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Stability Analysis of Laminated Composite Thin-Walled Beam Structures

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Abstract

The paper presents an algorithm for the buckling analysis of thin-walled laminated composite beam-type structures. A one-dimensional finite element is employed subject to the assumptions of large displacements, large rotation effects but small strains. The geometric stiffness matrix of thin-walled beam finite element is derived. Stability analysis is performed in a load deflection manner using the co-rotational formulation. The cross-section mid-line contour is assumed to remain not deformed in its own plane and the shear strains of the middle surface are neglected. The laminates are modelled on the basis of classical lamination theory. The results are validated on test examples.

Keywords: beam, stability, buckling, composite, laminate, critical load.

1 Introduction

Composite materials are very suitable for structural applications where high strength to weight and stiffness to weight ratios are required. Fibre reinforced laminates in the form of thin-walled beams have been increasingly used during the past few decades in engineering practice, particularly in aerospace engineering, shipbuilding and in the automobile industry. Such weight-optimised structural components are commonly very susceptive to buckling failure because of their slenderness so stability problems should be considered in their design [1].
A finite element model for stability analysis of 3D framed structures with thin-walled laminated composite cross section is presented in this paper. The beam cross-section geometry is discretized by quadratic monitoring areas and the structural discretization is performed throughout one-dimensional finite element. Model takes into account effects of large displacements on the response of space frames subjected to conservative and static external loads.

Classical lamination theory for thin fibre-reinforced laminates is employed in this paper. The model is applicable to any arbitrary laminate stacking sequence, shape of the cross section and boundary conditions. The shear strain of middle surface is assumed to be zero and the cross-section is not distorted in its own plane.
The stability analysis is performed in a load-deflection manner [2, 3]. The non-linear response of a load-carrying structure should be solved using numerical methods, e.g. the finite element method, and some of incremental descriptions like the total and updated Lagrangian ones, respectively, or the co-rotational description. Each description utilizes a different structural configuration for system quantities referring and results in the form of a set of non-linear equilibrium equations of the structure. This set can further be linearized and should be solved using some incremental-iterative scheme.

The co-rotational description, used in this work, is linear on element level and all geometrically non-linear effects are introduced through the transformation from the local to the global coordinate system. The local co-rotational system follows the element chord during the deformation and allows the use of simplified strain-displacement relations on the local element level. [4, 5]
Verification examples utilizing a numerical algorithm developed on the basis of abovementioned procedure are presented to demonstrate accuracy of this model [6].

2 Theoretical background

2.1 Displacements

In a local Cartesian coordinate system in which beam axe, that connects all cross sectional centres of gravity, coincides with z axe while x and y are principal axes, cross sectional rigid body displacements are:

$$\begin{aligned} &w_0 = w_0(z), \quad u_s = u_s(z), \quad v_s = v_s(z), \\ &\varphi_z = \varphi_z(z), \quad \varphi_x = -\frac{\mathrm{d}v_s}{\mathrm{d}z}, \quad \varphi_y = \frac{\mathrm{d}u_s}{\mathrm{d}z}, \quad \theta = \frac{\mathrm{d}\varphi_z}{\mathrm{d}z} \end{aligned} \tag{1}$$

In equation above, w_0 is translational displacement in z direction defined for cross-sectional centre of gravity; u_s and v_s are translational displacements in x and y directions, defined for shear centre; while φ_z, φ_x and φ_y are rotational displacement around z, x and y axes. Displacement θ is cross-sectional warping parameter.
Cross-sectional displacement field is defined as:

$$\boldsymbol{U}_{\mathrm{uk}}^{\mathrm{T}} = \{W \quad U \quad V\} = \{w + \tilde{w} \quad u + \tilde{u} \quad v + \tilde{v}\} \tag{2}$$

where w, u and v are the standard linear displacement field components, while $\tilde{w}, \tilde{u}$ and $\tilde{v}$ are second order components that arise from large rotation effects according to [2, 3].

In the contour coordinate system (z, n, s) on Figure 1, the part of the contour has been shown. Contour mid-line displacements are $\overline{w}, \overline{u}, \overline{v}$, while displacements of an arbitrary point of contour are:

$$w(z,s,n) = \overline{w} - n\frac{\partial \overline{u}}{\partial z}; \qquad v(z,s,n) = \overline{v} - n\frac{\partial \overline{u}}{\partial s}; \qquad u(z,s,n) = \overline{u} \tag{3}$$

where r is contour radius and q is the distance of pole P from normal n.

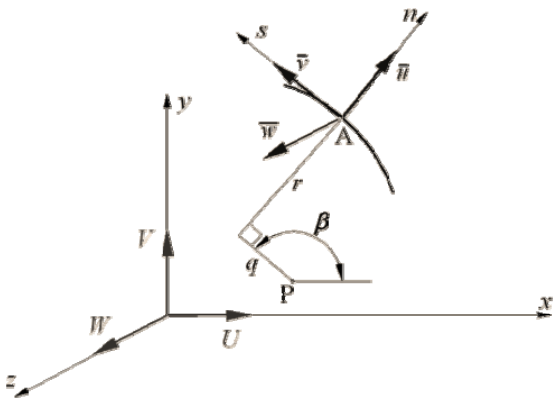

Figure 1: Cross-sectional contour displacement components.

Connection between the contour and beam displacements has been established as:

$$\begin{aligned} &\overline{w} = W(z,s,n); \\ &\overline{v} = U(z,s,n)\cos\beta + V(z,s,n)\sin\beta; \\ &\overline{u} = U(z,s,n)\sin\beta - V(z,s,n)\cos\beta \end{aligned} \tag{4}$$

2.2 Strains

Using nonlinear beam displacement field the strain tensor consists of three parts:

$$\varepsilon_{ij} = e_{ij} + \eta_{ij} + \tilde{e}_{ij}; e_{ij} = 0.5\left(u_{i,j} + u_{j,i}\right), \eta_{ij} = 0.5\, u_{k,i}\, u_{k,j}, \tilde{e}_{ij} = 0.5\left(\tilde{u}_{i,j} + \tilde{u}_{j,i}\right). \tag{5}$$

Supposing that the shair strain γ_{zs} of the middle surface is zero, and that the contour of the thin wall does not deform in its own plane, the non-zero strain components are:

$$e_{zz} = \frac{\partial w}{\partial z} = \frac{\partial \overline{w}}{\partial z} - n\frac{\partial^2 \overline{u}}{\partial z^2}; \; e_{zs} = \frac{\partial w}{\partial s} + \frac{\partial v}{\partial z} = -2n\frac{\partial^2 \overline{u}}{\partial s \partial z}; \tag{6}$$

$$\eta_{zz} = \frac{1}{2}\left[\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]; \quad \eta_{zs} = \frac{\partial w}{\partial z}\frac{\partial w}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial u}{\partial s} + \frac{\partial v}{\partial z}\frac{\partial v}{\partial s}; \tag{7}$$

$$\tilde{e}_{zz} = \frac{\partial \tilde{w}}{\partial z}; \quad \tilde{e}_{zs} = \frac{\partial \tilde{w}}{\partial s} + \frac{\partial \tilde{v}}{\partial z} \ . \tag{8}$$

2.3 Internal forces

The constitutive equations for one lamina are:

$$\begin{pmatrix} \sigma_z \\ \tau_{zs} \end{pmatrix} = \begin{pmatrix} \bar{Q}_{11}^{\ *} & \bar{Q}_{16}^{\ *} \\ \bar{Q}_{16}^{\ *} & \bar{Q}_{66}^{\ *} \end{pmatrix} \cdot \begin{pmatrix} \varepsilon_z \\ \gamma_{zs} \end{pmatrix} \tag{9}$$

where $\bar{Q}_{\mathrm{ii}}^{\ *}$ are so called transformed reduced stiffnesses [7]. Integrating over the laminate thickness n and the contour direction s, and transforming into the beam coordinate system follows the cross-sectional internal force components:

$$\begin{aligned} F_z &= \int_A \sigma_z \, dn \, ds; \quad M_z = \int_A \tau_{zs} n \, dnds; \quad M_\omega = \int_A \sigma_z (\omega - nq) dnds; \\ M_x &= \int_A \sigma_z (y - n\cos\beta) dnds; \quad M_y = \int_A \sigma_z (x + n\sin\beta) dnds; \end{aligned} \tag{10}$$

2.4 Beam finite element

In Figure 2 two-nodded beam finite element with eight degrees of freedom is presented.

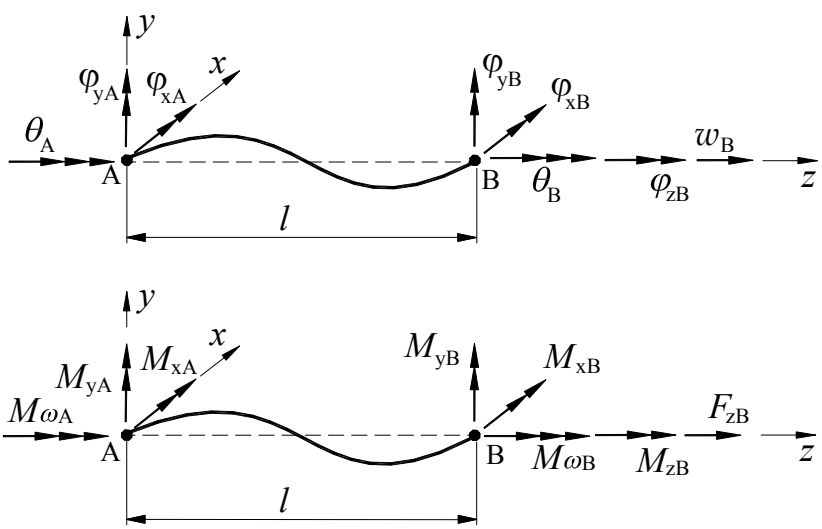

Figure 2: Two-nodded spatial beam element in local coordinate system.

The nodal displacement vector of the beam element is:

$$\left(\mathbf{u}^e\right)^{\mathrm{T}} = \left\{ w_{\mathrm{B}}, \varphi_{\mathrm{zB}}, \varphi_{\mathrm{xA}}, \varphi_{\mathrm{xB}}, \varphi_{\mathrm{yA}}, \varphi_{\mathrm{yB}}, \theta_{\mathrm{A,}}, \theta_{\mathrm{B}} \right\} \tag{11}$$

and an appropriate nodal force vector is:

$$\left(\mathbf{f}^{e}\right)^{\mathrm{T}} = \left\{F_{\mathrm{zB}}, M_{\mathrm{zB}}, M_{\mathrm{xA}}, M_{\mathrm{xB}}, M_{\mathrm{yA}}, M_{\mathrm{yB}}, \mathrm{M}_{\omega\mathrm{A}}, \mathrm{M}_{\omega\mathrm{B}}\right\}. \tag{12}$$

Incremental analysis supposes that a load-deflection path is subdivided into a number of steps or increments. This path is usually described using three configurations: the initial or undeformed configuration C_0; the last calculated equilibrium configuration C_1 and current unknown configuration C_2. Adopting co-rotational formulation, all system quantities should be referred to configuration C_2, [3].

Applying the virtual work principle and neglecting the body forces, the equilibrium of a finite element can be expressed as, [2, 3]:

$$\delta \mathrm{U} = \delta \mathrm{W} \tag{13}$$

in which U is potential energy of internal forces, W is the virtual work of external forces, while δ denotes virtual quantities.

After making the first variation of eq. (10), the following incremental equations can be obtained:

$$\delta \mathrm{W} = \left(\delta \mathbf{u}^{e}\right)^{\mathrm{T}} \mathbf{f}^{e}; \quad \delta \mathrm{U} = \left(\delta \mathbf{u}^{e}\right)^{\mathrm{T}} \mathbf{k}_{\mathrm{T}}^{e}\, \mathbf{u}^{e} \tag{14}$$

In Equation (14) $\mathbf{k}_{\mathrm{T}}^{\mathrm{e}}$ denotes the local tangent stiffness matrix of the e-th beam element which can be evaluated according to procedure explained in [4, 5]. Now the incremental equilibrium equation can be written in the following form:

$$\mathbf{k}_{\mathrm{T}}^{\mathrm{e}}\, \Delta \mathbf{u}^{\mathrm{e}} = \Delta \mathbf{f}^{\mathrm{e}} \tag{15}$$

The element global tangent stiffness matrix $\overline{\mathbf{k}}_{\mathrm{T}}^{e}$ can be obtained as follows:

$$\overline{\mathbf{k}}_{\mathrm{T}}^{e} = \mathbf{t}_{\mathbf{1}}^{e} \mathbf{k}_{\mathrm{T}}^{e} \mathbf{t}_{\mathbf{1}}^{e} + \mathbf{t}_{\mathbf{2}}^{e} \mathbf{f}^{e}. \tag{16}$$

Matrices $\mathbf{t}_{\mathbf{1}}^{e}$ and $\mathbf{t}_{\mathbf{2}}^{e}$ are standard transformation matrices from local co-rotational to global coordinate system explained in [4]. Matrix $\mathbf{t}_{\mathbf{1}}^{e}$ is of dimension 14×8 and contains first derivations of local with respect to global displacements while $\mathbf{t}_{\mathbf{2}}^{e}$ is 14×14×8 matrix containing second derivations. Matrix $\mathbf{t}_{\mathbf{2}}^{e}$ presents geometric stiffness contribution because it contains effects on global forces caused with change in geometry. Element force vector transformed from local to global coordinate system is:

$$\overline{\mathbf{f}}^{e} = \mathbf{t}_{\mathbf{1}}^{e} \mathbf{f}^{e}. \tag{17}$$

After the standard assembling procedure, the overall incremental equilibrium

equations can be obtained as:

$$\mathbf{K}_{\mathrm{T}}\mathbf{U} = \mathbf{P}, \quad \mathbf{K}_{\mathrm{T}} = \sum_{e} \bar{\mathbf{k}}_{\mathrm{T}}^{e}, \quad \mathbf{P} = {}^{2}\mathbf{P} - {}^{1}\mathbf{P}, \tag{18}$$

where $\mathbf{K}_{\mathrm{T}}$ is tangential stiffness matrix of a structure, while $\mathbf{U}$ and $\mathbf{P}$ are the incremental displacement vector and the incremental external loads of the structure. ${}^{2}\mathbf{P}$ and ${}^{1}\mathbf{P}$ are the vectors of external loads applied to the structure at C_2 and C_1 configurations, respectively.

3 Numerical examples

A cantilever and simply supported beam subjected to an axial force are considered, Figure 3. The beam and cantilever length is L = 2 m and thin-walled cross section is of I-type with dimensions 10 cm × 10 cm × 1 cm. The web and the flanges are considered to be four layered symmetric angle-ply laminates with stacking sequence [0, 0, 0, 0], [0, 90, 90, 0] and [45, -45, -45, 45]. The all plies are of the same thickness of 2.5 mm. The analysed material is graphite-epoxy (AS4/3501) whose properties are: E_1=144 GPa, E_2=9.65 GPa, G_{12}=4.14 GPa and ν_{12}=0.3. To initiate the flexural instability, the horizontal perturbation force ΔF of value $10^{-3}F$ is added in both cases at positions shown on Figure 3.

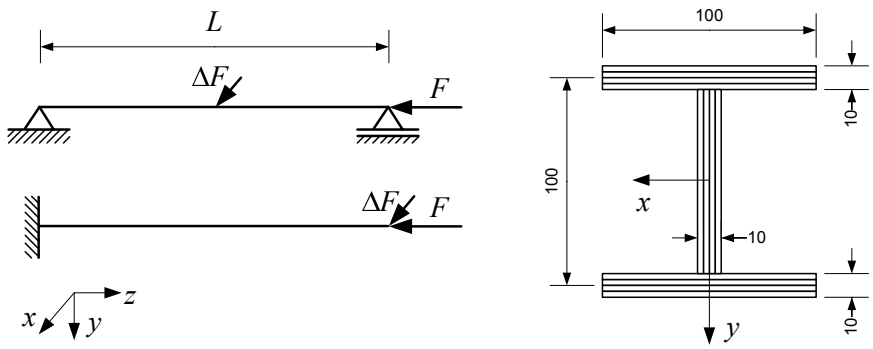

Figure 3: Simply supported beam, cantilever and cross-section geometry.

The Figures 4 and 5 illustrate the load-deflection curves for different laminate configuration for both cases of constraints, simply supported beam and cantilever. All the obtained results correspond to the flexural buckling modes in the *xz*-plane and used mesh consist of eight beam finite elements.

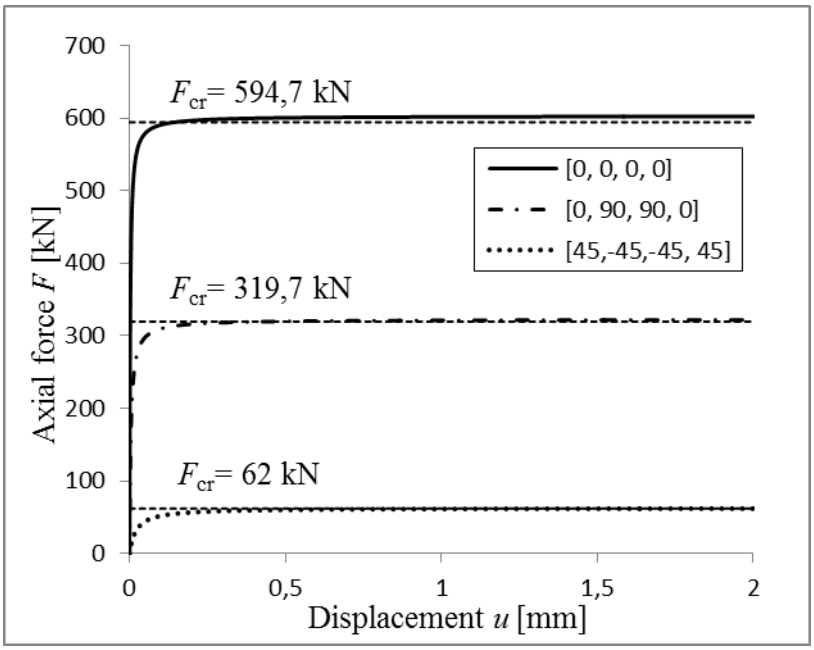

Figure 4: Load vs. displacement curves for simply supported beam.

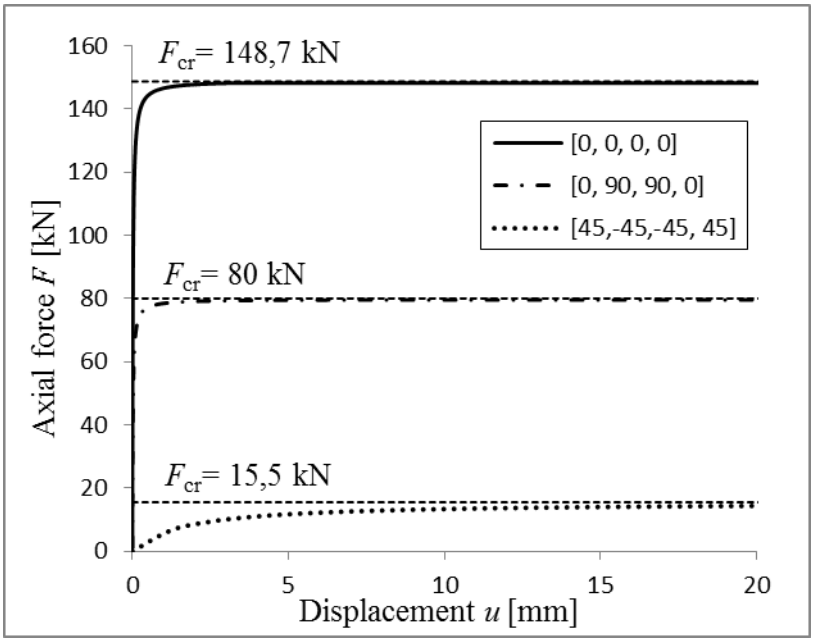

Figure 5: Load vs. displacement curves for cantilever.

The buckling response of this beam was considered previously by Cortinez and Piovan [6] where the critical buckling loads were obtained by eigenvalue analysis. Buckling predictions in this paper depicted on Figures 4 and 5 show that very good accuracy is achieved in comparison with their results, marked by dashed line.

4 Conclusion

The co-rotational formulation for the non-linear analysis of beam columns with laminated composite cross section is proposed. The governing incremental equilibrium equations of a two-node space beam element are developed using the linearised virtual work principle. The presented test examples suggest that the numerical model developed is an accurate tool for modelling composite cross section beam structure nonlinear behaviour.

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