

# The Influence of Elastic Boundary Conditions on the Stability of Steel Frames

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## Abstract

The behaviour of steel frame structures including soil-structure interaction, column base connection rigidity and diaphragm effect of the corrugated roof sheeting has been investigated. The frames are evaluated with common soil characteristics and column base configuration. The influence of vertical bracing stiffness is also studied. Second-order effects are taken into account using non-linear analysis. The present study is limited to single bay single storey frames subjected to static loads. It has been shown that using simplified calculation model might cause deviations from the real structural behaviour and in some cases results were non-conservative.

**Keywords:** elastic support, column base, soil-structure interaction, second-order effect, buckling length, semi-rigid, diaphragm action, non-linear.

## 1 Introduction

Usually steel building structures are designed as consisting of planar frame units. Behaviour of steel frame structures under statically applied loads has been extensively studied before. A large amount of previous works have aimed at aspects of accuracy of the structural model. Although in design practice it is still quite common to treat joints in frames as fully rigid or pinned, there is a lot of evidence that this simplification is not always justified. More accurate design model of supports and connections of frame elements is needed since the stiffness of joints may have substantial effect on the structural behaviour as indicated in [1] and [2]. The possible interaction between soil and structure at column base alters the stiffness of supports. Displacements and rotations at supports may influence the distribution of forces in structural elements and the stability of the frame as a whole and therefore should not be neglected without careful consideration. Most of the existing studies on frames with elastic joints assess the effect of beam-column connections on the global frame behaviour, while the base connection has been modelled either rigid or pinned. Although a number of existing works deal with column base stiffness [3], [4] and [5], only very limited information is available on

possible soil-structure interaction effects on the distribution of forces and the stability of the frame. It is clearly shown in [6] and [7] that soil-structure interaction effects cannot be neglected. Single-storey buildings typically incorporate steel sheeting as a roof deck, which can be applied as a diaphragm to transfer lateral loads to vertically braced bents. In design analysis the framework is often treated as consisting of planar frames and 3-dimensional interaction of frames with the sheeting is neglected or if diaphragm effect is accounted for, it is included in the model as a pinned support at the roof level, excluding lateral displacements of the column top. The analysis of roof diaphragm is thoroughly studied in [8] and [9], where also reduction factors on sway moments for each frame are given that can be added to two dimensional frame calculation model.

In reality the support at the column top is elastic with stiffness depending on the dimensions of the diaphragm and the distance of the particular frame from the vertically braced bent. The stiffness of the elastic spring support, provided by the diaphragm to the frame, influences the behaviour of the frame, which can vary to a great extent, ranging from sway to non-sway frame according to the well-known classification. Consequently the buckling of the frame may occur according to either sway or non-sway mode, depending on the interaction of different elements of the whole 3-dimensional structure – the frames, soil, roof sheeting and vertical bracing. The present paper focuses on effect of parameters that represent soil- structure effect, column base connection rigidity and diaphragm effect to the stability of framed building.

## 2 Structural model

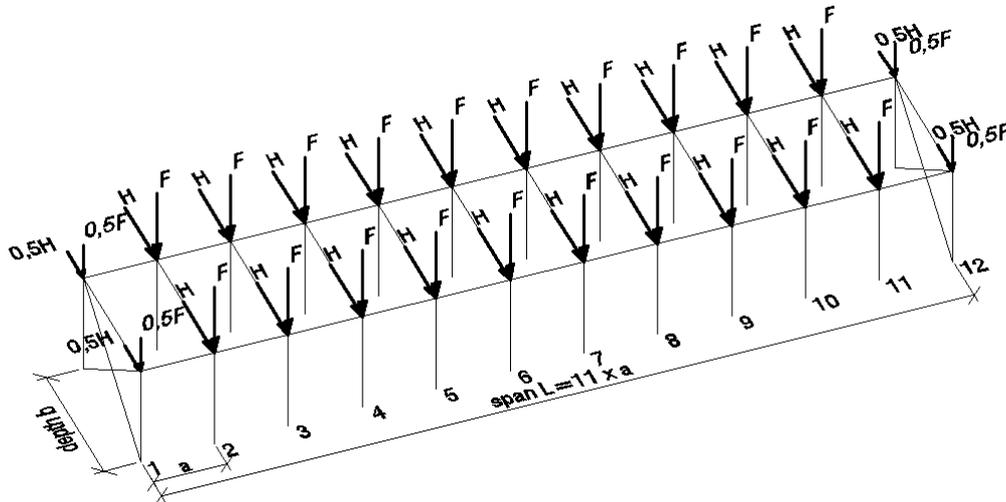


Figure 1: Structural model

In the present work the internal forces of the frames were calculated in a three dimensional model by finite element method using Bernoulli beam elements with six degrees of freedom per node. Horizontal and vertical loads corresponding by magnitude to the design loads in ultimate limit state were applied to the frame nodes

as indicated in Figure 1. The geometric imperfections of the frame elements were taken into account by the equivalent notional load method [10].

Stiffness of stressed skin diaphragm sheeting, spanning parallel to length of building, was calculated according to Bryan and Davies [8], [9] and the diaphragm was substituted by beam elements with equivalent stiffness in the 3-dimensional model.

The model of spread foundations was established in the drained conditions of soil with embedment depth of 1 meter. The settlements were determined for different values of soil deformation modulus – 15, 10 and 5 MPa. The spring model of the frame foundations was based on the resulting vertical displacements and rotations.

The stiffness of the column base connection is determined using Equation (1).

$$k_c = 15 \frac{EI_c}{L_c} \quad (1)$$

Where  $E$ ,  $I_c$  and  $L_c$  are the modulus of elasticity, moment of inertia and the length of the column. Rotational stiffness of the column base was combined with the relevant foundation stiffness and integrated into the model.

Finite element method with iterative solution scheme was used to account for the second order effects, total load was applied in five steps. For comparison the impact of the second order effects was also evaluated, using the well-known amplification factor approach[10].

The buckling length factor for columns was determined by several methods for comparison. First, the buckling length was calculated using the Wood method [11], accepted by the Eurocodes. Second, the approach proposed by Lui [12] was used for buckling length factor determination. Finally, buckling factor was determined by eigenvalue analysis in the finite element model.

Main characteristics of calculation models are presented in Table 1. Each number in three-digit number of model represents characteristic of foundation, column base and end frame respectively. All calculation models are numbered accordingly.

Number of model	Foundation	Column base	End frame
111	rigid	rigid	rigid
112	rigid	rigid	elastic
211/311/411	elastic (E=15/10/5 MPa)	rigid	rigid
212/312/412	elastic (E=15/10/5 MPa)	rigid	elastic
121	rigid	semi-rigid	rigid
122	rigid	semi-rigid	elastic
221/321/421	elastic (E=15/10/5 MPa)	semi-rigid	rigid
222/322/422	elastic (E=15/10/5 MPa)	semi-rigid	elastic

Table 1: Characteristics of calculation models

The end frame is made rigid by adding rigid lateral support to the nodes 1 and 2 in Figure 2. In Figure 2  $K_x$  represents diaphragm stiffening effect in two dimensional models,  $H_y$  is the rotational stiffness of the support and  $K_z$  is the vertical stiffness of the node.

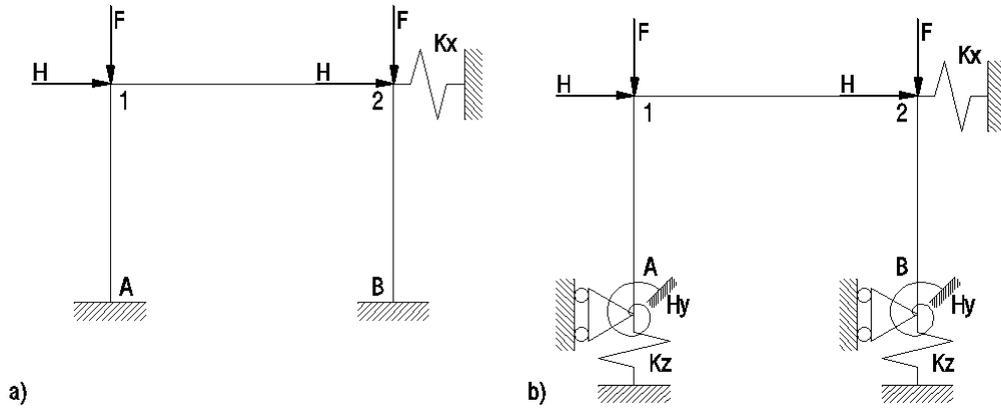


Figure 2: Calculation model with a) rigid foundation and b) elastic foundation

Frame stiffness  $K_{frame}$  describes frame lateral rigidity and is determined using Equation (2).

$$K_{frame} = \frac{H_{fic}}{\delta_{fic}} \quad (2)$$

Where  $H_{fic}$  is a fictitious force 100kN and  $\delta_{fic}$  is the horizontal displacement of top nodes, caused by fictitious force. Similar parameter is used by Bryan [9] to determine reduction factors for sway effects.

### 3 Numerical example 1

Frame dimensions are given in Figure 3. There are rigid joints connecting the beam with columns at nodes 1 and 2. The influence of joint elasticity at nodes A and B has been studied. All the columns of the frame have been made of steel grade S355 and profile HEA 160. Frame is loaded with vertical load  $F=76$  kN and horizontal load  $H=10,5$  kN as shown in Figure 1.

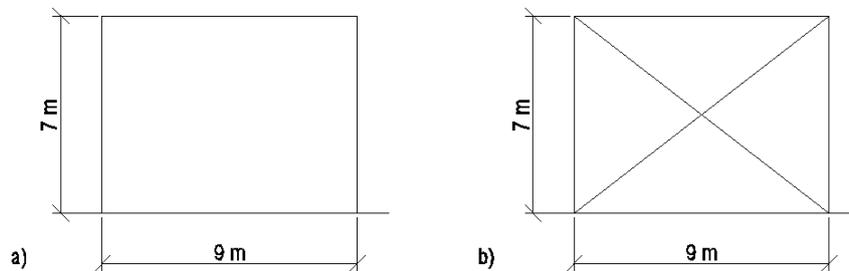


Figure 3: Shape of a) intermediate frame and b) end frame with bracing

Depending on the lateral stiffness and load characteristics the second order effects in frame analysis may be neglected or have to be taken into account.

It is useful to evaluate how essential parameters influence the frame. For intermediate frames the main parameter that influences lateral stiffness is the rotational rigidity of support node as seen in Figure 4 curve  $H_y$ . Lateral displacement remains unchanged due vertical stiffness of studied soil conditions as seen in Figure 4 curve  $K_z$ .

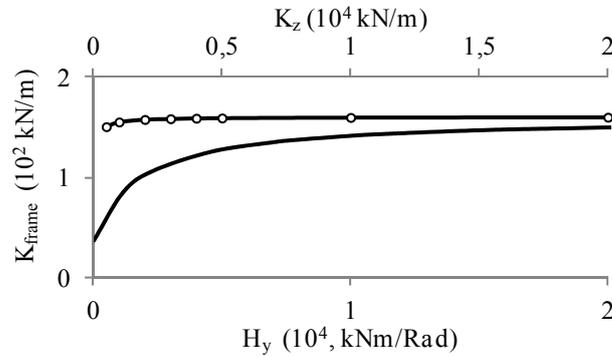


Figure 4: Effect of support conditions on frame stiffness

The main influencers to frame stiffness of end frames are vertical rigidity of support node and the bracing rigidity. It can be seen that influence of support vertical stiffness rises with the increase of the bracing section area as indicated in Figure 5. Also it is clearly visible that rotational rigidity of support node has a small influence on the stiffness of end frame.

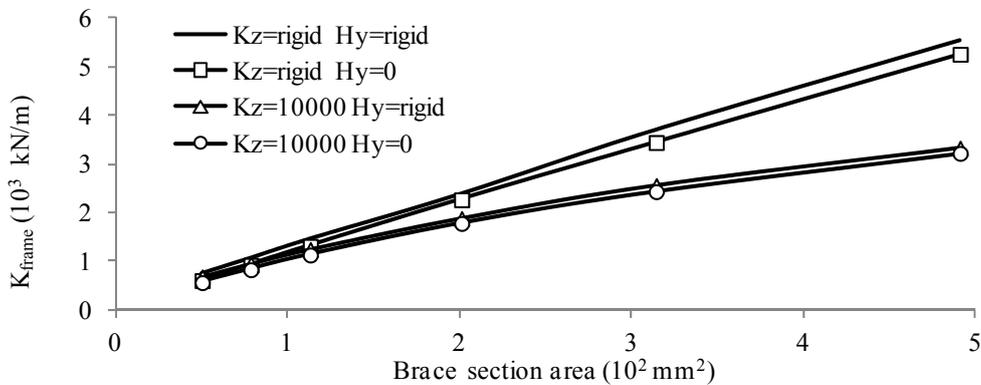


Figure 5: Effect of brace stiffness and support conditions on end frame stiffness

The factor  $\alpha_{cr}$  has been calculated to evaluate the necessity to account for the second order effects according to [10]. If the first order analysis is valid without the second order effects, the calculation model number 111 imitates the behaviour of a frame with simplified boundary conditions – rigidly fixed support at the bottom nodes and diaphragm effect with fully rigid end frames. Alternatively such a frame could also be calculated as a separate two dimensional frame, see Figure 2, because sheet stiffening effect can be substituted by spring stiffness factor  $K_x$ , taken from table by Bryan in [9] according to frame stiffness to sheeting stiffness ratio.

As shown in Figure 6, the lateral displacements of none of the frames of type 111 exceed the limit (labelled S.O. limit), which requires the second order analysis [6]. Therefore the frames of type 111 can be defined as non-sway.

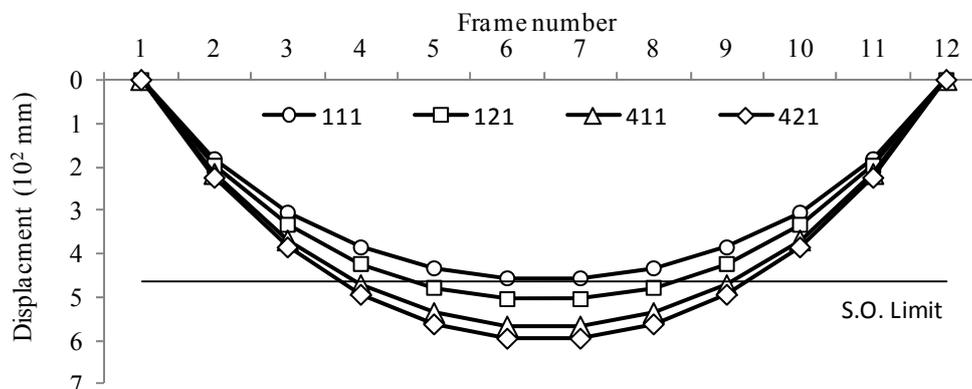


Figure 6: Horizontal displacements of node 1 and 2, caused by fictitious force  $H_{fic}$

Horizontal displacement due fictitious force is used to calculate  $\alpha_{cr}$  and evaluate second order effects according to [10]. In this case, applying the Wood method [11] the buckling length factor of the columns could be 0,58. This method does not take into account the horizontal displacement due to the elasticity of the roof diaphragm and therefore it should be used with certain care. According to Eurocode [10] the buckling length factor for non-sway frames can be taken equal to 1, which is a conservative value in many cases, but in some cases it may occur non-conservative. For more accurate results the Lui method and eigenvalue analysis have been used to evaluate the buckling length factor and the relevant results have been given Figure 7.

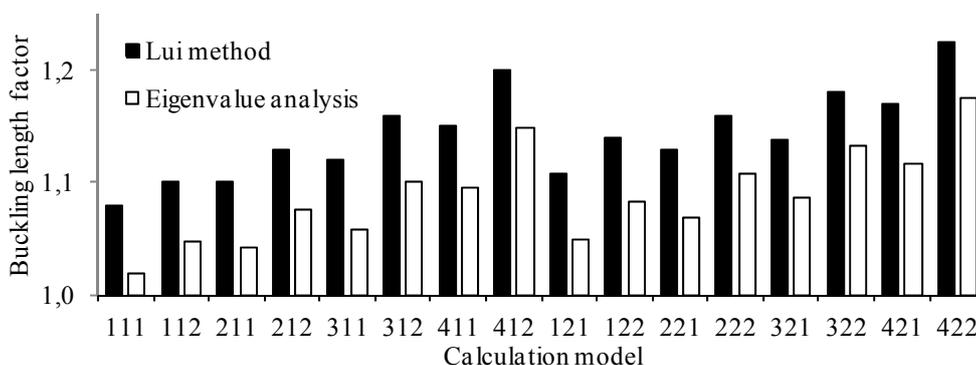


Figure 7: Column buckling length factor, frame number 6

The roof diaphragm was supported at end frames by the interaction of the vertical bracing system with the columns. The support reaction of the diaphragm, obtained from the calculation model 111 was applied to the planar model of the end frame and the dimensions of the bracing diagonals were determined to resist the reaction force. The elasticity of the end frames was introduced to the 3-dimensional calculation model and as a result the displacements of some of the intermediate frames of type 112 exceeded the second order limit as shown in Figure 8.

Consequently the frame had to be treated as a sway frame. The buckling length factor according to Wood method was 1,2, which was conservative in many cases, as can be seen in Figure 7.

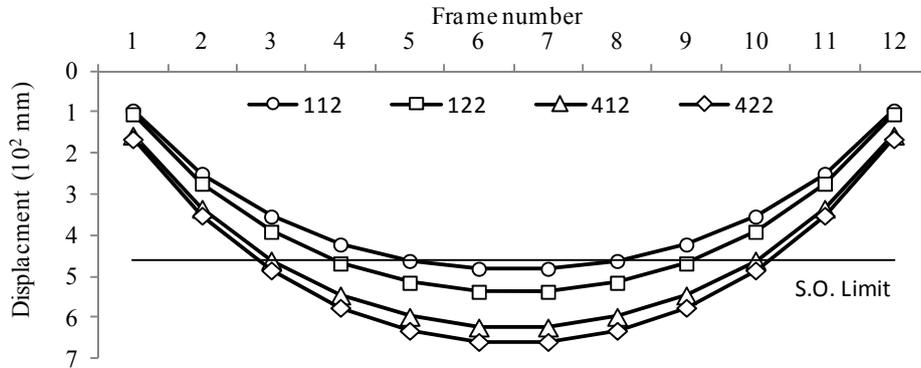


Figure 8: Horizontal displacements of node 1 and 2, caused by fictitious force  $H_{fic}$

Foundation and column base effect were added to the model. In Table 2  $H_{y,f}$  is the rotational stiffness of the support with only foundation taken into account and  $H_{y,tot}$  is the support rotational stiffness with combined effect of foundation and column base connection.

Soil	$K_{z,f}$ [kN/m]	$H_{y,f}$ [kNm/rad]	$H_{y,tot}$ [kNm/rad]
E=5Mpa	6217	2824	2054
E=10Mpa	12435	5655	3229
E=15Mpa	19017	10628	4407

Table 2: Vertical and rotational stiffness of support node

Influence of column base connection rigidity to calculation model is illustrated in Figure 6 with curves 121 and 421 and in Figure 8 with curves 122 and 422. It shows that frames 3-10 should be classified as sway frames.

Reducing the stiffness of boundary conditions causes bending moments to shift from the column base joints to the knee joints as indicated in Figure 9.

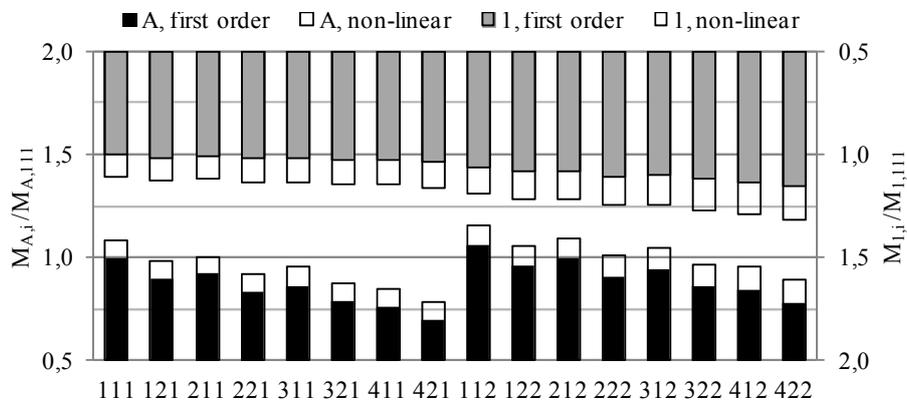


Figure 9: Bending moment value  $M_i$  of different calculation models relative to model 111 value  $M_{111}$ , nodes A and 1

## 4 Numerical example 2

Frame dimensions have been given in Figure 2. There are pinned joints connecting the beam with columns at nodes 1 and 2. The influence of joint elasticity at nodes A and B has been studied. All the columns of the frame have been made of steel grade S355 and profile HEA 220. Frame is loaded with vertical load  $F=87,7$  kN and horizontal load  $H=10,5$  kN as shown in Figure 1.

In Table 3  $H_{y,f}$  is the rotational stiffness of the support with only foundation taken into account and  $H_{y,tot}$  is the support rotational stiffness with combined effect of foundation and column base connection.

Soil	$K_{z,f}$ [kN/m]	$H_{y,f}$ [kNm/rad]	$H_{y,tot}$ [kNm/rad]
E=5Mpa	6737	5317	4363
E=10Mpa	13474	10634	7401
E=15Mpa	20211	15951	9636

Table 3: Vertical and rotational stiffness of support node

Influence of column base connection rigidity on the calculation model with absolutely rigid end frames is illustrated in Figure 10.

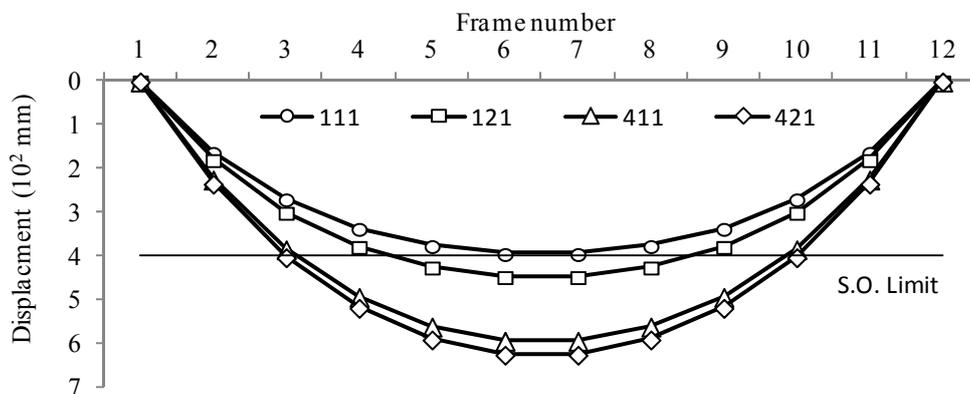


Figure 10: Horizontal displacements of node 1 and 2, caused by fictitious force  $H_{fic}$

Frames of type 111 can be defined as non-sway. In this case, applying the Wood method the buckling length factor of the columns could be 0,7. According to Eurocode the buckling length factor for non-sway frames can be taken equal to 1, which is a conservative value in many cases, but in this case it seems non-conservative. For the same case the Lui method and eigenvalue analysis relevant results have been given in Figure 11.

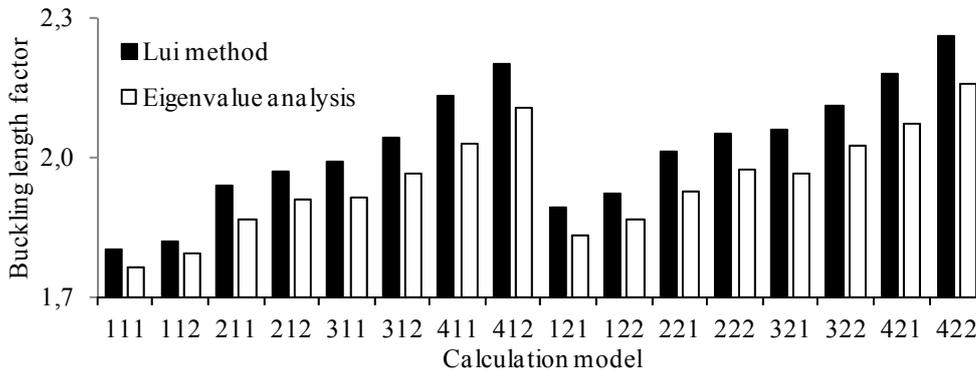


Figure 11: Column buckling length factor, frame number 6

Influence of elastic end frame on the calculation model is illustrated in Figure 12.

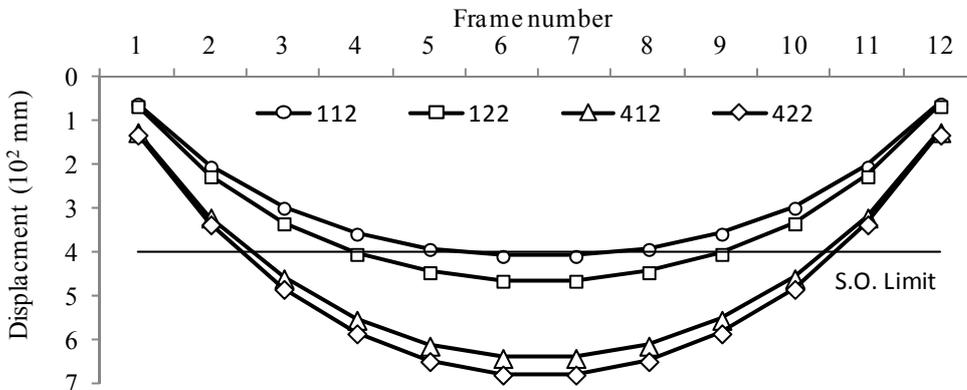


Figure 12: Horizontal displacements of node 1 and 2, caused by fictitious force  $H_{fic}$

The intermediate frames that exceeded the second order limit as shown in Figure 10 and Figure 12, had to be treated as a sway frame. The buckling length factor according to Wood method was 2, which is non-conservative, as can be seen in Figure 11.

## 5 Conclusions

In the present work we evaluated the inaccuracy, caused by the simplifying assumptions, applied in the design calculations of common steel frameworks. For this purpose a three dimensional model with semi-rigid supports has been studied.

The stiffness of the roof diaphragm, column base deformability and soil-foundation interaction are essential parameters, which affect the stiffness of the frames and force distribution in its elements.

The calculation models can be made very complex but adding every possible parameter can be overwhelming and time consuming task. Therefore, we need to study the effect of essential parameters in simpler environment.

Soil-foundation interaction has been integrated in the structural model as rotational and vertical spring stiffness at column base. It has been shown, that using planar

frames with simplified support conditions may cause deviations from the real structural behaviour and the results of simplified methods are in some cases non-conservative.

The roof diaphragm provides lateral support to the frame. Classification of frames as sway or non-sway depends strongly on this lateral support, which is treated as an elastic spring at column top. The stiffness of the lateral support depends on the location of the frame, on the stiffness of the diaphragm itself and the end frame.

In case of framework with roof diaphragm the impact of soil deformation is more pronounced, as vertical displacements at the column base of the end frame increase the deformability of the diaphragm support and consequently the stiffness of the diaphragm is modified.

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