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Free Vibration of a U-Type Liquid-Contained Rectangular Container with Simply Supported Boundary Condition

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Abstract

This paper presents an analysis method for free vibration of a rectangular container partially filled with an ideal liquid. It is assumed that the container open at the top is simply supported along the two edges and its side edges are fixed. Wet dynamic displacements of the container are approximated by combining the orthogonal polynomials satisfying the boundary condition. As the facing vertical rectangular plates are assumed to be geometrically identical, the vibration modes of the container can be divided into two categories: symmetric and antisymmetric modes with respect to the vertical plane passing through the centre of the container and perpendicular to the free liquid surface. The liquid displacement potential satisfying the Laplace equation and liquid boundary condition are derived, and the wet dynamic modal functions of a half of the container can be expanded by the finite Fourier transform for compatibility requirements along the contacting surfaces between the container and liquid. An eigenvalue equation is derived using the Rayleigh–Ritz method.

Keywords: free vibration, rectangular container, hydroelastic vibration, Rayleigh – Ritz method, orthogonal polynomials, mode shapes.

1 Introduction

Rectangular tanks have been used for liquid storage in many engineering fields. They should be designed to withstand a wide variety of external impacts. Many investigators have developed some approximate solutions to predict the changes in the natural frequencies when a rectangular plate is in contact with a liquid. Also an analytical method to calculate the natural frequencies of two identical rectangular plates coupled with a bounded fluid was suggested [1]. However, to date, there has been minimal research regarding a flexible rectangular tank filled with a liquid.

Although, Zhou et al. [2] and Jeong [3] recently developed a theory on the three–dimensional dynamic characteristics of flexible rectangular tanks partially filled with water using the Rayleigh–Ritz method, these studies did not include the flexible bottom plate in their theoretical models. This paper will present a theoretical formulation able to extract the wet natural frequencies of an open U-type rectangular container, which has the simply supported boundary condition and partially filled with an ideal liquid. The effect of the liquid depth on the natural frequencies will be discussed.

2 Analysis

2.1 Mathematical model for container

A liquid–filled open rectangular container has a height of *c*, length of *b*, width of *H* and thickness of *h* as depicted in Figure 1. The container is partially filled with an ideal liquid of the depth, *L*. Since the liquid storage container is composed of three rectangular plates, the vibration modes of the container can be classified into two types of modes according to whether the deformed shapes are symmetric or antisymmetric with respect to the centreline of the container as shown in Figure 2.

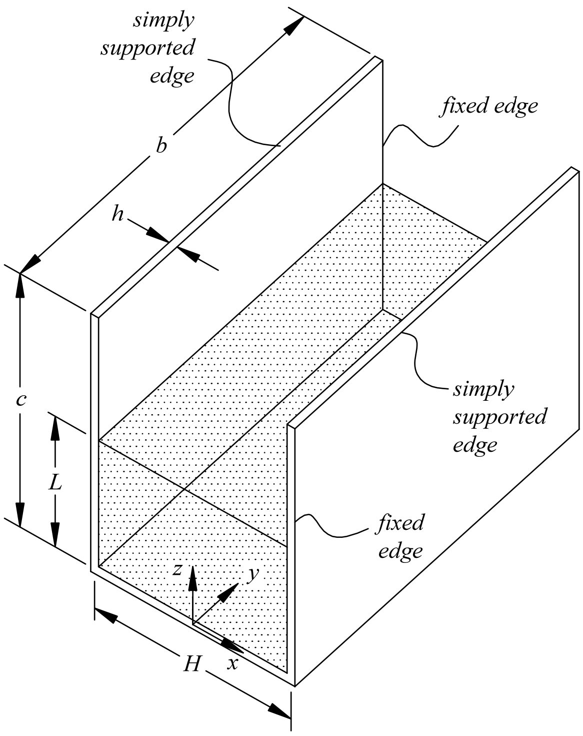


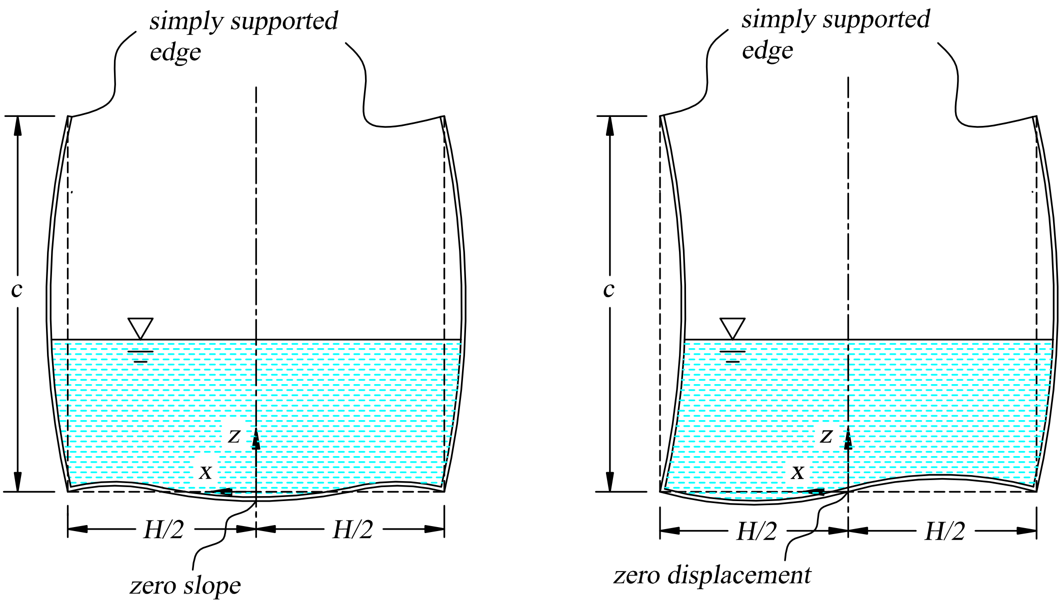
Figure 1: A rectangular flexible open U-type container partially filled with a liquid.

Therefore, the whole liquid–filled container can be analyzed by taking into account of a half of the liquid–filled rectangular container. The two semi–plates of the half can be regarded as a line–supported rectangular plate by introducing a new coordinate system, namely ξ–y plane as illustrated in Figure 3. Each mode shape of the plate in the ξ–y plane can be approximated by a combination of a finite number of admissible functions, $W_{mn}(\xi, y)$, and the corresponding unknown coefficients, q_{mn}.

$$w(\xi, y, t) = \sum_{m=1}^{M} \sum_{n=1}^{N} q_{mn} \, W_{mn}(\xi, y) \exp(\mathrm{i}\,\omega t). \qquad (1)$$

where $\mathrm{i} = \sqrt{-1}$ and ω is the circular natural frequency of the dry or wet container. The indices m and n indicate the m-th order polynomial in the ξ–direction and the n-th order polynomial in the y–direction, respectively. The transverse modal function can be defined by a multiplication of the ξ– and the y– directional admissible functions, $H_m(\xi)$ and $F_n(y)$.

$$W_{mn}(\xi, y) = G_m(\xi) \, F_n(y). \qquad (2)$$



(a) Symmetric mode (b) Antisymmetric mode

Figure 2: Mode category of a rectangular flexible open container partially filled with a liquid.

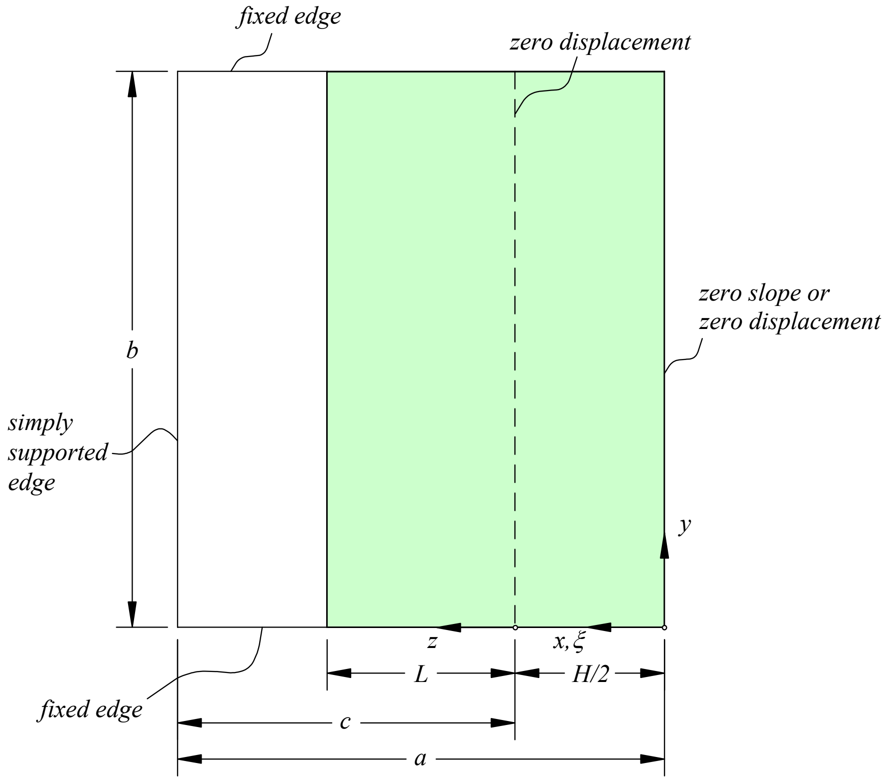


Figure 3: Theoretical model of a half of a rectangular flexible open container partially filled with a liquid.

When the container is simply supported at the top and bottom, the admissible functions are assumed as a set of orthogonal poliynomials, $H_m(\xi)$ and $F_n(y)$ satisfying the geometric and natural boundary conditions. For the symmetric modes,

$$G_1(\xi) = \frac{H^2}{4(H+c)a} - \frac{3H^2 + 6Hc + 4c^2}{4(H+c)a}\left(\frac{\xi}{a}\right)^2 + \left(\frac{\xi}{a}\right)^3 . \tag{3}$$

For the antisymmetric modes,

$$G_1(\xi) = \frac{H}{2a}\left(\frac{\xi}{a}\right) - \frac{(H+c)}{a}\left(\frac{\xi}{a}\right)^2 + \left(\frac{\xi}{a}\right)^3 . \tag{4}$$

The successive polynomials can be generated from the recursive formulas based on the the Gram–Schmidt process [3 ~ 5].

For the clamped boundary condition along the y–direction, the slope and deformation at the side edges must be zero, simultaneously. So, the first polynomial function of the y–direction will be

$$F_1(y) = \sqrt{45/104}\left[6\,y^2 - 4\,y^3 + y^4\right]. \qquad (5)$$

The successive polynomials of the y–direction can be also generated from the recursive formulas based on the the Gram–Schmidt process [3 ~ 5].

2.2 Natural frequencies of dry rectangular container

The symmetric dry or wet vibrational modes can only be obtained by a combination of the symmetric admissible functions. In the same way, the antisymmetric modes can only be constructed by a combination of the antisymmetric admissible functions. A sufficiently large finite number of terms, N and M, must be considered to obtain a converged solution, and a vector $\boldsymbol{q}$ of the unknown coefficients is introduced to perform numerical calculations.

$$\boldsymbol{q} = \{\, q_{11} \quad q_{12} \quad q_{13} \;\cdots\; q_{1N} \quad q_{21} \quad q_{22} \quad q_{23} \cdots \quad q_{MN} \,\}^T . \qquad (6)$$

where N indicates the number of the admissible functions of the vertical direction (y) to be considered and M represents those of the lateral direction (ξ) in Figure 3.

To begin with, the kinetic and potential energies of the system should be defined to apply the Rayleigh–Ritz method using the admissible functions previously mentioned. The reference kinetic energy T^* corresponding a half of the rectangular container can be obtained by using the orthogonal property of the admissible functions.

$$T^* = \frac{\rho\,h}{2}\,\boldsymbol{q}^T\,\boldsymbol{Z}\,\boldsymbol{q}\,, \qquad (7)$$

where ρ is the mass density of the rectangular container. The matrix $\boldsymbol{Z}$ in Eq. (7) is an $(M\,N)\times(M\,N)$ diagonal matrix which can be given as

$$Z = \int_0^b \int_0^a W_{mn}\,W_{uv}\,d\xi\,dy\,. \qquad (8)$$

The maximum potential energy V of a half of the rectangular container can be computed by integrating the derivatives of the admissible modal functions.

$$V = \frac{D}{2}\int_0^b\int_0^a \left[\left\{\frac{\partial^2 W_{mn}}{\partial \xi^2}\frac{\partial^2 W_{uv}}{\partial \xi^2}\right\} + \left\{\frac{\partial^2 W_{mn}}{\partial y^2}\frac{\partial^2 W_{uv}}{\partial y^2}\right\}\right.$$
$$\left. + \mu\left\{\frac{\partial^2 W_{mn}}{\partial \xi^2}\frac{\partial^2 W_{uv}}{\partial y^2} + \frac{\partial^2 W_{uv}}{\partial \xi^2}\frac{\partial^2 W_{mn}}{\partial y^2}\right\} + 2\,(1-\mu)\left\{\frac{\partial^2 W_{mn}}{\partial \xi\,\partial y}\frac{\partial^2 W_{uv}}{\partial \xi\,\partial y}\right\}\right] d\xi\,dy\,, \qquad (9)$$

where the flexural rigidity of the container is given as $D = E\,h^3 / 12\,(1-\mu^2)$; μ and E are the Poisson's ratio of the container and the modulus of elasticity, respectively. Inserting the orthogonal admissible functions into Eq. (9) gives the maximum potential energy of a half of the rectangular container as a matrix form.

$$V = \frac{D\,a\,b}{2}\,\boldsymbol{q}^T \boldsymbol{U}\,\boldsymbol{q}\,, \tag{10}$$

where $\boldsymbol{U}$ is also an $(M\,N)\times(M\,N)$ matrix which can be derived as

$$U = \Lambda 1_{mu}\;\Xi 1_{nv} + \Lambda 2_{mu}\;\Xi 2_{nv} + 2\,\mu\,\Lambda 3_{mu}\;\Xi 3_{nv} + 2\,(1-\mu)\,\Lambda 4_{mu}\;\Xi 4_{nv}\,, \tag{11}$$

and

$$\Lambda 1_{mu} = \int_0^a G_m''(\xi)\,G_u''(\xi)\,d\xi\,, \qquad \Xi 1_{nv} = \int_0^b F_n(y)\,F_v(y)\,dy\,, \tag{12, 13}$$

$$\Lambda 2_{mu} = \int_0^a G_m(\xi)\,G_u(\xi)\,d\xi\,, \qquad \Xi 2_{nv} = \int_0^b F_n''(y)\,F_v''(y)\,dy\,, \tag{14, 15}$$

$$\Lambda 3_{mu} = \int_0^a G_m''(\xi)\,G_u(\xi)\,d\xi\,, \qquad \Xi 3_{nv} = \int_0^b F_n(y)\,F_v''(y)\,dy\,, \tag{16, 17}$$

$$\Lambda 4_{mu} = \int_0^a G_m'(\xi)\,G_u'(\xi)\,d\xi\,, \qquad \Xi 4_{nv} = \int_0^b F_n'(y)\,F_v'(y)\,dy\,, \tag{18, 19}$$

The relationship between the reference kinetic energy multiplied by its square circular frequency and the maximum potential energy is used to extract the natural frequencies of the dry container. The Rayleigh quotient for the dry rectangular container is given as V / T^*. Minimizing the Rayleigh quotient with respect to the unknown parameters $\boldsymbol{q}$, the Galerkin equation yields

$$D\,\boldsymbol{U}\,\boldsymbol{q} - \omega^2 \rho\,h\,\boldsymbol{Z}\,\boldsymbol{q} = \{\boldsymbol{0}\}\,. \tag{20}$$

2.3 Displacement potential of the liquid

Since an ideal liquid is partially filled in the rectangular container, the velocity and displacement potentials of the liquid should satisfy the Laplace equation. The vertical liquid displacement potential at the top liquid surface should vanish because the liquid has a free surface at $z = L$ and the gravity effect on the system is neglected.

$$\phi(x, y, z)\big|_{z=L} = 0\,. \tag{21}$$

The symmetric modes requires the zero liquid displacement along $x = 0$. So,

$$\left.\frac{\partial \phi\,(x,y,z)}{\partial x}\right|_{x=0} = 0\,, \tag{22}$$

The displacement potential for the liquid satisfying both the Laplace equation and the boundary condition of Eqs. (21) and (22) can be written

$$\phi(x,y,z) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\Big[K_{rs}\cosh\left(\sigma_{rs}\,x\right)\cos\left(\lambda_r\,z\right)$$
$$+\,R_{rs}\cos\left(\alpha_r\,x\right)\left\{\cosh\left(\beta_{rs}\,z\right)-\sinh\left(\beta_{rs}\,z\right)/\tanh\left(\beta_{rs}\,L\right)\right\}\Big]\cos\left(\tau_s\,y\right), \tag{23}$$

where

$$\tau_s = \frac{(s-1)\,\pi}{b}\,, \qquad \lambda_r = \frac{(2\,r-1)\,\pi}{2\,L}\,, \qquad \sigma_{rs} = \sqrt{\tau_s^2+\lambda_r^2}\,, \tag{24–26}$$

$$\alpha_r = \frac{2\,(r-1)}{H}\,\pi\,, \quad \beta_{rs} = \sqrt{\tau_s^2+\alpha_r^2}\,, \qquad s, r = 1, 2, 3, \ldots. \tag{27, 28}$$

The unknown coefficients R_{rs} and K_{rs} can be determined by the compatibility condition at the wet surface between the container and the liquid.

As the transverse displacement of the container must be identical to the dynamic liquid displacement in the normal direction to the interface surface along the liquid–contacting surface, the compatibility conditions at the interfacing surfaces yield for the symmetric modes.

$$\sum_{m=1}^{M}\sum_{n=1}^{N} q_{mn}\,W_{mn}\left(\xi,z\right) = \left[\partial\phi\left(x,y,z\right)/\partial x\right]_{x=H/2} \text{ for } 0\le z\le L, \tag{29}$$

$$\sum_{m=1}^{M}\sum_{n=1}^{N} q_{mn}\,W_{mn}\left(\xi,z\right) = \left[\partial\phi\left(x,y,z\right)/\partial z\right]_{z=0} \text{ for } 0\le \xi\le H/2. \tag{30}$$

Substituting Eqs. (2) and (23) into Eqs. (29) and (30) results in

$$\sum_{n=1}^{N}\sum_{m=1}^{M} q_{mn}\,G_m(\xi)\,F_n(y) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} K_{rs}\,\sigma_{rs}\sinh\left(\frac{\sigma_{rs}\,H}{2}\right)\cos\left(\lambda_r\,z\right)\cos\left(\tau_s\,y\right),$$

$$\sum_{n=1}^{N}\sum_{m=1}^{M} q_{mn}\,G_m(\xi)\,F_n(z) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \frac{\beta_{rs}\,R_{rs}}{\tanh\left(\beta_{rs}\,L\right)}\cos\left(\alpha_r\,x\right)\cos\left(\tau_s\,y\right). \tag{31, 32}$$

Multiplication of $\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\cos\left(\lambda_r z\right)\cos\left(\tau_s y\right)$ to Eq. (31) and multiplication of $\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\cos\left(\alpha_r x\right)\cos\left(\tau_s y\right)$ to Eq. (32), and an integration along the wet surfaces, to perform the finite Fourier transform, give relationships between the unknown coefficients for the symmetric modes.

$$R_{rs} = -\frac{\tanh\left(\beta_{rs} L\right)}{\beta_{rs}\, J_s\, \Delta_r}\sum_{n=1}^{N}\sum_{m=1}^{M} q_{mn}\,\Theta_{mr}\,\Xi_{ns}\,, \qquad K_{rs} = \frac{\sum_{n=1}^{N}\sum_{m=1}^{M} q_{mn}\,\Gamma_{mr}\,\Xi_{ns}}{\sigma_{rs}\sinh\left(\sigma_{rs} H/2\right) J_s\, \Lambda_r}, \tag{33, 34}$$

where the coefficients, J_s , Δ_r , Θ_{mr} , Ξ_{ns} , Γ_{mr} and Λ_r can be obtained by the integrations. Now, the displacement potential can be described in terms of the unknown coefficients, q_{mn} of the admissible functions. For the antisymmetric modes, the similar formulation can be obtained.

2.4 Natural frequencies of wet rectangular container

As the reference kinetic energy of the liquid in the container for the symmetric mode can be evaluated from its boundary motion, the reference kinetic energy of the liquid is given as

$$\begin{aligned} T_o^* = &-\frac{1}{2}\rho_o \sum_{u=1}^{M}\sum_{v=1}^{N}\int_0^{H/2}\int_0^{b} W_{uv}\, q_{uv}\, \phi\left(x,y,0\right) dy\, dx \\ &-\frac{1}{2}\rho_o \sum_{u=1}^{M}\sum_{v=1}^{N}\int_0^{L}\int_0^{b} W_{uv}\, q_{uv}\, \phi\left(\frac{H}{2}, y, z\right) dy\, dz\,, \end{aligned} \tag{35}$$

where ρ_o is the mass density of the contained liquid. By substituting Eqs. (2), (23), (33) and (34) into Eq. (35), one can obtain

$$\begin{aligned} T_o^* = -\frac{1}{2}\rho_o \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{u=1}^{M}\sum_{v=1}^{N}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} q_{mn}\, q_{uv} & \left[\frac{\Gamma_{mr}\,\Xi_{ns}\,\Xi_{vs}}{\sigma_{rs}\,\Lambda_r\, J_s}\left\{\frac{\Omega_{urs}}{\sinh\left(\sigma_{rs} H/2\right)}\right.\right. \\ & \left.\left. -\frac{\Gamma_{ur}}{\tanh\left(\sigma_{rs} H/2\right)}\right\} + \frac{\tanh\left(\beta_{rs} L\right)\Xi_{ns}\,\Xi_{vs}\,\Theta_{mr}}{\beta_{rs}\,\Delta_r\, J_s}\left\{\left(-1\right)^r \Pi_{urs} - \Theta_{ur}\right\}\right] = \rho_o\, \boldsymbol{q}^T \boldsymbol{G}\, \boldsymbol{q}, \end{aligned} \tag{36}$$

where the coefficients, Ω_{urs} and Π_{urs} are the derived coefficients. By the same method, for the antisymmetric modes, the reference kinetic energy of the contained liquid can be formulated. The reference total kinetic energy multiplied by its square circular frequency should be equal to the maximum potential energy of the liquid–coupled system. Minimizing the Rayleigh quotient with respect to the unknown parameters $\boldsymbol{q}$, eventually the eigenvalue equation is obtained and the wet natural frequencies can be calculated by using Eq. (37).

$$D\,\boldsymbol{U}\,\boldsymbol{q} - \omega^2\left\{\rho\,\boldsymbol{Z} + \rho_o\,\boldsymbol{G}\right\}\boldsymbol{q} = \left\{\boldsymbol{0}\right\}. \tag{37}$$

3 Examples and discussion

3.1 Examples for theoretical calculation and finite element analysis

The eigenvalues of Eqs. (20) were calculated by using a commercial software, Mathcad (version 14) in order to find the natural frequencies of a dry rectangular container. The frequency equation derived in the previous sections involves infinite series expansions of algebraic terms and a finite number of admissible functions. In the theoretical calculation, the series expansion terms r and s were set at 30 and the numbers of admissible functions are 10 in ξ–direction and 10 in y–direction regardless of the symmetry of the vibration modes, respectively, which gave converged solutions.

In order to check the validity of the proposed theory, three dimensional finite element analyses were also carried out for the system by using a commercial computer code, ANSYS (release 12.0). Finite element models were constructed with the container geometry, boundary conditions and material properties used in the theoretical calculation. The aluminium container had 360 mm in height, 300 mm in length, 240 mm in width and 3 mm in thickness. The liquid depth in z–direction was considered for two water levels, that is, $L = 0$ mm (dry condition) and L = 180 mm (50% water level). The physical properties of the aluminum containers material were as follows: modulus of elasticity = 69.0 GPa, Poisson's ratio = 0.3 and mass density = 2700 kg/m^3. The liquid filled in the containers had a density of 1000 kg/m^3. The viscosity and the compressibility of water were neglected in the finite element analyses.

Finite element analyses using a commercial computer code, ANSYS software, were performed to obtain the natural frequencies and the mode shapes of the rectangular containers in partially filled with water for the simply supported boundary condition as mentioned above. Three–dimensional finite element models were constructed with contained fluid elements (FLUID80) and elastic shell elements (SHELL63) of ANSYS software. The facing rectangular plates of the container were divided into 13824 (2×72×96) elastic shell elements with the same size and the bottom rectangular plate were divided into 5760 (60×96) elastic shell elements with the same size. Therefore, the total number of the shell elements for the containers was 19584. On the other hand, the liquid region of the finite element model consisted of 207360 (60×36×96) fluid elements with an identical size to build the 50% liquid depth ratio. The total number of 50 modal frequencies and the corresponding mode shapes were extracted and plotted in the finite element analyses, which employs the Block Lanczos method.

3.2 Comparison of theoretical and finite element results

The theoretical natural frequencies of the rectangular container for the dry condition are listed and compared with the finite element analyses result in Table 1. The symbol “A” represents an antisymmetric mode, the symbol “S” a symmetric mode, the symbol “I” an in-phase mode, “O” an out-of-phase mode. The numbers indicate the number of nodal lines. It was found that the discrepancies between the theoretical and finite element analyses results are less than approximately 2% within the 10th serial mode for the dry modes. These result indicates that the combination of the orthogonal polynomials can approximate the mode shapes of the dry rectangular container excellently for such as the simply supported boundary condition. Since each mode shape is symmetric or antisymmetric with respect to $x = 0$, the admissible functions were selected from the corresponding symmetric and antisymmetric modes so that matrices size in the theoretical calculation could be reduced.

The natural frequencies of the rectangular containers filled with water for the liquid depth L = 180 mm are listed in Table 2 within the 10th serial modes. However, the theoretical wet natural frequencies were not extracted yet. In the finite element analysis, it was observed that the wet natural frequencies decrease with the water depth due to an increase of hydrodynamic mass, as is well known.

3.3 Mode shapes

The typical dry and wet mode shapes of the containers with 50% water level were illustrated in Figures. 4 and 5, respectively. It was observed that the lower dry mode shapes of each plate in the container are almost identical to those of a single plate. However, the relative deformations of connected neighboring plates are not identical to each other due to the difference in the flexural rigidity between the neighboring plates of the container as shown in the 3rd, 7th and 8th modes of Figure 4. It could be found that the wet mode shapes of the container are slightly distorted from the classical dry mode shapes of the rectangular plate due to the hydrodynamic effect of water as shown in the 2nd, 5th and 7th modes of Figure 5. Especially the modes

Serial mode	Mode		Natural frequency (Hz)		Discrepancy (%)
	Side plates	Bottom plate	Theory	ANSYS	
1	O (0,0)	S (0,0)	115.7	115.2	0.43
2	I (0,0)	A (1,0)	123.1	122.1	0.82
3	O (1,0)	S (0,0)	172.6	172.2	0.23
4	O (0,1)	S (0,1)	245.5	242.8	1.11
5	I (0,1)	A (1,1)	246.8	246.3	0.20
6	I (1,0)	A (1,0)	283.8	281.7	0.75
7	O (1,1)	S (0,1)	289.3	287.2	0.73
8	O (1,0)	S (2,0)	301.9	300.7	0.40
9	I (1,1)	A (1,1)	403.3	401.8	0.37
10	O (1,1)	S (2,1)	416.0	414.4	0.39

Table 1: Natural frequencies of a dry rectangular container with the simply supported boundary condition at both top edges.

Serial mode	Mode		Natural frequency (Hz) by ANSYS
	Side plates	Bottom plate	
1	O (0,0)	S (0,0)	44.4
2	O (0,0)	S (0,0)	59.7
3	I (0,0)	A (1,0)	61.9
4	O (1,1)	S (0,1)	88.6
5	O (0,1)	S (0,1)	119.9
6	I (0,1)	A (1,1)	121.2
7	I (1,0)	A (1,0)	156.7
8	O (1,0)	S (0,2)	166.3
9	I (1,0)	A (1,0)	187.3
10	O (1,0)	S (0,2)	188.3

Table 2: Natural frequencies of a wet rectangular container with the simply supported boundary condition at both top edges (50% liquid level).

shapes of the lateral plates are identical, but they are in−phase or out−of−phase pattern. In the rectangular container, two apparently similar modes in the vertical plates, such as the 1st and 2nd modes in Figure 1, were observed. These two mode shapes had zero nodal lines in the height and the length, but they had different phases. The fundamental mode shows an out−of−phase mode (symmetric mode), on the other hand, the 2nd mode shows an in−phase mode (antisymmetric mode). Generally, it was observed that the natural frequency of the in−phase modes were slightly larger than those of the corresponding out−of−phase modes.

3.4 Liquid depth effect

It is well known that the natural frequencies in the wet condition decrease from those of in the dry condition owing to the added mass effect of the contained liquid. The natural frequencies are plotted as a function of the liquid level in Figure 6. It was observed that the mode shapes and the natural frequencies of the liquid-filled container change according to the liquid level as shown in Figures 4 ~ 6. The wet natural frequencies of the container drastically decrease in the lower liquid level. Especially, the natural frequencies of the first, 4th and 6th modes decrease to about

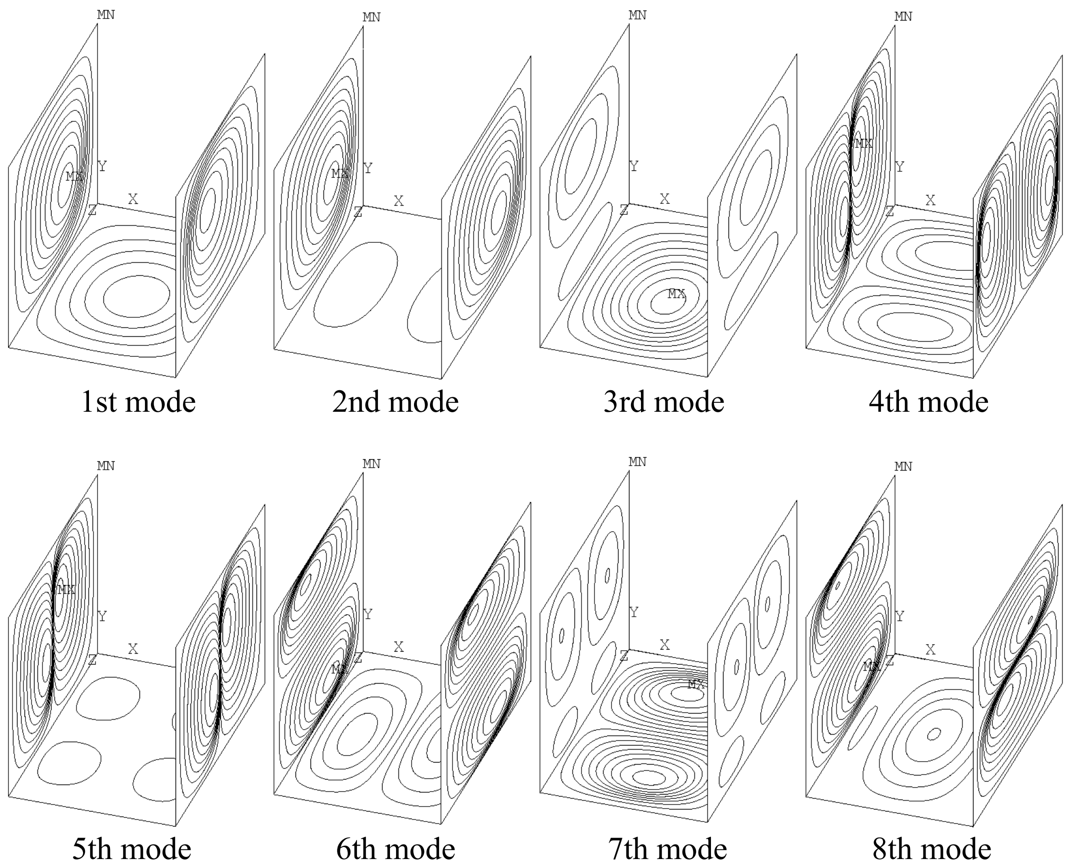


Figure 4: Mode shapes of a dry container.

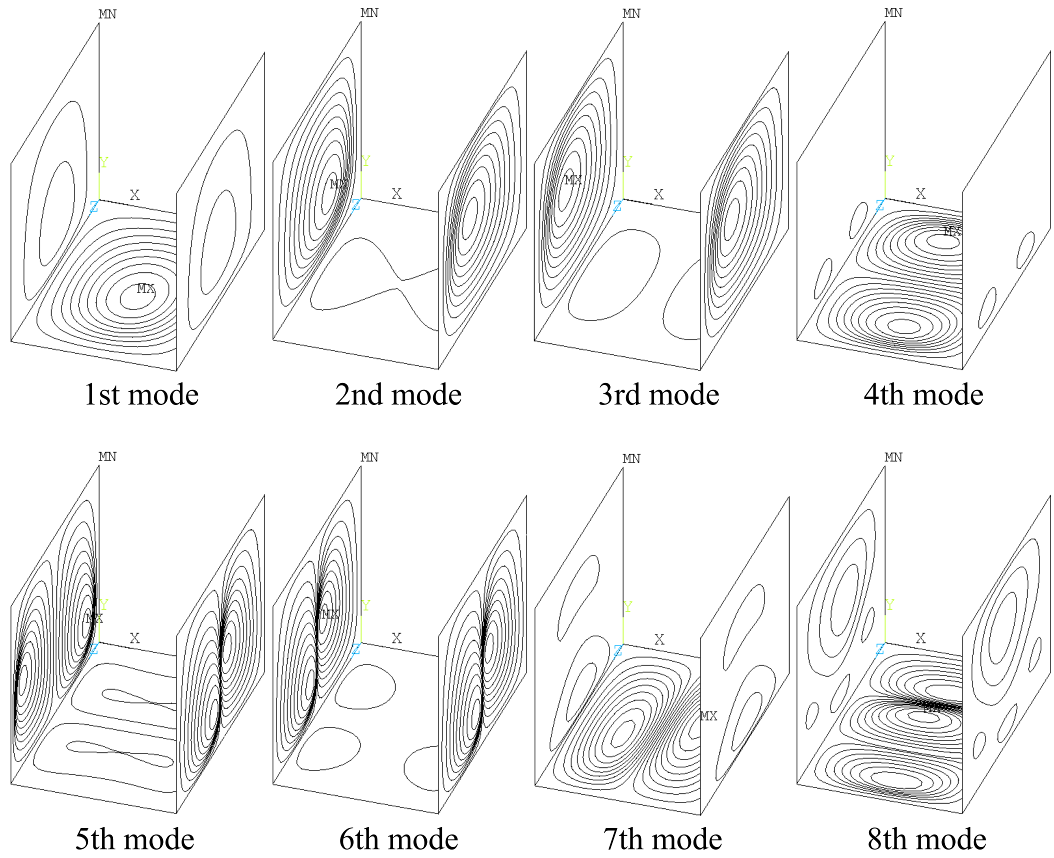


Figure 5: Mode shapes of a wet container with a half liquid level.

80% of the total decrease in the range of 0% ~ 20% liquid level as shown in Figure 6. Because the major dynamic deformation is concentrated on the bottom plate in these modes. Hence the added mass effect are very sensitive at the lower liquid level. On the contrary, as the 2nd and 5th modes show a little deformation in the bottom plate, the natural frequencies are less sensitive for the lower liquid level. The natural frequency of the in−phase modes are higher than those of the out−of−phase modes. The 2nd wet mode of Figure 5 is out−of−phase mode, but the 3rd wet mode is in−phase mode for the two lateral plates. Therefore, the natural frequency of the 2nd mode is less than that of 3rd mode. This is the same in the 5th and 6th modes.

4 Conclusions

An analytical formulation based on the Rayleigh−Ritz method for natural frequencies of a simply supported rectangular container partially filled with a liquid was developed. The wet dynamic displacement of the container was approximated by combining the orthogonal polynomial functions satisfying the boundary conditions at the top and side edges of the container.

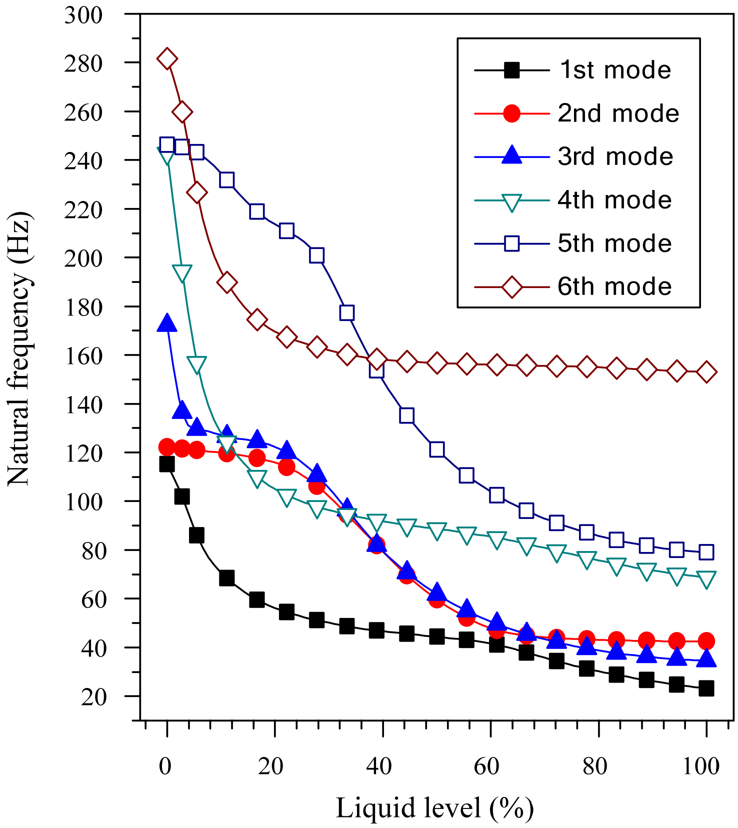


Figure 6: Effect of liquid level on the natural frequencies.

The liquid displacement potential satisfying the liquid boundary conditions and the Laplace equation were derived and the wet dynamic modal functions of the container were expanded in terms of the finite Fourier series for a compatibility requirement along the contacting surface between the container and the liquid. From the appropriate liquid displacement potential and admissible modal functions, the total kinetic and the potential energies of the liquid coupled system were extracted. Finally, the Rayleigh–Ritz method led to an eigenvalue equation to calculate the natural frequencies of the U-type rectangular container. The theoretical formulation for the dry condition was verified using the finite element analysis. It was found that the natural frequency of the modes with a large deformed bottom plate is very sensitive to the liquid height in the lower liquid level.

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