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Damage Detection of Truss Structures using an Improved Charged System Search Algorithm

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Abstract

In this paper, detection and assessment of structural damage using the changes in a structure's natural frequencies is addressed as an optimization problem. The damaged element(s) and the percentage of damage are considered as the problem variables. The objective is to set these variables such that the natural frequencies of the model correspond to the experimentally measured frequencies of the actual damaged structure.

This is a problem with several global optimal solutions each representing a probable state of damage. Obviously, unlike many other optimization problems, it is not sufficient to find one of these optimal solutions; it is important to find all such possible states and to compare them. On the other hand, meta-heuristic optimization algorithms tend to converge to a single solution in each run. These algorithms do not generally use any optimality criterion and evaluate the quality of solutions only by direct comparison. Experimental results show that some of the optimal solutions have a greater probability of being found by the algorithms than others. In fact the algorithm agents neglect some of the promising regions of the search space to the benefit of some others.

In this paper, the charged system search algorithm is improved and utilized to tackle the problem of finding as many global optimal solutions as possible in a single run.

Keywords: damage detection, charged system search, truss structures, change of frequencies.

1 Introduction

Structures can be damaged due to many different reasons in their lifespan. Finding the locations and the measurements of these damages, which is undeniably important to maintain the structural safety, is not always possible through visual inspection. Therefore, the responses of the structure and the changes occurred in them due to damage is viewed as a means to assess structural damage. Being accurately measurable and independent from the external excitation, natural frequencies of the structure are among the best response candidates for this purpose [1].

One of the most important aspects of evaluation of structural systems and ensuring their lifetime safety is structural damage detection [2]. Damages may be caused due to different reasons from manufacturing defects in structural materials to deterioration under service loads. These damages may endanger structure's integrity and functionality and need to be accurately detected.

Damage causes changes in structural parameters (e.g., the stiffness of a structural member), which in turn, alter the dynamic properties (such as natural frequencies and mode shapes) [3]. Among different structural responses that can be used as measures of structural damage, modal parameters enjoy the benefit of being independent form external excitation. Natural frequencies are more easily obtainable than mode shapes and less vulnerable to experimental errors. So, they have been used extensively in the formulation of inverse problems of damage detection. An inverse problem may be defined as determination of the internal structure of a physical system from the system's measured behavior or identification of the unknown input that gives rise to a measured output signal [4].

One of the earliest uses of natural frequencies for structural damage detection is due to Cawley and Adams [5]. Hassiotis and Jeong used an observation of the sensitivity of eigen frequencies to local stiffness reduction to detect the reduction in stiffness [6]. Nikolakopoulos et al. [7] used contour graph forms to show the dependency of the first two structural eigen frequencies on crack depth and location. Ruotolo and Surace utilized a genetic algorithm to address the problem of non-destructive location and depth measurement of cracks in beams formulated as an inverse optimization [8]. Cerri and Vestroni investigated the problem of finding damaged zones in beam models using the reduction of the stiffness occurring in the damaged region. They used natural frequencies to measure this stiffness reduction [9]. Liu and Chen [10] explored the problem in frequency domain introducing a computational inverse technique for identifying stiffness distribution on structures using structural dynamics response. Maity and Tripathy [11] used a genetic algorithm for the detection of structural damage by the use of changes in natural frequencies. Sahoo and Maity [12] proposed a hybrid neuro-genetic algorithm and considered both natural frequencies and strains as input parameters to address the problem of damage detection. Mehrjoo et al. [2] used artificial neural networks for the damage detection of truss bridge joints using both natural frequencies and mode shapes.

Charged System Search (CSS) is a population based meta-heuristic optimization algorithm which has been proposed recently by Kaveh and Talatahari [13]. In the CSS each solution candidate is considered as a charged sphere called a Charged Particle (CP). The electrical load of a CP is determined considering its fitness. Each CP exerts an electrical force on all the others according to the Coulomb and Gauss laws from electrostatics. Then the new positions of all the CPs are calculated utilizing Newtonian mechanics, based on the acceleration produced by the electrical force, the previous velocity and the previous position of each CP. Many different structural optimization problems have been successfully solved by the CSS [13-16].

In this paper an improved Charged System Search is utilized for the damaged detection of truss structures using changes in natural frequencies. This is an inverse optimization problem with several probable global optimal solutions and the improvements on CSS are directed toward the proper handling on these global optima.

The remainder of this paper is organized as follows: The formulation of the problem under consideration is briefly stated in Section 2. In Section 3, the optimization algorithm is presented. A brief background of the standard CSS is also represented. Numerical examples are studied in Section 4. Finally, the concluding remarks are provided in Section 5.

2 Problem Formulation

In this section, damage detection of structures using changes in natural frequencies is briefly described. Displacement based finite element equations are summarized first.

2.1 Finite Element Equations

A planar/spatial truss structure is modeled using two dimensional bar elements with two/three degrees of freedom at each end. From finite elements theory, the corresponding stiffness and mass matrices in element coordinate system can be expressed as [17]:

$$[\mathrm{k}] = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{1}$$

$$[\mathrm{m}] = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{2}$$

Where A, E, L and ρ are cross-sectional area, modulus of elasticity, length and density of the member, respectively. These matrices can be transformed into global coordinates using following relations:

$$[\mathrm{K}] = [\mathrm{T}]^{\mathrm{t}}[\mathrm{k}][\mathrm{T}] \tag{3}$$

$$[\mathrm{M}] = [\mathrm{T}]^{\mathrm{t}}[\mathrm{m}][\mathrm{T}] \tag{4}$$

in which t is the transformation matrix. For Planar truss the transformation matrix [T] can be written as:

$$[T] = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix} \tag{5}$$

Where c = $\cos\alpha$ and s= $\sin\alpha$, α being the angle between the element and the global axis X. Similarly, for a spatial truss the transformation matrix [T] can be written as:

$$[T] = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 & 0 & 0 & 0 \\ \eta_1 & \eta_2 & \eta_3 & 0 & 0 & 0 \\ \zeta_1 & \zeta_2 & \zeta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_1 & \xi_2 & \xi_3 \\ 0 & 0 & 0 & \eta_1 & \eta_2 & \eta_3 \\ 0 & 0 & 0 & \zeta_1 & \zeta_2 & \zeta_3 \end{bmatrix} \tag{6}$$

Where $\{\xi_1, \eta_1, \zeta_1\}$ are the direction cosines of the global axis X with respect to local xyz coordinate system. Similarly, $\{\xi_2, \eta_2, \zeta_2\}$ and $\{\xi_3, \eta_3, \zeta_3\}$ are direction cosines of global Y and Z axis with respect to *xyz* coordinate system respectively.

The dynamic equation which governs the behavior of an undamped structure is:

$$[M]\{\ddot{x}\}+[K]\{x\}= 0 \tag{7}$$

2.2 Damage Formulation

Here, damage is considered as a reduction in stiffness which is incorporated into the equations by a reduction factor β. When damage occurs in an element, the stiffness matrix of the element is modified as:

$$[k_{id}] = \beta_i\,[k_i] \tag{8}$$

Here, the parameter β ranges from 0.4 to 1 introducing a maximum of 60 percent damage in each element.

The mass matrix [M] of the structure is assumed to be unchanged. The *j*th eigenvalue equation of the damaged structure will be derived by substitution of the structure's stiffness matrix by that of the damaged one:

$$[K_d]\{\phi_{jd}\} - \lambda_{jd}[M]\{\phi_{jd}\} = \{0\} \tag{9}$$

in which, λ_{jd} and ϕ_{jd} are the *j*th natural frequency and the *j*th shape mode of the damaged structure, respectively.

2.3 Objective Function

The objective function of the optimization is considered as:

$$F(X) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(f_i^a - f_i^c)^2} \tag{10}$$

Where, X is the solution vector representing the state of damage; n is the number of natural frequencies involved in the objective function; f_i^a and f_i^c are the ith actual (measured) and computed natural frequencies, respectively.

3 Optimization Algorithm

Charged system search (CSS) algorithm introduced by Kaveh and Talatahari [13] with some modifications is used here as the optimization algorithm. In this section, standard CSS is first represented briefly. Then, the altered features are discussed using an illustrative example.

3.1 Standard CSS

CSS is a population based meta-heuristic algorithm proposed by Kaveh and Talatahari [13]. This algorithm is based on laws from electrostatics of physics and Newtonian mechanics.

The Coulomb and Gauss laws provide the magnitude of the electric field at a point inside and outside a charged insulating solid sphere, respectively, as follows [18]:

$$E_{ij} = \begin{cases} \dfrac{k_e q_i}{a^3} r_{ij} & if\ r_{ij} < a \\ \dfrac{k_e q_i}{r_{ij}^{\ 2}} & if\ r_{ij} \geq a \end{cases} \tag{11}$$

Where k_e is a constant known as the Coulomb constant; r_{ij} is the separation of the centre of sphere and the selected point; q_i is the magnitude of the charge; and a is the radius of the charged sphere. Using the principle of superposition, the resulting electric force due to N charged spheres is equal to [13]:

$$F_j = k_{eq}\sum_{i=1}^{N}\left(\frac{q_i}{a^3} r_{ij}.i_1 + \frac{q_i}{r_{ij}^{\ 2}}.i_2\right)\frac{r_i - r_j}{\left\| r_i - r_j \right\|} \quad \begin{cases} i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{cases} \tag{12}$$

Also, according to Newtonian mechanics, we have [6]:

$$\Delta r = r_{new} - r_{old} \tag{13}$$

$$v = \frac{r_{new} - r_{old}}{\Delta t} \tag{14}$$

$$a = \frac{v_{new} - v_{old}}{\Delta t} \tag{15}$$

Where r_{old} and r_{new} are the initial and final positions of the particle, respectively; v is the velocity of the particle; and a is the acceleration of the particle. Combining the above equations and using Newton's second law, the displacement of any object as a function of time is obtained as [18]:

$$r_{new} = \frac{1}{2}\frac{F}{M}.\Delta t^2 + v_{old} + r_{old} \tag{16}$$

Inspired by the above electrostatic and Newtonian mechanics laws, the pseudo-code of the CSS algorithm is presented as follows [15]:

Level 1: Initialization

Step 1. Initialization. Initialize the parameters of the CSS algorithm. Initialize an array of charged particles (CPs) with random positions. The initial velocities of the CPs are taken as zero. Each CP has a charge of magnitude (q) defined considering the quality of its solution as:

$$q_i = \frac{fit(i) - fit_{worst}}{fit_{best} - fit_{worst}} \qquad i = 1,2,\ldots.N \tag{17}$$

Where fit_{best} and fit_{worst} are the best and the worst fitness of all the particles; $fit(i)$ represents the fitness of agent i. The separation distance r_{ij} between two charged particles is defined as:

$$r_{ij} = \frac{\left\| X_i - X_j \right\|}{\left\| \frac{(X_i + X_j)}{2} - X_{best} \right\| + \varepsilon} \tag{18}$$

Where X_i and X_j are the positions of the ith and jth CPs, respectively; X_{best} is the position of the best current CP; and ε is a small positive to avoid singularities.

Step 2. CP ranking. Evaluate the values of the fitness function for the CPs, compare with each other and sort them in increasing order.

Step 3. CM creation. Store the number of the first CPs equal to charged memory size (CMS) and their related values of the fitness functions in the charged memory (CM).

Level 2: Search

Step 1. Attracting force determination. Determine the probability of moving each CP toward the others considering the following probability function:

$$P_{ij} = \begin{cases} 1 & \dfrac{fit(i) - fit_{best}}{fit(j) - fit(i)} > rand \vee fit(i) > fit(j) \\ 0 & else \end{cases} \tag{19}$$

and calculate the attracting force vector for each CP as follows:

$$F_{ij} = q_j \sum_{i,i \neq j} \left(\frac{q_i}{a^3} r_{ij} \, . \, i_1 + \frac{q_i}{r_{ij}^2} \, . \, i_2 \right) p_{ij} \left(X_i - X_j \right) \quad \begin{cases} j = 1,2,\ldots,N \\ i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{cases} \tag{20}$$

Where F_j is the resultant force affecting the jth CP.

Step 2. Solution construction. Move each CP to the new position and find its velocity using the following equations:

$$X_{j,new} = rand_{j1} \, . \, k_a \, . \, \frac{F_j}{m_j} \, . \, \Delta t^2 + rand_{j2} \, . k_v \, . \, V_{j,old} \, . \, \Delta t + X_{j,old} \tag{21}$$

$$V_{j,new} = \frac{X_{j,new} - X_{j,old}}{\Delta t} \tag{22}$$

Where $rand_{j1}$ and $rand_{j2}$ are two random numbers uniformly distributed in the range (1,0); m_j is the mass of the CPs, which is equal to q_j in this paper. The mass concept may be useful for developing a multi-objective CSS. Δt is the time step, and it is set to 1. k_a is the acceleration coefficient; k_v is the velocity coefficient to control the influence of the previous velocity. In this paper k_v and k_a are taken as:

$$k_a = c_1(1 + iter/iter_{max}), \quad k_v = c_2(1 - iter/iter_{max}) \tag{23}$$

Where c_1 and c_2 are two constants to control the exploitation and exploration of the algorithm; iter is the iteration number and $iter_{max}$ is the maximum number of iterations.

Step 3. CP position correction. If each CP exits from the allowable search space, correct its position using the HS-based handling as described by Kaveh and Talatahari [13].

Step 4. CP ranking. Evaluate and compare the values of the fitness function for the new CPs; and sort them in an increasing order.

Step 5. CM updating. If some new CP vectors are better than the worst ones in the CM, in terms of their objective function values, include the better vectors in the CM and exclude the worst ones from the CM.

Level 3: Controlling the terminating criterion

Repeat the search level steps until a terminating criterion is satisfied.

3.2 Improved CSS

Damage detection inverse optimization problems may have more than one global optimal solution. Since the objective is to detect the damage occurred in the structure, it is important to obtain all these optimal solutions and to compare them. On the other hand, meta-heuristic optimization algorithms, including CSS, generally seek for a single solution in each run. These algorithms evaluate the quality of the solutions through direct comparison and do not utilize any optimality criterion. Thus, having access to the optimal value of the objective function, the problem under consideration can be viewed as an opportunity to adapt optimization algorithms such that they can reach as many global optimal solutions as possible in a single run.

In order to further illustrate the idea, consider the example of a ten-bar planar truss shown in Figure 1. Since the structure is symmetric with respect to the horizontal axis, it is apparent that some different states of damage will give rise to similar changes in the natural frequencies (e.g. equal percentage of damage in elements 1 and 3). These different states result in an identical value of the objective function and the optimization algorithms will only attain one of them on a random basis in each run.

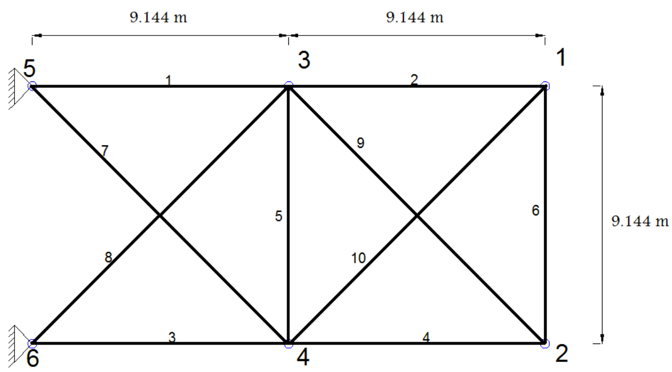

Figure 1. A ten bar truss

Although meta-heuristic algorithms are supposed to explore the search space randomly, it is observed that the chance of being found is not equal for the similar solutions mentioned above; i.e. the algorithm neglects some of the solutions to the benefit of some others most of the time. This is probably because of the characteristics of the search space which makes these solutions more easily accessible in comparison to the others. This more highlights the severity of the problem. In the case of paired elements 1 and 3 for example, the standard CSS finds element 3 as the damaged element almost always. This is probably because of element 1 being located at the boundary of the search space which makes it rather hard to access compared to element 3. This phenomenon also underlines the significant effect of side constraints handling and element numbering tasks on the results of the optimization procedures which may be addressed in a separate study.

In order to make the CSS algorithm capable of doing a more extensive search in the search space and to find all of the optimal solutions the following simple improvements are carried out:

1. When a global optimal solution is found, it is saved in a separate memory called Optimal Solution Memory. In this study, optimal solutions are characterized by their corresponding objective function value of zero; however, any other optimality criterion could be used depending on the nature of the problem. A value of 2.5×10^{-5} is used here instead of zero due to approximation errors.
2. Once an optimal solution is found, the charged memory is cleared in order to make the algorithm's effort for finding other solutions visible in the convergence curves.
3. The algorithm starts the next phase of the optimization process by producing new solutions randomly.
4. In order to keep algorithm away for the previously obtained solutions, their corresponding solutions vectors are eliminated from the search space i.e. the regions corresponding to these vectors are treated as prohibited zones.

4 Numerical Examples

In this section numerical results are presented to demonstrate the viability of the improved CSS algorithm to detect structural damage in truss structures. First three natural frequencies are used as the input response parameters to the inverse problem. A population of 20 CPs is used in all cases.

4.1 A ten-bar planar truss

The ten-bar planar truss of Figure 1 is considered as the first numerical example. A non-structural mass of 454.0 kg is attached to the free nodes. This structure has been used as an example in the field of structural optimization by several researchers (Grandhi and Venkayya [19], Sedaghati et al. [20], Lingyun et al. [22] and Kaveh and Zolghadr [16] among others). Table 1 represents the properties of this example.

Property/unit	Value
E (modulus of elasticity)/ N/m^2	6.98×10^{10}
ρ (material density)/ kg/m^3	2770.0
Added mass/kg	454.0
L (main bar's dimension)/m	9.144
A (cross-sectional area of the members)/m^2	0.0025

Table 1. material properties of the ten-bar planar truss

This example is considered in two cases:

Case 1: 50 percent damage in element 1; (50 percent of damage in element 3 will result in the same set of natural frequencies).

Case 2: 30 percent damage in element 2 and 60 percent damage in element 4; (30 percent damage in element 4 and 60 percent damage in element 2 will result in the same set of natural frequencies).

Each of the cases has a twin considering the structure's symmetry. Table 2 represents the values of the natural frequencies of the undamaged and damaged structures.

Frequency number	Undamaged structure	Damaged structure	
		Case 1	Case 2
1	6.6421	5.8908	6.5376
2	19.9916	18.1815	18.0504
3	21.4192	21.3799	20.0068
4	37.8057	37.5393	35.7983
5	43.2929	43.2520	42.2003
6	49.1099	48.1646	44.7563
7	50.6731	49.7354	47.8243
8	58.2766	57.7033	54.9885

Table 2. Natural frequencies of the undamaged and damaged structures (the ten-bar truss)

Each of these cases is solved 100 times using the standard CSS. In case 1, the standard CSS never detects element 1 as the damaged element and in all of the runs converges to the solution containing element 3, i.e. the twin optimal solution. As mentioned before, this is because of the location of element 1 in the search space. In case 2, the ratio of the convergence of the standard CSS to each of the optimal solutions is 41/59. Naturally, the standard CSS never obtains both of the optimal solutions in a single run. Variation of the objective function with number of iterations using the standard CSS for an arbitrary run is shown in Figure 2. It can be seem from the figure that the standard CSS does not try to find the other optimal solution after converging to the first one.

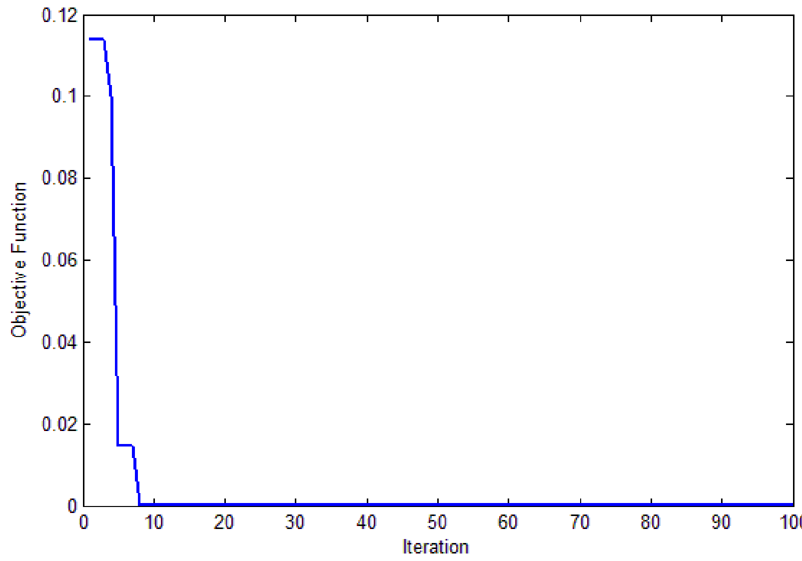

Figure 2. Variation of the objective function with number of iterations for the ten-bar truss using standard CSS (Case 1)

The improved CSS on the other hand, obtains both of the optimal solutions resulting in the same set of natural frequencies in each run. Figures 3 and 4 represent the variation of the objective function with the number of iterations for case 1 and case 2, respectively.

The sudden increase in the value of the objective function has happened after finding the first optimal solution and is due to the charged memory being cleared up. At the end of each run, both optimal solutions are detected and saved in the Optimal Solution Memory.

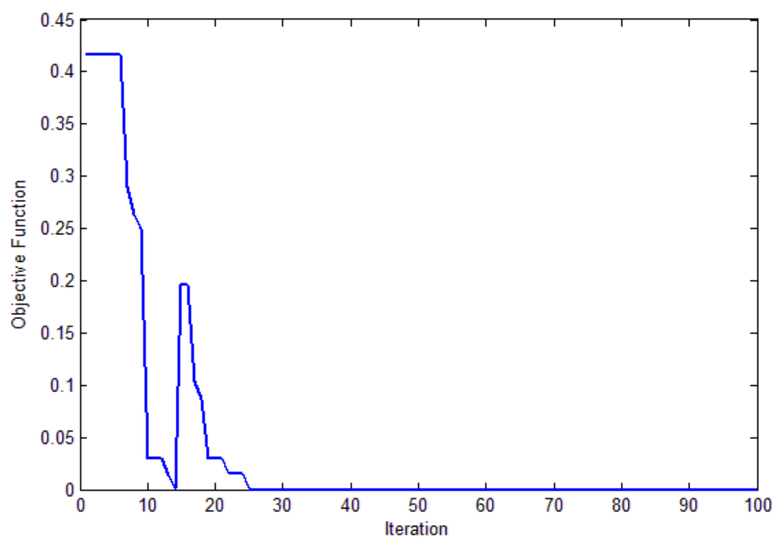

Figure 3. Variation of the objective function with number of iterations for the ten-bar truss using improved CSS (Case 1)

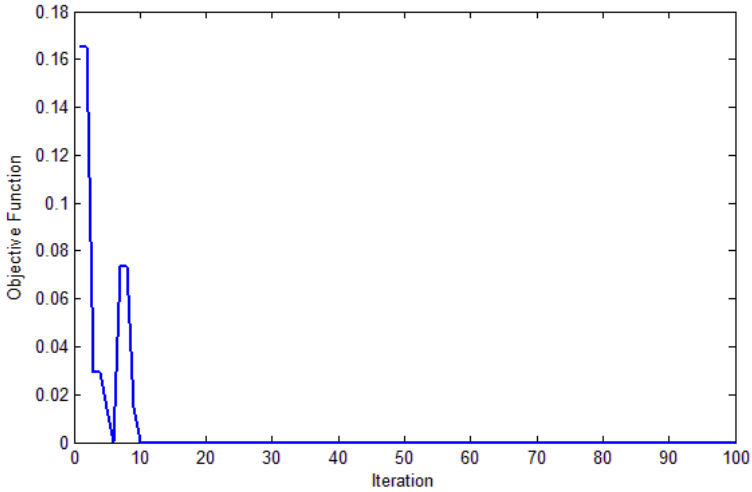

Figure 4. Variation of the objective function with number of iterations for the ten-bar truss using the improved CSS (Case 2)

4.2 A 72-bar spatial truss

A 72-bar spatial truss is considered as the second numerical example, Figure 5. Four non-structural masses of 2270 kg are attached to the nodes 1 through 4. This structure has also been investigated as an example in the field of structural optimization with frequency constraints by different researchers (Konzelman [22], Sedaghati [23] and Kaveh and Zolghadr [16] among others). Table 3 represents the properties of this example.

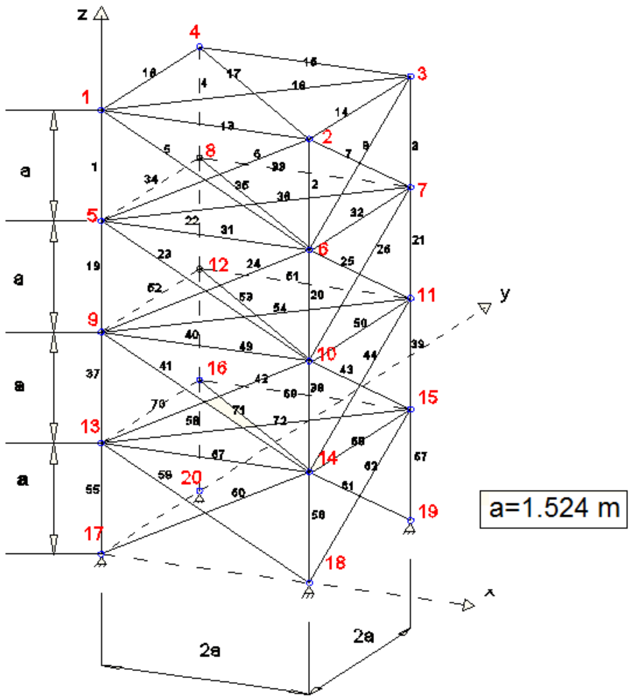

Figure 5. A 72-bar spatial truss

Property/unit	Value
E (modulus of elasticity)/ N/m^2	6.98×10^{10}
ρ (material density)/ kg/m^3	2770.0
Added mass/kg	2270
A (cross-sectional area of the members)/m^2	0.0025

Table 3. Properties of the 72-bar space truss

Three cases of damage are assumed for this structure:

Case 1: 40 percent of damage in element 55; (40 percent of damage in each of the vertical members of the first storey will result in the same set of natural frequencies)

Case 2: 20 percent of damage in element 37 and 60 percent of damage in element 39; (20 percent of damage in element 39 and 50 percent of damage in element 37 will result in the same set of natural frequencies)

Case 3: 20 percent of damage in element 5, 50 percent of damage in element 6, and 60 percent of damage in element 31; (rotations of 90, 180, and 270 degrees along the z axis will result in the same set of natural frequencies).

Table 4 represents the first 8 natural frequencies of the undamaged and damaged structures.

Frequency number	Undamaged structure	Damaged structure		
		Case 1	Case 2	Case 3
1	6.0434	5.7463	5.6205	5.9872
2	6.0441	6.0438	6.0439	6.0478
3	10.4627	10.4627	10.4634	10.3068
4	18.2275	17.9592	17.5178	18.2225
5	25.4466	25.4374	24.9517	25.2069
6	25.4510	25.4495	25.4524	25.4336
7	26.5189	26.5110	26.4362	26.4691
8	38.0799	38.0798	38.0770	37.2642

Table 4. Natural frequencies of the undamaged and damaged structures (the ten-bar truss)

Cases 1, 2, and 3 have 4, 2, and 4 optimal solutions, respectively. As an example, case 1 is first solved by the standard CSS to demonstrate that the algorithm does not attain all of the optimal solutions in a single run. Variation of the objective function with the number of iterations for this run is represented in Figure 6.

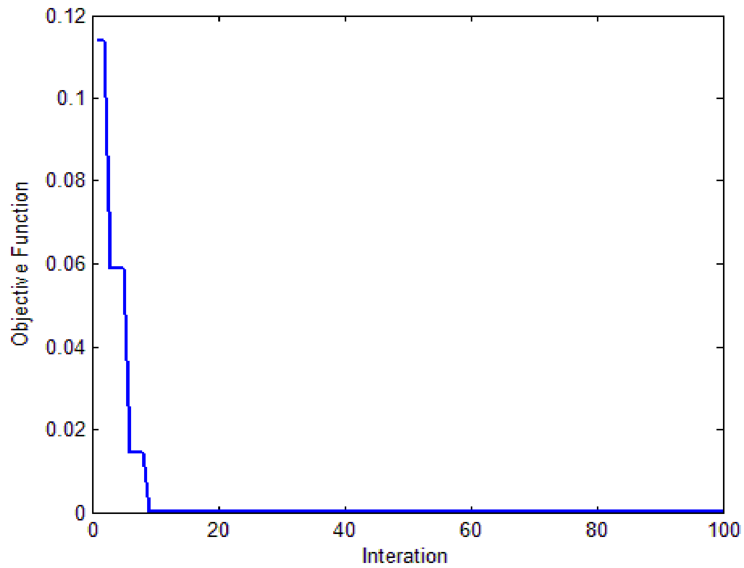

Figure 6. Variation of the objective function with the number of iterations for the 72-bar truss using the standard CSS (Case 1)

Figures 7, 8, and 9 represent the variation of the objective function with number of iterations using improved CSS for cases 1, 2, and 3, respectively.

In Figures 7 to 9, the capability of the improved algorithm for finding all of the global optimal solutions (damage states) is apparent. Every time that the algorithm finds a global optimal solution, the convergence curve touches the horizontal axis. Then, since the Charged Memory is cleared, the curve jumps up and starts another search process in order to find the next optimal solution.

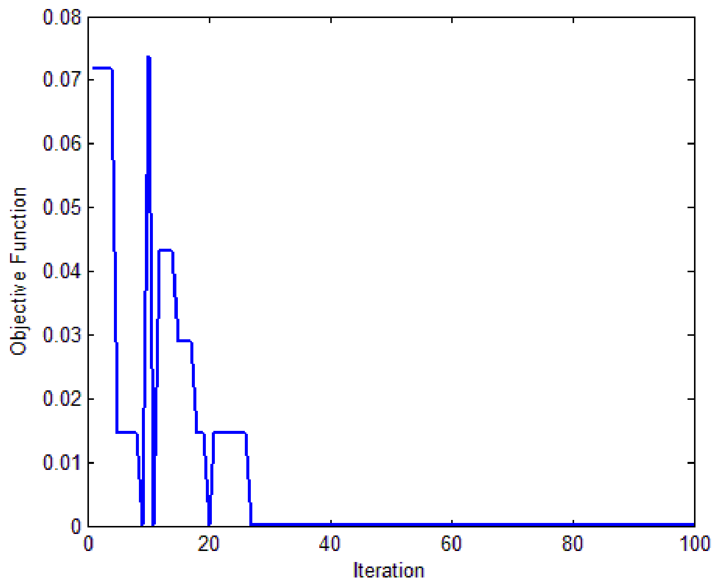

Figure 7. Variation of the objective function with the number of iterations for the 72-bar truss using the improved CSS (Case 1)

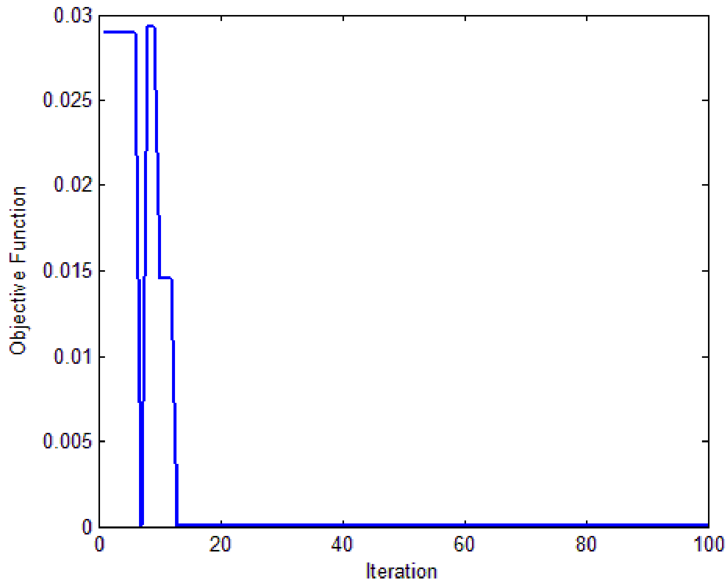

Figure 8. Variation of the objective function with the number of iterations for the 72-bar truss using the improved CSS (Case 2)

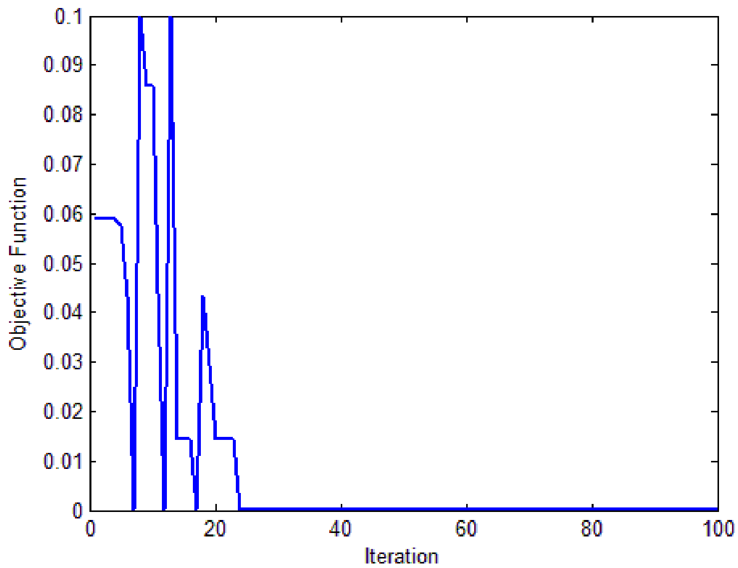

Figure 9. Variation of the objective function with the number of iterations for the 72-bar truss using the improved CSS (Case 3)

5 Concluding Remarks

Structural damage detection using changes in natural frequencies, formulated as an inverse optimization problem, is considered in this paper. This is a problem with several global optimal solutions in which finding all of the solutions is important. Here, this is viewed as a good opportunity to adapt a meta-heuristic optimization algorithm with the capability of finding all of the global optimal solutions in a single run.

The Charged System Search algorithm developed by Kaveh and Talatahari [13] is improved and utilized in order to find as many global optimal solutions as possible in a single run of the inverse optimization problem mentioned above.

Numerical examples are used to demonstrate the capability of the improved algorithm in finding all of the global optimal solutions (damage states) in each run. Comparison of the convergence curves show that while the standard CSS only finds one of the solutions, the improved version obtains all of them in a single run.

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