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A Finite Element-Boundary Element Approach for Sound and Vibration Reduction using Piezoelectric Shunt Damping

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Abstract

In this paper, we present a coupled finite element-boundary element method (FE-BE) for control of noise radiation and sound transmission of vibrating structure by active piezoelectric techniques. The system consists of an elastic structure (with surface mounted piezoelectric patches) coupled to external/internal acoustic domains. The passive shunt damping strategy is employed for vibration attenuation in the low frequency range.

Keywords: finite element method, boundary element method, vibroacoustics, piezoelectric patches, shunt damping technique, noise and vibration attenuation.

1 Introduction

During the last two decades there has been an accelerating level of interest in the control of noise radiation and sound transmission from vibrating structures by active piezoelectric techniques in the low frequency range. In this context, resonant shunt damping techniques have been recently used for interior structural-acoustic problems [1, 2]. The present work concerns the extension of this technique to internal/external vibroacoustic problems using a finite element-boundary element method (FEM-BEM) for the numerical resolution of the fully coupled electro-mechanical-acoustic system.

First, a finite element formulation of an elastic structure with surface-mounted piezoelectric patches and subjected to pressure load due to the presence of an external fluid is derived from a variational principle involving structural displacement, electrical voltage of piezoelectric elements and acoustic pressure at the fluid-structure interface. This formulation, with only one couple of electric variables per patch, is well adapted to practical applications since realistic electrical boundary conditions, such that equipotentiality on the electrodes and prescribed global electric charges, naturally appear. The global charge/voltage variables are intrinsically adapted to include any external electrical circuit into the electromechanical problem and to simulate the effect of resistive or resonant shunt techniques.

In the second part of this work, the direct boundary element method is used for modeling the scattering/radiation of sound by the structure coupled to an acoustic domain. The BEM is derived from Helmholtz integral equation involving the surface pressure and normal acoustic velocity at the boundary of the acoustic domain. The coupled FE-BE model is obtained by using a compatible mesh at the fluid-structure interface. The present coupling procedure is quite general and suitable for modeling any three-dimensional geometry for bounded or unbounded structural-acoustic radiation problems.

Finally, the efficiency of the proposed coupling methodology is demonstrated on one numerical example. Thus, the vibration reduction of an elastic plate backed by a closed acoustic cavity is analyzed. For this example, a complete FE method developed by the authors in [2] is compared to the present FEM-BEM approach.

2 Finite element formulation of elastic structure with piezoelectric shunt systems

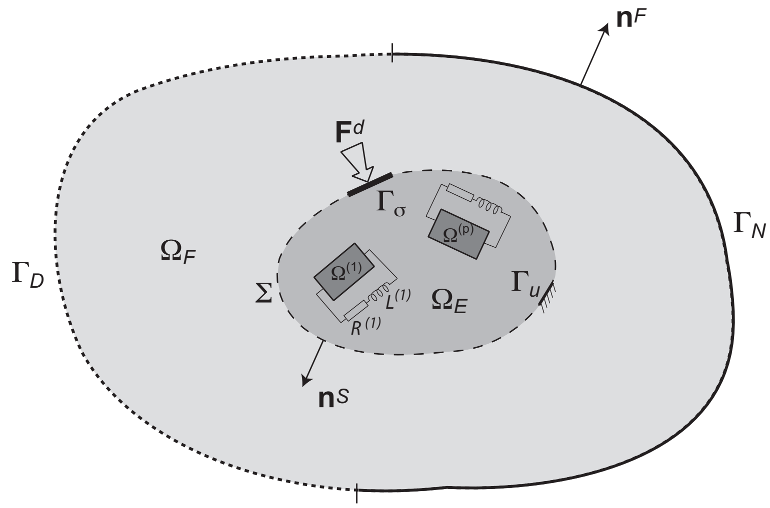

Figure 1: Vibrating structure with piezoelectric shunt systems coupled to an acoustic domain.

2.1 Harmonic equations

An elastic structure occupying the domain Ω_E is equipped with P piezoelectric patches and coupled to an inviscid linear acoustic fluid occupying the domain Ω_F (figure 1). Each piezoelectric patch has the shape of a plate with its upper and lower surfaces covered with a very thin layer electrodes. The pth patch, $p \in \{1, \cdots, P\}$, occupies a domain $\Omega^{(p)}$ such that $(\Omega_E, \Omega^{(1)}, \cdots, \Omega^{(P)})$ is a partition of the all structure domain Ω_S. In order to reduce the vibration amplitudes of the coupled problem, a resonant shunt circuit made up of a resistance $R^{(p)}$ and an inductance $L^{(p)}$ in series is connected to each patch [3, 1, 4].

We denote by Σ the fluid-structure interface and by $\mathbf{n}^S$ and $\mathbf{n}^F$ the unit normal external to Ω_S and Ω_F, respectively. Moreover, the structure is clamped on a part Γ_u and subjected (i) to a given surface force density $\mathbf{F}^d$ on the complementary part Γ_σ of its external boundary and (ii) to a pressure field p due to the presence of the fluid on its boundary Σ. The electric boundary condition for the pth patch is defined by a prescribed surface density of electric charge Q^d on $\Gamma_D^{(p)}$.

The linearized deformation tensor is $\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla^T \mathbf{u}\right)$ and the stress tensor is denoted by $\boldsymbol{\sigma}$. Moreover, $\mathbf{D}$ denotes the electric displacement and $\mathbf{E}$ the electric field such that $\mathbf{E} = -\nabla \psi$ where ψ is the electric potential. ρ_S is the mass density of the structure. The linear piezoelectric constitutive equations write:

$$\boldsymbol{\sigma} = \mathbf{c}\,\boldsymbol{\varepsilon} - \mathbf{e}^T\,\mathbf{E} \tag{1}$$

$$\mathbf{D} = \mathbf{e}\,\boldsymbol{\varepsilon} + \boldsymbol{\epsilon}\,\mathbf{E} \tag{2}$$

where $\mathbf{c}$ denotes the elastic moduli at constant electric field, $\mathbf{e}$ denotes the piezoelectric constants, and $\boldsymbol{\epsilon}$ denotes the dielectric permittivities at constant strain.

The local equations of elastic structure with piezoelectric patches and submitted to an acoustic pressure are [5]

$$\operatorname{div} \boldsymbol{\sigma} + \omega^2 \rho_S \mathbf{u} = \mathbf{0} \qquad \text{in } \Omega_S \tag{3a}$$

$$\boldsymbol{\sigma}\,\mathbf{n}^S = \mathbf{F}^d \qquad \text{on } \Gamma_\sigma \tag{3b}$$

$$\mathbf{u} = \mathbf{0} \qquad \text{on } \Gamma_u \tag{3c}$$

$$\boldsymbol{\sigma}\,\mathbf{n}^S = p\,\mathbf{n} \qquad \text{on } \Sigma \tag{3d}$$

$$\operatorname{div} \mathbf{D} = 0 \qquad \text{in } \Omega^{(p)} \tag{4a}$$

$$\mathbf{D} \cdot \mathbf{n}^S = Q^d \qquad \text{on } \Gamma_D^{(p)} \tag{4b}$$

where ω is the angular frequency.

For each piezoelectric patch, a set of hypotheses, which can be applied to a wide spectrum of practical applications, are formulated:

- The piezoelectric patches are thin, with a constant thickness, denoted $h^{(p)}$ for the pth patch;
- The thickness of the electrodes is much smaller than $h^{(p)}$ and is thus neglected;
- The piezoelectric patches are polarized in their transverse direction (i.e. the direction normal to the electrodes).

Under those assumptions, the electric field vector $\mathbf{E}^{(p)}$ can be considered normal to the electrodes and uniform in the piezoelectric patch [6], so that for all $p \in \{1, \cdots, P\}$:

$$\mathbf{E}^{(p)} = -\frac{V^{(p)}}{h^{(p)}}\mathbf{n}^{(p)} \quad \text{in} \quad \Omega^{(p)} \tag{5}$$

where $V^{(p)}$ is the potential difference between the upper and the lower electrode surfaces of the pth patch which is constant over $\Omega^{(p)}$ and where $\mathbf{n}^{(p)}$ is the normal unit vector to the surface of the electrodes.

2.2 Variational formulation

By considering successively each of the $P+1$ subdomains $(\Omega_E, \Omega^{(1)}, \cdots, \Omega^{(P)})$, the variational formulation of the structure/piezoelectric-patches coupled system can be written in terms of the structural mechanical displacement $\mathbf{u}$, the electric potential difference $V^{(p)}$ constant in each piezoelectric patch, and the fluid pressure p at the fluid-structure interface:

$$\int_{\Omega_S} [\mathbf{c}\boldsymbol{\varepsilon}(\mathbf{u})] : \boldsymbol{\varepsilon}(\delta\mathbf{u})\,\mathrm{d}v + \sum_{p=1}^{P} \frac{V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \mathbf{e}^T\mathbf{n}^{(p)} : \boldsymbol{\varepsilon}(\delta\mathbf{u})\,\mathrm{d}v - \omega^2\rho_S \int_{\Omega_S} \mathbf{u}\cdot\delta\mathbf{u}\,\mathrm{d}v$$
$$- \int_{\Sigma} p\,\mathbf{n}\cdot\delta\mathbf{u}\,\mathrm{d}s = \int_{\Gamma_\sigma} \mathbf{F}^d\cdot\delta\mathbf{u}\,\mathrm{d}s \quad \forall\delta u_i \in C_u^* \tag{6}$$

where the admissible space C_u^* is defined by $C_u^* = \{\mathbf{u} \in C_u \,|\, \mathbf{u} = 0 \,\text{on}\, \Gamma_u\}$. C_u being the admissible space of regular functions $\mathbf{u}$ in Ω_S.

$$\sum_{p=1}^{P} \delta V^{(p)} C^{(p)} V^{(p)} - \sum_{p=1}^{P} \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} [\mathbf{e} : \boldsymbol{\varepsilon}(\mathbf{u})]\cdot\mathbf{n}^{(p)}\,\mathrm{d}v = \sum_{p=1}^{P} \delta V^{(p)} Q^{(p)} \quad \forall\delta V^{(p)} \in \mathbb{R} \tag{7}$$

where $C^{(p)} = \epsilon_{33} S^{(p)}/h^{(p)}$ defines the capacitance of the pth piezoelectric patch ($S^{(p)}$ being the area of the patch and ϵ_{33} the piezoelectric material permittivity in the direction normal to the electrodes) and $Q^{(p)}$ is the global charge in one of the electrodes (see [6]).

2.3 Finite element formulation

After discretization of the previous variational formulation by finite element method and using the following additional relation between electrical potential differences and electric charges due to the shunt circuits:

$$-\omega^2\mathbf{L}\mathbf{Q} - i\omega\mathbf{R}\mathbf{Q} + \mathbf{V} = \mathbf{0} \tag{8}$$

we find the following matrix equation:

$$\begin{bmatrix} \mathbf{K}_u + \mathbf{C}_{uV}\mathbf{K}_V^{-1}\mathbf{C}_{uV}^T & \mathbf{C}_{uV}\mathbf{K}_V^{-1} & -\mathbf{C}_{up} \\ \mathbf{K}_V^{-1}\mathbf{C}_{uV}^T & \mathbf{K}_V^{-1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} - i\omega \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} + \\ - \omega^2 \begin{bmatrix} \mathbf{M}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \end{bmatrix} \quad (9)$$

where $\mathbf{Q} = (Q^{(1)}\, Q^{(2)} \cdots Q^{(P)})^T$ and $\mathbf{V} = (V^{(1)}\, V^{(2)} \cdots V^{(P)})^T$ are the column vectors of electric charges and potential differences; $\mathbf{R} = \mathrm{diag}\left(R^{(1)}\, R^{(2)} \cdots R^{(P)}\right)$ and $\mathbf{L} = \mathrm{diag}\left(L^{(1)}\, L^{(2)} \cdots L^{(P)}\right)$ are the diagonal marices of the resistances and inductances of the patches; $\mathbf{U}$ and $\mathbf{P}_\Sigma$ are the vectors of nodal values of $\mathbf{u}$ and p; $\mathbf{M}_u$ and $\mathbf{K}_u$ are the mass and stiffness matrices of the structure (elastic structure and piezoelectric patches); $\mathbf{C}_{uV}$ is the electric mechanical coupled stiffness matrix; $\mathbf{K}_V = \mathrm{diag}\left(C^{(1)}\, C^{(2)} \cdots C^{(P)}\right)$ is a diagonal matrix filled with the P capacitances of the piezoelectric patches; $\mathbf{C}_{up}$ is the fluid-structure coupled matrix; $\mathbf{F}$ is the applied mechanical force vector.

3 Boundary element formulation for external/internal acoustic fluid

3.1 Harmonic equations

In this section, the direct boundary element method for exterior/interior acoustic domain is presented. The governing equations of the acoustic fluid are [7, 8]

$$\Delta p + k^2 p = 0 \qquad \text{in } \Omega_F \quad (10a)$$

$$\frac{\partial p}{\partial n} = 0 \qquad \text{on } \Gamma_D \quad (10b)$$

$$\frac{\partial p}{\partial n} = \rho_F \omega^2 \mathbf{u} \cdot \mathbf{n} \qquad \text{on } \Sigma \quad (10c)$$

$$\frac{\partial p}{\partial r} + ikp = \theta(\frac{1}{r}) \qquad \text{for } r \to \infty \quad (10d)$$

Eq. (10a) represents the Helmholtz equation where $k = \omega/c$ is the wave number, i.e. the ration of the circular frequency ω and the sound velocity c; Eq. (10b) is the rigid boundary condition on Γ_D; Eq. (10c) is the kinematic interface fluid-structure condition on Σ; Eq. (10d) represents the Summerfield condition at infinity.

3.2 Boundary element formulation

The boundary element formulation for acoustic problems can be used for the interior and exterior problems. The Helmholtz equation is valid for the pressure p at the arbitrary collocation point $\mathbf{x}$ within the acoustic domain Ω_F. A weak form of this equation is obtained by weighting with the fundamental solution:

$$G(\mathbf{x},\mathbf{y}) = \frac{e^{ik|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|} \tag{11}$$

where $|\mathbf{x}-\mathbf{y}|$ denotes the distance between the arbitrary point $\mathbf{x}$ and the load source point $\mathbf{y}$.

Applying Green's second theorem, the Helmholtz equation can be transformed into a boundary integral equation, which can be expressed as follows

$$c(\mathbf{x})p(\mathbf{x}) = \int_{\partial\Omega_F} p(\mathbf{y})\frac{\partial G(\mathbf{x},\mathbf{y})}{\partial n_y}\mathrm{d}S - \int_{\partial\Omega_F} \frac{\partial p(\mathbf{y})}{\partial n_y}G(\mathbf{x},\mathbf{y})\mathrm{d}S \tag{12}$$

where

$$c(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \text{ in fluid domain} \\ \frac{1}{2} & \mathbf{x} \text{ on smooth boundary of fluid domain} \\ \frac{\Omega(\mathbf{x})}{4\pi} & \mathbf{x} \text{ on nonsmooth boundary of fluid domain} \\ 0 & \mathbf{x} \text{ outside fluid domain} \end{cases} \tag{13}$$

and

$$\Omega(\mathbf{x}) = 4\pi + \int_{\partial\Omega_F} \frac{\partial(|\mathbf{x}-\mathbf{y}|^{-1})}{\partial n_y}\mathrm{d}S$$

is the solid angle seen from $\mathbf{x}$. Note that the value $c(\mathbf{x}) = \frac{1}{2}$ is valid if the surface $\partial\Omega_F$ is assumed to be closed and sufficiently smooth, i.e. there is a unique tangent to $\partial\Omega_F$ at every $\mathbf{x} \in \partial\Omega_F$. For the general case, where nounique tangent plane exists at $\mathbf{x} \in \partial\Omega_F$, we use $c(\mathbf{x}) = \frac{\Omega(\mathbf{x})}{4\pi}$(for example, when $\mathbf{x}$ is lying on a corner or an edge).

The fluid boundary is divided into N quadrilateral elements ($\partial\Omega_F = \sum_{j=1}^{N} S_j$) and Eq. (12) is discretized. After using the relation between the acoustic pressure and the fluid normal velocity $\frac{\partial p}{\partial n} = -i\rho_F\omega v$ (where $v = \mathbf{v}^F \cdot \mathbf{n}$), the discrete Helmholtz equation can be written for any point $\mathbf{x}$ defined by the node i as

$$c_i p_i = \sum_{j=1}^{N}\int_{S_j} p_j(\mathbf{y})\frac{\partial G(\mathbf{x}_i,\mathbf{y})}{\partial n_y}\mathrm{d}S + i\rho_F\omega\sum_{j=1}^{N}\int_{S_j} v_j(\mathbf{y})G(\mathbf{x}_i,\mathbf{y})\mathrm{d}S \tag{14}$$

For each quadrilateral element j, the pressure $p_j(\mathbf{y})$ and the normal velocity $v_j(\mathbf{y})$ can be expressed as a function of their nodal values

$$p_j(\mathbf{y}) = \sum_{k=1}^{4} N_k p_j^k = \mathbf{N}\mathbf{p}_j \quad , \quad v_j(\mathbf{y}) = \sum_{k=1}^{4} N_k v_j^k = \mathbf{N}\mathbf{v}_j \tag{15}$$

and Eq. (14) becomes

$$c_i p_i = \sum_{j=1}^{N}\sum_{k=1}^{4}\int_{S_j} N_k\frac{\partial G(\mathbf{x}_i,\mathbf{y}_j)}{\partial n_y}\mathrm{d}S p_j^k + i\rho_F\omega\sum_{j=1}^{N}\sum_{k=1}^{4}\int_{S_j} N_k G(\mathbf{x}_i,\mathbf{y}_j)\mathrm{d}S v_j^k \tag{16}$$

or in the following form

$$c_i p_i = \sum_{j=1}^{N}\sum_{k=1}^{4} \widehat{H}_{ij}^{k} p_j^k + i\rho_F\omega \sum_{j=1}^{N}\sum_{k=1}^{4} G_{ij}^{k} v_j^k \tag{17}$$

where

$$\widehat{H}_{ij}^{k} = \int_{S_j} N_k \frac{\partial G(\mathbf{x}_i,\mathbf{y}_j)}{\partial n_y} \mathrm{d}S \quad , \quad G_{ij}^{k} = \int_{S_j} N_k G(\mathbf{x}_i,\mathbf{y}_j)\mathrm{d}S \tag{18}$$

We place now the point $\mathbf{x}_i$ at each of nodal points on the boundary $\mathbf{y}_j$ successively, which is know as ”collocation point”. We obtain

$$c_i \delta_{ij} p_j = \sum_{j=1}^{N}\sum_{k=1}^{4} \widehat{H}_{ij}^{k} p_j^k + i\rho_F\omega \sum_{j=1}^{N}\sum_{k=1}^{4} G_{ij}^{k} v_j^k \tag{19}$$

When the collocation scheme is repeated for all nodal points N_n of the boundary element mesh, a set of N_n expressions in the nodal field variables is obtained which can be assembled into the following matrix equation

$$\mathbf{HP} = i\rho_F\omega\mathbf{Gv} \tag{20}$$

where $\mathbf{P}$ and $\mathbf{v}$ are the vectors with sound pressure and velocity in the normal direction to the boundary surface at the nodal position of the boundary element mesh.

4 FE-BE formulation for the fluid-structure with shunt systems coupled problem

The fluid boundary domain $\partial\Omega_F$ is divided into two parts including Γ_D (where the rigid boundary condition is applied) and the interface Σ (for the fluid-structure interface) such as $\partial\Omega_F = \Gamma_D \cup \Sigma$ and $\Gamma_D \cap \Sigma = \emptyset$. The boundary conditions given in Eqs. (10b) and (10c) can be expressed in discretized form

$$\mathbf{v}_D = \mathbf{0} \qquad \text{on } \Gamma_D \tag{21a}$$
$$\mathbf{v}_\Sigma = i\omega\mathbf{TU} \quad \text{on } \Sigma \tag{21b}$$

where $\mathbf{T}$ is the global coupling matrix that transforms the nodal normal displacement of the structure to the normal velocity of the acoustic fluid at the interface. Substituting Eqs. (21) into the BE matrix expression (Eq. (20)) yields

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{P}_\Sigma \\ \mathbf{P}_D \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \begin{bmatrix} i\rho_F\omega^2\mathbf{TU} \\ \mathbf{0} \end{bmatrix} \tag{22}$$

By combining Eq. (9) with Eq. (22), we find the following coupled FE-BE matrix equation

$$\begin{bmatrix} \mathbf{K}_u + \mathbf{C}_{uV}\mathbf{K}_V^{-1}\mathbf{C}_{uV}^T - \omega^2\mathbf{M}_u & \mathbf{C}_{uV}\mathbf{K}_V^{-1} & -\mathbf{C}_{up} & \mathbf{0} \\ \mathbf{K}_V^{-1}\mathbf{C}_{uV}^T & \mathbf{K}_V^{-1} - i\omega\mathbf{R} - \omega^2\mathbf{L} & \mathbf{0} & \mathbf{0} \\ i\rho_F\omega^2\mathbf{G}_{11}\mathbf{T} & \mathbf{0} & \mathbf{H}_{11} & \mathbf{H}_{12} \\ i\rho_F\omega^2\mathbf{G}_{21}\mathbf{T} & \mathbf{0} & \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \\ \mathbf{P}_D \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{23}$$

5 Numerical examples

In order to validate the methodology, we present in this section the analysis of an interior damped structural-acoustic system using an inductive shunt damping technique, according to the FEM-BEM approach described in this paper. First, the FEM/FEM modal analysis of the electromechanical-acoustic problem is presented. Then, the FEM-BEM frequency response of the coupled system in short circuit and inductive shunt cases are compared in terms of attenuation of vibration.

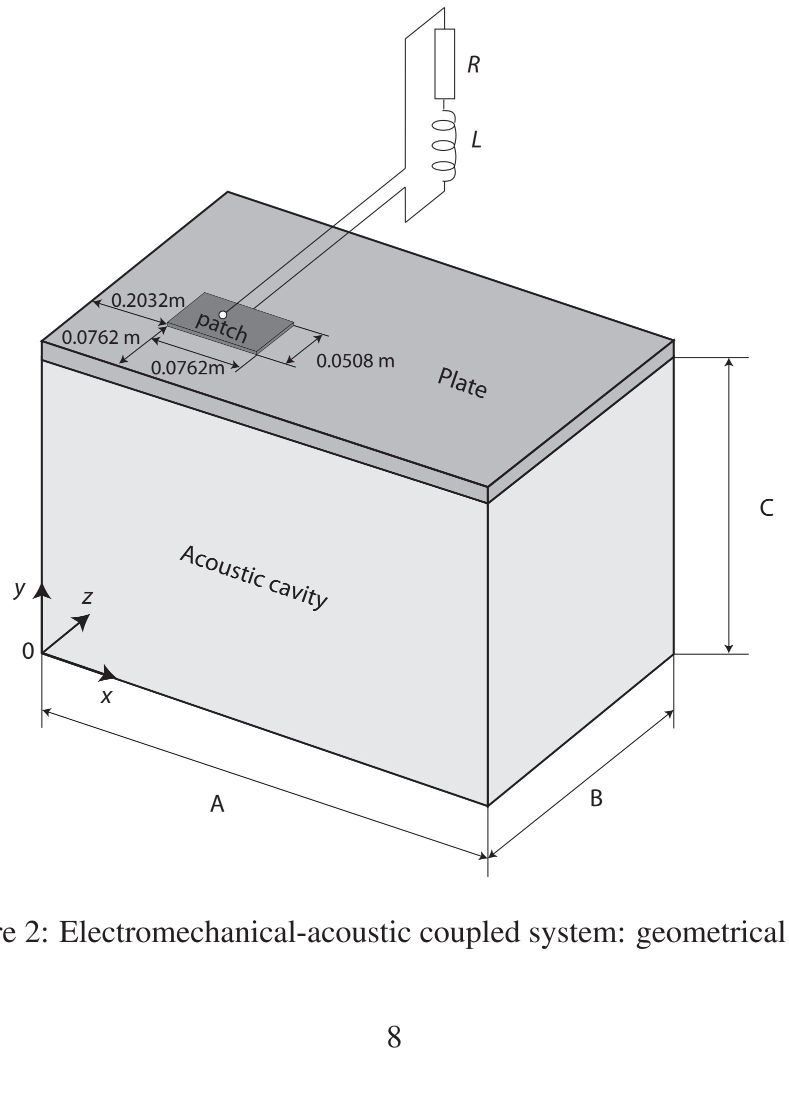

Figure 2: Electromechanical-acoustic coupled system: geometrical data.

We consider a 3D hexaedric acoustic cavity of size A= 0.3048 m, B=0.1524 m and C=0.1524 m along the directions x, y, and z, respectively. The cavity is completely filled with air (density ρ= 1.2 kg/m^3 and speed of sound c=340 m/s). The cavity walls are rigid except the top one, which is a flexible aluminum plate of thickness 1 mm. The density of the plate is 2690 kg/m^3, the Youngs modulus is 70 GPa and Poisson ratio 0.3. On the top surface of the plate, a PIC 151 patch is bonded, whose in plane dimensions are 0.0762$\times$0.0508 m^2 along x and y and 0.5 mm thick (see figure 2). The mechanical characteristics of the piezoelectric material PIC 151 are given in [2].

Concerning the FEM/FEM discretization, we have used, for the structural part, 72 plate elements. The portion of the plate covered by the piezoelectric patch and the patch itself has been modeled according to laminated theory [2]. Moreover, only one electrical degree of freedom is used to represent the electrical charge Q in the patch. The acoustic cavity is discretized using 12$\times$6$\times$6 hexahedric elements. The structural and acoustic meshes are compatible at the interface, and the fluid-structure coupling is realized through the $\mathbf{C}_{up}$ matrix.

For the FEM-BEM formulation, besides the FE discretization of the plate, only the boundary Σ of the fluid domain is discretized with boundary elements. Notice that the BE nodes on this part must coincide with FE nodes.

Table 1 presents the first ten eigenfrequencies in three cases: (i) the 3D rigid acoustic cavity, (ii) the plate with the patch in short and open circuited cases and (iii) the plate/acoustic-cavity coupled system in the short circuited case. All results are computed with the finite element formulation presented in [2] . The fourth and ninth frequencies are associated with the first two acoustic modes in the rigid cavity lower than 1128.2 Hz, while the other frequencies correspond to the first eight vibration modes of the structure. This can be confirmed by comparing the mode shapes in case (iii) with those obtained in case (i) or case (ii), which are not shown here for the sake of brevity. Moreover, as expected, the natural frequencies of the coupled modes (structure dominated) are lower than those for the structure in vacuum (except for the first mode) due to the added-mass effect of the fluid.

Fluid	Structure SC	Structure OC	Fluid-Structure
559.3	210.6	210.7	215.1
1128.1	329.5	330.9	327.1
1128.3	540.6	544.4	538.9
1128.5	722.2	722.2	561.7
1259.1	828.4	830.6	719.4
1259.5	834.3	834.7	826.5
1595.5	1023.8	1024.7	832.1
1595.6	1204.7	1204.7	1021.0
1595.7	1296.1	1296.7	1128.1
1690.8	1567.0	1567.0	1129.9

Table 1: Computed frequencies (Hz) of the structural-acoustic coupled system.

The plate is now excited by an unit distributed time harmonic pressure load. The coupled FEM-BEM results are compared with those obtained from a coupled FEM-FEM analysis. As can be seen in figure 3, the sound pressure level is calculated on the plate center. The results for the two methods are very similar at sound level peaks (resonance frequencies) which enable us to check the validity of the proposed FEM-BEM coupled formulation.

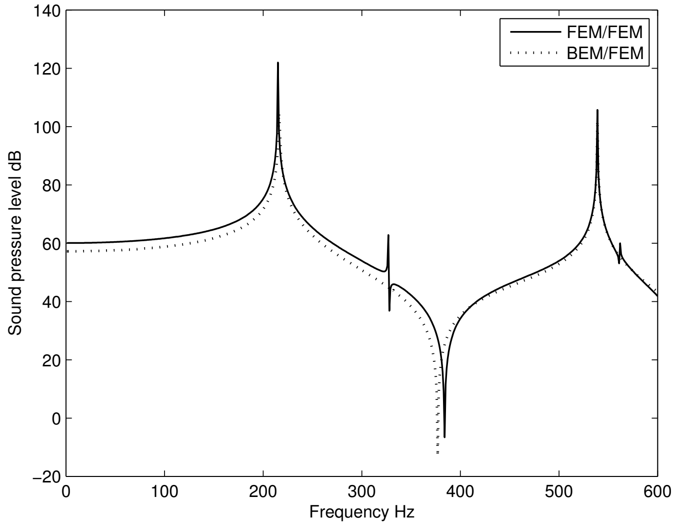

Figure 3: Sound pressure level on plate center: comparison between FEM-FEM and FEM-BEM approaches.

In order to achieve maximum vibration dissipation of the third coupled mode, the patch is tuned now to an RL shunt circuit. The optimal values of the shunt electrical circuit are taken R=348 Ω and L=0.61 H. The system vibratory response is obtained with the proposed BEM-FEM approach. Figure 4 presents the sound pressure level on plate center with and without shunt system. This figure shows that the resonant magnitude for the third mode has been significantly reduced due to the shunt effect. In fact, the strain energy present in the piezoelectric material is converted into electrical energy and hence dissipated into heat using the RL shunt device.

6 Conclusions

In this paper, a coupled finite element-boundary element method (FEM-BEM) for control of noise radiation and sound transmission of vibrating structure by active piezoelectric techniques is presented. The passive shunt damping strategy is employed for vibration attenuation in the low frequency range. Work in progress concerns the validation of the proposed approach for structure with piezoelectric patches coupled with an external fluid domain in order to validate the efficiency of the shunt technique for

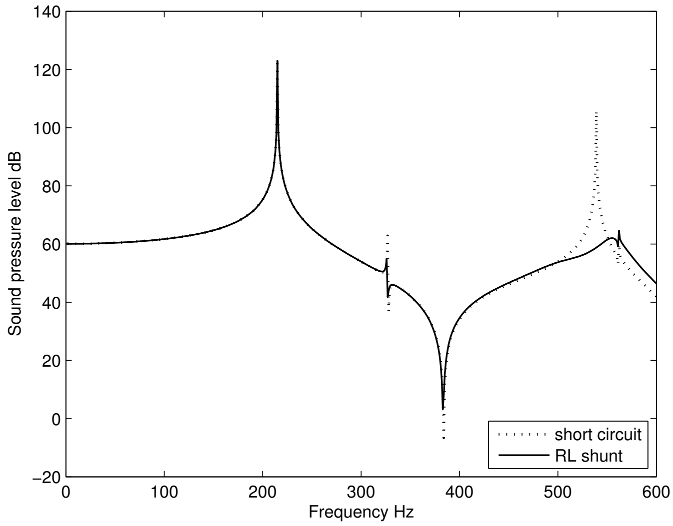

Figure 4: Sound pressure level on plate center with and without shunt system.

reducing the acoustic radiation power. Associated to this methodology, a reduced order model for the BEM-FEM coupled formulation is under development.

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