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Optimization of Arch Supported Tensile Roofs

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Abstract

This paper introduces some results of the optimization of tensile roofs suspended from arches by the help of different suspension systems. In the case of “conventional” suspension the individual suspension cables connect the breakpoints of the ridge cables of the tensile roof with the joints of the supporting arches directly. In the case of block and tackle suspension system the continuous suspension cables pass through a series of upper and lower pulleys along the supporting arches. The aim of the block and tackle suspension system is to minimize the bending moment of the supporting arches.
The aim of the current research is to optimize the structure of arch supported tensile roofs. The current paper deals with the effect of the curvature of the circular supporting truss arches on the internal forces of the arches and the displacements of the tensile roofs.

Keywords: tensile structure, block and tackle, dynamic relaxation, nonlinear analysis, friction, truss arch, suspension system, cable net.

1 Introduction

Tensile roofs supported by 3 and 4 truss arches have been analysed in the current research. Structures with different geometry and with different suspension systems have been compared. In the case of “conventional” suspension the short, individual suspension cables connect the breakpoints of the ridge cables to the joints of the truss arches directly. In the case of block and tackle suspension system the continuous suspension cables passes through a series of upper and lower pulleys along the supporting arches. The lower pulleys are secured to the ridge cable of the tensile roof; pairs of upper pulleys are secured to the supporting arch. In Figure 1 the side view of the block and tackle suspension can be seen. Since the force in the

continuous suspension cable is (almost) constant, the supporting arch can be designed to correspond to the pressure line of the arch loads.
The block and tackle suspension system has been invented by Árpád Kolozsváry to minimize the bending moment in the supporting arches of tensile roofs [1]. The invention and the first numerical results of the analysis based on idealised, frictionless pulleys were introduced in [2]. Later a more precise numerical procedure based on the well-known dynamic relaxation method [3, 4] has been developed by the author [5]. By the help of this procedure the friction between the pulley and its shaft can be also taken into account.
The aim of the current research is to optimize the structure of the roof and the supporting arches. In the current paper the effect of the curvature of the circular supporting truss arches has been analysed on the normal forces and bending moments of the arches, the maximum forces in the chord members and the displacements of the structure.

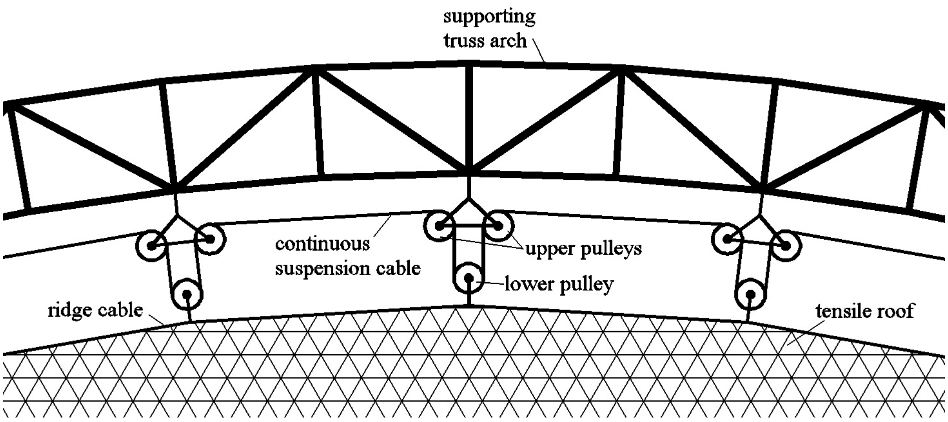

Figure 1: Side view of the block and tackle suspension system

2 The analysed structures

Hyperbolic cable net roofs supported by truss arches of constant curvature have been analysed. The longest diagonal of the covered area was 100 m. The angle α measured at the supports between the centreline of the arch and the horizontal plane has been varied between 30° and 50° in steps of 5° to analyse its effect on the internal forces of the arches and the displacements of the roof.
There were 3 supporting arches in the case of model 1. In Figure 2 the floor plan, in Figure 3 the axonometric view of model 1 can be seen. The roof of model 2 was suspended to 4 arches. In Figure 4 the floor plan, in Figure 5 the axonometric view of model 2 can be seen.
The truss arches had one lower and two upper chords with the same cross-sectional area, A_{chord}=216.5 cm^2 in the case of model 1, A_{chord}=176.8 cm^2 in the case of model 2. The arches were supported by universal hinges. The depth of the arches was 3 m,

the width was 2.5 m. The roofs were suspended at 14 points to every arch. All models have been analysed with individual suspension cables and with block and tackle suspension system. In the second case the ratio of the radius of the pulley (R) and the radius of its shaft (r) was supposed to be R/r=5. The coefficient of friction between the pulley and its shaft was supposed to be μ=0.01. The individual suspension cables and the suspension cables of the upper pulleys in the case of the block and tackle suspension system are radial in the unloaded structures.

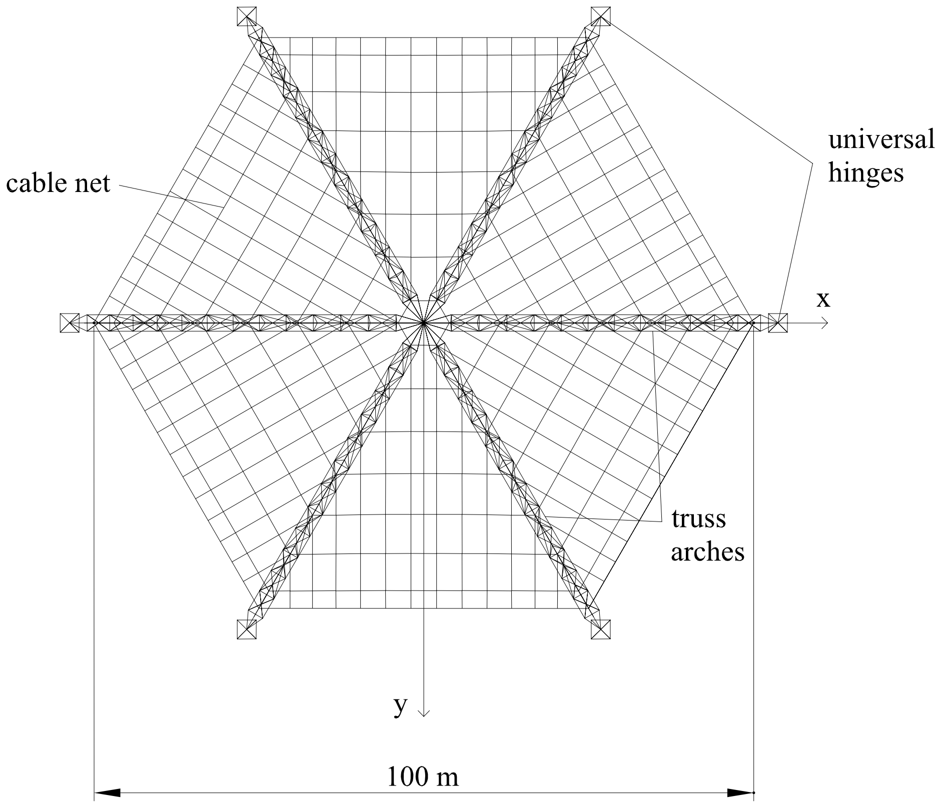

Figure 2: Floor plan of model 1

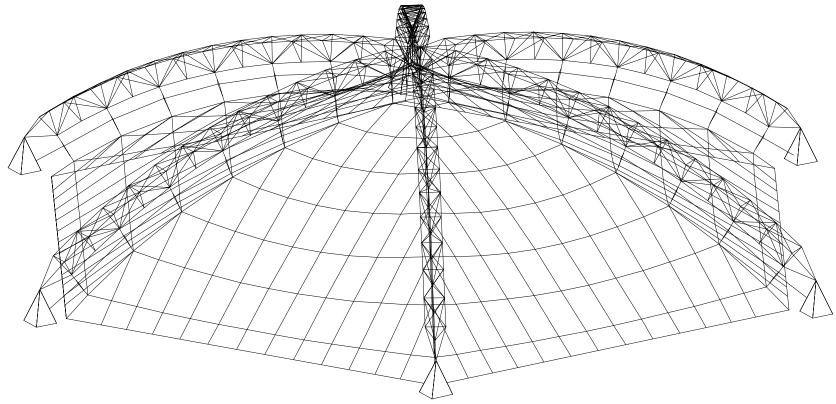

Figure 3: Axonometric view of model 1

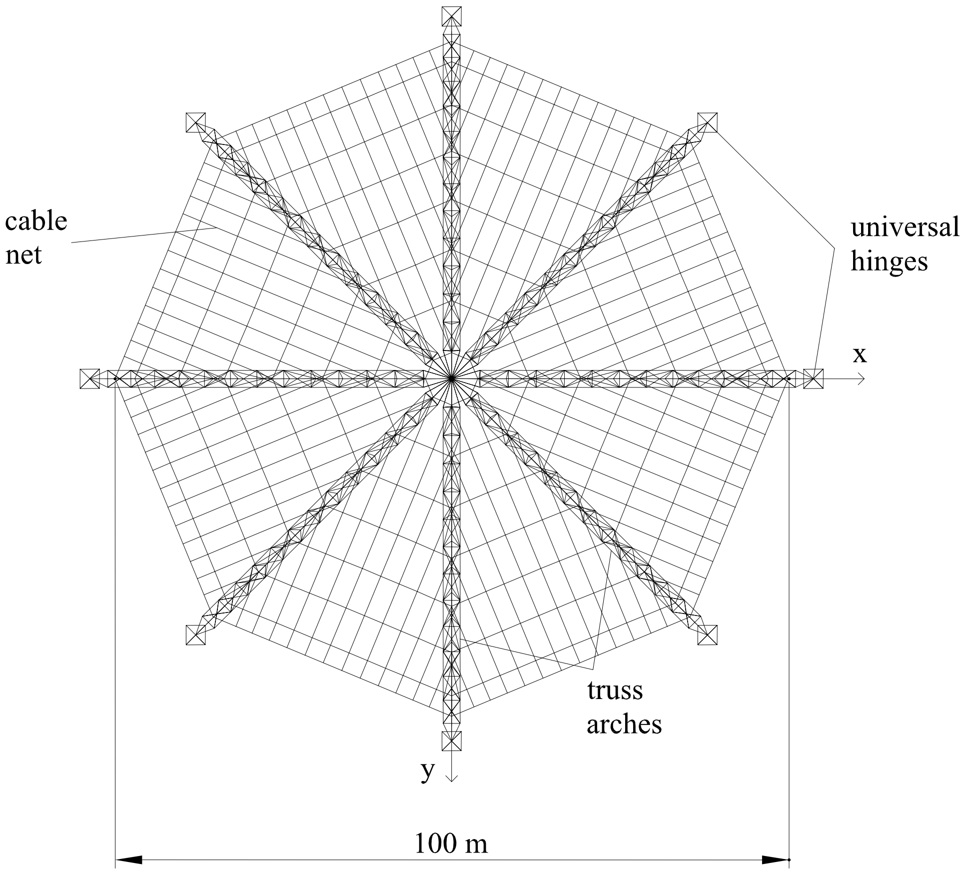

Figure 4: Floor plan of model 2

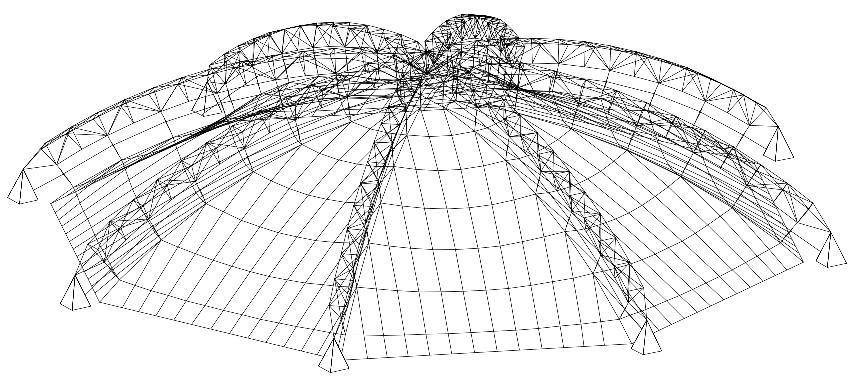

Figure 5: Axonometric view of model 2

All models have been analysed under two load cases:

- total snow load, 1 kN/m^2 vertical load on the whole roof,
- partial snow load, 1 kN/m^2 vertical load on the half of the roof, where x>0.

3 The numerical results

During the dynamic relaxation the forces in the different cable and truss members have been calculated. The normal and shear forces and the bending moments of the arches have been calculated between the suspension points on the basis of the forces in the truss members.

3.1 Results of model 1

Figure 6 shows the maximum arch load due to total snow load (TSL) and partial snow load (PSL). The results show that the maximum arch loads are 15% to 30% smaller in the case of block and tackle suspension system (BTSS) than in the case of individual suspension cables (ISC). In the case of smaller angle α the arch loads are smaller because the directions of the arch loads are closer to vertical and the roof loads were independent of angle α during the analysis. (In practice the roof load probably smaller in the case of larger angle α because of the steeper surface of the roof.)

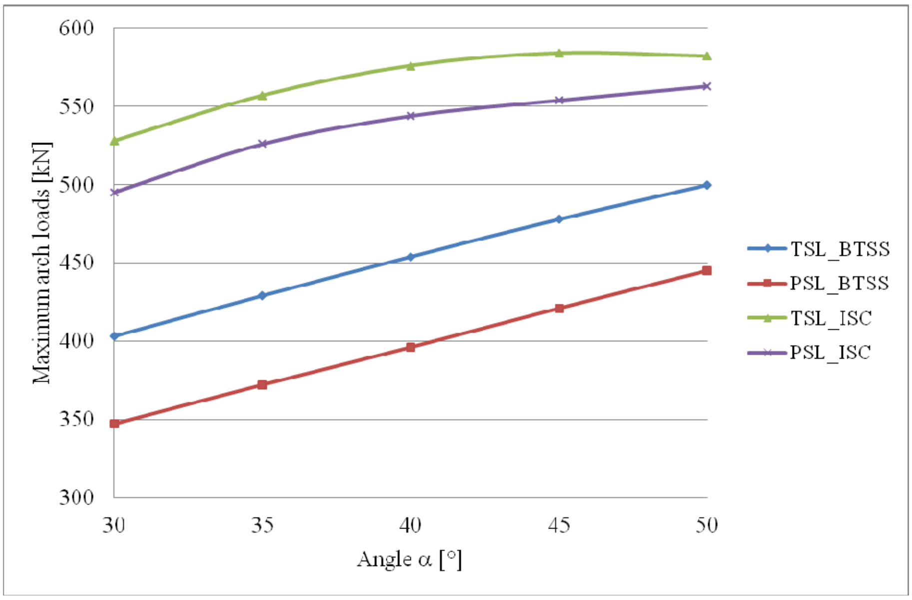

Figure 6: The maximum arch loads of model 1 in the case of arches with different angle α

In Figure 7 the maximum normal force in the arches can be seen. The results show that the larger curvature of the arches (in the case of larger angle α) results in smaller normal forces (besides larger arch loads). On the other hand the maximum normal force is 17% to 36% larger in the case of BTSS than in the case of ISC.
Figure 8 shows the maximum in-plane bending moments of the arches. (The out-of-plane bending moments are zero in the case of TSL and very small in the case of PSL.) By using the BTSS the maximum bending moment of the arches can be

reduced by 73% to 88%. The bending moments due to PSL are 59% to 291% larger than the bending moments due to TSL (in the case of the same suspension system and same angle α).

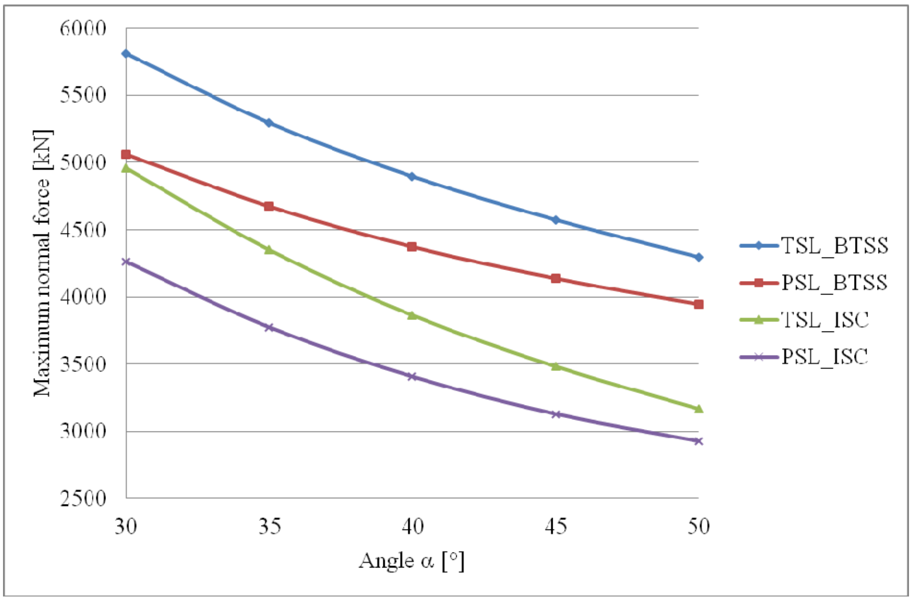

Figure 7: The maximum normal force in the arches of model 1 in the case of arches with different angle α

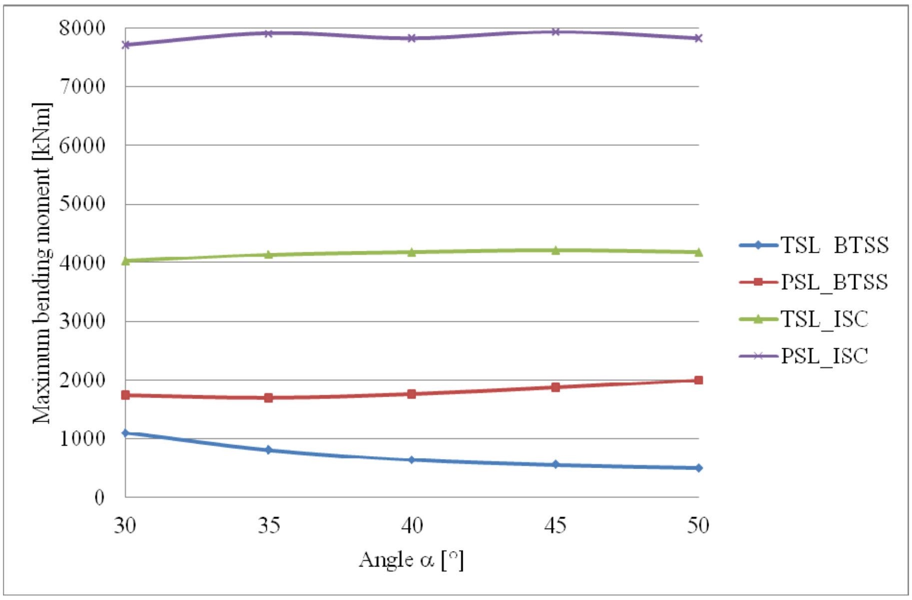

Figure 8: The maximum bending moment in the arches of model 1 in the case of arches with different angle α

In Figure 9 the maximum force in the chord members can be seen. In the case of ISC the PSL results in approximately 13% larger maximum forces in the chord members.

In the case of BTSS and angle α smaller than approximately 38° the maximum force in the chord members caused by the TSL is larger than the maximum force in the chord members caused by the PSL. On the other hand the use of BTSS results in 21% to 30% smaller maximum force in the chord members depending on angle α.

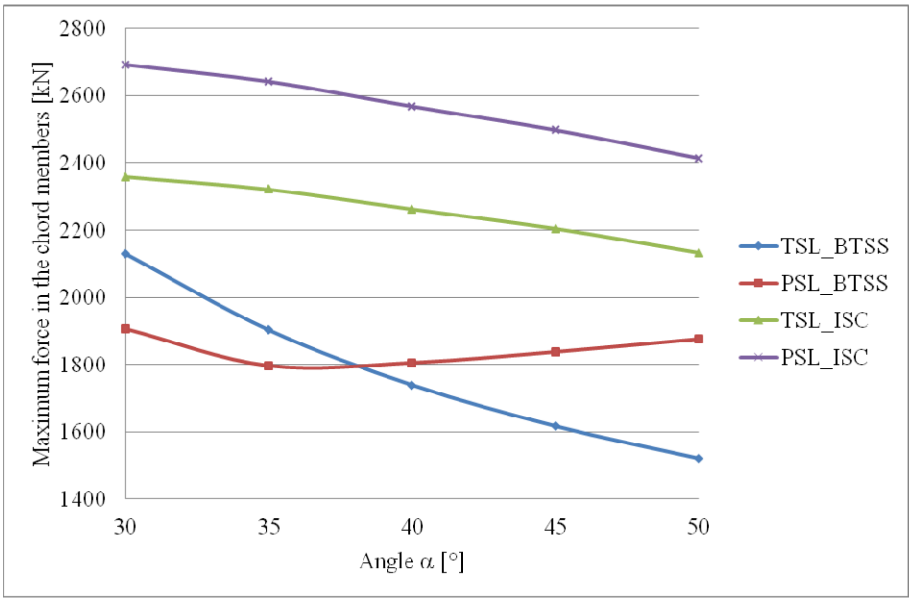

Figure 9: The maximum force in the chord members in the arches of model 1 in the case of arches with different angle α

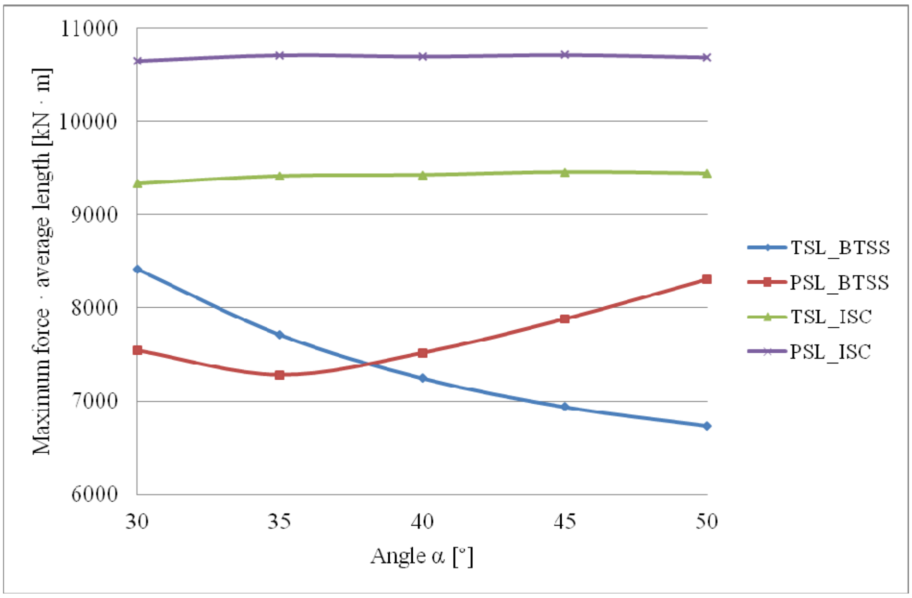

Figure 10: The maximum force in the chord members of model 1 multiplied by the average length of the chord members

To determine an optimal angle α the maximum forces in the chord members were multiplied by the average length of the chord members (for a given angle α), see Figure 10. In the case of ISC the results are almost independent of angle α. In the case of BTSS there is a significant minimum point near 38° at the intersection of the two curves according to TSL and PSL.

Figure 11 shows the forces in the truss members due to partial snow load in the case of ISC and α =40°. The bending moments in the arch are large enough to cause tension in the lower chord of the arch. The maximum tension in the chord members is 1558 kN, the maximum compression is 2569 kN.

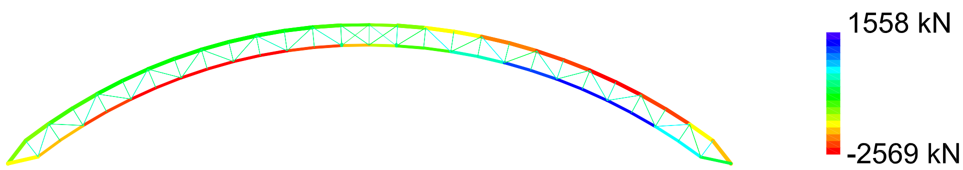

Figure 11: The forces in a truss arch of model 1, due to partial snow load (on the right side of the arch), in the case of individual suspension cables and α =40°

Figure 12 shows the forces in the truss elements in the case of BTSS. The maximum compression in the chord members is 1805 kN, 30% smaller than in the case of ISC. The minimum compression is 824 kN, there is no tension in the chord members.

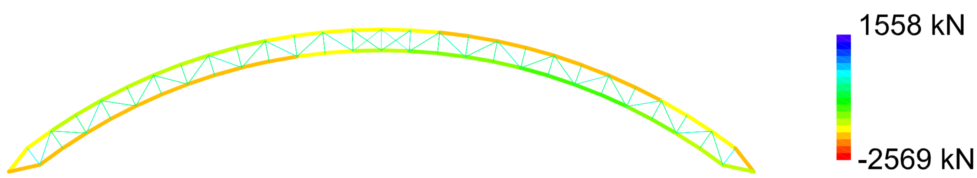

Figure 12: The forces in a truss arch of model 1, due to partial snow load (on the right side of the roof), in the case of BTSS and α =40°

Figure 13 shows the maximum displacements of the roof. The displacements are 58% to 137% larger in the case of BTSS than in the case of ISC, caused by the motion of the pulleys. On the other hand the larger maximum arch loads in the case of larger α (see Figure 6) results in larger displacements of the pulleys.

The displacements of the roof due to PSL can be seen in Figure 14.

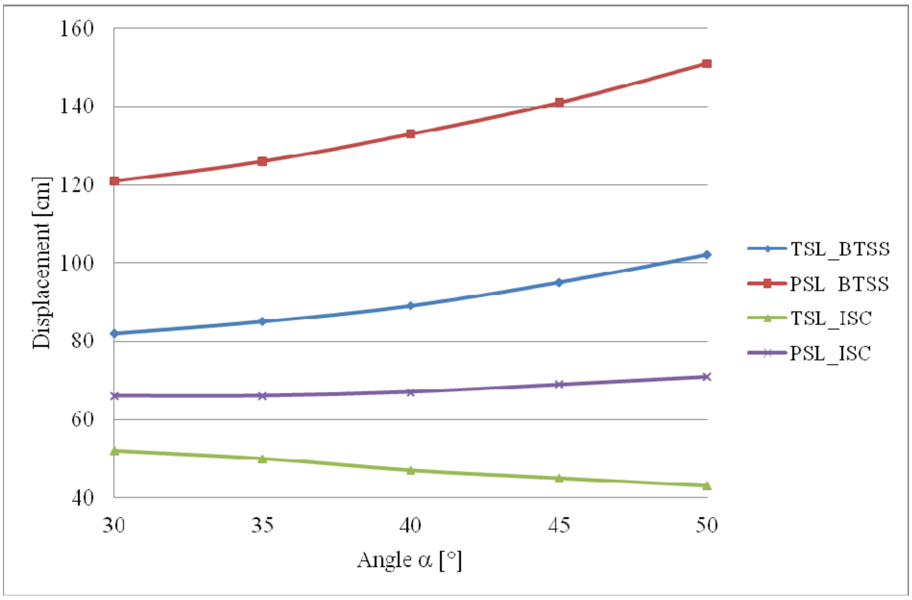

Figure 13: The maximum displacements of the roof in the case of model 1

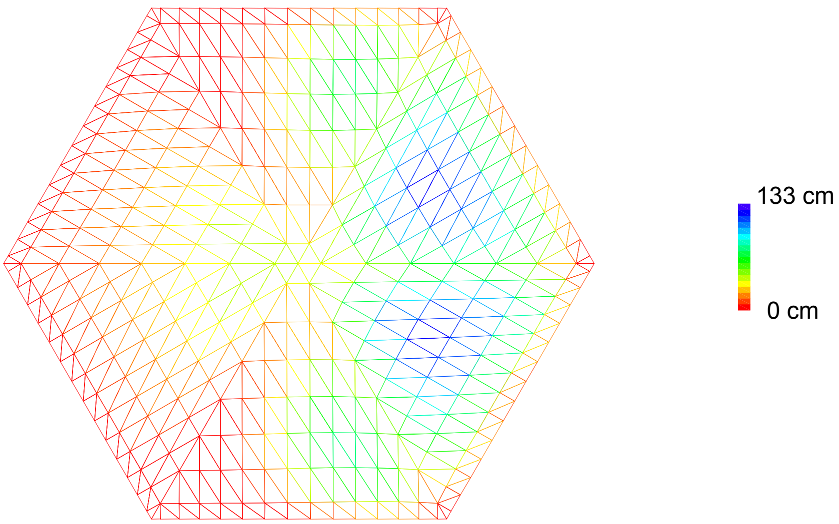

Figure 14: The displacements of the roof due to PSL in the case of BTSS

3.2 Results of model 2

In Figure 15 the maximum normal force in the arches can be seen. The maximum normal force is 21% to 43% larger in the case of BTSS than in the case of ISC. Figure 16 shows the maximum in-plane bending moments of the arches. The results show that by the help of BTSS the maximum bending moment of the arches can be reduced by 72% to 86%.

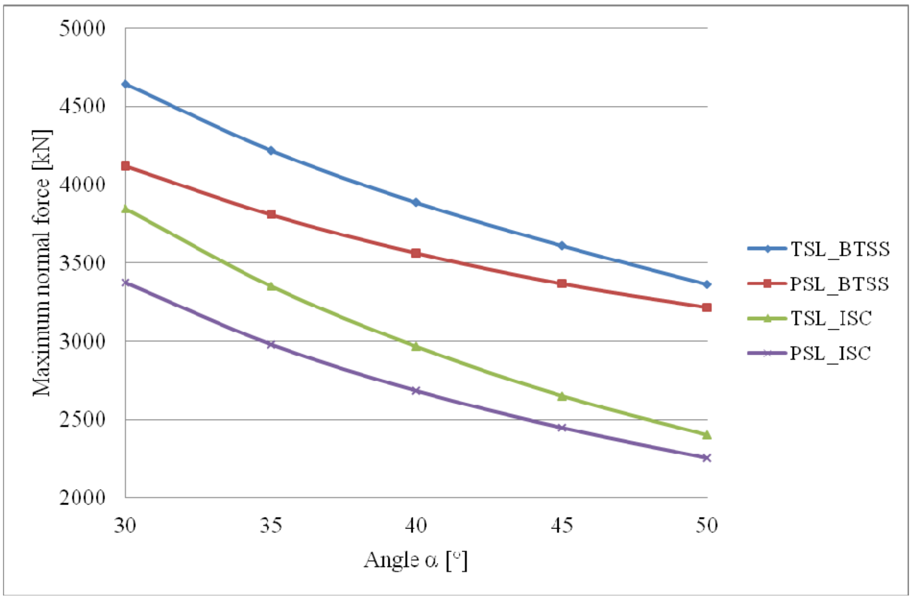

Figure 15: The maximum normal force in the arches of model 2 in the case of arches with different angle α

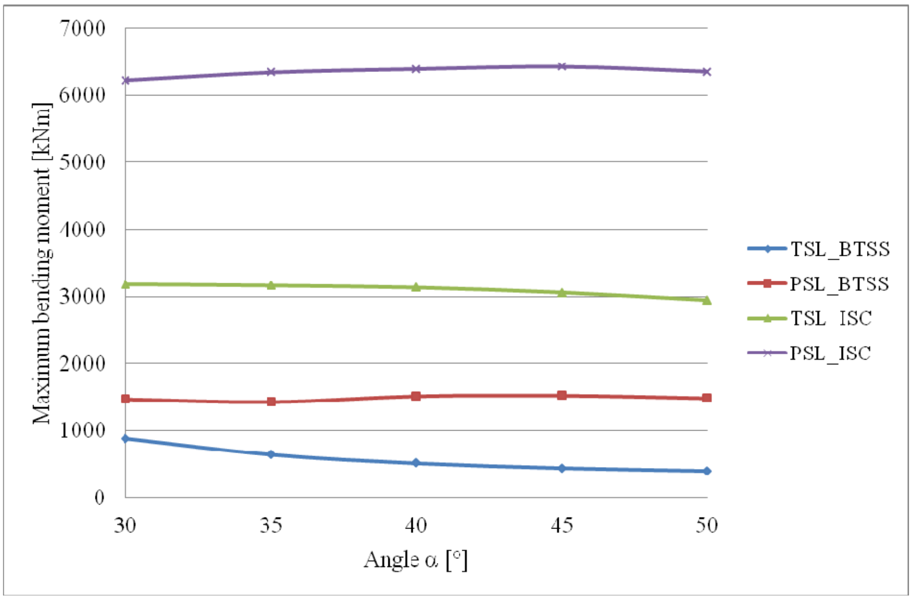

Figure 16: The maximum bending moment in the arches of model 2 in the case of arches with different angle α

In Figure 17 the maximum force in the chord members can be seen. In the case of ISC the partial snow load results in 21% to 36% larger maximum forces in the chord members than the total snow load. In the case of BTSS and angle α smaller than approximately 35° the maximum force in the chord members caused by the TSL is larger than the maximum force in the chord members caused by the PSL. On the other hand the use of BTSS results in 22% to 30% smaller maximum force in the chord members depending on angle α.

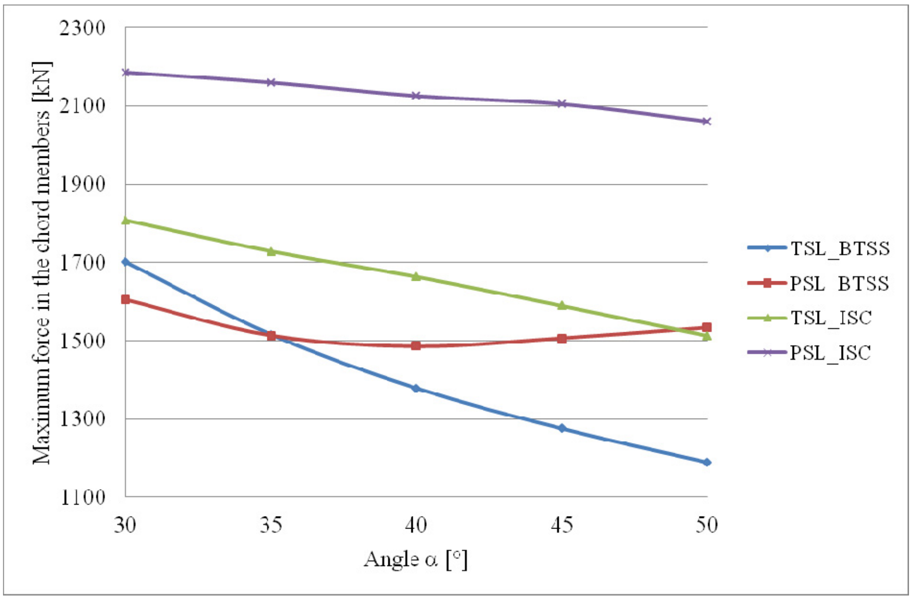

Figure 17: The maximum force in the chord members in the arches of model 2 in the case of arches with different angle α

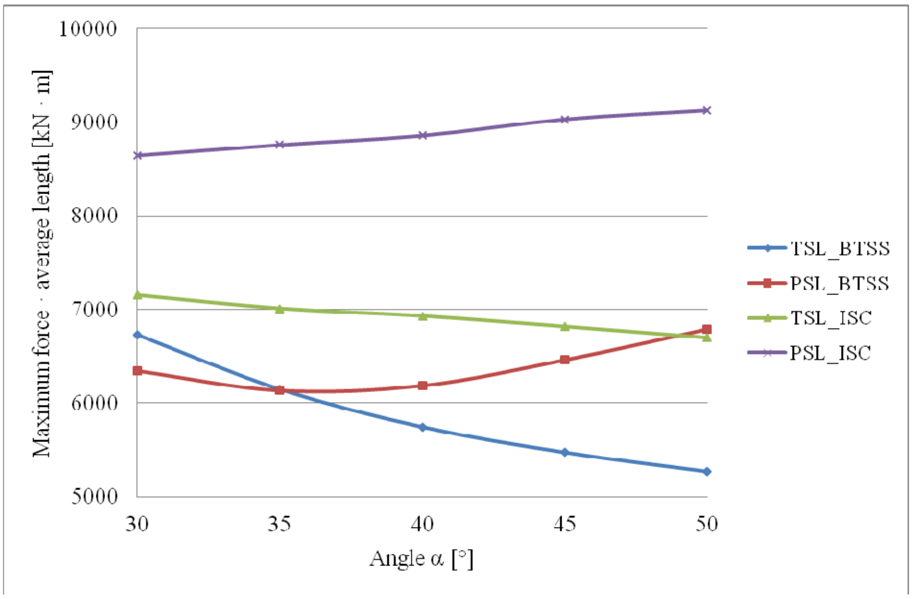

Figure 18: The maximum force in the chord members of model 2 multiplied by the average length of the chord members

To determine an optimal angle α the maximum forces in the chord members were multiplied by the average length of the chord members (for a given angle α). Figure 18 shows the results. In the case of ISC the PSL results in larger maximum force in the chord members than the TSL. The results show that in the case of ISC the smaller angle α results in smaller values (in the analysed region). In the case of BTSS there is a significant minimum point near 35° at the intersection of the two curves according to TSL and PSL. This means that from this point of view arches with α=35° are close to the optimum in the case of BTSS.

Figure 19 shows the forces in the truss members due to partial snow load in the case of ISC and α =40°. The bending moments in the arch are large enough to cause tension in the lower chord of the arch. The maximum tension in the chord members is 1279 kN, the maximum compression is 2127 kN.

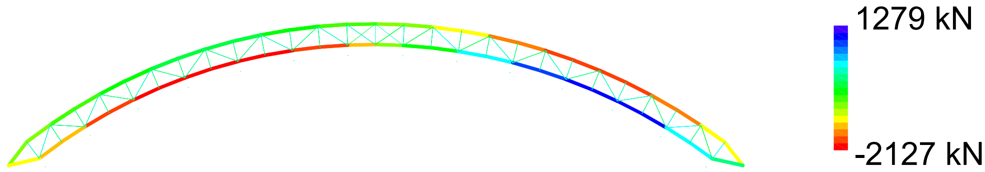

Figure 19: The forces in a truss arch of model 2, due to partial snow load (on the right side of the roof), in the case of individual suspension cables and α=40°

Figure 20 shows the forces in the truss elements in the case of BTSS. The maximum compression in the chord members is 1486 kN, 30% smaller than in the case of ISC. The minimum compression is 645 kN, there is no tension in the chord members.

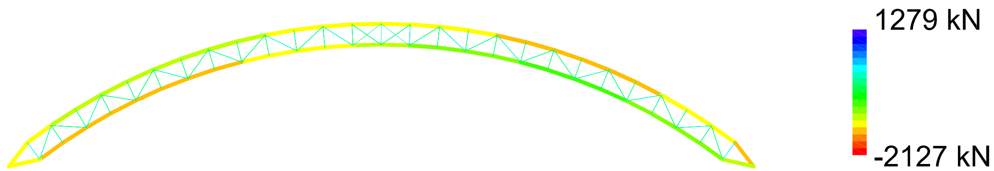

Figure 20: The forces in a truss arch of model 2, due to partial snow load (on the right side of the roof), in the case of BTSS and α=40°

Figure 21 shows the maximum displacements of the roof. The displacements are 117% to 181% larger in the case of BTSS than in the case of ISC, caused by the motion of the pulleys. The effect of the BTSS on the displacements is more significant than in the case of model 1. More arches results in smoother surface and less significant ridge with smaller stiffness in radial direction and larger displacements of the pulleys in the case of BTSS.

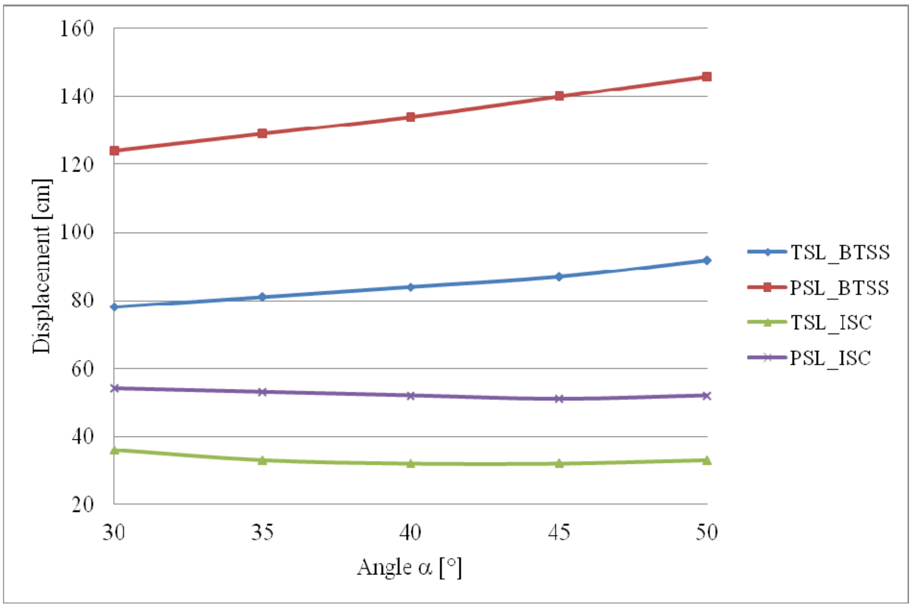

Figure 21: The maximum displacements of model 2 in the case of arches with different angle α

4 Conclusions

Tensile roofs supported by three and four truss arches of constant curvature have been analysed with individual suspension cables and block and tackle suspension system. The curvature of the arches has been varied to analyse its effect on the behaviour of the structure. The results show that the curvature of the arches has a strong effect on the arch loads and the normal forces in the arch, but its effect on the bending moments is less significant. The optimal geometry of the arches has been determined on the basis of the minimization of the maximum force in the chord members of the truss arches multiplied by the average length of the chord members. On the other hand the results show that by the help of the block and tackle suspension system the bending moment of the supporting arches can be reduced radically, independently of the curvature of the circular arches.

Acknowledgements

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