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Paper 88

The Passive Electric Potential Computed Tomography Method for Identification of a Part-Through Three-Dimensional Crack

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Abstract

The present authors proposed the passive electric potential computed tomography (CT) method for crack identification using the electric potential distribution on the surface of a piezoelectric film pasted on a cracked member. In the present study the passive electric potential CT method was applied to the identification of part-through three-dimensional cracks. The least residual method was used in the inverse analysis, in which the combination of crack parameters minimizing the residual between the measured and calculated electric potential distributions was sought. It was found that the passive electric potential CT method was effective for the estimation of the crack location and crack size of the part-through cracks.

Keywords: non-destructive evaluation, crack identification, electric potential method, piezoelectric film, passive method.

1 Introduction

Increase in a number of aged plants and their components gives rise to increase in importance of non-destructive evaluation and monitoring. The online monitoring has an advantage that it does not require shutdown of the plants. The electric potential method was developed for the identification of cracks using electric potential distribution under electric current application to the cracked body [1, 2]. By applying the inverse analysis schemes [3-7] to the electric potential distribution appearing on the surface of a cracked body, the present authors [8-10] proposed the active electric potential CT (Computed Tomography) method for quantitative identification of cracks.

The piezoelectric film was applied to the damage monitoring in composites by Gelea et al [11] and Tin et al [12]. When the piezoelectric film is pasted on the surface of cracked body subjected to a mechanical load, the electric potential distribution develops on the film due to the direct piezoelectric effect. The present authors [13-23] proposed the passive electric potential CT (Computed Tomography) method using the electric potential distribution on the piezoelectric film. It was found that a characteristic electric potential distribution appeared on the piezoelectric film when a crack existed in the body. The crack can be identified from the electric potential distribution. An inverse analysis was conducted for crack identification based on the comparison of the electric potential distribution measured on the piezoelectric film and that obtained by the finite element method (F.E.M.).

In the present study the passive electric potential CT method is applied to the identification of a part-through three-dimensional crack. Numerical simulations are made on the applicability of the method to the identification of the part-through crack.

2 The passive electric potential CT method

2.1 Specimen and piezoelectric film

For the passive electric potential CT method, a piezoelectric film is pasted on the structural material as shown in Figure 1. When the structure is subjected to a mechanical load, the strain distribution on the piezoelectric film induces the electric potential distribution on the piezoelectric film due to the direct piezoelectric effect. If a crack exists in the structure, there are changes in the strain distribution and the electric potential distribution depending on the crack size and location. The passive electric potential CT method uses the change in the measured electric potential distribution to identify the crack using the inverse analysis.

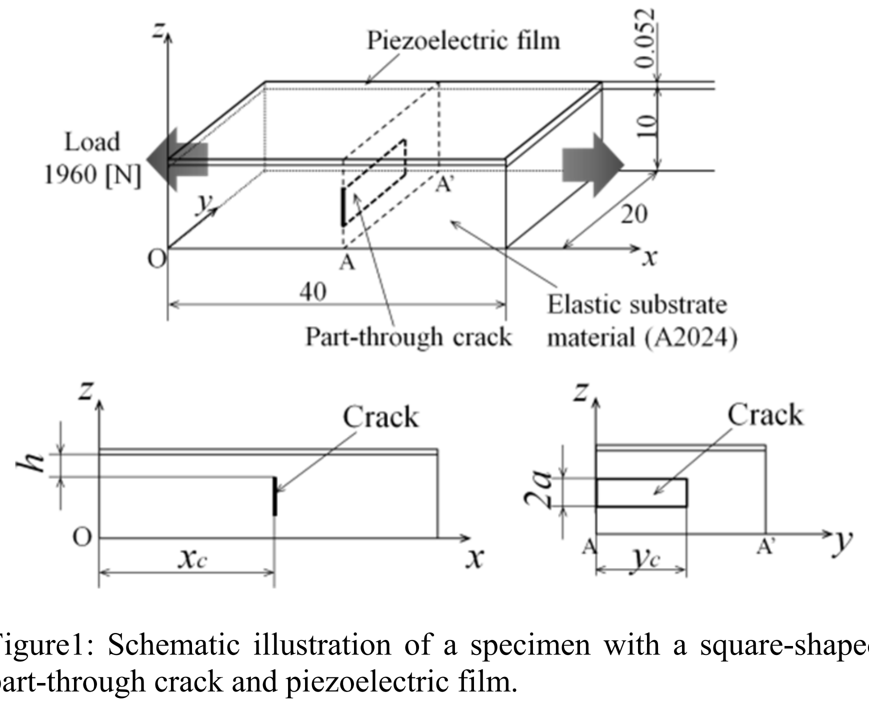

Figure1: Schematic illustration of a specimen with a square-shaped part-through crack and piezoelectric film.

2.2 Inverse analysis method of the passive electric potential CT method

When piezoelectric film is glued on the surface of cracked plate subjected to mechanical load, a change in electric potential distribution is observed on the surface of piezoelectric film. The F.E.M. computer analysis scheme for coupled elastic and electric potential problem was developed to investigate the relationship between crack parameters and electric potential distribution on the piezoelectric film [13]. The governing equations of the coupled strain field and electric field of piezoelectric material can be written as:

$$\{\sigma\} = [C]\{\varepsilon\} - [e]^T\{E\} \tag{1}$$

$$\{D\} = [e]\{\varepsilon\} + [g]\{E\} \tag{2}$$

Here $\{\sigma\}$ and $\{\varepsilon\}$ are stress vector and strain vector, [C], [e] and [g] are stiffness matrix, piezoelectric coefficient matrix and dielectric constant matrix, respectively. {E} is electric field vector. {D} is electric displacement vector. Based on Equations (1) and (2), the static F.E.M. equations are obtained as:

$$[\boldsymbol{K}_{uu}]\{\boldsymbol{d}\} + [\boldsymbol{K}_{u\phi}]\{\boldsymbol{\phi}\} = \{\boldsymbol{F}\} \tag{3}$$

$$[\boldsymbol{K}_{u\phi}]\{\boldsymbol{d}\} + [\boldsymbol{K}_{\phi\phi}]\{\boldsymbol{\phi}\} = \{\boldsymbol{Q}\} \tag{4}$$

Here $[\boldsymbol{K}_{uu}]$, $[\boldsymbol{K}_{u\phi}]$ and $[\boldsymbol{K}_{\phi\phi}]$ are the mass matrix, displacement electric stiffness matrix and electric stiffness matrix, respectively. {F} and {Q} are the mechanical load vector and the electric load vector, respectively.

As the inverse analysis method for the identification of crack, the least residual method [10] was applied. The electric potential is computed by using the F.E.M. taking the piezoelectric coupled effect between mechanical strain and electric field into account. The computed electric potential values $\phi^{(c)}$ are compared with the measured values $\phi^{(m)}$ to determine the most plausible crack location and size. As a criterion for crack identification the following square sum R of residuals is calculated.

$$R\,(a, h, x_c, \ldots) = \sum_{i=1}^{M} \left(\phi_i^{(c)}(a, h, x_c, \ldots) - \phi_i^{(m)}\right)^2 \tag{5}$$

Here $\phi_i^{(m)}$ denotes the measured electric potential value at the i-th measuring point, and $\phi_i^{(c)}(a, h, x_c, \ldots)$ denotes the electric potential values at the i-th measuring point computed by the F.E.M. for assumed crack length a, crack depth h and crack location in the x-direction x_c, and other crack parameters. M is the total number of measuring points. The combination of crack location and size, which minimizes R, is employed as the most plausible one among all the assumed combinations of the crack location and size. For effective identification of the crack, a hierarchical calculation scheme is introduced as will be described later.

3 Identification of a square-shape part-through crack

3.1 Specimen with a square-shaped crack and electric potential distribution

The passive electric potential CT method was applied to the identification of a square-shape part-through crack shown in Figure 1. As the parameters describing the crack, crack length *a,* crack penetration length in *y*-direction y_c, crack depth *h,* and crack location x_c were used. The cracked specimen is subjected to mechanical load of 1960 (N).

Electric potential distribution of the piezoelectric film was calculated by the F.E.M. taking the coupled elastic and electric effect into account. Considering the measurement error, artificial random noise of the level of 2 % of remote electric potential was added to the distribution using uniform random function. The electric potential distributions on several constant *y* values are shown in Figure 2. It is seen in the figure that the electric potential distributions show two peaks around the location of crack. The measured potential distribution obtained by using the smart-layer agrees well with those obtained by the F.E.M. analyses [15]. The simulated noisy electric potential distributions shown in Figure 2 were used in the identification of the crack.

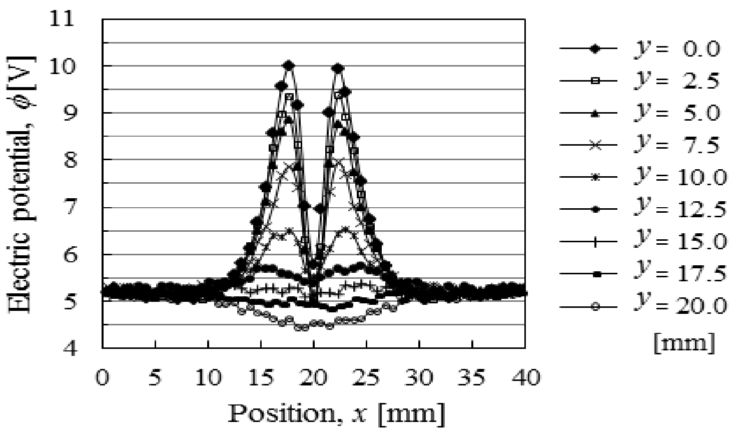

Figure 2: Electric potential with 2 % noise on piezoelectric film for a square-shape crack with $(a, y_c, h, x_c) = (3, 10, 2, 20)$.

3.2 Hierarchical inverse analysis scheme

For the identification of the crack a hierarchical inverse analysis scheme [16] was used, which was consisted of a rough estimation and a detailed estimation.

3.2.1 Step 1: rough estimation of crack parameters

It is known that crack location x_c agrees well with the point of local minimum of the electric potential [16]. The location of local minimum is taken as a rough estimate of the crack location x_c. It is also known that depth h is nearly equal to the half of the distance between two peaks in the electric potential distribution [16]. The half of the distance is taken as a rough estimate of crack depth h.

As for the effect of a and y_c, residual R is approximated by the following quadratic equation from value of R for several combinations of a and y_c.

$$R(a, y_c) = A + Ba + Cay_c + Da^2 + Ey_c + Fy_c^2 \tag{6}$$

A rough estimate of a and y_c, is obtained by minimizing R described by Equation (6).

3.2.2 Step 2: detailed estimation of crack parameters

The modified Powell optimization [24] is applied to obtain a detailed estimate of crack parameters. The result of the rough estimate is used as the initial values of crack parameters.

3.3 Result of crack identification

The result of identification of the square-shaped crack is shown in Table 1. In the table S denotes the area of crack. It is seen that all the crack parameters are estimated within the error of 1 % from the potential distribution with noise level of 2 %.

	a[mm]	y_c[mm]	S[mm^2]	h[mm]	x_c[mm]	R
Actual	3.0000	10.0000	60.0000	2.0000	20.0000	-
Start of search	3.3593	12.6221	84.8028	2.2500	20.0000	58.1
Estimated	3.0074	9.9128	59.6235	1.9954	20.0048	1.73
Error	+0.0074	-0.0872	-0.3765	-0.0046	+0.0048	-
	(0.25%)	(0.87%)	(0.63%)	(0.23%)	(0.024%)	-

Table 1 Estimated crack parameters for square-shaped crack (a, y_c, h, x_c) = (3, 10, 2, 20).

4 Identification of a round-tipped part-through crack

4.1 Specimen with a round-tipped crack and electric potential distribution

The passive electric potential CT method was applied to the identification of a round-tipped part-through crack shown in Figure 3. The tip shape of the crack is assumed to be semi-elliptical with axes of a and b. Then the parameters describing the crack are a, y_c, b, h and x_c.

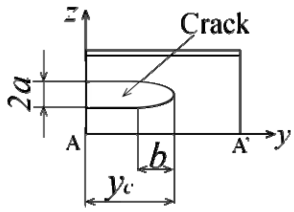

Figure3: A round-tipped part-through crack.

The electric potential distribution of the piezoelectric film was calculated by the F.E.M. Random noise of the level of 2 % of remote electric potential was added to the distribution using uniform random function. The electric potential distributions on the lines of several y values used for the crack identification are shown in Figure 4.

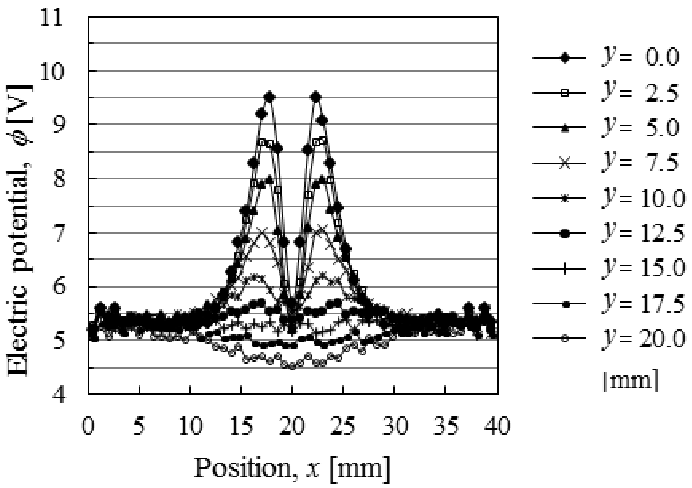

Figure 4: Electric potential with 2 % noise on piezoelectric film for a round-tipped crack with $(a, y_c, b, h, x_c) = (3, 10, 7, 2, 20)$.

4.2 Hierarchical inverse analysis scheme

For the identification of the crack a hierarchical inverse analysis scheme is used, which is consisted of a rough estimation and detailed estimation.

4.2.1 Step 1a: rough estimation of crack parameters assuming a square-shaped crack

Using a process described in 3.2.1 a rough estimation of crack parameters is made assuming that the crack is square-shaped.

4.2.2 Step 1b: rough estimation of crack parameters assuming a semi-circular-tipped crack

Using a process described in 3.2.1 a rough estimation of crack parameters is made assuming that the tip shape of crack is semi-circular.

4.2.3 Step 2: detailed estimation of crack parameters

The modified Powell optimization [24] is applied to obtain a detailed estimate of crack parameters. The R value obtained in Step 1a assuming a square-shaped crack and that obtained in Step 1b assuming semi-circular tipped crack are compared. The crack giving smaller R value is used as the initial crack in the optimization.

4.3 Result of crack identification

The result of identification of the round-tipped part-through crack is shown in Table 2. The estimated crack shape is shown in Figure 5. It is seen that although large error is including in the estimation of axis b other parameters are estimated reasonably.

	a[mm]	y_c[mm]	b[mm]	S[mm^2]	h[mm]	x_c[mm]	R
Actual	3.0000	10.0000	7.0000	50.9700	2.0000	20.0000	-
Start of search	3.0842	8.9592	3.0842	51.1736	2.0782	20.0046	2.84
Estimated	3.0562	9.2715	4.1493	51.2182	2.0702	20.0037	2.69
Error	+0.0562	-0.7285	-2.8507	+0.2482	+0.0702	+0.0037	-
	(1.9%)	(7.3%)	(41%)	(0.49%)	(3.5%)	(0.019%	-

Table 2 Estimated crack parameters for a round-tipped crack with $(a, y_c, b, h, x_c) = (3, 10, 7, 2, 20)$.

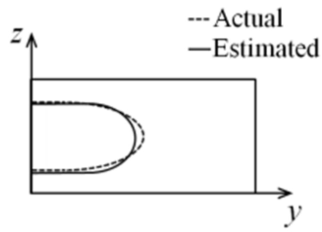

Figure 5: Estimated crack shape for a round-tipped crack with
(a, y_c, b, h, x_c) = (3, 10, 7, 2, 20).

5 Conclusions

The passive electric potential CT (Computed Tomography) method was applied to the identification of square-shape part-through crack and a round-tipped part-through crack. The cracks were identified based on the least residual between the measured and calculated electric potential distributions on the piezoelectric film pasted on the cracked specimen. It was found from numerical simulations that the passive electric potential CT method was effective for the estimation of the crack location and crack size of the part-through cracks even in the existence of 2 % potential noise.

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