# Numerical Evaluation of Chord Failure Resistance for Multi-Intersection Tubular Joints with Non-Regular Geometry: A Case Study 

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#### Abstract

Recent architectonic trends in the design of truss systems with multiple structural members, as well as design and construction requirements for the evaluation of the ultimate strength of hollow section walls at the location of the joints (chord failure resistance), can sometimes lead to cases in which tubular joints cannot be managed in a systematic manner in accordance with current codes and guidelines. Often, they are not easy to compare with technical recommendations and those from the literature. As a result, this means that the designer has to use alternative criteria for evaluating the ultimate strength of the structural joints, including, as stated here, numerical analyses which are specifically intended to discover the ultimate design loads. This work refers precisely to the use of the finite element method for numerical analyses, adopted as an additional and comparative means for the European Standard EN_1993-1-8 (Design of Joints) [1] for research into chord failure resistance of multi-intersection tubular joints with non-regular geometry.


Keywords: steel structures, joint, tubular intersection, non-linear mechanical response.

## 1 Introduction

The New Headquarters of the Faculty of Law and Political Science of the University of Turin, which is currently being built, is used here as a structural example, since it is a good example of the type of structure discussed in this work. The complex is characterized by a large vaulted roof (designed by the architects Foster \& Partners), created using a pre-stressed membrane system. The image in Figure 1 shows the overall construction system, using a model (courtesy of the University of Turin).


Figure 1: photo of the model of the system (courtesy of the University of Turin)
The roof is approximately $16,700 \mathrm{~m}^{2}$, with a similar-toroidal planimetric layout, which creates a repetitive wave effect along the structure, generated by the structural membrane that is stretched over 54 curved steel tubular frames (Figure 2).


Figure 2: tubular systems supporting the structural membrane
It must be noted that, although the wave system may appear regular, the membrane surface is characterized by double-curve geometry that is constantly different in every part of the structure. As a result the steel structures supporting the membrane,
later described in further detail, also have a non-regular geometry. Figure 3 shows the first structures while they were being installed.


Figure 3: the pre-assembly phase of the steel tubular system to support the structural membrane (Courtesy of Studio Ossola)

More specifically in terms of the theme of this work, particular attention was paid to the aspects of verification, design and construction of the truss system joints. These phases involved in-depth management of the problems linked to the number of converging structural members, the number of angles of intersection between the structural members and the joints and the coupling of thicknesses which were often very varied, with the appendages and natural extensions in terms of the potential for metallurgical welding and inspection. As an example, the images in Figure 4 show some of the most geometrically complex joints in terms of the above criteria.


Figure 4: joint with 3 converging structural members (left) and 6 structural members converging at the chord (right)

As far as the construction design and verification of the connections is concerned, it is well known that a typical problem of similar static systems is that of the correlation of the nominal load on the structural member (nominal stress in the framework) with the local effects generated by stress concentration on the structural member walls where the joints are. This phenomenon, when it is critical (chord failure) is considered particularly risky when it occurs at the same time as the following two conditions:

- the use of thin-walled structural profiles. This refers to the series of profiles such as hollow section walls for structural work, and the use of members thickness where $\mathrm{t} \leq 6 \div 8 \mathrm{~mm}$. It is well known from the literature and from construction experience that the use of thicknesses which are less than the above-mentioned threshold intrinsically involves the problem of the calculation of a relationship (or ratio) of chord failure ( $\chi_{\text {cf }}$ ) between the nominal stress on the bracing structural member ( $N_{E d}$ ) and local resistance of the chord ( $N_{R d . j}$ ). This relationship must be carefully observed, as the condition $\chi_{\mathrm{cf}}>1.00$ may often occur. When thicknesses greater than 8 mm are used, this problem is often of secondary importance in comparison to normal criteria for the nominal verification of the profiles.
- the absence of joint reinforcements. This is when it is impossible or difficult to construct joint geometries characterized by the interruption of the chord sections or using auxiliary welded plates or strengthened chord sections with increased thickness in the area of the joint, etc.

When the structural profile involves the use of thin-walled structures, which are not locally strengthened, as described above, the designer must evaluate the chord failure ratio ( $\chi_{\mathrm{cf}}$ ), and may need to intervene by improving the thickness of the chord, or installing suitable strengthening systems on the chord. In this way the following approaches can be used:

- EN-1993-1-8 (2005) [1]: paragraphs 7.2 - 7.7, where the various sections show criteria for calculating the design strength of the chord wall according to the 6 characteristics for the potential collapse of the joint: face failure (plastic collapse of the wall as a result of localized bending, web failure (generalized collapse, as a result of bending or local buckling, of the core of the chord wall), shear failure (generalized collapse as a result of shear of the chord wall), punching shear failure (collapse of the areas surrounding the welding as a result of shear), brace failure (collapse of the bracing member), and local buckling (collapse as a result of local instability of the walls of members). The section began with Japanese studies (models by Togo et al), and is the result of years of European and Japanese research (although not without American influence), carried out from the 1960s onwards. Today, thanks to the significant coordination by the CIDECT, the most recent knowledge from the sector has been put into practice (see the years of experience of Wardenier, Kurobane, Makino, Yeomans, Van Der Vegte et al), thus providing important technical support for current structural design.
- the use of non-linear finite element analysis. This has the great advantage of being able to obtain evaluations relating to particular, specific geometric cases, which are often not included by guidelines and codes, however highlighting the burden of numerical modelling which must be considered in terms of activity management.

In the specific case described here, the large number of geometries to inspect, the significant geometrical variability as a result of the number of structural members, as well as the angles of the structural members themselves, has resulted in the implementation of a campaign for numerical tests aimed at providing parameters for the chord failure resistance of the structural members, revealing interesting profiles for comparison, both in terms of the European code EN_1993-1-8 [1] and with reference to the aforementioned conditions of collapse.

## 2 The numerical modelling of joints: two-dimensional mesh systems and three-dimensional mesh systems

The generation of numerical meshes was obtained by means of two different approaches: three-dimensional meshes (with solid finite elements) for the analysis of detail and a preliminary study of the influence of the welding thickness in the distribution of stress states, which are capable of providing qualitative validation of the mechanical response; and two-dimensional meshes (with surface finite elements) to provide a parametric response regarding the thickness of chord walls. For example, when focusing on two connections among those which were the subject of numerical research (one with three-dimensional mesh and the other with twodimensional mesh), Figure 5 shows the mathematical models implemented using automatic meshing procedures.


Figure 5: mathematical models of the meshes with finite elements used for the study of sample 1 (left) and sample 2 (right)

The F.E.M. models which form the basis of the mechanical analyses reported here were obtained using three-dimensional and two-dimensional finite elements of the finite element software packages Strand7 Non Linear, Tetra4 (4 faces, 4 nodes) with linear isoparametric shape functions and 3 degrees of freedom (translation only) for each joint, and Quad4 (4 sides, 4 nodes) with linear isoparametric shape functions and 6 degrees of freedom (translation and rotation) for each joint. Figure 6 shows two finite elements projected in the natural reference system, while as far as the mathematical formulation of the shape functions is concerned, the natural reference system uses Tetra4 for the solid element,
$\begin{array}{ll}\mathrm{Ni}\left(\xi_{1}, \xi_{2}, \xi_{3}\right)=1-\xi_{1}-\xi_{2}-\xi_{3} & \text { for } i=1 \\ \mathrm{Ni}\left(\xi_{1}, \xi_{2}, \xi_{3}\right)=\xi_{\mathrm{i}-1} & \text { for } i=2,3,4\end{array}$
and for the surface element Quad4:
$\mathrm{Ni}\left(\xi_{1}, \xi_{2}\right)=0.25 *\left(1+\xi_{1 \mathrm{i}} * \xi_{1}\right) *\left(1+\xi_{2 i} * \xi_{2}\right) \quad$ for $i=1,2,3,4$
while the peak values of the basic functions of the joints refer to [2] and [3].


Figure 6: the finite elements Tetra4 (brick element) and Quad4 (plate element), graphic representation

It must be noted that the mathematical properties displayed, when referred to finite elements from the Strand7 Non Linear software package, are all in line with other finite element software packages currently in use and widespread in professional practice, from which, with non-linear solvers, the analyses conducted here are intended to have a merely applicative value (professional point of view).

## 3 The numerical response

The numerical solutions obtained from the parametric analyses, given the use of current S355J0 alloys and the relative bilinear elastoplastic diagram (with ideal plasticity), non-linear geometrical response of the solution also included, were obtained with reference to the possible static methods for loading the joints, in accordance with the design load conditions. Moreover, an appropriate optimization of the load cases was carried out in a certain way in order to remove the load conditions with reduced stress response and less susceptibility (in terms of probability) to mechanical collapse from the global set of analyses, as well as to reduce the computational burden. The analyses, due to the application of resolution criteria by displacement control, have always shown sufficient numerical stability, highlighting adequate achievement in the post-elastic phases. For sample 1, for example, the monitoring of responses for each of the structural members intersected at the joint showed distinct safety loads, with response always observed in a precritical phase (elastic phase). For a given thickness taken from those in the design, Figure 7 shows the design point $\left(\mathrm{N}_{\mathrm{Ed}}\right)$ compared to the critical thresholds for the outlinearity point and breaking point. In order to determine the threshold of the outlinearity point $\left(\mathrm{N}_{\mathrm{yd}}\right)$, where a significant deterioration in the mechanical response of the joint traditionally occurs, in terms of stiffness, the stiffness curves of the individual structural members meeting at the joint were checked beforehand, as shown in Figure 8 (left), leading to targeted evaluations regarding:

- iso or hyper-staticity of the structures as a whole (layout configuration of the system);
- position of the joint within the overall system and plausible kinematic mechanism of collapse with the hypothesis of the first failure conferred by the analyzed joint and formation of a single plastic hinge.
- influence of the joint deformation in the post-critical phase on the overall displacement of the static systems (response of the tensile membrane first and foremost);

A specific value of $0.70 * \mathrm{E}_{\mathrm{j} .0}$ has been determined as the critical stiffness of the joint (Figure 8, right), to establish the threshold which is considered for the out linearity of the overall response of the joint. As far as the definition of the ultimate strengths of the joint $\left(\mathrm{N}_{\mathrm{ud}}\right)$ is concerned, these were conventionally attributed using minimization between:

- $21 \%$ fibre strain ( $\varepsilon_{u}=0.21$ );
- interruption of the numerical solution with values lower than the nominal ultimate strain $\left(\varepsilon_{u}<0.21\right)$;
- maximum load established by the load-displacement curve, where the first critical point with zero gradient is.


Figure 7: sample 1, typical mechanical response


Figure 8: sample 1, structural brace-member stiffness and average stiffness of the joint

Furthermore, it is interesting to observe the plastic mapping during the incremental loading sequences (Figure 9), the loss of joint stiffness can often be associated with the typical phenomenon of propagation of the plastic areas located at different points on the walls of the chord, to the extent that these areas can connect with each other (plastic links).

On the other hand, when focusing on the group of joints represented by sample 2 (Figure 5), particularly interesting responses emerged, in that they were characterized by the appearance of local buckling phenomena below a given thickness, as a function of the stress ratio $\left(\mathrm{n}_{\mathrm{p}}\right)$ of the chord. The images in Figures 10 and 11 show a phenomenon of buckling of a part of the chord wall (along the ridge line), of the $3^{\text {rd }}$ type (snap-through, with progressive geometrical transition of an area of the chord wall and modification of the mechanical response of the walls) following an increase of the above-mentioned stress ratio to the value $\mathrm{n}_{\mathrm{p}}=0.80$.


Figure 9: sample 1, plastic mapping (step 30: overall linear-elastic response, step 95: incipient ultimate strength of the joint)


Figure 10: sample 2, load-displacement curves


Figure 11: sample 2, local buckling of the ridge line and branches with unstable path

## 4 The design resistance of the joint

As far as the design resistance of the joint is concerned ( $\mathrm{N}_{\mathrm{Rd} . \mathrm{j}}$ ) the writers of this paper adopted, for plastic chord failure (mechanism statistically resulted the most common and characterized by typical post-critical ductile response), the following threshold as limit:
$\mathrm{N}_{\mathrm{Rd} . \mathrm{j}}=\mathrm{N}_{\mathrm{yd}}+\chi_{\mathrm{y}}\left(\mathrm{N}_{\mathrm{ud}}-\mathrm{N}_{\mathrm{yd}}\right)$ design limit for plastic chord failure, with $\chi_{\mathrm{y}}=0.20$
where $\mathrm{N}_{\mathrm{yd}}$ represents the loss of linear response of the joint (at $\mathrm{E}_{\mathrm{t}}=0.70 * \mathrm{E}_{\mathrm{j} .0}$, according to the previously mentioned criteria), with $\mathrm{N}_{\mathrm{ud}}$ as the ultimate strength of the joint and $\chi_{\mathrm{y}}$ as the non-linear response coefficient of the joint, obtained by the hypothesis of assigning a safety coefficient for breaking point (ultimate resistance) equal to $\gamma_{\mathrm{M} . \mathrm{pl}}=\mathrm{N}_{\mathrm{ud}} / \mathrm{N}_{\text {Rd.j }} \approx 1.35$, where the joint has a ductility parameter of $\xi_{\mathrm{pl}}$ $=\mathrm{N}_{\mathrm{ud}} / \mathrm{N}_{\mathrm{yd}}=1.50$.

In terms of non-plastic chord failure, or where the collapse conditions cannot be unequivocally identified such as plastic chord failure, the absence of the important phenomenon of plastic dumping produced by the plastic bending of the chord walls and their relative change in shape, led to the use of a safety coefficient calibrated each time according to the post-critical response obtained for the joint (semi-ductile response or failure due to local buckling), thus guaranteeing suitable safety factors compared to the ultimate strength of the joint $\left(\mathrm{N}_{\mathrm{ud}}\right)$, regardless of the values of $\mathrm{N}_{\mathrm{yd}}$ (loss of the linear response of the joint), using the following criteria:
$\mathrm{N}_{\mathrm{Rd} . \mathrm{j}}=\mathrm{N}_{\mathrm{ud}} / \gamma_{\mathrm{M} . \mathrm{br}}$ design limit for non-plastic chord failure
The value of the coefficient $\gamma_{\text {M.br }} \geq 1.40$ (safety coefficient due to post-critical semiductile or fragile behaviour), was calibrated each time on the basis of the trend shown during the post-critical phase.

## 5 Conclusions

Comments were made regarding a series of non-linear F.E.A. numerical tests, aimed at the systematic extrapolation of chord failure resistance of the structural joints. The results were used as an additional and comparative means for the indications that can be deduced from EN_1993-1-8 (Design of Joints) [1], relating to standard geometrical configurations ( $\overline{\mathrm{X}}$ joint, T joint, etc). Two indicative examples, although they are not discussed in detail, were also reported. The analysis of the results appeared, in the opinion of the writers, to be doubly interesting in design terms, for the following reasons:
a) definition of the minimum resistance threshold (chord failure resistance), used for the optimization of member thicknesses;
b) analysis and observation of the post-critical phases, with particular reference to the possibility of interpretation of the collapse type (ductile, semi-ductile, brittle);

Furthermore, this paper shows that the adoption of non-linear numerical analysis techniques can sometimes be considered as a valid aid for designers when it comes to obtaining extra information and/or validation in addition to the adducible standards or codes.

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## References

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