite-length signal, the slow dynamics is less repeated than the fast dynamics. It explains the best identification of high frequencies.

The subspace fitting with and without prior knowledge are compared by using the CPU time taken for identify the eigenvalues for the beam with different numbers of elements (Table 2).

The main advantage of the prior knowledge based approach is that it enables the

Number of elements	2	3	4	5
CPU time with prior knowledge (sec)	0.87	1.83	2.94	9.98
CPU time without prior knowledge (sec)	6.76	33.61	110.90	960.82

Table 2: Comparison of CPU times for subspace fitting with and without prior knowledge.

CPU times to be largely decreased. In the case of the beam of five elements, the CPU time is reduced by more than $95\ \%$.

6 Characterization of damages

There are a number of approaches to the modeling of damages in beam structures. The simplest way to model a damage consists in modifying locally the stiffness of one element composing the whole FEM [8]. The damage can therefore be written:

$$\mathbf{K}_{\mathbf{damaged}}(p,s) = \mathbf{K}_{\mathbf{healthy}} - \mathbf{\Delta K}(p,s), \tag{27}$$

$$\begin{cases} \mathbf{\Psi}_{\mathbf{damaged}}(p,s) \equiv \mathbf{\Psi}_{\mathbf{healthy}}, \\ \mathbf{\Lambda}_{\mathbf{damaged}}(p,s) = \mathbf{\Lambda}_{\mathbf{healthy}} - \mathbf{\Delta \Lambda}(p,s), \end{cases} \tag{28}$$

where p is the damage position and s the percentage of stiffness reduction.

The theoretical damaged observability matrix is then written:

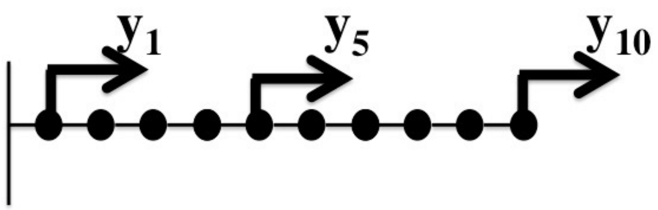
$$\mathbf{\Gamma}^{th}_{damaged} = \begin{bmatrix} \mathbf{C_{mod}} \\ \mathbf{C_{mod}} \mathbf{\Lambda}_{\mathbf{damaged}} \\ \vdots \\ \mathbf{C_{mod}} \mathbf{\Lambda}^{i-1}_{\mathbf{damaged}} \end{bmatrix} = \begin{bmatrix} \mathbf{C_{mod}} \\ \mathbf{C_{mod}} \left(\mathbf{\Lambda}_{\mathbf{healthy}} - \mathbf{\Delta \Lambda}(p,s) \right) \\ \vdots \\ \mathbf{C_{mod}} \left(\mathbf{\Lambda}_{\mathbf{healthy}} - \mathbf{\Delta \Lambda}(p,s) \right)^{i-1} \end{bmatrix}, \tag{29}$$

where $\mathbf{C_{mod}} = \mathbf{C}\boldsymbol{\Psi}_{\mathbf{healthy}}$ and $\boldsymbol{\Lambda}_{\mathbf{healthy}}$ are known for the healthy FEM and $\boldsymbol{\Delta\Lambda}$ is the problem unknown, that depends only to p and s. The subspace fitting method, for the damaged problem, is then reduced to the minimization of $\boldsymbol{\Delta\Lambda}$ through p ans s, i.e:

$$\boldsymbol{\Delta\hat{\Lambda}} = \arg\min_{p,s}\left[\mathrm{trace}(\mathbf{P}_{\perp}(\boldsymbol{\Delta\Lambda})\hat{\boldsymbol{\Gamma}}_{\mathbf{damaged}}\hat{\boldsymbol{\Gamma}}^{H}_{\mathbf{damaged}})\right]. \tag{30}$$

The method is applied to the Euler Bernoulli cantilever beam made of 10 elements. The eigenvectors and eigenvalues are extracted from the healthy FEM. Then a damage is introduced. Displacements of the beam under random excitation are recorded with an sampling frequency of $10\ kHz$ for the three nodes 1, 5, and 10 and noise is added . The damaged observability matrix ($\hat{\boldsymbol{\Gamma}}_{\mathbf{damaged}}$) is obtained with the MOESP algorithm in agreement with the output matrix constructed in Figure 3.

The residues resulting of the subspace fitting with prior knowledge are plotted for



$$\mathbf{C} = \left[\begin{array}{cccccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \mathbf{0}_{3\times 10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \end{array}\right]$$

Figure 3: Output matrix for the beam with outputs recorded at nodes 1, 5 and 10.

different experimental damage localizations (elements 2 and 8) and severities (5% and 15%).

As displayed in Figure 4, minimums are obtained in accordance with experimental damage localizations and severities.

7 Conclusion

In this paper a method in which prior knowledge from modal data of a coarse finite element model is introduced in a subspace fitting procedure. It was shown that subspace fitting improves the identification of modal frequencies compared to the shift invariance for signals corrupted by noise.

Incorporating a prior knowledge from the eigenvectors of undamaged coarse finite element model in a subspace fitting method largely decreases the CPU coasts.

The efficiency of this method is used numerically for identification, localization and estimation of damage in mechanical system and very good results are obtained.

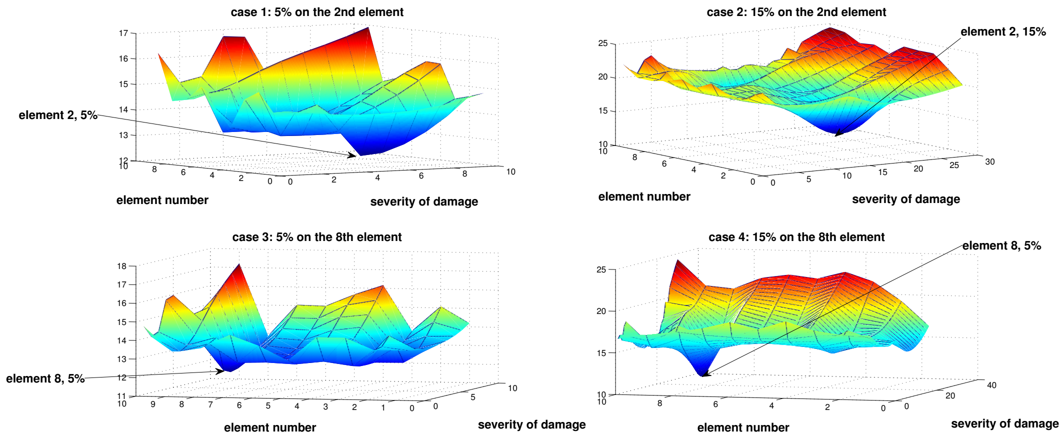


Figure 4: Residues plotted for the four experimental cases.

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