

Optimization of Materials Selection in a Multi-Part Environment with Direct Multi-Search

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Abstract

Materials selection is a matter of great importance to engineering design and software tools are valuable to inform decisions in product development. When a pool of alternative materials is available for different parts, the question of what optimal material mix to select for a set of parts is not simple.

In engineering problems, the designer decides about the part shape and material and their decision largely determines the part cost and weight. However, cost and weight are only possible to calculate when the part geometrical attributes and production process elements are defined.

As a result of the large number of permutations, exhaustive search is not possible to justify the use of an optimization procedure to determine the optimal solution. Another aspect of the optimization procedure is that it needs to deal with non-differentiable objective functions and constraints. To solve this multi-objective optimization (MOO) problem, a new routine based on the direct multisearch (DMS) algorithm [1] is proposed.

An example from industry has been solved using this new methodology based on DMS. Results from the Pareto front can help the designer to align the material selection for a complete set of materials with product attribute objectives, depending on the relative importance of each objective. The results illustrate the capacity of this DMS model to solve the optimization problem with reasonable computational time.

Keywords: materials selection, multiple parts, automotive, direct search, multiobjective optimization.

1 Introduction

Materials are the embodiment of products. Materials selection is the task of adapting material properties to specifications of that product or part. In engineering projects, most problems are related with weight of the product versus its cost of manufacturing. When choosing materials for an application it is important to evaluate alternatives in more than one objective, and the trade-off is often between the costs to produce a part versus its technical performance. Another issue in materials selection is when a set of parts is up for selection. The number of alternatives is high and the number of evaluations can be cumbersome.

Good cost estimation is a necessary condition for the financial success of the company. Estimating high can result on loss of business and market share and low estimations can result on selling with a lower price resulting in bad results for the enterprise [2]. Dewhurst established some metrics for early cost estimation in product design intended to reduce costs in manufacturing, [3]. The attraction of cost as a metric for decision-making is the apparent simplicity of it: “*an economic measure of the resources employed to undertake a set of actions, typically to yield a good or service*” [4]. However, the estimation of a product cost is neither trivial nor a straightforward activity in the design process in particular in the conceptual phases of product design [5]. In this phase, there is a need for informed life cycle cost estimations [6]. A common rule of thumb is that about 70 to 90% of the cost of a product is determined by the design. In these early phases, engineers and designers are usually far less comfortable with cost when tasked with relating it to a set of specific technical or design changes. The difficulty arises from the fact that cost has traditionally been associated with accounting rather than the engineering field [4].

If one wants to estimate cost for bidding purposes, cost estimation must be absolute and any small inaccuracy in the estimation can make the difference between profit and loss. However, in an engineering design environment cost is a design dependent variable used for comparing and evaluating alternatives. Cost estimation can then be understood in a relative context. When the purpose is to guide the decision making process through the design phases the estimates can be less accurate [7].

In this paper we use process based cost modeling (PBCM) which is a tool to assess the resources needed for the manufacture of a component. The advantage of these methods is that they can be predictive, but, as they rely on the modeling of the manufacturing process, their use in the late stages of product development is preferable. The key for using process based cost models is to identify relevant cost elements, establish contribution factors and correlate process operations to the cost of the used factors [8].

In the automotive industry the final weight of the vehicle is of most importance, so another fundamental aspect of materials selection is the weight of the set of parts resulting from materials selection. For simple applications, with a small number of parts, traditional materials selection techniques can be employed to aid the decisions. But if the number of parts increases, along with the number of alternative materials for each, a more automated process for materials selection must be

arranged. In this paper, multiple parts are evaluated at the same time inside a MOO process to provide non dominated solutions for multiple parts selection.

An example from industry has been solved using this methodology based on DMS, a case study for a sheet metal progressive stamping process. It solves the problem for 55 small stamping automotive parts, each with a given geometry and material grade already suggested by an experienced designer. The optimization tool must find, from a pool of different cold rolled material grades and different thicknesses available, the optimum material selection of each part over two objectives, cost and weight, with constraints relative to each part manufacturability, strength and stiffness.

In this paper, the formulation necessary to calculate the objective functions and constraints is presented in the next section, followed by the optimization procedure, numerical results and conclusions.

2 Formulation

In this section, the objective functions are calculated using the process based cost modeling (PCBM) technique and the constraints are calculated using a design model, explained bellow.

2.1 The process based cost model

Process based cost models are a predictive way of estimating the production costs of a part, by assessing the major cost drivers and their relation with a product description, process and financial parameters.

Each model is different, for each process, but all models share the same construction: an estimation of the necessary labor, energy, material, equipment, tooling, maintenance and building to produce a part in a process. All inputs and outputs are computed and product, process and financial inputs are worked to deliver an economic cost [4].

The initial set together with its cost and weight will create the baseline solution for this case.

The annual manufacturing cost MC , of the set of parts is the sum of the manufacturing costs of each part, MC_i calculated through the PCBM for sheet metal forming:

$$MC = \sum_i (MC_i \cdot PV_i) \quad (1)$$

where PV_i is the annual production volume of each part. The manufacturing unit cost of each part is given by:

$$MC_i = \frac{C_{i_Material} + C_{i_labor} + C_{i_energy} + C_{i_equipment} + C_{i_tools} + C_{i_maintenance} + C_{i_building}}{PV_i} \quad (2)$$

The costs drivers C_i are the allocated costs of labor per year, energy, equipment, tools, maintenance and building for the total production volume.

The objective of the model is to feed the optimization routine with the manufacturing costs of all parts in the iterative search for the best solution.

2.2 The design model

The design model is where constraints are calculated. A design baseline solution is extremely important for the functionality of the model. The better the initial set of materials for each part regarding the technical performance is, the better results are expected. As with all optimization procedures, the optimization procedure follows a predefined objective to maximize or minimize subject to constraints. In this case, the constraints are design requirements verified, at each objective function evaluation, by comparing the performance of the set of parts proposed by the DMS algorithm, with the baseline set.

The design model presented regulates the optimization process, not allowing the model to generate new reference sets that do not comply with the design model and is fully dependent on the initial set of materials provided. The optimization framework does not evaluate the technical performance of each part as regards to its conformity to technical specifications. However, simple engineering functions can be used to establish a relationship between the performance of the initial part, which is a design solution that fulfills the specifications, and the performance of the new part generated and evaluated at each evaluation of the optimization.

Unlike PCBM model, the design model regulates each part independently. The model should be able to accept any technical constraint for each part individually and be able to compare the behavior of the new parts as regards the new materials and thickness suggestions with the initial ones. When a reference from the available pool of materials is presented, the design model evaluates if that part also suits the part specifications.

Since the design is frozen to part shape changes, except for thickness and material properties, and it is assumed that the baseline solution fulfills the requirements, only calculations of equivalent stiffness, strength and stampability need to be performed, providing relative performance measures. In other words, the new generated parts are validated not because they fill a specification but because their technical behavior is similar to the behavior of the initial ones. In addition, the designer can insert any constraints applicable to each part. For this case, the following three inequality design constraints were used: part strength, part stampability and part stiffness.

The strength of part in bending around the neutral axis is given by:

$$\sigma_{\max} = \frac{Mc}{I} \leq \frac{YS}{SF} \Leftrightarrow M \leq \frac{YS}{SF} \cdot \frac{I}{c} \quad (3)$$

where M is the applied moment, c is the maximum perpendicular distance to the neutral axis and I is the second moment of area about the neutral axis. YS is the

yield stress and SF is a safety factor. For a rectangular flat section $c = t/2$ and $I = 1/12 bt^3$. So:

$$M \leq \frac{YS}{SF} bt^2 \quad (4)$$

where t and b are the part thickness and width, respectively. If the part material and the part thickness can change from a generic iteration i to a generic iteration j , for the same loading and safety factor an equivalent strength is achieved, if:

$$YS_j \cdot t_j^2 \geq YS_i \cdot t_i^2 \quad (5)$$

In these conditions, the part in iteration j has an equivalent strength higher than the same part in the incumbent solution i , even if materials and thicknesses have changed. Remember that the word incumbent is chosen because the method bases all constraints on a validated initial solution.

A factor of how much the designer wants the new material to be better than the incumbent can easily be added for each part simply by the multiplication of a factor r_i^m , where index m represents the constraint type. For C , I or hat shaped beams the same exercise can be created easily.

For the equivalent stiffness, the exercise is also simple. The stiffness of a beam is given by:

$$\frac{P}{\delta} = \frac{C_1 EI}{L^3} \quad (6)$$

where P is the load, δ is the correspondent displacement, C_1 is the constant for the load supports, E is the Young modulus and L is the length of the beam. Then when comparing two different materials, if

$$E_j t_j^3 \geq E_i t_i^3 \quad (7)$$

then, the part made of the new material achieved in iteration j has better stiffness than the initial part solution based on the incumbent material (iteration i).

To analyze the stampability of the new parts an analytical model can also be devised taking advantage of material comparisons using forming limit diagrams (FLD). A FLD is a graphical representation of the forming capability of a material where the stretching limits are a function of major and minor strains in the part [9-11]. The shape of the curve is approximately constant for all steels, the curve moves up or down the major strain axis depending on the strain hardening exponent n characteristic of each material and thickness of the metal sheet. FLD_0 is the value of the major strain at plain strain conditions and can be calculated by:

$$FLD_0 = \frac{n(23.3 + 14.1t)}{0.21} \quad (8)$$

where n is the strain hardening exponent and t is the thickness of the sheet metal stamping in mm .

Using this information, a forming limit diagram is drawn for each specific steel sheet material. Considering material i and material j the first has better stretching capability¹ if:

$$n_i(23.3 + 14.1 \cdot t_i) \geq n_j(23.3 + 14.1 \cdot t_j) \quad (9)$$

For this paper, three constraints models were presented for design and process purposes, but the model is able to process much more if necessary. These constraints can also be introduced as objective functions in MOO, but since these constraints are used for every part, the number of objective functions would increase to the number of parts times the number of constraints plus the original objective functions.

In this model, the user can introduce new constraints following the same procedure above, and can also relax existing constraints with the factor r_i^m . The relaxing of some constraints for some parts is dependent on the decision of the designer/engineer. For example, a lightweight solution can be accepted if the strength of the part is kept equal to the incumbent one, meaning that for the strength constraint r_i^m needs to be defined as 1. This factor can be applied to all parts and to each specific constraint. For the examples below,

Table 1 shows the constraint factors used in the model. For this example, the designer is willing to accept, if necessary, a 20% reduction in stampability and strength and no reduction in stiffness. The objective is to get rid of sets that will not perform in terms of manufacturability or part performance under use conditions, while allowing the accommodation of some design flexibility.

Case	Constraints r_i^m
Test	$r_i^m = \begin{cases} \text{Stamping} \\ \text{Strenght} \\ \text{Stiffness} \end{cases} = \begin{cases} 0.8 \\ 0.8 \\ 1 \end{cases}$

Table 1 – Constraint relaxation factor for each attribute.

3 Multi-objective optimization using Direct Multisearch

Materials selection can be formulated as a MOO problem, as it involves the minimization of several conflicting objectives. In this case, the minimization of the

¹ Note that for cup drawing this equation is not valid.

manufacturing costs of multiple parts and the weight of the set of parts is constructed.

This problem is combinatorial by nature and a MOO tool that does not rely on derivatives is needed. One of the methodologies available for this type of problems is the DMS, a solver for derivative-free multi-objective optimization, inspired by the search/pool paradigm of direct search methods of directional type and uses the concept of Pareto dominance to maintain a list of non-dominated points (see [11] for a more complete treatment)

A constrained nonlinear MOO problem takes the form [12]:

$$\text{find } x = \begin{Bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_s \end{Bmatrix} \quad (10)$$

which minimizes:

$$\min F(x) \equiv (f_1(x), f_2(x), \dots, f_k(x))^T \quad (11)$$

subject to:

$$\begin{aligned} g_{l_1}(x) &\leq 0, & l_1 &= \{1, 2, \dots, m_1\} \\ h_{l_2}(x) &= 0, & l_2 &= \{1, 2, \dots, m_2\} \end{aligned} \quad (12)$$

where s is the number of design variables, k is the number of objective functions to be minimized and m_1 m_2 are the number of constraint equations. Any or all functions $f_k(X)$, $g_{l_1}(X)$ and $h_{l_2}(X)$ can hold a nonlinear nature. In general, since in MOO there are often conflicting objectives for each objective function, the concept of Pareto dominance is used to characterize global and local optimality [1]. A feasible solution of X is called a Pareto optimal if there exists no other feasible solution Y such that $f_i(Y) \leq f_i(X)$ for all $i = \{1, 2, \dots, k\}$ with $f_j(Y) < f_j(X)$ for at least one j , $j \in \{1, 2, \dots, k\}$.

For this case, the process based cost model generates the values for the objective functions. Constraints are calculated inside the design module in each evaluation for validation purposes. In the global optimization model, Equations (10) to (12) take the form:

$$\text{find } X = \left\{ \left\{ \begin{matrix} t_1 \\ m_1 \end{matrix} \right\} \quad \left\{ \begin{matrix} t_2 \\ m_2 \end{matrix} \right\} \quad \cdot \quad \cdot \quad \left\{ \begin{matrix} t_i \\ m_i \end{matrix} \right\} \right\} \quad (13)$$

where i is the number of parts understudy, which minimize

$$f_1(X) = \sum_i MC_i(t_i, m_i) = \text{sum of the manufacturing cost of each part} \quad (14)$$

$$f_2(X) = \sum_i W_i(t_i, m_i) = \text{sum of the weight of each part} \quad (15)$$

At a given iteration, the DMS produces new values for X variables. For each part, the two objectives are calculated using the PCBM technique and the design model with the respective constraints. These constraints compare the technical performance of the new parts (with new X values) with the original set of parts (with the initial X values). Figure 1 shows the optimization procedure for each evaluation.

An initial vector with design variables of all the parts is introduced, as the incumbent solution, and the initial value of the objective function is calculated. For each X , created by DMS, the design module determines the correspondent valid model solution and the PCBM module computes the value of the objective functions on each design valid solution. The design module is constructed with information from the baseline solution composed of all the incumbent references of material and thickness. If accepted, this new vector calculates the objective function inside the PCBM module and, if not, a penalization function is used. The optimization iterates and a Pareto front is presented.

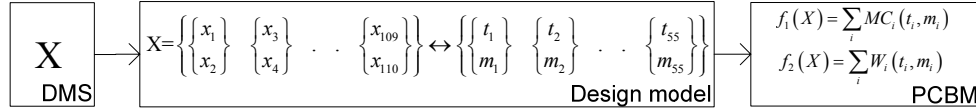


Figure 1 – For each X , created by DMS, the Design module determines the correspondent valid model solution, the PCBM model compute the value of the objective functions on this valid model solution.

4 Numerical example

This section presents the results achieved when the initial set of parts of the example case (the baseline solution) is subject to the optimization procedure aiming to associate new material references (material grades and thicknesses) to the parts that minimize the manufacturing costs and total weight, while simultaneously keeping the technical requirements of each part. Table 2 shows some parameters necessary to estimate the manufacturing cost of each part by PCBM.

The design variables are the material reference grade and thickness. The DMS provides a text file list of X values:

$$X = [x_s] \text{ with } x_s \in [0,1], s = 1, \dots, 110 \quad (16)$$

A correspondence to the physical model is given by the design module:

	Part Number	1	55
Design variables	Material Reference	7	31
	Gage [mm]	0.85	0.75
Parameters and inputs necessary for part cost calculation	Complexity [1 / 2 / 3]	2	1
	Part width [mm]	350	922
	Part length [mm]	350	1106
	Final Surface Area [m ²]	0.1225	1.019
	Proj. Surface Area [m ²]	0.1470	1.223
	Cutting Perimeter [mm]	240	638
	Number of hits	5	3
	Material Class	2	1

Table 2 – Parts description inputs for the PCBM.

$$X = \left\{ \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \begin{Bmatrix} x_3 \\ x_4 \end{Bmatrix} \dots \begin{Bmatrix} x_{109} \\ x_{110} \end{Bmatrix} \right\} \leftrightarrow \left\{ \begin{Bmatrix} t_1 \\ m_1 \end{Bmatrix} \begin{Bmatrix} t_2 \\ m_2 \end{Bmatrix} \dots \begin{Bmatrix} t_{55} \\ m_{55} \end{Bmatrix} \right\} \quad (17)$$

in which t_i and m_i are the coil (part) thickness and material grade for each part, respectively. The material grades are the ones available in the market for cold rolled steel. In this case, 36 different materials (different grades) are available:

$$m_i \in \{1, \dots, 36\} \quad (18)$$

The thickness for each part can vary from 0.5 to 2 mm in steps of 0.05 mm. From 1.5 to 2 mm, the step is 0.1 mm:

$$t_i \in \{0.5, 0.55, \dots, 1\} \cup \{1.6, 1.7, \dots, 2\} \quad (19)$$

The number of alternatives available for each part is then 936, a matrix of 36 by 26. After this translation made by Equations (17) to (19), the design module can deal with the proposed value of X and validate each of the 55 parts. If a part is validated, then the value of X is unchanged, if not, then for the same thickness is kept, the next closest material is chosen and the new pair (t_i, m_i) is evaluated. This procedure goes till a satisfactory pair is selected. If not found, the pair will resume to the incumbent solution, which is self-satisfactory. In the end, a valid model of design variables (materials and thicknesses for each part) can be evaluated by the PCBM module.

The valid model solution are calculated in Design Model, the PCBM compute the objective functions from Equations (18) and (19). Results for that evaluation are written in a text file and sent back to the DMS procedure for a new evaluation.

The non-dominated solutions computed by DMS, for the example, are presented in Figure 2 and values for selected sets are presented in Table 3. The comparison

between the incumbent solution and the Pareto front generated from the optimization routine is of the most importance. The incumbent solution represents the current materials selection techniques and considers the sum of the manufacturing costs of the materials for the initial set of parts. The results show that alternatives can provide a less costly solution, maintaining the weight of the set almost unchanged.

Part set	Cost [€]	Weight [Ton]
Incumbent	€ 25,153,891	20,378
Min Cost	€ 24,158,739	20,503
Min Weight	€ 24,220,570	20,378

Table 3 – Pareto front results for different objectives, 1st case, bold indicates minimum results in one objective

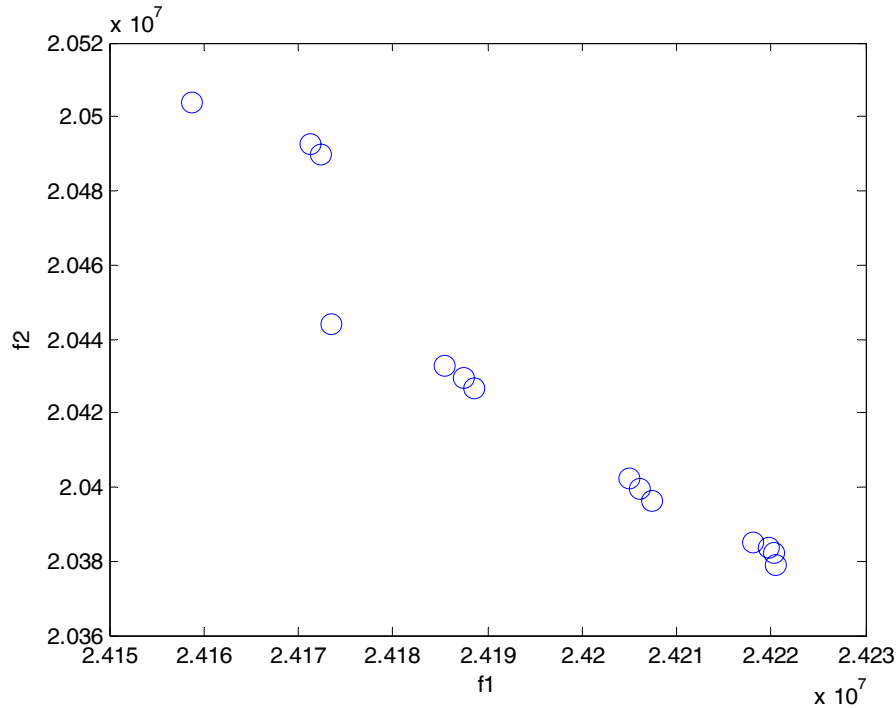


Figure 2 – Pareto front for the numerical test. Cost f^1 in € and weight f^2 in kg.

5 Conclusions

In this paper, an application based on the DMS algorithm is proposed for the solution of multiple part materials selection considering the manufacturing costs and weight of the whole set of parts. The solutions obtained shows that, from an original incumbent solution, different materials combinations to each part can alter the

materials selection procedure for each part. From the Pareto front, the designer can align its materials selection procedure for a complete set of materials with product attribute objectives, depending on the relative importance of each objective. The results also illustrate the capacity of this DMS model to solve the optimization problem with reasonable computational time.

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