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Influence of Incomplete Modal Data on the Performance of a Vibration-Based Damage Detection Methodology

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Abstract

The use of changes in the dynamic response of a structure after damage has become very popular in the scientific community to formulate methodologies that permit assessing the integrity of a structure. In this sense, a finite-element model (FEM) that represents the damaged structure can be obtained by minimizing the difference between the dynamic response of this model and the response of the current structure obtained experimentally. The optimization variables are composed of the stiffness reduction factor of each element that belongs to the FEM of the undamaged structure. This paper proposes the use of an adaptive Particle Swarm Optimizer to solve the optimization problem associated with the detection of the damage in a multi-supported beam structure and studies how incomplete data affect the performance of the proposed algorithm. Adaptation is implemented to avoid the definition of the PSO parameters - cognitive and social parameters - by trial and error. Natural frequencies and mode shapes were selected as the dynamic characteristics to be used in the objective function. As this research was developed by using numerical simulations, information about only a few first modes in specific degrees of freedom was considered available in order to take into account the incomplete measurement issue. Results have shown that a minimum quantity of modal data is necessary to guarantee the success of the damage detection methodology and that the ability to locate and quantify damage may not be improved by using excessive information. It has also been observed than simple damage scenarios can be more reliably detected than multiple ones.

Keywords: damage detection, dynamic parameters and particle swarm optimization.

1 Introduction

Civil, mechanical and aeronautical industries have been interested in developing non destructive damage detection methodologies to assess the integrity of their structures. Some of these methodologies are based on changes that occur in the dynamic parameters after damage. However, the application of the former concept to damage detection could be limited by issues, such as low sensitivity of the dynamic parameters in relation to the damage, high complexity level of the technique used to determine the damage scenario, possibility of the dynamic parameters being affected by factors different to the damage and incomplete measurements [1]. Concerning the latter issue, and if modal parameters are used, the incompleteness is related to the fact that it is not possible to excite all vibration modes of the structure and measure the mode shapes in all degrees of freedom, DOFs, in the finite-element model that represents the undamaged structure [2].

Some indications related to the influence of incomplete measurements on the performance of vibration-based damage detection methodologies have been reported in the literature.

Law et al. [3] proposed an energy-based damage detection methodology. The application of this formulation required complete mode shapes, which were obtained by using a mode shape expansion technique. This type of technique was preferred instead of a model reduction as it preserves the connectivity of the structure. The authors concluded that the smaller the number of measured DOFs, the larger the set of falsely damaged elements.

Kim and Bartkowicz [4] implemented a two-level damage detection approach based on updating and design sensitivity methods. The mismatch between the analytical and experimental models was solved by using a hybrid reduction/expansion technique. Numerical simulations showed that the quantity of DOFs with measured information influenced the correct identification of some damage scenarios.

Araújo dos Santos et al. [5] observed that the quality of the results of a damage detection methodology was determined by the technique used to deal with the incomplete measurements. Such an effect was more pronounced as the damage extent increased.

Reza and Medhi-pour [6] proposed to locate damage using a subspace rotation method and, then, quantify the damage extent by using some concepts from the control theory. They concluded that the increment in the number of measurements resulted in an increment in the successful damage cases found.

Raich and Liszkai [7] used a genetic algorithm with implicit redundant representation to take into account that the number and position of the damaged elements were not known *a priori*. The objective function was based on changes in the frequency response function matrix of the structure. They affirmed that an increment in the number of sensors used might not facilitate the damage identification in all cases, but that, in general, localization and quantification of damage would be performed more accurately.

Yun et al [8] proposed implementing the damage detection process in two stages. The authors worked with the first stage, which consisted in reducing the set of elements by identifying probably damaged zones with a subset selection method based on the residual vector force. It was observed that when the measurements were complete and noise-free then the damaged elements in a beam structure were located no matter the number of mode shapes used. If the measurements contained noise, then more reliable results were obtained when more mode shapes were used.

Meruane and Heylen [9] projected a real-coded genetic algorithm to detect damage and tested it in a spatial truss structure. They carried out an analysis of the influence of the quantity of sensors in the structure on the performance of the proposed methodology and found that the higher the number of damage locations, the higher the quantity of measured DOFs required for a successful detection.

Fan and Qiao [10] carried out a review of damage detection methodologies based on vibration and studied some topics that influenced the damage identification in a beam-type structure, including sensor spacing. They observed that if the sensor spacing was large a damage detection methodology based on the curvature of mode shapes performed better when the curvature was measured and not computed.

Several numerical researches on vibration-based damage detection have simulated the issue of incomplete measurements by assuming that a pre-determined quantity of modal data is available for the analyzed structure. For example, measurements are taken in all [11, 12] or some [13, 14] vertical DOFs in the case of beam structures. Conversely, some of these methodologies require complete mode shapes and, therefore, implement computational techniques, such as reduction techniques, mode expansion techniques or a combination of both to match the size of the numerical and experimental models [15, 16]. The use of these techniques introduces numerical errors in the model [17]; consequently, it is desirable to propose damage detection methodologies that use only the measured information.

The above paragraphs permitted establishing the context in which this research was developed. Thereby, the main objective of this research was to propose a damage detection methodology based on modal information and study how the incompleteness of the data could influence its performance. The problem was formulated as an optimization one and solved by a particle swarm optimizer (PSO). Adaptation was proposed as the performance of the original PSO [18] to solve an optimization problem could be affected by the values chosen for the cognitive and social parameters. The version of PSO used in this research is the one proposed by Shi and Eberhart in 1998 [19], which includes a weight factor that will be deterministically computed. The objective function was formulated in terms of natural frequencies and modes shapes. The proposed function avoids the utilization of any technique to match the DOFs of the numerical and experimental models as it uses the measured information only. A beam structure discretized into 34 elements under different damage scenarios was analyzed. A set of different quantities of modal information was proposed to study the effect of incomplete measurements, permitting establishing the optimal quantity of modal information.

The paper is divided into six sections, starting with the above introduction. Section 2 presents the basic theory on the particle swarm optimizer and the proposed technique to set the PSO parameters. The modeling of the damage and how it influences the dynamic parameters of a structure are presented in Section 3. The proposed methodology and the assumptions are summarized in Section 4. Section 5 presents the analyzed structure and the damage scenarios to be studied together with the set of modal data that will be tested. Finally, the main conclusions of this research are established.

2 Adaptive Particle Swarm Optimizer

As previously mentioned, the damage detection problem is formulated as an optimization problem, therefore it is necessary to define the optimization technique to solve it. In this research, the Particle Swarm Optimizer (PSO) was selected; it is a population-based stochastic algorithm used to find an optimal or almost-optimal solution to maximization or minimization problems. PSO was proposed by Kennedy and Eberhart in 1995 [18] under the assumption of a simple interaction model among individuals, in which the solution to a common problem is to be found by the collaboration of the whole community.

PSO considers a swarm of particles flying through the search space characterized by a position, a velocity and a cost. The current position of each particle on the search space corresponds to a possible solution to the analyzed problem and the velocity is a property that permits orienting the particle's flight. The cost indicates the quality level of the solution found by the particle and is computed from the objective function previously defined for the problem. The velocity and position of each particle are updated iteratively by using the knowledge of the swarm about the search space. In this sense, each particle is able to remember the best position visited (pbest) and transmits this information to the swarm in order to determine the best current position (gbest). Acceleration parameters are used to take into account the weight of the information acquired in previous iterations, permitting assessing the particle's trust in its own knowledge and in the knowledge of the swarm. As the iterations increase, the swarm converges to a region in the search space where the optimal solution is expected to be located. The algorithm is stopped if either the swarm converges to a solution or a pre-specified number of iterations is reached. Due to the stochastic characteristics of the PSO, it is necessary to execute the algorithm a specific number of times to find the final solution to the problem. That solution can be specified by either combining a specific number of the best solutions or, simply, choosing that one with the highest cost among all the runs. On the other hand, the expressions for the updating of velocities, v_i, and positions, x_i, given in reference [17] were used in this study

$$v_i^{t+1} = w \times v_i^t + c_1 \times r_1^t \times (pbest_i - x_i^t) + c_2 \times r_2^t \times (gbest_i - x_i^t), \tag{1}$$

$$x_i^{t+1} = x_i^t + v_i^{t+1}, \tag{2}$$

where i is the i-th particle in the swarm, t is the current iteration, c_1 and c_2 are the acceleration parameters (cognitive and social parameters, respectively), r_1 and r_2 are random values between 0 and 1, and w is the inertia weight.

Different techniques have been used to set some of the PSO parameters, such as fuzzy systems [20], self-adaptation [21], deterministic adaptation based on pbest and gbest [22] and Nelder-Mead Simplex [23]. This paper presents a simple and efficient technique to adapt the PSO parameters, which considers that each particle in the swarm has its own acceleration parameters. The cognitive parameter for the i-th particle at the iteration t+1, c_{1i}^{t+1}, is computed as follows

$$c_{1i}^{t+1} = c_{1i}^{t} + r_3 \times (c_{1best}^{t} - c_{1i}^{t}), \tag{3}$$

where c_{1best} is the value of c_1 for the best particle in the swarm in the previous iteration $t\text{-}1$ and r_3 is a random value between 0 and 1. The initial values for c_{1i} are defined as random numbers generated from a uniform distribution in the [1.8-2.2] range based on the value used in reference [24]. Regarding the social parameter, we followed the recommendation given in reference [25], which states that if the sum of the acceleration parameters is superior to four then it is necessary to use the constriction factor to control the amplitude of the velocity. Consequently, to avoid the use of a constriction factor we defined c_{2i}^{t+1} as

$$c_{2i}^{t+1} = 4 - c_{1i}^{t+1}, \tag{4}$$

To compute the weight factor, it is desirable to increase the global search ability of the PSO at the beginning of the iterative process and to perform a more local search at its end. Therefore, the weight factor can be obtained as proposed in [24]:

$$w^{t} = (w_2 - w_1) \times \left(\frac{t_{max} - t}{t_{max}}\right) + w_1, \tag{5}$$

where t_{max} is the maximum number of iterations permitted. The values for w_2 and w_1 are 0.9 and 0.4, respectively [24].

Finally, to avoid the so called explosion phenomena [25] the velocity is limited by the following expression:

$$v_{max}^{i} = \frac{X^{h} - X^{l}}{5}, \tag{6}$$

where h and l are the boundary values for the variable X.

3 Influence of the Damage on the Dynamic Parameters

As previously mentioned, the dynamic parameters of a structure change with the presence of the damage due to the alterations in the structural properties. In the following paragraphs we describe how the damage is modeled and how it influences the dynamic parameters, supposing that a finite-element model that represents the behavior of the undamaged structure is available. The structure is considered undamped, thus it is necessary to compute the stiffness, K_{str}, and mass matrices, M_{str}, of the structure, as follows

$$K_{str} = \sum_{i=1}^{NElem} k_i, \tag{7}$$

$$M_{str} = \sum_{i=1}^{NElem} m_i, \tag{8}$$

where m_i and k_i are the stiffness and consistent mass matrices for the *i*-th element, respectively, and *NElem* is the number of elements in the structure. The dynamic parameters of an undamped structure can be computed by the following equation

$$\left(K_{str} - \omega^2 M_{str}\right)\phi \quad = 0, \tag{9}$$

where ω is the natural frequency and ϕ is the mode shape.

To model the damage, the mass matrix was considered constant after damage and that damage could be represented as a reduction in the stiffness matrix of the damaged elements given by

$$K_{Dam} = \sum_{i=1}^{NElem} (1 - \beta_i)k_i, \tag{10}$$

where K_{Dam} is the damaged stiffness matrix and βi is the stiffness reduction factor for the i-th element. The element does not present damage if the β factor assumes a value equal to 0, and it is completely damaged if this factor is equal to 1.

Finally, the dynamic parameters of the damaged structure can be determined by

$$\left(K_{Dam} - \omega_d^2 M_{str}\right)\phi_d = 0, \tag{11}$$

where *d* refers to the damaged condition. It can be observed that the dynamic parameters will be different of those obtained by using Equation 8.

4 Damage Detection Methodology

In this study, the localization and quantification of the damage will be performed by using concepts from the dynamic behaviour of the structure. Vibration-based damage detection methodologies are based on the fact that the dynamic properties of a structure are connected to the structural properties. Any change in the matrices of the system – stiffness, mass and damping matrices – produces a change in the natural frequencies and mode shapes of the system. One of the ways to use this concept in damage detection is by formulating an optimization problem in which the difference between the dynamic parameters of a numerical model that represents the damaged structure and those obtained in a real dynamic test in the structure is minimized.

Concerning the above formulation, it is necessary to define the optimization variables, the solver and the objective function to use. Herein, the optimization variables correspond to each stiffness reduction factor in the structure in a total equal to the number of elements in the structure. The selection of the algorithm that solves the optimization problem is one of the key aspects to ensure a correct detection of the damage. Thus, an adaptive particle swarm optimizer was selected, as shown in Section 2. The initial position of the particles in the swarm is heuristically determined in order to orient the search for non severe damage scenarios with a few damaged elements. For each variable in each particle, a random number is assigned in the [0, 1] range. If this number is smaller than 0.5, the variable assumes a random value between 0 and 0.5; otherwise, a value equal to zero is given to the variable. The parameters selected to form the objective function have to be sensitive to the damage in the structure, therefore in this study the objective function was based on natural frequencies and mode shape and will be shown later. It is worth mentioning that the above philosophy of formulating the damage detection problem has been studied by many researchers in the last decades - one of the first researches published by Mares and Surace in 1996 [11].

An algorithm that describes the guidelines to solve the damage detection problem is presented in Figure 1. The first step in the proposed methodology consists in defining a finite element model for the structure that permits representing the undamaged condition. The model for the damaged condition is obtained by updating the model in Step 1, in this case the stiffness matrix. In the second step, the damaged dynamic parameters have to be experimentally determined. As this research was numerically developed, those parameters were obtained from Equation 10 by computing the damaged stiffness matrix for the damage scenario searched for. In an attempt to simulate the conditions found in a real test, these parameters were numerically perturbed by 1% for natural frequencies and 3% for mode shapes [26] to consider the presence of noise in the measurements. Also, only few incomplete mode shapes were available in a determined number of DOFs in the structure. Steps 3 and 4 were already discussed in the above paragraph. The PSO is executed and the best particle in the swarm is chosen as the solution to the problem. The damage scenario found is shown in Step 5 and contains the elements that presented a damage extent higher than a specific threshold. Elements presenting lower values are considered undamaged.

Begin

1. Define the finite element model, FEM, of the structure without damage.
2. Obtain the damaged dynamic parameters: natural frequencies and mode shapes.
3. Define the objective function.
4. Apply the Adaptive Particle Swarm Optimizer.
5. Show the results.

End

Figure 1. Damage detection methodology.

A total of 10 runs of the PSO is carried out and the solution with the highest cost will be selected as the damage scenario searched for. At this point, it is important to recall that heuristic techniques do not guarantee that they will find the correct answer. Thus, a form to assess the reliability of the proposed methodology is to count the number of runs in which the algorithm finds a damage scenario close to the real one.

As previously mentioned, the objective function was based on natural frequencies and mode shapes and is given by [27]

$$G = \sum_{j=1}^{nm} \frac{a_1}{a_2 + F_j}, \tag{12}$$

with

$$F_j = \left| \frac{\omega_j^{pso} - \omega_j^{ex}}{\omega_j^{ex}} \right| + W \times \sqrt{\frac{\sum_{i=1}^{ndf} \left(\phi_{ij}^{pso} - \phi_{ij}^{ex} \right)^2}{\sum_{i=1}^{ndf} \left(\phi_{ij}^{ex} \right)^2}}, \tag{13}$$

where *nm* refers to the number of vibration modes considered, *ndf* is the number of degrees of freedom with available information, superscript *pso* refers to a value found by the particle swarm optimizer and superscript *ex* indicates the experimental results. ω_j is the j-th natural frequency and ϕ_{ij} is the value of the j-th mode shape for the i-th degree of freedom. a_1 and a_2 are constants with values 200 and 1, respectively, and $W = 2.0$ is the weight factor. The objective function used does not require complete modal information, avoiding the use of either model reduction techniques or mode expansion techniques.

5 Numerical Examples

A beam structure (Figure 2) with multiple supports was used to demonstrate the ability of the proposed methodology to locate and quantify damage and to determine the influence of the incomplete modal data. The beam was 8 meters long and discretized into 34 elements containing cross-section area A= 0.001 m^2; moment of inertia I= 0.00005 m^4; density ρ=7800 kg/m^3 and elasticity module, E= 200 x 10^9 N/m^2. A Bernoulli-type beam finite element was employed to model the structure, which has 2 nodes and two DOFs per node - one rotational and one vertical DOF. Figure 2 shows how the nodes were enumerated. Concerning the setting of the PSO characteristics, the population comprised 200 particles and a maximum number of 200 iterations was permitted. As previously mentioned, the other acceleration parameters were allowed to adapt through the iterative process and the weight factor was computed in a deterministic way.

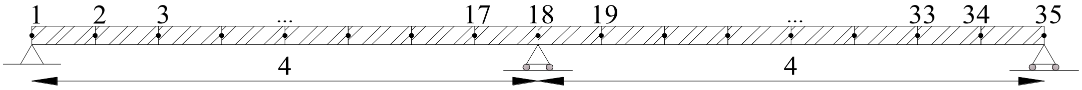

Figure 2. Beam-type structure.

Table 1 shows the different cases of incomplete modal information that were analyzed. These cases consist in the percentage of vertical DOFs measured and the position of the sensors in the beam. The sensors were uniformly distributed across the beam because the study of the effect of the sensor positions on the performance of the proposed methodology was not a goal of this study. Information about rotational DOFs is not used due to the difficulty of obtaining it in a real dynamic test. The incompleteness of the measurements also refers to the possibility of measuring only few modes of the structure. Thus, the performance of the proposed methodology when the number of measures modes ranges between 2 and 12 is studied.

The performance of the proposed methodology was assessed by the detection of the simple and multiple damage scenarios shown in Tables 2 and 3, respectively, and under the consideration that the modal information was incomplete. The simple damage scenarios were chosen in such a way that the methodology could detect damaged elements from all zones of the beam - near the supports and the center. Multiple damage scenarios were chosen to take into account spread and uniform damage. The algorithm was applied to each damage scenario in a total of ten runs and the number of runs in which the algorithm found the real damaged elements was determined (Tables 4-11). The optimal quantity of modal information was defined as the one that permits finding the real damage scenario in at least three runs. It is important to mention that the results shown in Tables 4 to 11 were obtained by computing the average of the results from five different groups of runs for each damage scenario. As shown, only a few scenarios were tested to determine the optimal quantity of modal information as the computational effort involved was very high. However, a total of other 50 random cases was simulated to prove that the above quantity was adequate.

Case	% of Vertical measured DOFs	Nodes (numeration starts from the fixed support of the beam)
I1	100	Corresponding to all of the free vertical DOF
I2	75	3, 4, 5, 6, 8, 9, 10, 11, 13, 14, 15, 16, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33
I3	50	3, 5, 7, 9, 10, 12, 14, 16, 20, 22, 24, 26, 27, 29, 31, 33
I4	25	4, 8, 11, 15, 21, 25, 28, 32

Table 1. Position of the nodes where there is information in the corresponding vertical DOF.

Scenario S1		Scenario S2		Scenario S3		Scenario S4	
Element	β	Element	β	Element	β	Element	β
2	0.250	6	0.350	15	0.170	15	0.420

Table 2. Simple damage scenarios.

Scenario M1		Scenario M2		Scenario M3		Scenario M4	
Element	β	Element	β	Element	β	Element	β
4	0.200	12	0.150	3	0.200	4	0.270
20	0.200	13	0.150	14	0.250	8	0.230
		14	0.150	20	0.400	27	0.180
						28	0.180

Table 3. Multiple damage scenarios.

% of Vertical	Number of measured modes					
DOFs	2	4	6	8	10	12
25	2	5	6	6	9	8
50	1	5	7	7	9	7
75	0	4	7	9	8	7
100	4	7	8	9	10	8

Table 4. Results for simple damage scenario S1.

Tables 4 - 7 show the results of the proposed methodology to detect simple damage scenarios. In general, it is observed that the methodology´s performance was slightly improved when the quantity of sensors was increased and that the higher the number of used mode shapes, the higher the number of successful runs of the algorithm. Therefore, the desired performance can be reached when either the first four mode shapes are measured in the 25% of the vertical DOFs or the two first mode shapes are measured in the total of the vertical DOFs. However, if it is possible to measure the six first mode shapes, then a performance superior to 60% in the number of successful runs can be obtained and, practically, superior to 90% when the ten first mode shapes are used. The use of 12 mode shapes can produce a decrement in the methodology performance in comparison to that one obtained with fewer measurements. This result shows that there exists a maximum quantity of measurements to be used before the performance level can be prejudiced by the excess of information. On the other hand, Tables 6 and 7 illustrate the ability of the methodology to detect damage in element 15 when the damage extent of the single damaged element is varied. When the damage extent is higher, the reliability of the proposed methodology increases no matter the quantity of modal information used.

% of Vertical	Number of measured modes					
DOFs	2	4	6	8	10	12
25	2	7	7	8	10	6
50	3	6	9	8	9	7
75	3	7	7	9	9	7
100	6	7	9	9	8	7

Table 5. Results for simple damage scenario S2.

% of Vertical	Number of measured modes					
DOFs	2	4	6	8	10	12
25	0	3	6	5	9	6
50	2	4	7	8	8	6
75	2	2	5	8	8	6
100	4	6	8	8	9	7

Table 6. Results for simple damage scenario S3.

% of Vertical DOFs	Number of measured modes					
	2	4	6	8	10	12
25	3	7	8	9	9	8
50	2	6	7	9	10	8
75	3	6	8	9	10	7
100	7	8	9	9	10	7

Table 7. Results for simple damage scenario S4.

The results concerning the application of the proposed methodology to detect multiple damage scenarios are reported in Tables 8 -11. For damage scenarios M1 and M3, the methodology meets the desired performance by measuring the six first mode shapes in 25% of the vertical DOFs. If eight or more mode shapes are used, the performance is kept constant at around 80% for scenario M1 and 40% for scenario M3. For a reliable detection of Scenario M2, it is necessary to measure eight mode shapes in any of the measurement configurations shown in Table 1. From the results for damage scenario M2, it can be pointed out that the use of measurements in all vertical DOFs helps to locate uniform damage scenarios more reliably; however, obtaining measurements in all vertical DOFs can be impossible for technical and economical reasons. Moreover, if less than 8 mode shapes are measured the algorithm does not converge to the correct solution. The assessment of this scenario type seems to be the most challenging for the proposed methodology. Finally, scenario M4 can be detected with the expected reliability level by using any of two combinations of measurements: 1) the first eight mode shapes in the 50% of the vertical DOFs or 2) the first ten mode shapes in the 25% of the vertical DOFs. The use of more than the ten first mode shapes did not improve the performance of the methodology considerably for any of the analyzed multiple damage scenarios.

As previously mentioned, a total of 50 scenarios were tested in order to prove that the quantity of modal information used is enough. In this case the least possible number of measured points was selected; therefore we decided to use measurements in 25% of the vertical DOFs and 10 mode shapes. Table 12 summarizes the results for the above damage scenarios, which were divided in function of the number of damaged elements. The minimum quantity of successful runs aforementioned was guaranteed and there was a low number of misidentified elements defined as those undamaged elements that presented damage extents higher than 0.03. Also, the maximum difference between the real and computed damage extension was found to be very low. Moreover, it is worth mentioning that a better performance of the proposed methodology was obtained when only one element was damaged. These results allow concluding that the quantity of modal data selected was adequate to detect damage in the beam.

% of Vertical DOFs	Number of measured modes					
	2	4	6	8	10	12
25	1	1	3	6	8	8
50	1	1	4	7	8	8
75	1	1	5	6	9	9
100	2	3	8	9	9	9

Table 8. Results for multiple damage scenario M1.

% of Vertical DOFs	Number of measured modes					
	2	4	6	8	10	12
25	0	0	0	3	3	4
50	0	0	0	3	4	4
75	0	0	0	3	2	3
100	0	1	3	4	4	5

Table 9. Results for multiple damage scenario M2.

% of Vertical DOFs	Number of measured modes					
	2	4	6	8	10	12
25	0	2	3	4	5	3
50	0	1	3	5	4	5
75	0	1	1	4	5	5
100	0	2	5	7	7	6

Table 10. Results for multiple damage scenario M3.

% of Vertical DOFs	Number of measured modes					
	2	4	6	8	10	12
25	0	0	0	1	3	3
50	0	0	1	3	3	3
75	0	0	1	1	3	4
100	1	1	2	4	3	6

Table 11. Results for multiple damage scenario M4.

Characteristics	Damage scenario Type	
	Simple	Multiple
Minimum number of successful runs	7	3
Maximum number of misidentified elements	3	4
Maximum difference in the damage extent	0.055	0.065

Table 12. Results for the 50 damage scenarios.

As shown in the above results, the methodology does not converge to the correct answer in all runs, thus it is necessary to guarantee that the best solution corresponds to any of those successful runs. This fact was not verified for one of the cases analyzed in this research. This case corresponds to a damage scenario that includes two adjacent elements that are near the supports. The five best solutions for this scenario, among the 10 runs, are reported in Table 13. Only the elements with a β value higher than 0.050 were presented. It can be observed that all damaged elements different to element 2 were identified and that the difference in the cost of these solutions was smaller than 6. Moreover, in runs 1 and 3 an approximate damage scenario was found and element 3, instead of element 2, was identified as damaged. Table 14 shows the results for the same scenario when measurements were free of noise. In this case, the methodology was successful in three runs and achieved the maximum value for the cost (Costmax=2000). Thus, the presence of noise in the measurements makes the space search become more complex, and solutions with a different set of damaged elements present high value for the cost, prejudicing the convergence of the proposed PSO to the real damage scenario.

Damaged Element	Damage Extent β					
	Real	1	2	3	4	5
1	0.200	0.544	0.193	0.521	0.543	0.179
2	0.220	- - -	0.238	- - -	- - -	0.238
3	- - -	0.081	- - -	0.098	- - -	- - -
7	- - -	- - -	- - -	0.082	0.138	- - -
10	- - -	- - -	0.059	- - -	- - -	- - -
24	- - -	0.064	- - -	0.062	0.097	- - -
25	0.190	0.182	0.227	0.206	0.191	0.197
30	0.260	0.270	0.287	0.275	0.263	0.259
Cost		1965.7	1963.3	1962.08	1962.0	1960.6

Table 13. Example of a wrong damage identification.

Damaged Element	Damage Extent β					
	Real	1	2	3	4	5
1	0.200	0.200	0.200	0.200	0.157	0.362
2	0.220	0.220	0.220	0.220	0.224	0.167
21	- - -	- - -	- - -	- - -	0.094	0.083
25	0.190	0.190	0.190	0.190	- - -	- - -
28	- - -	- - -	- - -	- - -	0.054	- - -
29	- - -	- - -	- - -	- - -	- - -	0.076
30	0.260	0.260	0.260	0.260	0.269	0.242
Cost		2000	2000	2000	1973.9	1972.4

Table 14. Example of a correct identification for free-of-noise measurements.

Tables 15 and 16 show a complete description of the damage scenarios found by the methodology when applied to detect the damage scenario presented in Tables 2 and 3. The number of sensors covered 25% of the vertical DOFs in the structure and the number of mode shapes was permitted to vary. These results correspond to the best execution in the first set of runs, recalling that five sets of 10 runs were carried out. The value in parentheses corresponds to the error in the computation of the damage extent for the real damaged elements. Also, only the elements that presented a β value higher than 0.050 are presented. If only 6 modes are used, the real damaged elements may not be found for damage scenarios S1 or M4, or may produce errors in the quantification of the damage extent higher than 60%. A similar performance was observed when either 10 or 12 modes were measured, producing an error lower than 20%. This result shows that a great improvement in the damage detection is not achieved by increasing the quantity of mode shapes used. Also, the algorithm has no tendency to either underestimate or overestimate the damage extent and the values found depend on the dynamics of the execution. Moreover, the results showed that only a few elements were misidentified with low values of damage and that, in general, the methodology was more reliable to detect simple damage scenarios than multiple ones.

ID Scenario	Element	Damage Extent β			
		Real	8 modes	10 modes	12 modes
S1	1	- - -	0.600	0.077	- - -
	2	0.250	- - - (100)	0.239(4)	0.258 (3)
	12	- - -	0.125	- - -	- - -
	23	- - -	0.091	- - -	- - -
S2	6	0.350	0.353 (1)	0.352 (1)	0.345 (1)
S3	1	- - -	0.114	- - -	- - -
	3	- - -	- - -	0.051	- - -
	11	- - -	0.048	- - -	- - -
	15	0.170	0.155 (9)	0.172 (1)	0.163 (4)
	18	- - -	- - -	0.071	- - -
	21	- - -	- - -	0.057	- - -
	24	- - -	0.066	- - -	- - -
S4	9	- - -	0.143	- - -	- - -
	12	- - -	- - -	0.070	- - -
	15	0.420	0.392 (6)	0.424 (1)	0.399 (5)
	17	- - -	0.088	- - -	- - -
	18	- - -	- - -	- - -	0.050
	29	- - -	- - -	0.095	- - -

Table 15. Application of the proposed methodology to detect simple damage. Measurements in 25% of the vertical DOFs.

ID Scenario	Element	Damage Extent β			
		Real	8 modes	10 modes	12 modes
M1	4	0.200	0.091 (55)	0.172 (14)	0.225 (13)
	5	- - -	0.099	- - -	- - -
	8	- - -	0.112	- - -	- - -
	17	- - -	0.065	- - -	- - -
	20	0.200	0.195 (3)	0.194 (3)	0.211 (5)
	28	- - -	- - -	- - -	0.050
M2	7	- - -	0.060	- - -	- - -
	12	0.150	0.056 (63)	0.174 (16)	0.136 (9)
	13	0.150	0.242 (61)	0.143 (5)	0.144 (4)
	14	0.150	0 (100)	0.155 (3)	0.146 (3)
	15	- - -	0.086	- - -	- - -
M3	3	0.200	0.185	0.165 (18)	0.217 (8)
	4	- - -	- - -	0.069	- - -
	14	0.250	0.154	0.243 (3)	0.250 (0)
	20	0.400	0.199	0.405 (1)	0.376 (6)
	22	- - -	0.358	- - -	- - -
	23	- - -	0.102	- - -	0.044
M4	4	0.270	0.203 (25)	0.274 (1)	0.292 (8)
	8	0.230	0.193 (16)	0.230 (0)	0.213 (7)
	12	- - -	0.086	- - -	- - -
	18	- - -	0.127	- - -	- - -
	27	0.180	- - - (100)	0.184 (2)	0.184 (2)
	28	0.180	0.227 (21)	0.183 (2)	0.161 (11)
	31	- - -	0.060	- - -	- - -

Table 16. Application of the proposed methodology to detect multiple damages. Measurements in 25% of the vertical DOFs.

Some observations about the performance of the proposed methodology to detect damage using incomplete modal information have been established:

- Different damage scenarios can be detected with different reliability levels for a same quantity of modal information available. In general, simple damage scenarios are detected more reliably than the multiple ones.
- There is an optimal quantity of modal information that permits detecting most of the possible damage scenarios. If a lower quantity is used, then the methodology can fail to detect the real damage scenario, but if a higher quantity is used, then there will be waste of resource and time in carrying out a more complete dynamic test for unnecessary information.
- The computational cost involved in the determination of the optimal quantity of modal information is high because many combinations of modal information have to be tested. It is important to observe that this set of values is limited by technical conditions, such available number of sensors and quantity of excitable modes.
- As noise can prejudice the performance of the proposed methodology, it is very important to obtain the clearest measurements.

6 Conclusions

An adaptive particle swarm optimization has been proposed to detect damage in beam structures under the assumption that the measurements are noisy and incomplete. Each damage detection methodology proposed requires a study to determine the minimum quantity of modal data that guarantees reliable damage detection in a specific structure. Also, the utilization of abundant information does not guarantee an improvement in the computation of the damage extension. The methodology obtained all damage scenarios with a low error in the computation of the damage extent and a few misidentified elements. It failed for only one damage scenario, but it found an approximate damage scenario. However, some improvements in the methodology have to be performed in order to diminish the number of measured points in the beam that are necessary to guarantee the correct damage detection. The influence of the position of the sensors on the structure will be studied in future works.

References

[1] J. Humar, A. Bagchi, H. Xu, “Performance of vibration-based techniques for the identification of structural damage”, Structural Health Monitoring, 5, 215-227, 2006.

[2] W. X. Ren, G. De Roeck, “Structural damage identification using modal data. II: test verification”, Journal of Structural Engineering, 128 (1), 96-104, 2002.

[3] S. S. Law, Z. Y. Shi, L. M. Zhang, “Structural damage detection from incomplete and noisy modal test data”, Journal of Engineering Mechanics, 124 (11), 1280-1288, 1998.
[4] H. M. Kim, T. J. Bartkowicz, “An experimental study for damage detection using a hexagonal truss”, Computers and Structures, 79, 173-182, 2001.
[5] J. V. Araújo dos Santos, C. M. Mota, C. A. Mota, N. M. M. Maia, “ Structural damage identification: influence of model incompleteness and errors”, Composite Structures, 62, 303-313, 2003.
[6] M. Reza, Y. Mehdi-pour, “Detection and assessment of damage in 2D structures using measured modal response”, Journal of Sound and Vibration, 306, 803-817, 2007.
[7] A. Raich, T. Liszkai, “Improving the performance of structural damage detection methods using advanced genetic algorithms”, Journal of Structural Engineering, 133(3), 449-461, 2007.
[8] G. Y. Yun, K. Ogorzalek, S. Dyke, W. Song, “A parameter subset selection method using residual force vector for detecting multiple damage locations”, Structural Control and Health Monitoring, 17, 48–67, 2010.
[9] V. Meruane, W. Heylen, “An hybrid real genetic algorithm to detect structural damage using modal properties”, Mechanical Systems and Signal Processing, 25, 1559–1573, 2011.
[10] W. Fan, P. Qiao, “Vibration-based damage identification methods: A review and comparative study”, Structural Health Monitoring, 10(1), 83-111, 2011.
[11] C. Mares, C. Surace, “An application of genetic algorithm to identify damage in elastic structures”, Journal of Sound and Vibration, 195 (2), 195-215, 1996.
[12] F. Au, Y. S. Cheng, L. G. Tham, Z. Z. Bai, “Structural damage detection based on a micro-genetic algorithm using incomplete and noisy modal test data”, Journal of Sound and Vibration, 259 (5), 1081-1094, 2003.
[13] R. Perera, R. Torres, “Structural Damage detection via modal data with genetic algorithms”, Journal of Structural Engineering, 132 (9), 1491–1501, 2006.
[14] R. Perera, A. Ruiz, “A Multistage FE updating procedure for damage identification in large-scale structures based on multi-objective evolutionary optimization”, Mechanical Systems and Signal Processing, 22, 970–991, 2008.
[15] K. Moslem, R. Nafaspour, “Structural damage detection by genetic algorithms”, AIAA Journal, 40 (7), 1395-1401, 2002.
[16] H. Li, J. Wang, S. L. James, “Using incomplete modal data for damage detection in offshore jacket structures”, Ocean Engineering, 35 (17-18), 1793-1799, 2008.
[17] H. P. Chen, N. Bicanic, “Assessment of damage in continuum structures based on incomplete modal information”, Computers and Structures, 74, 559-570, 2000.
[18] J. Kennedy, Eberhart R. “Particle swarm optimization”, In: Proceedings of the IEEE international conference on neural networks, vol. 4, 1942–1948, 1995.
[19] Y. Shi, R. C. Eberhart, “A modified particle swarm optimizer”, In: Proceedings of the IEEE International Conference on Evolutionary Computation, IEEE Press, Piscataway, NJ, 69-73, 1998.

[20] Y. Shi, R. C. Eberhart, “Fuzzy adaptive particle swarm optimization”, In: Proceedings of the IEEE International Conference on Evolutionary Computation, IEEE Press, Seoul, vol. 1, 101-106, 2001.
[21] K. E. Parsopoulos, M. N. Vrahatis, “Parameter selection and adaptation in unified particle swarm optimization”, Mathematical and Computer Modelling, 46, 198-213, 2007.
[22] M. Senthil, M. V. C. Rao, “On the improved performances of the particle swarm optimization algorithms with adaptive parameters, cross-over Operators and root mean square (RMS) variants for computing optimal control of a class of hybrid systems”, Applied Soft Computing, 8, 324-336, 2008.
[23] O. Begambre, J. E. Laier, “A hybrid particle swarm optimization – simplex algorithm (PSOS) for structural damage identification. Advances in Engineering Software”, 40, 883–891, 2009.
[24] Y. Shi, R. C. Eberhart, “Empirical study of particle swarm optimization, In: Proceedings of the IEEE International Congress on Evolutionary Computation, Washington, DC, 1945–1950, 1999.
[25] M. Clerc, J. Kennedy, “The particle swarm—explosion, stability, and convergence in a multidimensional complex space”, IEEE Transactions on Evolutionary Computation, 6 (1), 58-73, 2002.
[26] B. Chen, S. Nagarajaiah, “Flexibility-based structural damage identification using Gauss- Newton method”, In: Proceedings of SPIE- Sensors and smart structures, technologies for civil, mechanical, and aerospace systems, San Diego, USA, Vol 6529 Part 1, paper 65291L, 2007.
[27] J. D. Villalba, J. E. Laier, “Localising and quantifying damage by means of a multi-chromosome genetic algorithm”, Unpublished, 2012.