Paper 18

Proceedings of the Eleventh International Conference
on Computational Structures Technology,
B.H.V. Topping, (Editor),
Civil-Comp Press, Stirlingshire, Scotland

Structural Design of Frames Able to Prevent Element Buckling

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Abstract

Two formulations of a special multicriterion optimal design problem devoted to elastic perfectly plastic steel frame structures subjected to different combinations of static and dynamic loads are presented. In particular, two minimum volume design problem formulations are proposed: in the first one the structure is designed so as to be able to elastically behave for the assigned fixed loads, to elastically shakedown for serviceability seismic load conditions and to prevent the instantaneous collapse for suitably chosen combinations of fixed and ultimate seismic loadings; in the second one the structure must also satisfy further appropriate constraints related to element buckling. The actions that the structure must suffer are evaluated by making reference to the actual Italian seismic code and the dynamic response of the structure is performed by utilizing a modal technique. The applications are devoted to flexural steel frames, the different designs obtained are compared and the sensitivity of the structural response has been investigated on the grounds of the determination and interpretation of the Bree diagrams of the obtained optimal structures.

Keywords: multicriterion design, steel frames, dynamic loads, buckling.

1 Introduction

As it is well known, the formulation of a structural optimization problem requires, at first, the definition of a suitable objective function which involves appropriate relevant structural parameters (design variables) concerning the geometry and/or the elastic properties and/or the topology of the structure, etc. Yet, introduced the appropriate state equations, it is then usually necessary to adopt some suitable admissible ranges for the design variables as well as define appropriate admissibility criteria, written in terms of behavioural constraints for the structure which very often express mechanical conditions and which usually identify with different structural limit conditions.

The structural optimization problems are very often formulated as search for the minimum structural weight which substantially provides a quantity proportional with the minimum cost which must be suffered for the structure construction.

On the other side, the choice of the optimal design admissibility conditions is usually very complex and specific of the particular optimization problem which must be formulated. These conditions are substantially represented by inequalities identifying one or more limit behaviour related to the material or to the structure. If reference is made to ductile materials and, as a consequence, to elastic plastic structures, the (mechanical) limit conditions can characterize the limit of the purely elastic behaviour, the limit of the elastic shakedown behaviour and the limit of the plastic shakedown behaviour and/or of the incremental collapse beyond which the structure suffers an instantaneous collapse.

In addition, further admissibility conditions can be requested, yet in terms of mechanical behaviour. In general, these conditions don’t depend on the material resistance characteristic but on the elastic properties, on the particular structure geometry, on the load condition and on the structural response in terms of displacements. These conditions are related with possible P-Delta effects and/or with possible buckling of some structural element and they represent very dangerous limit states for the structure.

In the last decades several efforts have been devoted to the study of the optimal design of structures subjected to quasi-static loads and the fundamental results are reported in several books (see, e.g., [1-10]) where many formulations have been proposed characterized by different objective functions, different admissibility conditions as well as different approaches.

Further and specific improvements have been proposed for the elastic optimal structural design (see, e.g. [11]), for the elastic shakedown optimal design (see, e.g. [12-16]) and for the standard limit design (see, e.g. [17, 18]).

Each one of these criteria takes into account just the corresponding structural limit state, disregarding the observance of suitable safety factors for the other possible limit states. As a consequence, other formulations, the so-called multicriterion optimal design formulations, have been proposed (see, e.g., [19-23]).

Furthermore, for load conditions above the elastic shakedown limit, an alternating plasticity behaviour (plastic shakedown) is certainly preferable with respect to a ratchetting one, and (see, e.g. [24-25]) a first formulation of the so-called plastic shakedown (no ratchet) optimal design of FE structures subjected to a combination of fixed and cyclic loads has been proposed. Improved formulations and further applications (see, e.g., [26, 27]) have been also proposed.

More recently, some further formulations have been proposed in which the dynamic behaviour of the structure is taken into account and where the results obtained by a rigorous application of the Italian code are critically examined and several contributions are provided aimed at the improving of the optimal design (see, e.g. [28-29]).

Anyway, whatever the special formulation is utilized, substantially depending on the special limiting criterion imposed on the structure behaviour, it is very useful to know if the optimal structure, at the prescribed limit state, fulfils special limits on its functionality (see, e.g., [30]). Among such bounds, and in particular making reference to frame structures, an effective limit is related to the buckling of the elements (see, e.g., [31, 33]).

Aim of the present paper is to propose two formulations of a special multicriterion optimal (minimum volume) design problem devoted to elastic perfectly plastic frame structures subjected to a combination of fixed loads and seismic actions: the first formulation is devoted to the search for an optimal structure which behaves in a purely elastic manner for the assigned fixed loads, does not violate the elastic shakedown limit in serviceability seismic conditions and prevents the instantaneous collapse for ultimate seismic load conditions; the second one is devoted to the search for an optimal structure which, besides the already required features, must also prevent the risk of element buckling for all the above considered load combinations. The second formulation is obtained by the first one introducing appropriate further mechanical constraints on the element buckling.

The numerical applications are devoted to the search for the minimum volume designs (obtained by the two proposed formulations) of a flexural four floors elastic perfectly plastic steel frame. The behavioural features of the obtained optimal structures for different load conditions are compared and emphasized through the determination and interpretation of the related Bree diagrams.

2 Fundamentals and structural model

As widely described in the previous section, the fundamental aim of the present paper is the formulation of two appropriate multicriterion minimum volume design problems for elastic perfectly plastic steel frame structures subjected to different combinations of fixed and dynamic (seismic) loadings properly combined together and each amplified by suitably selected parameters. As previously specified the two proposed formulations different because the second (improved) one impose that the structure be even safe against element (pillar) buckling. In order to appropriately perform the above referenced formulations, some fundamentals must be introduced mainly regarding the definition of some appropriate model both for the frame structure and for the acting loads.

As known, the classical formulation of the static linear elastic analysis problem for frames constituted by beam type elements described by the Navier kinematical model is given as follows:

$$\boldsymbol{d} = \boldsymbol{C}\boldsymbol{u} \tag{1a}$$

$$\boldsymbol{Q} = \boldsymbol{D}\boldsymbol{d} + \boldsymbol{Q}^* \tag{1b}$$

$$\tilde{\boldsymbol{C}}\boldsymbol{Q} = \boldsymbol{F} \tag{1c}$$

where $\boldsymbol{d}$ is the element nodal displacement vector, $\boldsymbol{C}$ is the compatibility matrix, $\boldsymbol{u}$ is the frame nodal displacement vector, $\boldsymbol{Q}$ is the generalized stress vector evaluated at the element nodes, $\boldsymbol{D}$ is the frame internal stiffness matrix, $\boldsymbol{Q}^*$ is the perfectly clamped element generalized stress vector and $\boldsymbol{F}$ is the frame nodal force vector. The solution to problem (1) is given by:

$$\boldsymbol{u} = \boldsymbol{K}^{-1}\boldsymbol{F}^{*} \tag{2a}$$

$$\boldsymbol{Q} = \boldsymbol{DCu} + \boldsymbol{Q}^{*} = \boldsymbol{DCK}^{-1}\boldsymbol{F}^{*} + \boldsymbol{Q}^{*} \tag{2b}$$

in terms of displacements and generalized stresses, respectively, with $\boldsymbol{K} = \tilde{\boldsymbol{C}}\boldsymbol{DC}$ frame external stiffness matrix and $\boldsymbol{F}^{*} = \boldsymbol{F} - \tilde{\boldsymbol{C}}\boldsymbol{Q}^{*}$ is the equivalent (in terms of structure node displacement response) nodal force vector, where the over tilde means the transpose of the relevant quantity.

According with the guidelines of the greater part of international codes, in particular with the Italian one, the design of the relevant structure must be performed taking into account a fixed action, mainly related with the gravitational loads and a dynamic perfect cyclic load related to seismic actions, suitably combined. In the framework of the present paper the wind actions are not considered cause, usually, their effects are lower than the seismic ones, except for structure characterized by special geometry.

Making reference to the seismic actions, let us consider the relevant frame as a shear plane frame just subjected to an horizontal ground acceleration $a_g(t)$. It is modeled as a Multi-Degree-Of-Freedom (MDOF) structure, such that the total number of degrees of freedom is equal to the number of floors n_f.

The dynamic equilibrium equations can be written in the following form:

$$\boldsymbol{M}\,\ddot{\boldsymbol{s}}(t) + \boldsymbol{A}\,\dot{\boldsymbol{s}}(t) + \boldsymbol{K}_s\,\boldsymbol{s}(t) = \boldsymbol{f}(t) \tag{3}$$

with $\boldsymbol{f}(t) = -\boldsymbol{M}\boldsymbol{\tau}\,a_g(t)$, being $\boldsymbol{\tau}$ the ($n_f \times 1$) influence vector. $\boldsymbol{s}$ represents the displacement vector related to the structure dynamic degrees of freedom and the following initial conditions $\boldsymbol{s}(0) = \boldsymbol{0}$, $\dot{\boldsymbol{s}}(0) = \boldsymbol{0}$ hold.

In equation (3) $\boldsymbol{M}$ and $\boldsymbol{A}$ are the mass and damping matrices (with dimensions $n_f \times n_f$), $\boldsymbol{K}_s = \tilde{\boldsymbol{E}}\boldsymbol{KE}$ is the dynamic stiffness matrix of order n_f related just to the horizontal floor displacements, being $\boldsymbol{E}$ an appropriate condensation compatibility matrix. $\boldsymbol{M}$, $\boldsymbol{A}$ and $\boldsymbol{K}_s$ are assumed to be positive matrices. Furthermore, $\dot{\boldsymbol{s}}(t)$ and $\ddot{\boldsymbol{s}}(t)$ are the velocity and the acceleration ($n_f \times 1$) vectors of the system, respectively, and the over dot means time derivative of the relevant quantity.

As it is usual, the dynamic characteristics of the structural behaviour are identified in terms of natural frequencies as well as damping coefficients. In this framework, as usual, the following coordinate transformation is adopted:

$$\boldsymbol{s}(t) = \boldsymbol{\Phi}\boldsymbol{z}(t) \tag{4}$$

being $\boldsymbol{z}(t)$ the modal displacement vector and $\boldsymbol{\Phi}$ the so-called modal matrix of order ($n_f \times n_f$), normalized with respect to the mass matrix and whose columns are the eigenvectors of the undamped structure, given by the solution to the following eigenproblem:

$$\boldsymbol{K}^{-1}\boldsymbol{M}\,\boldsymbol{\Phi} = \boldsymbol{\Phi}\,\boldsymbol{\Omega}^{-2} \tag{5a}$$

$$\tilde{\boldsymbol{\Phi}}\,\boldsymbol{M}\,\boldsymbol{\Phi} = \boldsymbol{I}_{n_f} \tag{5b}$$

$$\tilde{\boldsymbol{\Phi}}\,\boldsymbol{K}\,\boldsymbol{\Phi} = \boldsymbol{\Omega}^{2} \tag{5c}$$

In equations (5a,c), besides the already known symbols, $\boldsymbol{I}_{n_f}$ represents the $\left(n_f \times n_f\right)$ identity matrix while $\boldsymbol{\Omega}^2$ is a diagonal matrix listing the square of the natural frequencies of the structure.

Once the modal matrix $\boldsymbol{\Phi}$ has been determined, the structure can be defined as a classically-damped one if $\tilde{\boldsymbol{\Phi}}\,\boldsymbol{A}\boldsymbol{\Phi} = \boldsymbol{\Xi}$ is a diagonal matrix and such that the element Ξ_{jj} is equal to $2\zeta_j\,\omega_j$, being ω_j and ζ_j the j^{th} natural frequency and the j^{th} damping coefficient, respectively.

According to the Italian code the study is performed taking into account all structural modes and assuming a constant damping coefficient equal to 0.05.

Making reference to the response spectrum $S_d\left(T\right)$ defined in the relevant code and once the natural frequencies and the modal matrix are known, the displacement vector due to the j^{th} mode can be determined as follows:

$$\boldsymbol{s}_j = \boldsymbol{\Phi}_j\,\frac{\boldsymbol{\Phi}_j^T\boldsymbol{M}\boldsymbol{\tau}\,S_d\left(T_j\right)}{\omega_j^2} \tag{6}$$

According to the above referred guidelines the displacements $\boldsymbol{s}$ and the related elastic generalized stresses $\boldsymbol{Q}$ are combined in a full quadratic way following the equation:

$$E_\ell = \sqrt{\sum_k \sum_j \rho_{jk}\,E_{j\ell}\,E_{k\ell}} \tag{7}$$

being E_ℓ the ℓ^{th} component of the combined effect of the relevant quantity, $E_{j\ell}, E_{k\ell}$ the ℓ^{th} component of the effect due to j^{th} and k^{th} modes, respectively, and ρ_{ij} the correlation coefficients between j^{th} and k^{th} modes expressed by the equation:

$$\rho_{jk} = \frac{8\zeta^2\,\beta_{jk}^{3/2}}{\left(1+\beta_{jk}\right)\left[\left(1-\beta_{jk}\right)^2 + 4\xi^2\,\beta_{jk}\right]} \tag{8}$$

in which $\beta_{jk} = T_k/T_j$ being T_j, T_k are the periods of the j^{th} and k^{th} mode.

Always according with the guidelines of the Italian code, seismic loadings have to be evaluate for two different conditions: the serviceability conditions, representing the limit for which the full usability of the building must be ensured, and the exceptional one in which the structure finds itself in an impending instantaneous collapse condition. Clearly, the intensity of seismic loadings is very different between the above referenced conditions and it strictly depends on the up-crossing probability of selected intensity levels during the lifetime of the structure.

Therefore, for the aim of the present paper and taking into account the referenced Italian code, we now assume that the actions are represented by three different appropriate combinations of the above referred loads each of which related to different limit conditions. The first combination is characterized by the presence of the full fixed loads $\boldsymbol{F}_0^*$, the second combination is defined as the superimposition of appropriate reduced fixed loads $\boldsymbol{F}_{0e}^*$ and (reduced) seismic actions related to the response spectrum S_d^S (serviceability conditions), function of a suitably selected up-crossing probability in the lifetime of the structure; the third combination is characterized by the superimposition of the fixed loads $\boldsymbol{F}_{0e}^*$ and seismic actions related to the response spectrum S_d^I (ultimate conditions), function of a different (lower) suitably selected up-crossing probability in the lifetime of the structure.

Obviously, the structure must be able of suffering the above described load combinations according to different limit conditions; in particular, it must possess a purely elastic behaviour when subjected to the first load condition, it must respond in an elastic manner (elastic shakedown) when subjected to the second load combination, it must prevent the instantaneous collapse when subjected to the third load combination.

In the above defined combinations, $\boldsymbol{F}_{0e}^*$ is a special combination of gravitational loads as prescribed by the referenced code, S_d^S and S_d^I are the response spectra related to serviceability and instantaneous collapse conditions, respectively.

Clearly, since the design problem under investigation is a minimum volume search one, the structural geometry is not definitely known a priori and, therefore, let the typical v^{th} element geometry be fully described by the m components of the vector $\boldsymbol{t}_v \left(v = 1, 2, ..., n \right)$ so that $\tilde{\boldsymbol{t}} = \left[\tilde{\boldsymbol{t}}_1, \tilde{\boldsymbol{t}}_2, ..., \tilde{\boldsymbol{t}}_v, ..., \tilde{\boldsymbol{t}}_n \right]$ represents the $n \times m$ supervector collecting all the design variables.

3 The optimal design problem formulation

The optimal design formulation for a given structure strongly depends on two elements: the selected objective function and the imposed limit states characterizing the required optimal structural behaviour. It is important to emphasize that usually the optimal structure is regarded as the lowest weight one the latter being also the lowest cost one.

Therefore, let us consider an elastic perfectly plastic frame structure as above described and, according to the Italian code and to the assumed loading model, let it be subjected to fixed mechanical loads and perfect cyclic dynamic (seismic) loads. The multicriterion (minimum volume) design problem formulation for the structure without any constraint on the element buckling, where suitable constraints are imposed on the purely elastic behaviour, on the elastic shakedown behaviour, and on the instantaneous collapse, can be written as follows:

$$\min_{\left(\boldsymbol{t},\boldsymbol{u}_0,\boldsymbol{u}_{0e},\boldsymbol{s}^S_{jce},\boldsymbol{s}^I_{jce},\boldsymbol{u}^S_{jce},\boldsymbol{u}^I_{jce},\boldsymbol{Y}^S_0,\boldsymbol{Y}^I_{0ie}\right)} V \tag{9a}$$

$$\boldsymbol{t} - \bar{\boldsymbol{t}} \geq \boldsymbol{0} \tag{9b}$$

$$\boldsymbol{H}\,\boldsymbol{t} - \bar{\boldsymbol{h}} \geq \boldsymbol{0} \tag{9c}$$

$$\boldsymbol{Q}_0 = \boldsymbol{DCu}_0 + \boldsymbol{Q}^*_0, \quad \boldsymbol{Ku}_0 - \boldsymbol{F}^*_0 = \boldsymbol{0} \tag{9d}$$

$$\boldsymbol{Q}_{0e} = \boldsymbol{DCu}_{0e} + \boldsymbol{Q}^*_{0e}, \quad \boldsymbol{Ku}_{0e} - \boldsymbol{F}^*_{0e} = \boldsymbol{0} \tag{9e}$$

$$\boldsymbol{s}^S_{jce} = \boldsymbol{\Phi}_j \frac{\tilde{\boldsymbol{\Phi}}_j \boldsymbol{M\tau}\, S^S_d\left(T_j\right)}{\omega_j^2}, \quad \boldsymbol{Q}^S_{jce} = \boldsymbol{DCu}^S_{jce}, \quad Q^S_{ce\ell} = \sqrt{\sum_j \sum_k \rho_{kj}\, Q^S_{kce\ell} Q^S_{jce\ell}} \tag{9f}$$

$$\boldsymbol{s}^I_{jce} = \boldsymbol{\Phi}_j \frac{\tilde{\boldsymbol{\Phi}}_j \boldsymbol{M\tau}\, S^I_d\left(T_j\right)}{\omega_j^2}, \quad \boldsymbol{Q}^I_{jce} = \boldsymbol{DCu}^I_{jce}, \quad Q^I_{ce\ell} = \sqrt{\sum_j \sum_k \rho_{kj}\, Q^I_{kce\ell} Q^I_{jce\ell}} \tag{9g}$$

$$\boldsymbol{\varphi}^E \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p \boldsymbol{Q}_0 - \boldsymbol{R} \leq \boldsymbol{0}, \tag{9h}$$

$$\boldsymbol{\varphi}^S_{ie} \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p \boldsymbol{Q}_{0e} + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p \boldsymbol{Q}^S_{ce} - \boldsymbol{SY}^S_0 - \boldsymbol{R} \leq \boldsymbol{0}, \quad \boldsymbol{Y}^S_0 \geq \boldsymbol{0} \tag{9i}$$

$$\boldsymbol{\varphi}^I_{ie} \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p \boldsymbol{Q}_{0e} + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p \boldsymbol{Q}^I_{ce} - \boldsymbol{SY}^I_{0ie} - \boldsymbol{R} \leq \boldsymbol{0}, \quad \boldsymbol{Y}^I_{0ie} \geq \boldsymbol{0} \tag{9j}$$

where equations (9i,j) hold for $i = 1,2$ while $j = 1,2,.....,n_{sm}$, being n_{sm} the number of structural modes and $\ell = 1,2,.....,6 \cdot n$.

In equations (9b,c) $\boldsymbol{t}$ is the design variable vector while $\bar{\boldsymbol{t}}$ represents the vector collecting the imposed limit values for $\boldsymbol{t}$, $\boldsymbol{H}$ is a suitably defined technological constraint matrix with $\bar{\boldsymbol{h}}$ is a suitably chosen technological vector.

In equations (9d-g) $\boldsymbol{u}_0$ and $\boldsymbol{Q}_0$, $\boldsymbol{u}_{0e}$ and $\boldsymbol{Q}_{0e}$, $\boldsymbol{u}^S_{jce} = \boldsymbol{Es}^S_{jce}$ and $\boldsymbol{Q}^S_{jce}$, $\boldsymbol{u}^I_{jce} = \boldsymbol{Es}^I_{jce}$ and $\boldsymbol{Q}^I_{jce}$ are the purely elastic response to the assigned full fixed loads, to the appropriately reduced fixed loads to join with seismic actions, to the reduced dynamic loads related to the j^{th} structural mode, to the full dynamic loads related to the j^{th} structural mode, respectively, in terms of structure node displacements and element node generalized stresses.

Finally, in equations (9h,i,j) $\boldsymbol{\varphi}^E$, $\boldsymbol{\varphi}^S_{ie}$ and $\boldsymbol{\varphi}^I_{ie}$ are the plastic potential vectors related to the purely elastic limit (apex *E*), to the elastic shakedown limit (apex *S*) and to the instantaneous collapse limit (apex *I*), respectively, while $\boldsymbol{Y}^S_0$ and $\boldsymbol{Y}^I_{0ie}$ are the fictitious plastic activation intensity vectors related to the elastic shakedown limit and to the impending instantaneous collapse, respectively. In addition, $\tilde{\boldsymbol{N}}$ is the matrix of the external normals to the elastic domain whose boundary is assumed as constituted by a discrete number of sides, $\tilde{\boldsymbol{G}}_p$ is an appropriate equilibrium matrix which applied to element nodal generalized stresses provides the generalized stresses acting upon the plastic nodes of the elements, $-\boldsymbol{S}$ is a time independent symmetric structural matrix which transforms the plastic activation intensities into the plastic potentials and $\boldsymbol{R}$ is the relevant plastic resistance vector.

As already stated, very often and especially for structures constituted by slender elements, as it is to expect for optimal frames, it is advisable to suitably take into account the risk of buckling. In the present case, the same problem (9) can be suitably improved in order to take into account the buckling effect on the pillars. First of all, it is necessary to obtain the relevant critical load for the special case under examination. Under the hypothesis of shear type behaviour for the frame (Fig. 1a), the critical load of the typical pillar can be obtained by referring to the following scheme (see Fig. 1b):

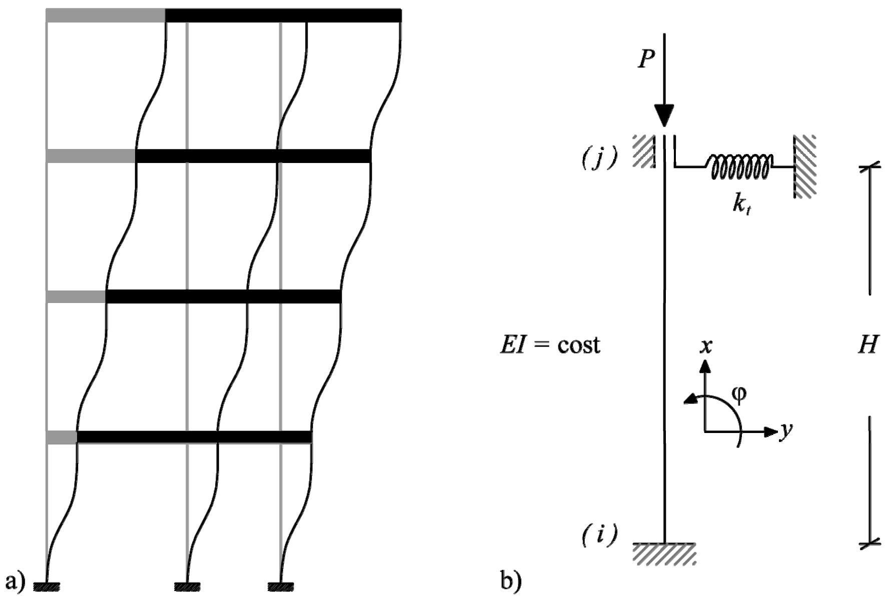

Figure 1: a) typical shear type plane frame;
b) chosen scheme for the evaluation of the critical load.

in which it is assumed that the j^{th} extreme of the pillar can suffer a transversal elastic displacement v, k_t is the spring stiffness computed as the total floor shear stiffness, E is the material Young's modulus, and I_{min} is the minimum moment of inertia of the relevant cross section.

The total potential energy functional of the typical pillar sketched in Fig. 1 can be written as follows:

$$\boldsymbol{V}^{\bullet} = \frac{1}{2}\int_0^H \left(EI_{min} v''^2 - P v'^2 \right) dx \,, \tag{10}$$

By imposing the stationariness of the above defined functional, the following differential equation is obtained:

$$EI_{min} v'''' + P v'' = 0 \,. \tag{11}$$

The general integrals of (11), here skipped for the sake of brevity, can be particularized by imposing the following boundary conditions:

$$v(0)=0;\ v'(0)=0;\ v'(H)=0\,, \tag{12a}$$

$$v'''(H)+\alpha^2 v'(H)-\frac{k_t}{EI_{min}}v(H)=0\,, \tag{12b}$$

in which $\alpha^2 = P/EI_{min}$, as usual.

The resulting equation system can provide a non trivial solution (in terms of integration constants) only if the related coefficient matrix is a singular one. Therefore, imposing the cited singularity and neglecting the trivial solution $\alpha = 0$ the following relation holds:

$$\cot\beta - \sec\beta = \frac{2k_t}{\dfrac{EI_{min}}{H^3}\beta^3 - k_t\beta}\,, \tag{13}$$

where $\beta = \alpha H$ has been defined.

Equation (13) can not be solved in a closed form so a parametric approach has been performed in order to solve the following equation:

$$\cot\beta - \sec\beta = \frac{2\gamma}{\beta^3 - \gamma\beta}\,, \tag{14}$$

obtained from equation (13) multiplying both numerator and denominator of the right hand side by H^3/EI_{min} and putting $\gamma = k_t H^3/EI_{min}$.

It has been observed that for values of γ greater than 20, usual for the case of frames, the zero of equation (13) is given for $\beta \cong 1.15$. It follows that the critical load for the pillar plotted in Fig. 1 is given by:

$$P_{cr} = 1.32\frac{EI_{min}}{H^2}\,. \tag{15}$$

As a consequence the improvement of problem (9) can be simply obtained by adding to the already imposed constraints the following ones:

$$\boldsymbol{\varphi}_{cr}^{E} \equiv \boldsymbol{\mathcal{N}}_0 - \frac{\boldsymbol{P}_{cr}}{\eta} \le \mathbf{0}\,, \tag{16a}$$

$$\boldsymbol{\varphi}_{icr}^{S} \equiv \boldsymbol{\mathcal{N}}_{0e} + (-1)^i\,\boldsymbol{\mathcal{N}}_{ce}^{S} - \frac{\boldsymbol{P}_{cr}}{\eta} \le \mathbf{0}\,, \tag{16b}$$

$$\boldsymbol{\varphi}_{icr}^{I} \equiv \boldsymbol{\mathcal{N}}_{0e} + (-1)^i\,\boldsymbol{\mathcal{N}}_{ce}^{I} - \frac{\boldsymbol{P}_{cr}}{\eta} \le \mathbf{0}\,. \tag{16c}$$

where equations (16b,c) hold for $i = 1,2$, $\boldsymbol{\mathcal{N}}_0$, $\boldsymbol{\mathcal{N}}_{0e}$, $\boldsymbol{\mathcal{N}}_{ce}^{S}$ and $\boldsymbol{\mathcal{N}}_{ce}^{I}$ are the axial force vectors on the pillars due to the full fixed loads (extracted by $\boldsymbol{Q}_0$), the reduced fixed loads (extracted by $\boldsymbol{Q}_{0e}$), the reduced seismic actions (extracted by $\boldsymbol{Q}_{ce}^{S}$) and the full seismic actions (extracted by $\boldsymbol{Q}_{ce}^{I}$), respectively, $\boldsymbol{P}_{cr}$ is the vector collecting the critical loads of all the pillars, while η is a suitably chosen safety factor.

Furthermore, for the sake of generality, in order to take into account the buckling effect on the cross bracing elements, if present, the following constraint can be introduced:

$$\pi^2 E\hat{\boldsymbol{L}}\,\hat{\boldsymbol{I}}_{min} - \eta_{cb}\hat{\boldsymbol{A}}\sigma_y \geq \boldsymbol{0} \tag{17}$$

where, besides the already defined symbols, $\hat{\boldsymbol{L}}$ is a diagonal square matrix collecting terms as $1/\ell_r^2$, $r \in I(n_{cb})$, being ℓ_r the length of the r^{th} cross bracing element and n_{cb} their total number, $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{I}}_{min}$ are the cross-section area and the related minimum moments of inertia vectors of the cross bracing elements, σ_y the material yield stress and η_{cb} a suitable chosen safety factor.

It is worth noticing that the meaning of equations (16) and (17) is substantially different; actually, constraints (16) admit the presence of slender pillars but, however, ensure that the relevant critical load is never reached, while constraints (17) impose that no cross bracing can be slender. Such an approach is certainly acceptable, actually the cross bracing elements of an optimal frame are often utilized as receptors of plastic deformations and as elements able to dissipate a great part of plastic energy.

4 Numerical applications

The optimal designs of plane steel frames have been obtained referring to the formulations previously proposed. At first, the multicriterion design problem (9) has been solved for the four floor frame plotted in Fig. 2a constituted by rectangular box cross section elements (Fig. 2b) with $b = 200$ mm and $h = 400$ mm, whose the constant thickness t is assumed as design variable. Furthermore, $L_1 = 600$ cm, $L_2 = 400$ cm and $H = 600$ cm, Young modulus $E = 21\ \mathrm{MN/cm^2}$, yield stress $\sigma_y = 23.5\ \mathrm{kN/cm^2}$.

Two rigid perfectly plastic hinges are located at the extremes of all the elements, considered to be purely elastic (Fig. 2c), and an additional hinge is located in the middle point of the longer beams. The interaction between bending moment *M* and axial force *N* has been taken into account. In Fig. 2d the dimensionless rigid plastic domain of the typical plastic hinge is plotted in the plane (N/N_y, M/M_y), being N_y and M_y the yield generalized stress corresponding to N and M, respectively.

The structure is subjected to a fixed uniformly distributed vertical load on the beams, $q_0 = 50$ kN / m and to seismic actions. We assume that the seismic masses are equal for each floor, $m = 40.77\ \mathrm{kN \cdot sec^2 / m}$, and located in the intermediate node at each floor, (Fig. 2a). The value assumed for the seismic masses depends on the remark that during the earthquake not all the gravitational loads are considered as acting on the structure. The selected response spectra for serviceability conditions (up-crossing probability in the lifetime 81%) and instantaneous collapse (up-crossing probability in the lifetime 5%) are those corresponding to Palermo, with a soil type B, life time 100 years and class IV.

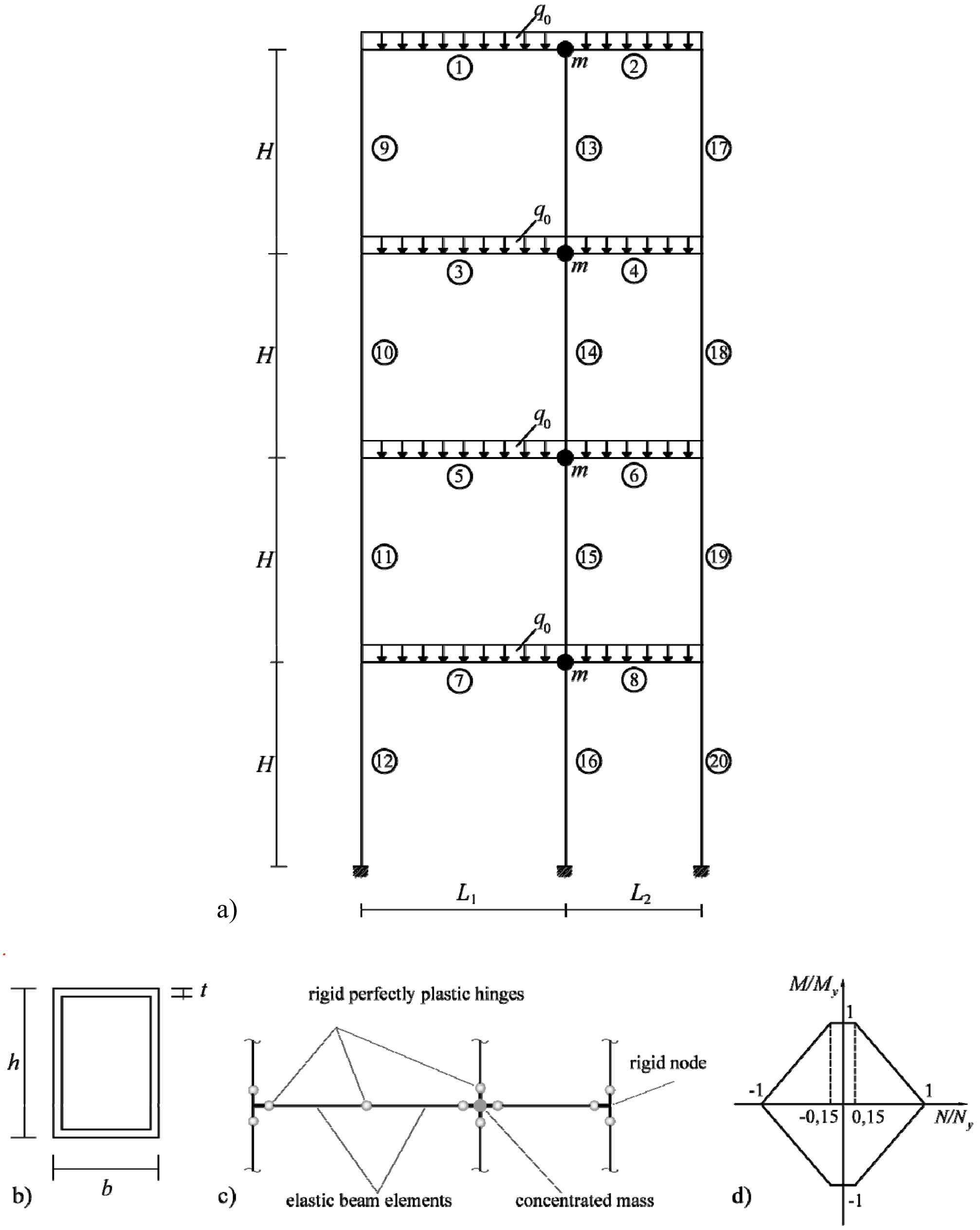

Figure 2: Four floor flexural steel frame: a) geometry and load conditions;
b) typical rectangular box cross section of the elements;
c) structural scheme of the relevant beams;
d) rigid plastic domain of the typical plastic hinge.

The optimal multicriterion design has been computed solving problem (9), assuming $\boldsymbol{F}_{0e}^{*} = 0.8\boldsymbol{F}_{0}^{*}$ deduced by a suitably reduced distribution q_{0e} of q_0, always according with the previously referenced Italian code.

The obtained results expressed in terms of element thicknesses are reported in Table 1 and the relevant optimal volume has been deduced $V = 1.641\ \mathrm{m}^3$.

El.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
s	4.38	2.81	10.91	10.66	15.20	20.17	18.05	36.90	4.38	6.15
El.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
s	10.32	8.19	6.85	14.94	20.45	33.83	2.84	7.56	12.60	19.71

Table 1: Optimal element thicknesses (mm) for the frame of Fig. 2.

In order to investigate the features of the obtained design the relevant Bree diagram has been determined and plotted in the plane ξ_0, ξ_c (see Fig. 3), where ξ_0 and ξ_c are the multipliers of the fixed and cyclic load, respectively.

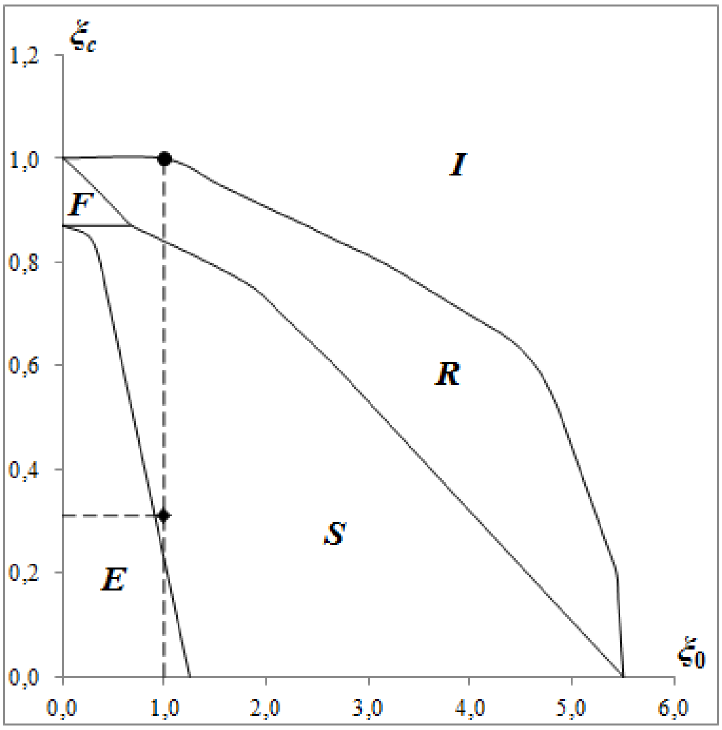

Figure 3: Bree diagram of the optimal frame obtained by solving problem (9).

As it is easy to observe, even if, as expected, the optimal structure does not violate in any case the imposed safety limit behaviours for the prescribed load combinations, it exhibits a dangerous condition of ratchetting even for cyclic load multipliers lower than the prescribed one; by analysing the referenced Bree diagram it suffices to impose $\xi_c \geq 0.83$ for determining a very dangerous incremental collapse condition. In other papers (see, e.g. [29,34]) the same authors faced the cited problem proposing different approaches in order to improve the safety structural behaviour; in the present study such a problem is disregarded focusing the attention just to the problem of the element buckling.

As already stated, for the chosen plane frame it is necessary to suitably take into account the risk of buckling. In the present case, the same problem (9) has been utilized but improved by adding constraints (16a-c), with $\boldsymbol{P}_{cr}$ deduced from equation (15) and setting $\eta = 1.25$.

The obtained results expressed in terms of element thicknesses are reported in Table 2 and the relevant optimal volume has been deduced $V = 1.712\ \mathrm{m}^3$.

El.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
s	4.39	2.79	10.33	11.01	16.90	20.46	26.50	24.71	4.39	6.27
El.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
s	10.32	15.74	7.28	14.22	23.40	37.96	2.80	8.23	11.51	14.71

Table 2: Optimal element thicknesses (mm) for the frame accounting for buckling.

As usual the features of the obtained new design can be interpreted by studying the relevant Bree diagram plotted in Fig. 4.

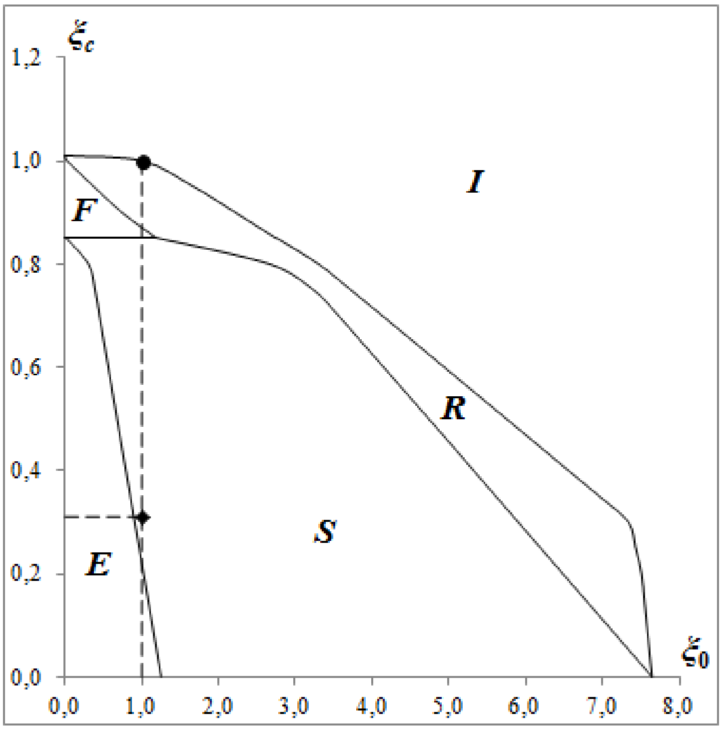

Figure 4: Bree diagram of the optimal frame safe against buckling.

As it was expected, again the optimal structure does not violate in any case the imposed safety limit behaviours for the prescribed load combinations, but yet it exhibits a dangerous condition of ratchetting even for cyclic load multipliers lower than the prescribed one; by analysing the referenced Bree diagram it suffices to impose $\xi_c \geq 0.86$ for determining a very dangerous incremental collapse condition. In this case however it guarantees that all the elements prevent the phenomenon of buckling.

5 Conclusions

This paper has been devoted to the optimal design of plane frames constituted by elastic perfectly plastic material and subjected to suitably defined load combinations characterized by the simultaneous presence of fixed and dynamic (seismic) actions. The optimal design problem has been formulated as the search for the minimum volume structure on the grounds of a statical approach and three different resistance limits have been simultaneously considered: the purely elastic limit, the elastic shakedown limit and the instantaneous collapse limit. In the proposed formulation, in order to define the loading combinations, reference has been made to the most recent Italian code related to the structural analysis and design; in particular, three different load combinations have been taken into account: the basic load combination has been defined as constituted by the solely assigned fixed loads, the serviceability load condition has been defined as the combination of suitably reduced fixed loads and reduced seismic actions, the ultimate limit load condition has been defined as the combination of suitably reduced fixed loads and full dynamic (seismic) actions.

Two different formulations of the minimum volume design have been proposed: the first one is devoted to the optimal design of the structure with constraints on the purely elastic behaviour related to the basic load condition, on the elastic shakedown behaviour related to serviceability conditions and on the instantaneous collapse related to suitable combination of fixed loads and dynamic actions; the second one is devoted to the optimal design of the structure under the same limits as before described but introducing new appropriate constraints aimed to prevent to the risk of element buckling. These last constraints consist in appropriate limits imposed on the axial forces suffered by the pillars and the resistance vector elements are suitable rates of the appropriate critical loads of the relevant elements.

A four floor plane steel frame has been investigated and the relevant minimum volume structure has been obtained solving both the described optimization problems. The features of the optimal structures obtained have been deduced by the interpretation of the related Bree diagrams. It has been found that the structures obtained by the improved optimal design formulation guarantee a decisively more safe behaviour with respect to all the imposed limit states. Therefore, the results obtained are encouraging and, furthermore, they show that the improved design is characterised by just a very modest cost increment (about 5%) with respect to the cited safety improvement.

References

[1] J. Atkočiūnas, Optimal shakedown design of elastic-plastic structures, VGTU Publishing House Technika, Vilnius, 2011

[2] N.V., Banichuk, Introduction to optimization of structures. New York: Springer-Verlag, 1990.

[3] P. Brousse, Optimization in mechanics: problems and methods. The Nederlands, Amsterdam: Elsevier Science Publishers, 1988.

[4] A. Gajewski; M. Zyezkowski, Optimal structural design under stability constraints. Kluwer, Dordrecht, 1988.

[5] R.H. Gallagher; O.C., Zienkiewicz, Optimum structural design. London: John Wiley & Sons, 1973.

[6] E.J. Haug, K.K. Choi, V. Komkov, Design Sensitivity Analysis of Structural Systems, Academic Press, New York, 1986.

[7] C.E. Massonet and M. Save, Plastic analysis and design, Blaisdell Publishing Company, New York, 1965.

[8] G.I.N. Rozvany, Optimal design of flexural systems. Oxford: Pergamon Press, 1976.

[9] M.A. Save, W. Prager, Structural optimization. New York: Plenum Press, 1985.

[10] W. Stadler, Multicriteria Optimization in Engineering and in the Sciences, Plenum Press, New York, 1988.

[11] C. Cinquini, G. Guerlement, D. Lamblin, "Finite element iterative methods for optimal elastic design of circular plates", Comp. & Struct. 12(1), 85-92, 1980.

[12] M.Z. Cohn, S.R. Parimi, "Optimal design of plastic structures for fixed and shakedown loadings", J. Appl. Mech. 595-599, 1973.

[13] F. Giambanco, L. Palizzolo, L. Cirone, "Computational methods for optimal shakedown design of FE structures", Structural Optimization 15(3/4), 284-295, 1998.

[14] F. Giambanco, L. Palizzolo, C. Polizzotto, "Optimal shakedown design of beam structures", Struct. Optimization 8, 156-167, 1994.

[15] F. Giambanco, L. Palizzolo, C. Polizzotto, "Optimal shakedown design of circular plates", Journal of Engineering Mechanics 120(12), 2535-2555, 1994.

[16] J. A. König, "On optimum shakedown design". In: Sawczuk, A.; Mroz, Z. (ed.) Optimization in structure design, pp. 405-414, Berlin: Springer-Verlag, 1975.

[17] G. Maier, R. Srinivasan, M. Save, "Optimal plastic design", J. Structural Mechanics, 4, 349-378, 1976.

[18] G. Maier, A. Zavelani Rossi, D. Benedetti, "A finite element approach to optimal design of plastic structures in plane stress", Int. J. Num. Meth. Eng. 4, 455-473, 1972.

[19] L. Cirone, F. Giambanco, L. Palizzolo, "Multicriteria optimal design of frames", ASEM '99, Seoul, Korea, August 10– 3, 1999.

[20] F. Giambanco, "Multicriteria optimal design of circular plates". IASS–IACM 2000, Fourth International Colloquium on Computation of Shell & Spatial Structures, Chania-Crete, June 4-7, 2000.

[21] F. Giambanco, L. Palizzolo, A Caffarelli, "Optimal design of elastic plastic structures: elastic and plastic shakedown formulations", "Intensive Workshop" – Optimal Design of Materials and Structures, Paris, France, November 25-27, 2002.
[22] L. Palizzolo, "Optimization of continuous elastic perfectly plastic beams", Computers & Structures, 82, 4-5, 2004, pp. 397-411, 2004.
[23] L. Palizzolo, A. Caffarelli, M. Zito, "Optimal design of circular plates according to different criteria", Lightweight Structures in Civil Engineering (LSCE 2005) International Colloquium, Micropublisher, Warsaw, Polonia, September 12-14, 2005.
[24] F. Giambanco, L. Palizzolo, A. Caffarelli, "An optimal plastic shakedown design", The Sixth International Conference on Computational Structures Technology, Prague, Czech Republic, September 4-6, 2002.
[25] L. Palizzolo, "Non-ratchet design of beam structures", Proceedings of IASS–IACM 2000, Fourth International Colloquium on Computation of Shell & Spatial Structures, Chania-Crete, 2000.
[26] F. Giambanco, L. Palizzolo, A. Caffarelli, "Computational procedures for plastic shakedown design of structures", Journal of Structural and Multidisciplinary Optimization, 28, 5, pp. 317-329, 2004.
[27] L. Palizzolo, "Optimal design of trusses according to a plastic shakedown criterion", J. of Appl. Mech., 71, 2, pp. 240-246, 2004.
[28] S. Benfratello, F. Giambanco, L. Palizzolo, "Shakedown design of structures under dynamic loading", The Fourth International Conference on Structural Engineering, Mechanics and Computation, SEMC 2010, Cape Town - South Africa, September 6-8, 2010.
[29] S. Benfratello, F. Giambanco, L. Palizzolo, "P-Delta effect in structural optimization", 2nd International Conference on Engineering Optimization, EngOpt 2010, Lisbona - Portogallo, September 6-9, 2010.
[30] F. Giambanco, L. Palizzolo, A. Caffarelli, "Optimum design of structures with limited ductility", Ninth International Conference on Computer Aided Optimum Design in Engineering (OPTI 2005), WIT Press (S. Henrnandez and C.A. Brebbia Eds.) Skiathos, Grecia, May 23-25, 2005.
[31] F. Giambanco, S. Benfratello, L. Palizzolo, P. Tabbuso, "Multicriterion design of frames with constraints on buckling", Atti del XX Congresso Nazionale AIMETA, Bologna, September 12–15, 2011.
[32] W. Szyszkowski, L.G. Watson, "Optimization of the buckling load of columns and frames", Engineering Structures, 10, 4, 249-256, 1988.
[33] C.M. Wang, S. Kitipornchai, V. Thevendran, "Optimal designs of I-Beams against lateral buckling", Journal of Engineering Mechanics, 116, 9, 1902-1923, 1990.
[34] S. Benfratello, F. Giambanco, L. Palizzolo, "Optimal design of structures under dynamic loading", Atti del XIX Congresso Nazionale AIMETA, Ancona, 14 – 17 September 2009.