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Improvement in Damage Localisation using Speckle Shearography

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Abstract

This paper proposes a damage localisation method based on the analysis of perturbations in the second, third and fourth order derivatives of experimental modal displacement field. The high spatial resolution modal rotation fields are measured using speckle shearography with stroboscopic illumination and time phase modulation. The mode shape derivatives up to the fourth order are obtained by numerical differentiation. A new numerical differentiation strategy is proposed with the objective of minimising the propagation of experimental noise. This study is performed on a laminated composite plate where internal damage at two locations was created by two low energy impacts. A comparative analysis of the results between this method and the one proposed in previous studies is presented. The low experimental noise obtained in the measurements and the new differentiation method has led to a significant improvement in the damage localisation.

Keywords: damage identification, speckle shearography, modal response, high order spatial derivatives

1 Introduction

In recent decades there has been an increased use of composite materials in lightweight structural applications such as those found in the automotive and aeronautics industries. Laminated composite structures present types of defects and damages mechanisms different from those of metals. The low damage tolerance of these structures and the lack of effective and global non-destructive inspection techniques have motivated, mainly in the last two decades, the search for new methodologies. The complexity of the problem and the difficulty in finding a robust solution has led to the proposal of different approaches [1-13]. However, the most promising technique referred in literature by different authors is based on the analysis of perturbation or discontinuities in modal curvatures or strain fields [1, 3, 7, 9, 14]. More recently, the use of higher order modal derivatives has been proposed [15-17]. In practice, these high order spatial derivatives can only be obtained by numerical differentiation of experimental displacements and rotations fields. Moreover, the accurate measurement of full-field data is required in order to minimize the amplification and propagation of experimental noise, through the numerical differentiation process [18, 19].

This paper presents a damaged localisation method based on the analysis of second and third order spatial derivative of experimentally measured modal rotation fields, which correspond to the third and fourth spatial derivative of modal displacements fields, respectively. The full-field modal rotations of a multi-damaged laminated composite plate are measured using speckle shearography with laser amplitude modulation. The direct measurement of the deformation gradient field, which is a good approximation to the rotation field for small displacements, has the advantage of reducing the order of the numerical differentiations by one. Moreover, these experimental measurements present a lower signal-to-noise ratio when compared to speckle shearography with pulsed laser, as was used in previous studies [19]. The high order spatial derivatives of the modal response are computed by the application of central finite differences and low-pass filters to the experimental phase maps. The damages are directly identified through the analysis of perturbations in the second, third and fourth order spatial derivatives of the out-of-plane modal displacement fields, without need of previous knowledge of the undamaged structure behaviour.

2 Methodology

2.1 Speckle Shearography

Experimental modal analysis is used to identify the modal parameters such as natural frequencies, mode shapes, modal damping coefficients, mass distribution and structural stiffness. Indeed, the structural damage localisation requires the accurate measurement of the structure modal displacement or rotation field and the computation of their high order spatial derivatives by numerical differentiation. In the conventional experimental modal analysis, the application of accelerometers, the gluing materials and the supply of connecting cables will results in the addition of mass to the system. Depending on the masses ratio and its location relatively to the modal amplitude, this can significantly change the dynamic behaviour of the structure. On the other hand, speckle interferometry techniques, such as electronic speckle pattern interferometry (ESPI) and speckle shearography, allow full-field, non-contact and high sensitivity measurements of the modal displacement and modal rotation fields on the surface of the structure, respectively [20]. In relation to the structural damage identification, the main limitations of the ESPI technique arise from the high density of fringes obtained from measurements of displacements fields, including the rigid-body displacements, which makes difficult the interpretation of the fringe patterns [21-23]. On the other hand, the speckle shearography allows to measure displacement gradients, for which reason it is practically insensitive to the rigid-body motion. In addition, it requires a simpler optical interferometer setup and a laser with low coherence length, from which more compact systems can be built, which are also more robust to external perturbations. This interferometer uses the principle of the speckle interference between two wavefronts reflected by the surface of the object, which are laterally shifted, i.e. sheared**.** This shift can be created through a glass-shaped wedge placed in the front half of the lens, a rotation of two glass plates, a Wollaston prism or a Michelson optical interferometer setup with a slight rotation of one of the mirrors [20]. The last option is preferred, since is the only that allows the easy adjustment of the amount of shearing.

Most applications of speckle shearography are dedicated to measuring the static rotation field, because of its simple experimental arrangement. Instead, the measurement of modal response requires the use of more complex illumination and synchronisation systems and, therefore, is more difficult to adjust. As a result, the first reports on measurements of vibration responses using speckle shearography were only published in the last decade [22, 24, 25]. Until then, an approximation to the modal rotation field was measured using the time-average method [20]. This has the advantages of using the same optical interferometer setup used in the static measurements and allows the observation of the vibration contour fringes at video rate. The method is based on subtraction of speckle interference patterns produced by stationary harmonic motion of objects during several cycles of vibration. In this case, the recording time is very long compared to the period of vibration. The black intensity fringes are observed as contours of equal amplitude of vibration, being the fringe intensity modulated by a Bessel function J_0, where the contrast decreases with the increase of the fringes order associated with the amplitude of vibration. Only recently, the introduction of the spatial phase modulation and the temporal phase modulations methods to speckle shearography made possible a quantitative evaluation of the phase distribution or phase map and the accurate measurement of the modal rotation field [24, 25].

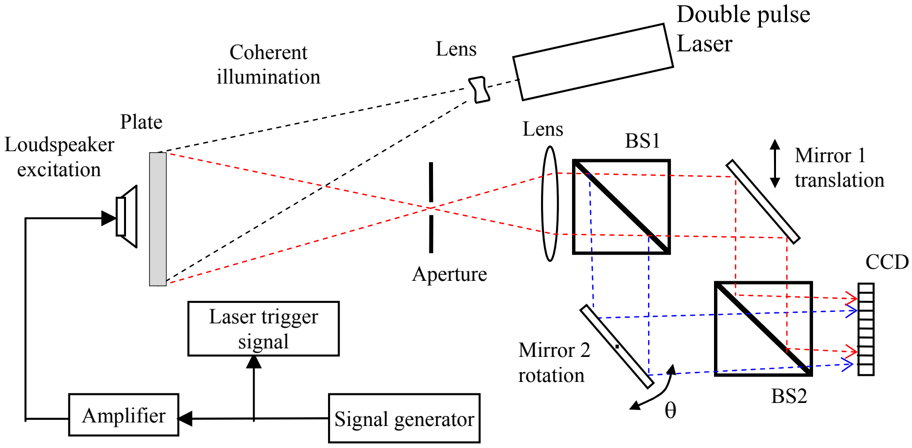

Figure 1: Schematic diagram of the speckle shearography system with the double pulse laser illumination used for the measurement of modal rotation fields

A spatial phase modulation and a double pulse laser illumination recording system are combined with speckle shearography to measure the modal rotation fields [19]. A spatial carrier is introduced in the interference pattern through a small rotation between the two wavefronts, in order to subsequently extract the phase map. The introduction of this spatial carrier is made possible by using the Mach-Zehnder interferometry optical setup, as shown in Figure 1.

In this configuration, the speckle pattern created and reflected by the rough and diffuse surface is divided by the first beam splitter (BS1) in two optical paths, reflects in mirror 1 and mirror 2, and is recombined in the second beam splitter (BS2). The amount of shearing can be controlled through the translation of mirror 1 and the spatial carrier by the rotation θ of mirror 2. The recording of the spatial carrier requires the use of small optical apertures, which limits the frequency of the measurement to 1/6 of the number of pixels of the CCD array [26]. Also, with this interferometer we cannot obtain uniform distributions of the spatial carrier, which leads to increased noise levels in the measurement. The determination of the interference phase involves the isolation of the spectral information around the spatial carrier and can be more easily performed through the application of forward and inverse fast Fourier transforms [27]. The intensity of the interference in wave number domain $I(u,v)$ can be described by [20]:

$$I(u,v) = A(u,v) + C(u,v) + C^{*}(u,v) \tag{1}$$

where $A(u,v)$ represents the background intensity, $C(u,v)$ and $C^{*}(u,v)$ the intensity of the phase interference modelled by the carrier phase, being u and v the order of the wave number in the horizontal and vertical directions, respectively. After demodulation of the spatial carrier, the phase of the interference can now be calculated by:

$$\Phi(\mathrm{x},\mathrm{y}) = \arctan\frac{\mathrm{Im}[\mathrm{C}(\mathrm{x},\mathrm{y})]}{\mathrm{Re}[\mathrm{C}(\mathrm{x},\mathrm{y})]} \tag{2}$$

where x and y are coordinates in the spatial domain. The double pulse laser and the vibration amplitude are controlled from an external synchronisation signal in order to capture the modal rotation amplitude in two different instants, defined has reference and deformation states. The phase map of the modal rotation fields $\Delta\phi(x,y)$ is extracted by subtracting the deformed interference phase $\Phi_{\mathrm{D}}(x,y)$ from the reference interference phase $\Phi_{\mathrm{R}}(x,y)$, according to the following equation:

$$\Delta\phi(\mathrm{x},\mathrm{y}) = \begin{cases} \Phi_{\mathrm{D}}(\mathrm{x},\mathrm{y}) - \Phi_{\mathrm{R}}(\mathrm{x},\mathrm{y}) & \text{if} \quad \Phi_{\mathrm{D}}(\mathrm{x},\mathrm{y}) \geq \Phi_{\mathrm{R}}(\mathrm{x},\mathrm{y}) \\ \Phi_{\mathrm{D}}(\mathrm{x},\mathrm{y}) - \Phi_{\mathrm{R}}(\mathrm{x},\mathrm{y}) + 2\pi & \text{if} \quad \Phi_{\mathrm{D}}(\mathrm{x},\mathrm{y}) < \Phi_{\mathrm{R}}(\mathrm{x},\mathrm{y}) \end{cases} \tag{3}$$

For measurements with a sensitivity vector perpendicular to the measurement surface, a relation between the gradient of the out-of-plane displacement field $w(x,y)$ and the measurement phase map can be established [20]:

$$\Delta\phi(\mathrm{x},\mathrm{y}) \approx \frac{2\pi\Delta\mathrm{x}}{\lambda}\frac{\partial \mathrm{w}(\mathrm{x},\mathrm{y})}{\partial \mathrm{x}} \tag{4}$$

being, Δx the shearing value in the x direction, λ the laser wavelength and $\partial w(x, y)/\partial x$ is the first spatial derivative of the out-of-plane displacement field in the x direction, which can be taken has good approximation of the rotation field for small deformation amplitude.

As an alternative methodology, time phase modulation and stroboscopic laser illumination are combined with speckle shearography to measure the modal rotation fields. The introduction of the stroboscopic laser illumination, synchronized with vibration excitation of the object, allows to freeze in time the speckle pattern [23]. Thus, temporal phase modulation can be applied to a quantitative determination of the phase map. In this case, a speckle shearography system is built based on the optical Michelson interferometer setup, the same used for static measurements. The stroboscopic illumination or intensity modulation can be generated from a continuous-wave laser either by using an electro-optic modulator or an acousto-optic modulator. In the first case, the short stroboscopic illumination pulses are created by switching the polarisation of a Pockel cell crystal by $\pi/2$, being its duration controlled by a high voltage electrical signal. This produces a more efficient illumination than the use of an acousto-optic modulator, but requires a more expensive system. Therefore, the acousto-optic modulator is normally used to generate the illumination pulses. In this case, the continuous-wave laser beam passes through a crystal were travelling sound waves are generated by a piezoelectric actuator. This produces periodic variations in the refractive index of the crystal. The light beam is then deflected laterally by selecting the grating first order diffraction, through the adjustment of the light beam incidence angle θ, as shown in Figure 2(a). A spatial filter is mounted in front of the acousto-optic modulator for the purpose of isolating the stroboscopic pulses. The pulses width should be narrow to frozen the interference speckle pattern and wide enough to illuminate the object surface. The stroboscopic illumination is produced by the modulation of the piezoelectric excitation signal with the generated pulse signal, which is synchronized with the harmonic vibration excitation, as depicted in Figure 2(b).

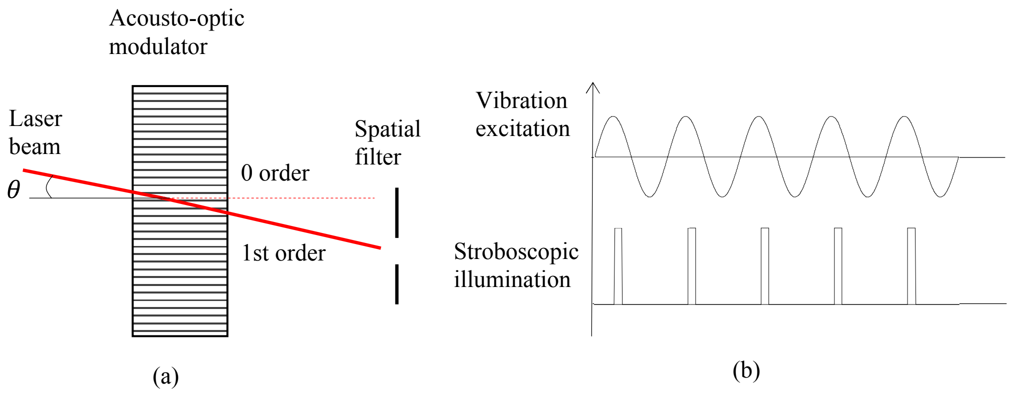

Figure 2: (a) Principle acousto-optic modulator with spatial filter, and (b) vibration excitation and stroboscopic illumination synchronous signals

Since the speckle pattern is seen as a static phenomenon by the CCD camera, we can used a speckle shearography system based on the Michelson optical interferometer and temporal phase modulation, for the quantitative determination of the interference phase, as shown in Figure 3. Indeed, the speckle pattern generated on the surface of the object is split in two by the beam splitter, and the slight rotation in one of mirrors is used to laterally shift the two intensity paths and create the interference phenomenon. The phase temporal modulation, also known as phase shifting or phase stepping, is created by translation of one of the mirrors using a piezoelectric actuator. The four intensity distribution with a constant phase step of $\pi/2$ is the most common used method. In this method, the phase of the speckle pattern for the reference and deformation states is a function of the four intensity distributions, I_0, $I_{\pi/2}$, I_π and $I_{3\pi/2}$:

$$\Phi(x, y) = \arctan\left[\frac{I_{3\pi/2}(x,y) - I_{\pi/2}(x,y)}{I_0(x,y) - I_\pi(x,y)}\right] \quad (5)$$

The relative phase map corresponding to the modal rotation field can be calculated, as describe in Equation (3), by subtracting the two interference phases, being the relation between the phase and the rotation field given by Equation (4).

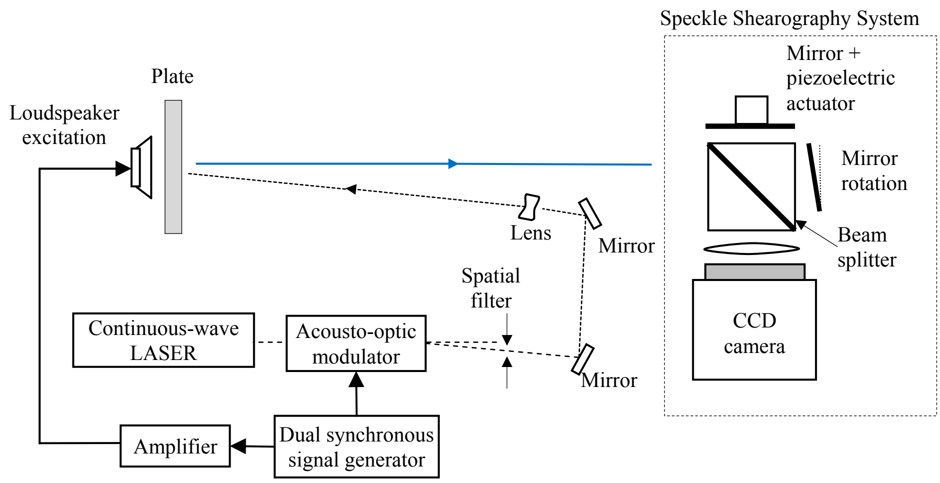

Figure 3: Schematic diagram of the speckle shearography system with stroboscopic illumination used for the measurement of modal rotation fields

2.2 Damage localisation method

The proposed damage localisation method is based on the analysis of perturbations or discontinuities in second, third, and fourth order spatial derivatives of the structural modal response. Indeed, the modal rotation fields are directly measured using speckle shearography with stroboscopic illumination. Since the rotation field corresponds to the first spatial derivative of out-of-plane displacements, the application of numerical differentiation techniques for determination of the higher derivatives is reduced in one order. In practice, the application of numerical differentiation to experimental data will lead to the amplification and propagation of experimental noise, particularly in higher frequencies. A method based on the combination of differentiation and low-pass filters techniques enables to mitigate some of these effects [19]. However, in this process are also eliminated the higher-order signal components, which are essential for the representation of high-order derivatives, including the ones associated to the local damage. The decoupling of the signal components from the high frequency noise can be more easily accomplished by increasing the spatial resolution of the measurement and by improving the signal-to-noise ratio of the experimental data. Also, the number of numerical operations applied to experimental data should be reduced to the minimum, in order to avoid the noise multiplication. For this reason and as an alternative approach to the method proposed in previous works [19, 28], the numerical differentiation is perform in the phase maps. Central finite differences and low-pass filters are successively applied for the calculation, up to the fourth order, of the spatial derivatives of the modal phase maps. Finally, the continuous distribution of the modal rotation fields is obtained by removing the phase discontinuities, through the application of unwrapping algorithms [29].

The spatial derivatives of the phase maps are obtained by lateral shifting the map and subtracting the phases. The phase map spatial derivative of order n of the mode shape i, can be approximated by:

$$\Delta\phi_i^n(x,y) = \arctan\left[\frac{\sin\left(\Delta\phi_i^{n-1}(x+\Delta x/2,y)-\Delta\phi_i^{n-1}(x-\Delta x/2,y)\right)}{\cos\left(\Delta\phi_i^{n-1}(x+\Delta x/2,y)-\Delta\phi_i^{n-1}(x-\Delta x/2,y)\right)}\right]/\Delta x \tag{6}$$

where Δx is the lateral shift size in the x direction.

The high frequency noise is removed by applying the average filtering technique to the phase map. However, to apply this technique is necessary first to transform the map in a continuous field, by shifting the map to the complex domain $\widetilde{\Delta\phi}_i^n(x,y) = e^{j\Delta\phi_i^n(x,y)}$, being $j = \sqrt{-1}$. The filtering is performed using the image convolution:

$$\overline{\widetilde{\Delta\phi}_i^n}(x,y) = \widetilde{\Delta\phi}_i^n(x,y) \otimes h(m,n) \tag{7}$$

being $\otimes$ the convolution operator, $h(m,n)$ the filter array, were m and n represent the horizontal and vertical dimensions of the filtering window. After the application of the filtering technique, the filtered phase map $\overline{\Delta\phi_i^n}(x,y)$ is obtained using the following equation:

$$\overline{\Delta\phi_i^n}(x,y) = \arctan\left(\overline{\widetilde{\Delta\phi}_i^n}(x,y)\right) \tag{8}$$

Finally, the modal derivatives fields are reconstructed by applying the Goldstein unwrapping algorithm [29].

3 Experimental Measurements

The modal rotation fields of a multi-damaged laminated composite plate, 276.5 mm long, 198 mm wide and 1.825mm thick, were measured using speckle shearography and stroboscopic laser illumination. The two internal damages were produced in the plate by two low-speed impacts, through the free fall of a steel sphere, being the energy of the first impact 13.5 J and the second 26.2 J [19]. After the impact, no damages were observed on the impacted surface of the plate. Figure 4 presents the localisation of the two impacts.

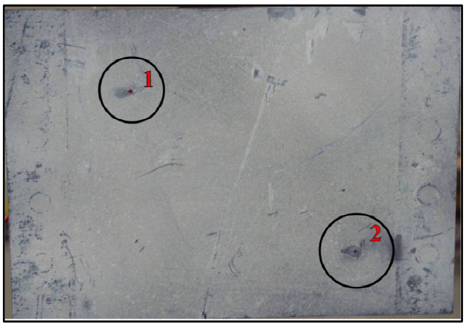

Figure 4: The localisation of the two impacts in the plate

The plate was suspended by flexible rubber bands to create an almost free-free condition and excited at its natural frequencies using a loudspeaker, mounted on the back of measurement surface. The surface was coated with a small layer of white powder to produce a uniform illumination, also shown in Figure 4. A continuous-wave laser from Coherent®, model Verdi with an wavelength $\lambda = 532nm$, is used to illuminate the measurement surface. The dual signal generator, Tektronix model AFG320, is used to produce two synchronous signals, the harmonic signal and the pulse signal. The harmonic signal is used with the amplifier and loudspeaker to excitate the plate at its natural frequencies, as presented in Figure 3. The pulse signal is applied to an acousto-optic modulator to generate the stroboscopic illumination. The speckle interference pattern is capture by a digital camera, Dalsa Falcon model 4M30 with the frame rate of 30 fps and CCD array of 4 megapixels (2352×1728), which gives a high spatial resolution of 4 million measurement points.

Time modulation is applied for the quantification of the interference phase. Four intensity patterns are recorded in the reference and deformation states, with a constant phase-shift of π/2. This is accomplished through the translation of one of the mirrors of the interferometer, actuated by a piezoelectric, see Figures 3 and 5. The piezoelectric is controlled using the National Instruments® PCI 6722 card along with amplifier Burleigh model PZ70. In the reference state, no excitation frequency is applied to the plate. After the vibration excitation is applied and the plate response becomes stationary, the information of the deformation state is taken. The observation in real time (frame rate) of the raw fringes of the phase map is used to control the amplitude of the modal rotation field, by adjusting the amplitude of the harmonic signal and/or the phase of the pulse signal.

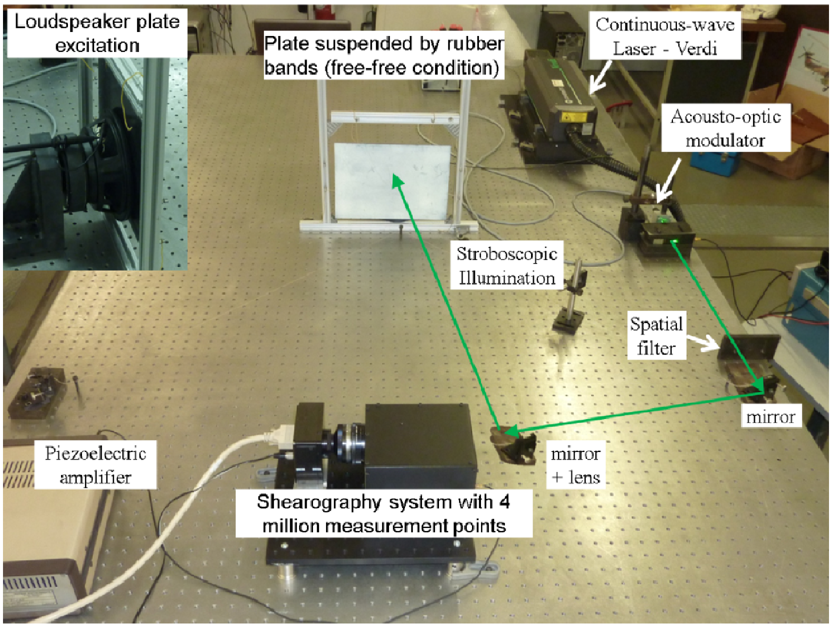

Figure 5: General view of the experimental setup

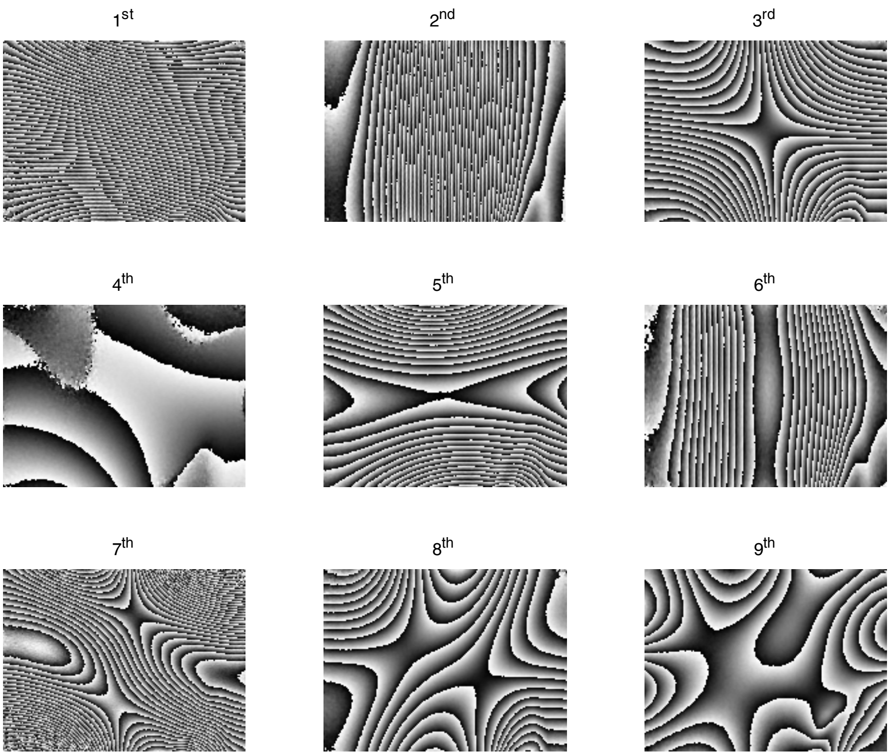

Figure 6: Phase maps of the first nine modal rotation field obtained by speckle shearography with stroboscopic illumination and temporal phase modulation

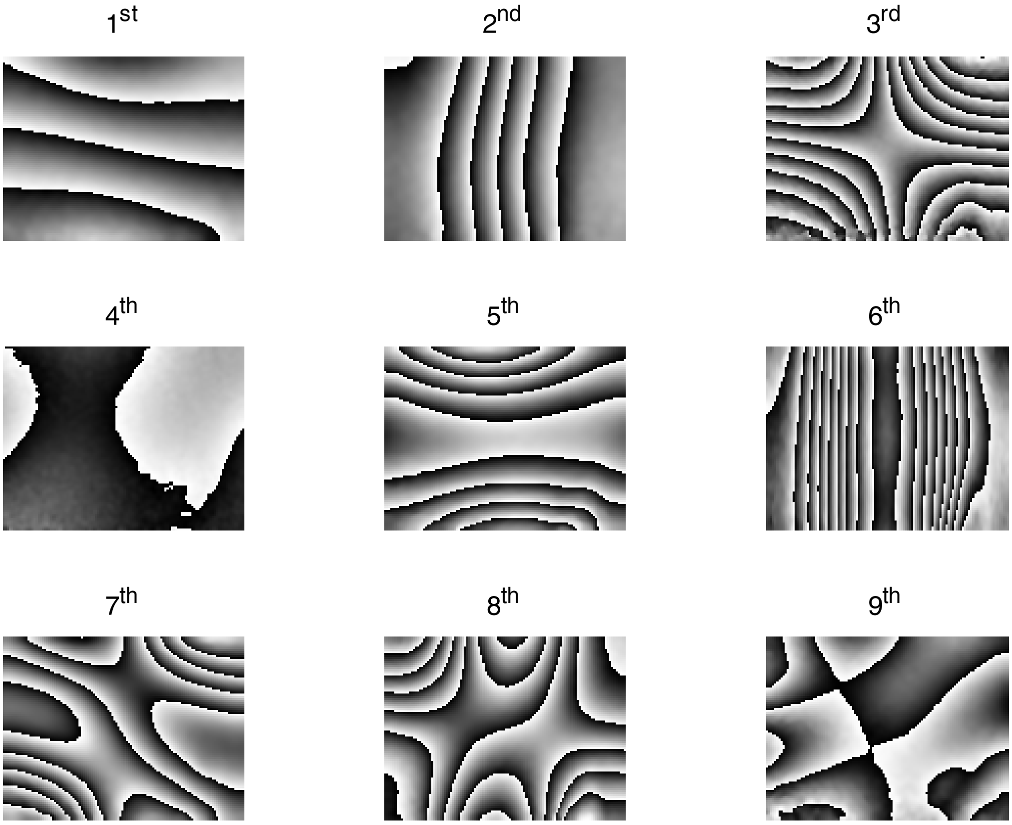

Figure 7: Phase maps of the first nine modal rotation fields obtained by speckle shearography with pulsed illumination and spatial phase modulation

In Figure 6 are shown the filtered phase maps of the plate first nine modal rotation fields relative to the horizontal direction, using a shearing value of $\Delta x = 10$ mm. In these maps are observed, with the exception of the fourth mode phase, a high density of fringes, as a result of the high spatial resolution of the measurement and the low signal-to-noise ratio. The analysis of the fringes distribution in Figure 6 reveals small perturbations close to regions where the second impact was made, i.e. near the lower right edge. This shows a significant improvement in the quality of the measurements relatively to the pulsed laser illumination with spatial phase modulation used in previous studies [19], which are shown in Figure 7.

4 Results

The identification of structural damage is performed by analysing the disturbances or discontinuities in the second, third and fourth derivatives of the modal displacements fields, which are obtained by the first, second and third order numerical differentiation of experimental modal rotation fields, respectively. In this case, the numerical differentiation is performed in horizontal direction by applying central finite differences to the phase maps, using lateral shift size of $\Delta x = 2.7$ mm

(Equation (6)). The phase maps spatial derivatives of the first five mode shapes and the localisation of the two impacts are shown in Figure 8. As can be seen, all the mode shapes present a local perturbation in the fringe pattern near the region of the second impact, which were earlier identified in the phase maps of the Figure 6.

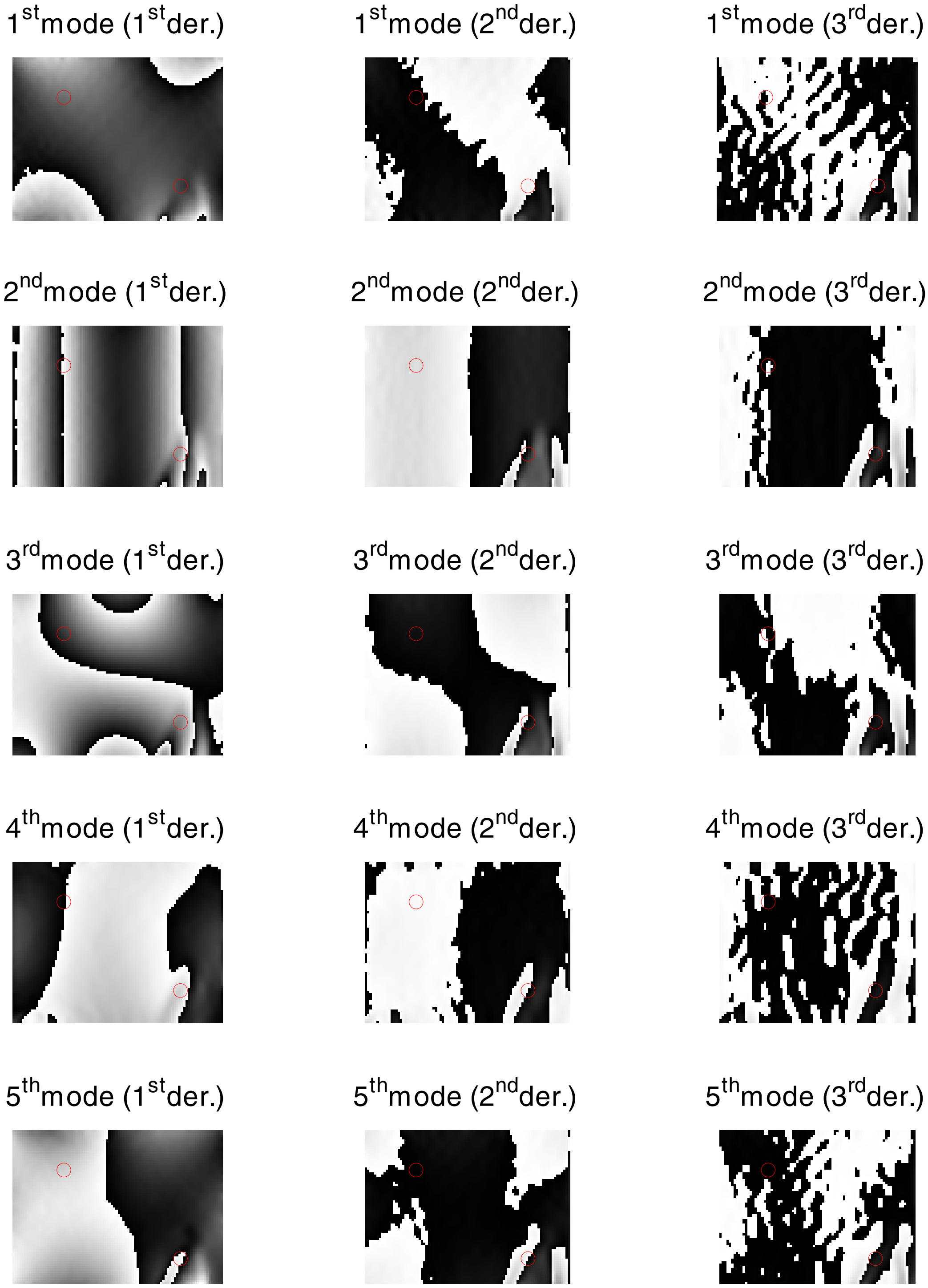

Figure 8: First, second and third order derivatives of the phase maps of the first five mode shapes

However, the damage can be more easily identified after unwrapping the phase maps. Indeed, the full-field of first, second and third order spatial derivatives of the plate first nine modal rotation fields are depicted in the Figures 9, 10 and 11, respectively. For the nine mode shapes analysed, the local discontinuity in these maps clearly shows the localisation of the damage created by the second impact. These perturbations are amplified and spread to the neighbouring region, due to the numerical differentiation process, and present the highest magnitudes for the fourth spatial derivative of the mode shapes. This result confirms the better performance of this new methodology in relation to the one presented in previous studies [19], were the damage could only be identified from the difference between the magnitudes of the damaged and undamaged modal curvature fields of the eighth mode shape. Also, this methodology has the advantage of not requiring the modal response of the undamaged structure model.

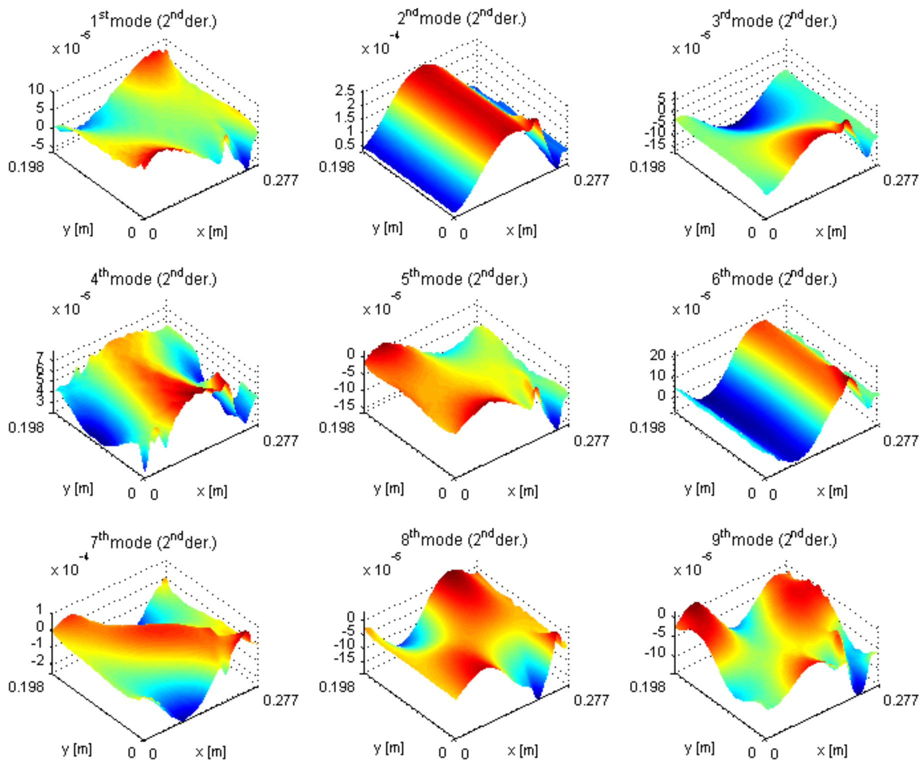

Figure 9: Second order spatial derivative of the first nine modal displacement fields

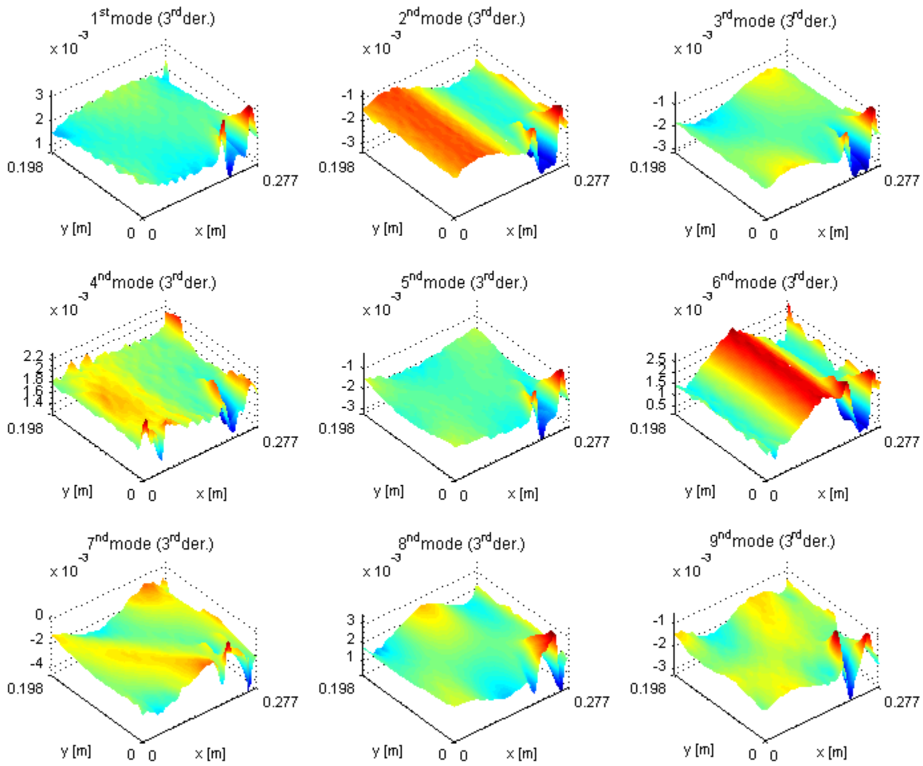

Figure 10: Third order spatial derivative of the first nine modal displacement fields

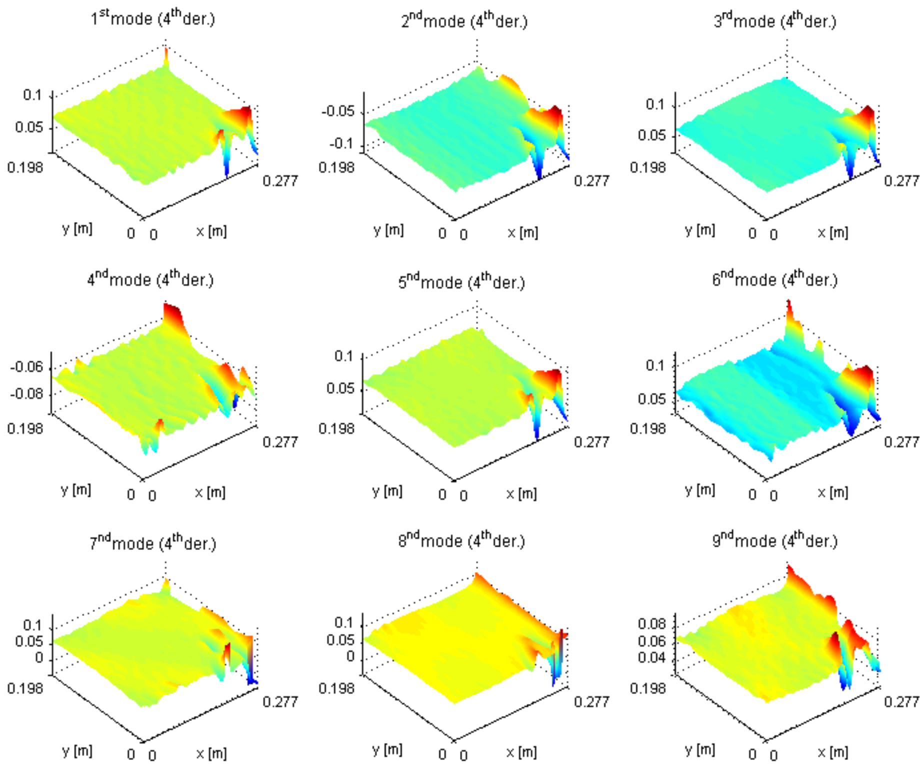

Figure 11: Fourth order spatial derivative of the first nine modal displacement fields

5 Conclusions

This paper describes two speckle shearography systems used for the measurement of modal rotation fields, which are used for the localisation of damage in a laminated composite plate. The first system is based on the Mach-Zehnder optical interferometer and combines double pulse illumination with spatial phase modulation. As to the second, based on the Michelson optical interferometer, stroboscopic illumination and time modulation are used. The phase maps of the first nine modal rotation fields of a laminated composite plate were successfully measured using the second system. A comparative analysis between the measured phase maps and the ones obtained in previous studies, using the first system, are presented. The results show the superior quality of the experimental measurements obtained with the second speckle shearography system.

Internal damage was created in a plate by low velocity impacts at two different locations. The damage localisation method is based on the analysis of the perturbations in the spatial derivatives, up to the fourth order, of the modal displacement field. No information is needed from the undamaged structure. A new differentiation methodology is proposed in order to minimise the experimental noise propagation through the numerical differentiation process. Contrary to previous works, the differentiation is applied to the phase maps, therefore avoiding the propagation of noise caused by theirs post-processing.

The combination of modal rotation fields, measured with speckle shearography with stroboscopic illumination, and the new differentiation methodology allows to obtain the second, third and fourth order derivatives of the first nine mode shapes. Based on the analysis of the modal spatial derivatives of these mode shapes, it was possible to identify the position of the damage created by the second impact. Finally, the results presented prove that the proposed methodology is more effective for damage localisation relatively to the one proposed in previous studies.

Acknowledgments

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