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Resolution of Different Length Scales by an Efficient Combination of the Finite Element Method and the Discrete Element Method

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Abstract

The objective of this paper is to resolve different length scales in structure analysis using an interface coupling the discrete element method (DEM) with the finite element method (FEM). This approach distinguishes itself from other methods in so far that no overlapping domains between finite and discrete elements exist. Both domains are separated in the physical space and the numerical simulation domain. The proposed approach is relevant to almost all engineering applications that deal with granular matter such as storage in hoppers, transport on conveyor belts or the displacement of granular material as in mixers or the excavation of the soil. For these applications an engineering device such as mixer blades or cutting tools are in contact with granular matter. Contacts with individual particles generate contact forces that act on both the engineering device and the granular material. The latter experiences a displacement of individual particles whereby the engineering structure responds with deformation and stresses. In order to predict and optimise both the behaviour and motion of granular material and the structures in contact, numerical simulation tools are increasingly employed. Simulations are popular especially because experiments are often expensive, time-consuming and sometimes even dangerous. The continuous increase in computing power is now enabling researchers to implement numerical methods that do not focus on the granular assembly as an entity, but rather deduce its global characteristics from observing the individual behaviour of each grain.

An interaction between granular media and a structure relies on a transfer of forces between them. Granular media consists of an ensemble of particles of which a number of particles may be in contact with a surface *e.g.* walls as surfaces of solid structures. The contact is resolved similar to inter-particle contacts by a representative overlap. It defines the position of impact as well as the force acting on the particle at this position. The same force, however, in the opposite direction defines a mechanical load for the structure. In order to determine the effect of forces on the solid structure, it is discretised using finite elements. The impact of the force is transferred to the nodes

of the respective surface element and appears as a load for the finite element system. Hence, integrating particle dynamics and the response of the solid structure arising from particle impacts advances both the new position of particles and the corresponding deformation as well as the stress of the solid structure in time.

Developing flexible software which is capable of performing simulation in different applications will enable the scientists to focus entirely on their specific problem and hence save them a lot of valuable time. This concept is supported by the software tools of the discrete particle method and the commercial multi-physics software package Diffpack. Hence, the solid structure is analysed using the finite element method under load as a result of the impact of individual particles that changes both in time and space. For this purpose traditional formulations of the finite element method are employed that are available in Diffpack. It represents an object-oriented hierarchy of classes that provide an excellent interface to introduce external loads from particle impact on the finite element structure. Diffpack is an object-oriented development environment, which comes as a rich set of C++ classes, for the numerical modelling and solution of arbitrary differential equations. User applications cover a wider range of engineering areas and span from simple educational applications to major product development projects.

The behaviour of granular material is represented by the advanced software package of the discrete particle method (DPM), which is based on the discrete element method. It is designed to relieve users from the underlying mathematics and software design and allows them to focus on physics and their applications. The software uses object oriented techniques that support objects representing three-dimensional particles of various shapes such as cylinders, discs or tetrahedrons for example, size and material properties. This makes it a highly versatile tool dealing with a large variety of different industrial applications of granular matter. Various force models for the inter-particle and particle-wall contacts are also available. A minimal user interface easily allows extending the software further by adding user-defined impact models or material properties to an already available selection of materials and properties. Thus, the user is relieved of underlying mathematics or software design, and therefore, is able to direct their focus entirely onto the application. The discrete particle method is written in the C++ programming language and works both in the Linux and XP environments.

Keywords: extended discrete element method, finite element method, discrete element method, modelling.

1 Objective

The objective of this contribution is to resolve different length scales in structure analysis by an interface coupling the discrete element method (DEM) with the finite element method (FEM). This approach distinguishes itself from other methods in so far that no overlapping domains between finite and discrete elements exist. Both domains are separated in physical space and numerical simulation domain. The proposed approach is relevant to almost all engineering applications that deal with granular matter such as storage in hoppers, transport on conveyor belts or displacement of granular material as in mixers or excavation of soil. For these applications an engineering device such as mixer blades or cutting tools are in contact with granular matter. Contacts with individual particles generate contact forces that act on both the engineering device and the granular material. The latter experiences a displacement of individual particles whereby the engineering structure responds with deformation and stresses. In order to predict and optimize both the behaviour and motion of granular material and the structures in contact, numerical simulation tools are increasingly employed. Simulations are popular especially because experiments are often expensive, time-consuming and sometimes even dangerous. The continuous increase in computing power is now enabling researchers to implement numerical methods that do not focus on the granular assembly as an entity, but rather deduce its global characteristics from observing the individual behaviour of each grain.

2 Introduction

2.1 Background and Scientific Context

A broad range of engineering applications are facing multi-scale problems. A large number of these problems involve heterogeneous materials such as granular media. Application in fracture mechanics, soil-structure interaction, fluidized particle beds and tire-terrain interaction are major fields to name when it comes to dealing with different length scales. The ever increasing computation power allows to account for these problems by different numerical simulation techniques.
The combination of discrete and continuum approaches (CCDM - combined continuum and discrete model) is a powerful tool to account for different scales within problems. Hereafter, different methods of combined discrete and continuum models are review and discussed. Traditional numerical methods, such as the finite element method (FEM), describe materials as continuous entities. This assumption allows an increasing number of engineering problems to be solved conveniently at the macroscopic scale by means of numerical simulations. But this approach inherits one fundamental drawback the averaging of all individual characteristics of the grain scale. However, high performance computer technique now enables the employment of methods, such as the discrete element method (DEM), able to account for the individual behaviour of each grain within a granular assembly. This allows to derive the macroscopic characteristics from the behaviour observed at each single grain. But since the discrete approach requires for contact detection, calculation of all contact reactions and a high resolution of the time scale, the method inherits large computation time as a significant disadvantage. Hence, the idea seem quite natural to utilize the advantages of both the continuum and the discrete approach and thereby compensating the shortages of

each method.
Thus, numerical methods of coupling continuum and discrete approaches are under constant development with the purpose to resolve different scales within engineering applications. The field of coupling between continuum and discrete methods can be separated into two parts, methods with overlapping and methods with separated physical domains.

One typical field of application connected to overlapping domains are computations of fracture and fragmentation of materials and structures. For instance, Morris et al. [1] simulated crack propagation within rock structures of tunnels. A combined DEM - FEM method was employed by blending in discrete regions as necessary. If within the FEM domain yield stresses are reached the crack region and new generated fracture surface were handled by DEM like description. Also to solve multiscale problems Nikita et al. [2] derived a two scale approach to predicted behaviour of heterogeneous materials. The problem dependence on different scales was resolved by computing the micro-scale behaviour by DEM and the macro-scale with the FEM method. The different length scales were linked by a homogenisation method of the micro response. The resultant stress was then fed into the macroscopic quantities of the FEM description. Thus, the macro-scale models dependent on the discrete micro variables. The approach of discrete particles - FEM coupling with overlapping domains finds its application also in the field of hypervelocity impacts on structures. Beissel et al. [3] predicted the burst of materials into a huge amount of particle-like pieces caused hypervelocity impacts. Thereby, the developed method relies on the conversion of finite elements into particles. After the conversion the particle-like pieces further propagate based on a meshfree Lagrangian description. Further effort comes from research of fluid flows in interaction with solid structure or with floating solid parts. Generally in the field of fluid flows, when it comes to continuum-discrete coupling, algorithms with an interface for transport phenomena are employed rather then overlapping domains. But PFEM is a Lagrangian formulation to calculate Fluid-Solid Interaction and an exception in this research field. The method discretizes any domain by means of FEM, but the mesh nodes are tracked like individual particles [4]. The Lagrangian formulation allows to handle the nodes like solid particles. This enables to compute the separation of fluid particles from the main domain.
As mentioned before, in the field of coupling methods with non-overlapping domains major contribution evolve from the research of fluid flows with solid parts and flows through porous media. The purpose of these methods is to share or transport quantities, like drag forces and heat flux, between the different domain through the particle-fluid interface. Coupling method with separated physical domains logically involve an interface, which provides the flexibility to apply different models for the exchange of information. This encompasses a natural link to coupling models for contact problems. This field of research is also addressed within this current study. To analyse different length scales in structural mechanics the coupling between discrete element method presenting solid particles and finite element method reflecting a deformable structure is most suitable. Valuable effort comes from the field of soil mechanics and terramechanics where soil structures have to be reinforced or solid bodies are built

into the underground. Villard et al. [5] presented a coupling approach between a finite element model used to describe a geosynthetic sheet and the discrete element method used to describe the behaviour of granular soil. They paid special attention to friction laws between the finite and discrete element in contact. In this approach both methods are governed by the Newtonian laws of motion. The numerical model has been validated by means of a analytical solution and experimental results. Their FE mesh consist of triangular elements describing the geosynthetic sheet as a membrane like geometry. In an iterative approach the static equations of large deformations are solved within the coupling method. To describe the granular material they employed a discrete element method based on spherical particles. Each grain shape was described by a cluster of two spheres. The particle contact force is predicted using a elastic-perfectly-plastic law. The normal contact force between a finite and discrete element is predicted based on the overlap of both. The tangential force is kept under a certained limit of the normal force, but apart from that independent predicted. The interface model of the tangential force depends on the incremental relative displacement, a micro-mechanical stiffness modulus of the interface and the influence area of a contact computed by means of the particle radius. Applications of DEM - FEM coupling with a more dynamical interaction come from vehicle - soil or tire - terrain interaction. Nakashima et al. [6] [7] employed a two dimensional FE-DE to describe tire-soil interaction and the traction dependences of a tread. For this purpose the derived their model in an similar matter as described above. Both domain where govern by the fully time dependent discretization of newtons second law. The contact force developed at the interface between particle and surface element was also based on the overlap. The counter part of the contact force is interpolated to the nodal points of the surface element based on virtual work equivalent. Horner et al. [8] used the same work equivalent to map force between discrete and finite element. Both employed linear triangular elements to approximate the surface. But Horner et al. [8] discussed his approach in the view of large parallel computations of applications for vehicle - soil interaction. Despite the advantage these coupling approaches inherited in many fields it is still more common to approximate the behaviour of the granular media with a continuum model.

2.2 Structure

Before the coupling algorithm will be presented both simulation techniques, the discrete and the continuous approach, are presented as standalone utilities. The first section introduces the extended discrete element method. The method does not only allow predicting the dynamics of particulate media, it is extended by the thermodynamics state of the each particle. Within the second section, the finite element method is shortly described. Hooke's generalized law to describe linear isotropic elasticity and thermal expansion and contraction due to heating and cooling is incorporated into Newton's second law. Hence, the finite element formulation is derived for a fully time-dependent deformation.

Thereafter, the coupling method between DEM and FEM is presented. First, the main algorithm in relating the single modulus and the information sharing is described. Further, the key parts of the algorithm will be attacked in more details. The final section contains applications and simulations currently undertaking by the introduced method.

3 Extended Discrete Element Method (XDEM)

Phenomena including a particulate phase may basically be modelled by two approaches: An ensemble of particles is treated as a continuum on a macroscopic level or is resolved on a particle-individual level. The former is well suited to process modelling due to its computational convenience and efficiency. However, detailed information on particle size, shape or material is lost due to the averaging concept. As a consequence, this loss of information on a particle scale has to be compensated for by additional constitutive or closure relations.

In following the newly extended discrete element method (XDEM) is introduced. Contrary to the continuum mechanics approach, XDEM considers the solid phase consisting of a finite number of individual particles similar to the discrete element method. Its prediction of the dynamics of a particulate system is extended by the thermodynamics state of each particle, and therefore, is referred to as the extended discrete element method. Thus, the shortcomings of the discrete element method that does not provide results on the thermodynamic state of particles are alleviated. The thermodynamic state of a particle may simply include an internal temperature distribution, but may also contain transport of species due to diffusion or convection in a porous matrix in conjunction with thermo-/chemical conversion due to reaction mechanisms. Hence, the Extended Discrete Element Method opens a broad domain for application in process engineering, food industry and solid reactor engineering as addressed in the current contribution. Differential conservation equations for energy, mass, species and momentum within a particle describe the thermodynamic state, and thus, applying it to each particle furnishes particle resolved results of a packed bed. Due to a discrete description of the solid phase, constitutive relations are omitted, and therefore, lead to a better understanding of the fundamentals. It offers a significantly deeper insight into the underlying physics as compared to the continuum mechanics concepts for a packed bed, and thus, advances extensively knowledge for analysis and design. In order to keep CPU time requirements within acceptable margins, fast and efficient algorithms have to be developed , which are preferably combined with high performance computing (HPC).

Since, prediction of both temperature and species distribution inside a particle require boundary conditions, a natural link to fluid-dynamics and structural mechanics evolves. Computational fluid dynamics (CFD) may employed to predict conditions in the vicinity of a particles for heat and mass transfer, while the finite element analysis (FEA) yields temperature distributions in reactor walls to estimate heat loss of particles in contact with the walls. Furthermore, forces predicted during the motion of particles may be transferred onto solid structures assessing strain and deformation by the finite element method (FEM).

The extended discrete element method presented in the present contribution considers a particle within a packed bed as an individual entity to which motion and thermodynamics are attached. By evaluating both position/orientation and thermodynamic state of each particle in a packed bed the entire process may be summarised by the following symbolic formula:

Entire Process = $\sum$ (Particle-Motion + Particle-Thermodynamics)

Results obtained by this concept inherently contain a large degree of detail, and thus, through analysis reveal the underlying physics of the processes involved. As abovementioned an ensemble of discrete and moving particles offers the highest potential to describe transport processes. Each of the particles is asumed to have different shapes, sizes and mechanical properties. Shapes such as barrel, block, cone, cube, cylinder, disc, double-cone, ellipsoid, hyperboloid , parallel-epiped, sphere, tetrahedron, torus, wall and pipe are available.

The motion of particles is characterised by the motion of a rigid body through six degrees of freedom for translation along the three directions in space and rotation about the centre-of-mass as depicted in fig. 1.

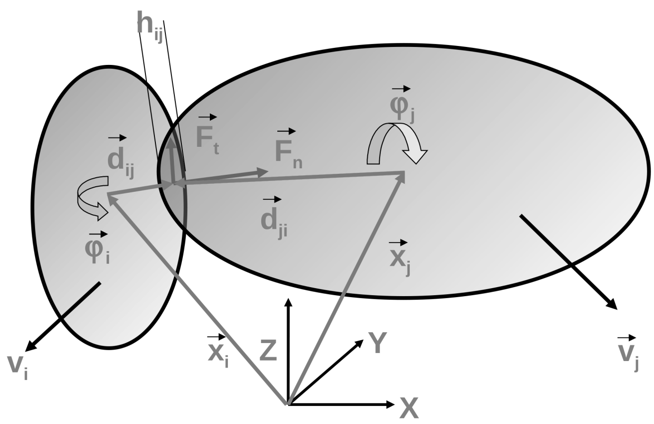

Figure 1: Particles in contact

By describing these degress of feedom for each particle its motion is entirely determined. Newton's Second Law for conservation of linear and angular momentum describe position and orientation of a particle i as follows:

$$m_i \frac{d^2\vec{r}_i}{dt^2} = \sum_{i=1}^{N} \vec{F}_{ij}(\vec{r}_j, \vec{v}_j, \vec{\phi}_j, \vec{\omega}_j) + \vec{F}_{extern} \quad (1)$$

$$\bar{I}_i \frac{d^2\vec{\phi}_i}{dt^2} = \sum_{i=1}^{N} \vec{M}_{ij}(\vec{r}_j, \vec{v}_j, \vec{\phi}_j, \vec{\omega}_j) + \vec{M}_{extern} \quad (2)$$

where $\vec{F}_{ij}(\vec{r}_j, \vec{v}_j, \vec{\phi}_j, \vec{\omega}_j)$ and $\vec{M}_{ij}(\vec{r}_j, \vec{v}_j, \vec{\phi}_j, \vec{\omega}_j)$ are the forces and torques acting on a particle i of mass m_i and tensorial moment of inertia $\bar{I}_i$. Both forces and torques depend on position $\vec{r}_j$, velocity $\vec{v}_j$, orientation $\vec{\phi}_j$, and angular velocity $\vec{\omega}_j$ of neighbour particles j that undergo impact with particle i. The contact forces comprise all forces as a result from material contacts between a particle and its neighbours. Forces may include external forces due to moving grate bars, fluid forces and contact forces between the particles in contact with a bounding wall. This results in a system of coupled non-linear differential equations which usually cannot be solved analytically.

The discrete element method (DEM), also called a distinct element method, is probably the most often applied numerical approach to describe the trajectories of all particles in a system. Thus, DEM is a widely accepted and effective method to address engineering problems in granular and discontinuous materials, especially in granular flows, rock mechanics, and powder mechanics. Pioneering work in this domain has been carried out by Cundall [9], Haff [10], Herrmann [11] and Walton [12]. The volume of Allen and Tildesley [13] is perceived as a standard reference for this field. For a more detailed review the reader is referrd to Peters [14].

As above-mentioned particles of a granular media exert forces on each other only during mechanical contact. Once the particles are in contact the repulsive force increases with a sharp gradient due to the rigidity of the particles. Therefore, a rather small time step has to be chosen to resolve the impact between particles accurately. Hence, the discrete element method is computationally intensive, which limits either the number of particles or the length of a simulation.

Therefore, in a computational approach, the deformation of two particles in contact may be approximated by a representative overlap h [15] as depicted in fig. 2.

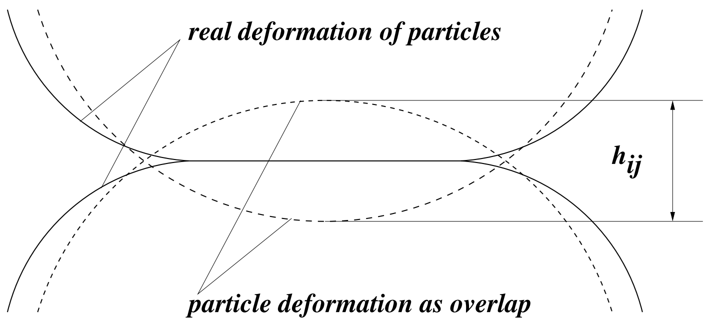

Figure 2: Approximation of particle deformation during impact through a respective overlap area

The resulting force $\vec{F}_{ij}$ due to contact may be decomposed into its normal and tangential components

$$\vec{F}_{ij} = \vec{F}_{n,ij} + \vec{F}_{t,ij} \quad (3)$$

where the components additionally depend on displacements and velocities normal and tangential to the point of impact between the particles. In a simple approach the analogy to a spring for an ideal elastic impact serves to determine the contact force as $\vec{F}_{ij} = k_{ij}\vec{h}_{ij}$. However, for more sophisticated applications additional influences such as non-linearity, dissipation or hysteresis need to be taken into a coount. For a detailed discussion of inter-particle forces the reader is referred to Peters [16, 17]

4 Finite Element Method (FEM) for Elastic Body Deformation

The Finite Element method is a highly practical and common tool in engineering and research faculties. The amount of problems attacked with the help of this method became as numerous as articles and books in this field. Langtangen et al. [18], Felippa [19], Betten [20] and Bathe [21] are just some of a plethora of respectable works.
This section covers the description of a fully time dependent model of elastic deformation with thermal expansion effects. The mathematical model of elastic deformation of solid bodies is also based on Newton's second law, which has already served within here as the foundation of the discrete element method. Additionally, for the description of deformation a constitutive relation between stress and strain has to incorporated. Thereafter, the finite element formulation is introduced as a general numerical approach to discretize the derived mathematical models and to solve the established PDEs. The continuum description of elastic deformation by means of finite element method played a key role and was a the biggest success while establishing mathematical modelling and scientific computing into the engineering world.
The deformation of an elastic continua is described by the differential form of Newton's second law (4). Contrary to the discrete approach this equation is valid in every volume point of the continuum.

$$\rho \frac{\partial u_r^2}{\partial t^2} = \frac{\partial \sigma_{rs}}{\partial x_s} + \rho\, b_r \tag{4}$$

Notice within this context u_r describes the displacement where in the discrete form the position $\vec{r}$ is placed. The indicial notation is use as widespread in solid mechanics. Thus, r, s and q are indices of the spatial dimensions. In Equation (4) ρ is the density, σ_{rs} reflects the internal forces due to stresses and the final term represents body forces. The main interest usually is to study the stresses. If the stresses exceed certain limits, the material might rupture or the validity limits of the elastic model are reached and needs to be replaced. As the density and body forces are prescribed, six unknowns reman within the symmetric stress tensor by three equations. To yield closure of the system the stresses and deformations are linked by Hooke's generalized thermo-elastic law.

$$\sigma_{rs} = \lambda \frac{\partial u_q}{\partial x_q}\, \delta_{rs} + \mu \left(\frac{\partial u_r}{\partial x_s} + \frac{\partial u_s}{\partial x_r} \right) - \alpha\,(3\lambda + 2\mu)(T - T_0)\, \delta_{rs} \tag{5}$$

The law describes the isotropic linear elasticity as well as isotropic expansion and contraction in case of heating and cooling respectively.

4.1 The Finite Element Formulation

The problem and the governing Equation (4) are fully time dependent. Hence, the discretization has to be temporal and spatial.
To discretize in time a three-point centred difference is employed by introducing the superscript l reflecting the time level. The time discrete equations of Equation (4) becomes

$$\rho \frac{u_r^{l-1} - 2u_r^l + u_r^{l+1}}{\Delta t^2} = \frac{\partial \sigma_{rs}^l}{\partial x_s} + \rho\, b_r^l \tag{6}$$

where all quantities superscripted with l are time dependent and Δt sets the time step length.
For spatial approximation the straightforward Galerkin method is applied to the governing equation. Thereby, the displacement field is approximated at n discrete nodal points by

$$\hat{u}_r = \sum_{j=1}^{n} u_j^r \cdot N_j(x_r) \tag{7}$$

where N_j are the finite element trial functions and u_j^r contains n times dimensions coefficients. As the Galerkin method inserts the approximation $\hat{u}_r$ for u_r a residual results, which is multiplied with the trial function and is required to vanish.
The final discretized form of the governing Equation (4) to compute isotropic elastic deformation showes Equation (8).

$$\mathbf{u}^{l+1} = 2\mathbf{u}^l - \mathbf{u}^{l-1} + \mathbf{M}^{-1}\Delta t^2 \cdot \left(-\mathbf{K}\mathbf{u}^l + \mathbf{b}^l\right) \tag{8}$$

with $\mathbf{M} = M_{ij}^{rs}$ being the mass matrix of the system. Further, $-\mathbf{K}\mathbf{u}^l + \mathbf{b}^l$ is the dicretized form of $\sigma_{rs,s}^l + \rho b_r^l$ where $\mathbf{K}$ equals the stiffness matrix of the system with the entries K_{ij}^{rs} and the term $\mathbf{b}$ holding all nodal forces. In the discretized form the force term $\mathbf{b}$ also holds the forces due to thermal effects, which have been introduced with the Hooke's model Equation (5).

The time-stepping scheme of Equation (8) gains the new value $\mathbf{u}^{l+1}$ by vector addition, matrix-vector and scalar multiplication. Hence, the scheme is an explicit procedure assuming that $\mathbf{u}^l$ and $\mathbf{u}^{l-1}$ are already predicted. The computational algorithm starts at $t = 0$ by solving the stationary elasticity problem $\mathbf{K}\mathbf{u}^0 = \mathbf{b}^0$ implicitly before it enters the time loop. The solution provides $\mathbf{u}^0$. Then $\mathbf{u}^0$ and $\mathbf{u}^l$ are equalized which implies the entire system is stationary unless $\mathbf{b}^l$ changes, which is a reasonable assumption.

5 Efficient DEM - FEM Coupling Method

In the following the coupling procedure is described within the time loop. Further, key parts of the coupling are presented in detail. The first key part presented is the efficient algorithm for a fast prediction of potential contact pairs between particle and surface element. After the prediction of potential contact pair the actual intersection of particle and surface element of the FE mesh is explained. Finally, as the major key of the coupling the interpolation of the contact force onto the element nodes is derived.

Within the context of explicit time integration the procedure of the DEM - FEM coupling algorithm is relatively straight forward. The procedure is schematically depict inf Figure 3. Before the procedure enters the time loop to predict the motion of particles and the deformation of the solid body an initiation phase is required to establish the foundation for later information exchange. While generating the mesh of the solid body one has to take care of meshing the surface of the body with linear triangular finite elements. The efficient contact detection is base on the intersection between spheres and triangles as particles and surface elements respectively.

```
Initiation
    ↦ solve → Ku⁰ = b⁰
    ↦ mirror displaced surface elements
Time Loop → t₀ + n · Δt
    ↦ integrate time step  tₙ  of Equation (1) & (2)
    ↦ transfer contact forces
        ↦ predict potential contact pairs
        ↦ computed contact forces
        ↦ interpolate forces to finite element
    ↦ integrate time step  tₙ of Equation (8)
    ↦ update boundary shapes
```

Figure 3: General Procedure of the Coupling Algorithm

As depicted in Figure 3 the initiation starts with solving the stationary problem with in the FEM domian. This step provides two informations to the procedure. First, it established the matrices and initial displacement values for Equation (8) of the FEM

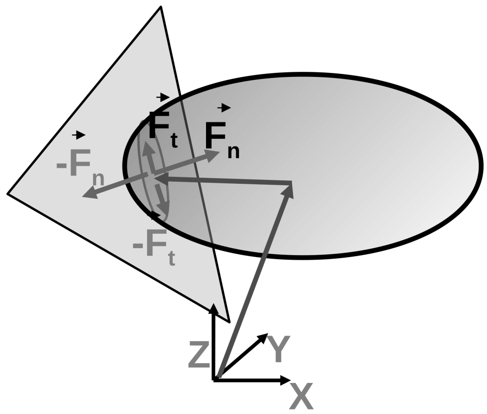

Figure 4: Contact between Surface Element and Particle

domain, as already discussed in Section 4. On the other hand it serves with the first deformed structure for the DEM domain. The second step of the initiation phase mirrors the deformed surface elements of the FEM mesh into DEM domain. Hence, at all time the surface geometry of the deformed body exists in the similar way inside the DEM domain. The DEM domain sees the deformed surface elements of the finite element mesh as geometrical boundary conditions. On this basis the coupling algorithm is linking the particular surface element with the according boundary shape between FEM and DEM domain respectively.

The loop over time is separated into four major parts. Thereby, the order of the computations follows the physical events logically. First, the motion of the granular assembly is integrated. Following this step, the impacts between particles and the elastic body are predicted. Thereafter, the deformation of elastic body due to the impacting forces can be solved by the FEM scheme. The last part of the procedure updates position of the boundary shapes before the particle motion is repredicted within the new time step.

Within a single time loop the motion of the granular material is predicted first. This step integrates the position and orientation of particles according to Equation (1) and (2). Thereby, the change in the dynamic quantities can also be due to forces of previous interaction between particles and solid body.

Logically, between the integration of the motion of particles and the integration of the deformation of the elastic body contact forces need to be computed and transferred. Therefore, the step of transfer of contact forces aligns between them at the second position within the time loop. This step is partitioned into three sub steps. First the potential contact pairs are detected by means of an efficient contact detection algorithm. The algorithm spared valuable computation time by a fast separation of the important particle - surface element pairs from the possibly huge number of contact pairs able with the system. The foundation of the algorithm is binary tree storing bounding volumes, which encapsulate a decreasing number of surface elements. The

next sub step computes contact forces for all pairs provided by the contact detection algorithm. The contact force between particle and triangular element, see Figure 4, is derived in a similar manner as described for particle - particle contact in Section 3. The contact force is decomposed into normal and tangential components which depend on displacements and velocities normal and tangential to the point of impact between the particle and surface element. The contact force is add to the particle forces and the counter force is transfer to the according surface element of the FE mesh. Thus, the third sub step executes the interpolation counter force onto the nodal forces of the finite element.
After the transfer of contact forces the computations of elastic deformation are executed. This step incorporates the insertion of interpolated contact forces into the force vector b^l of the finite element formulation. Then, the new displacement value $\mathbf{u}^{l+1}$ will be gain by solving Equation (8).
Finally before the new loop starts over, each the boundary shape within the DEM domain is updated according to the appropriate surface element. Therefore, the position and displacement vector of every nodal point of the surface mesh are added and the appropriate vertex of the triangular boundary shape is equalized with the result.

6 Applications

The interface coupling between the discrete element method (DEM) and the finite element method (FEM) is applicable to almost all engineering applications that deal with granular matter. Within this section two applications from entirely different engineering domains are introduced. But both application are challenged by similar physical problems. For both applications engineering devices are in contact with individual particles of a granular assembly. Hence, contact forces are generate that act between the engineering device and the granular material. In both cases, the impact propagates into different length scales. Thus, the DEM - FEM coupling then enables to resolve the different scale responses.
Figure 6 shows different time steps of the DEM - FEM simulation of a belt conveyer. Applications of a conveyer belts are widespread through all engineering companies concerned with the transport of industrial and agricultural materials, typical bulk material transported are grain, coal and ores. The upper part of Figure 6 depicts the a conveyer belt at the beginning of the loading with a particle assembly. Under further loading of particles the belt experiences deformation, presented by colors in Figure 6, and responses with stress, which act on the individual particle as repulsive force.

The interaction between terrain and tire tread is investigated within the second application, depict in Figure 5. The purpose of those computation is to study the traction mechanisms between the tire and the granular ground. The traction force not only the integral force from the friction of each individual grain in contact with the tire surface. The traction is also influenced by interlocking mechanisms between tread parts and granular in contact. To resolve this scale and the deformation of the tire a DEM -

FEM coupling simulation suits very well.

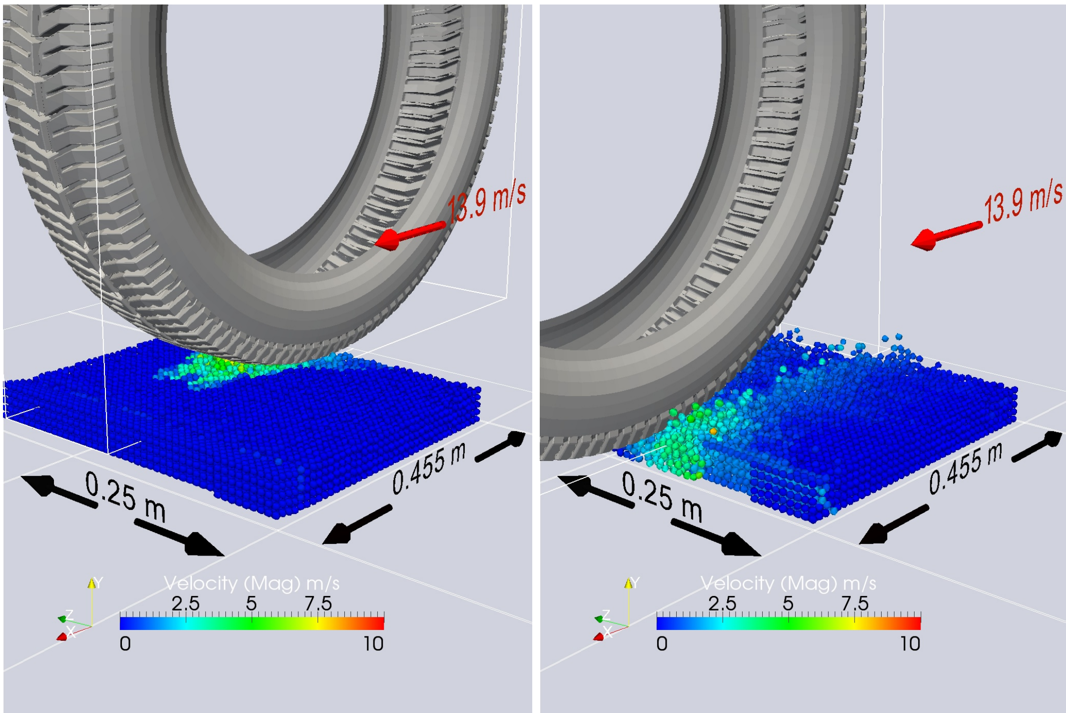

Figure 5: Tire Tread whiles Rolling Through Granular Media.

7 Conclusion

The discrete element method coupled with the finite element method can be used to resolve different length scale within almost all engineering problems dealing with granular assemblies in contact with a deformable body of an engineering device.

The extended discrete element method thereby describes the motion and contact forces of each individual grain within the granular assembly. The finite element method on the other hand, efficiently predicts deformations and the responding stress of the engineering device.

At the contact interface the developed impact propagates into the different length scales. Thus, the combined discrete and continuum approach now enables the tracking of both responses by the appropriate resolution. Each grain of the assembly in contact with the solid body generates a contact force and experiences a repulsive force which it reacts on individually. The contact forces sum up on the interface and cause the solid body to deform. This results in stresses which again the assembly recognised as repulsive response.

The coupling method utilizes quite naturally the advantages of both the continuum and the discrete approach and thereby compensating the shortages of each method.

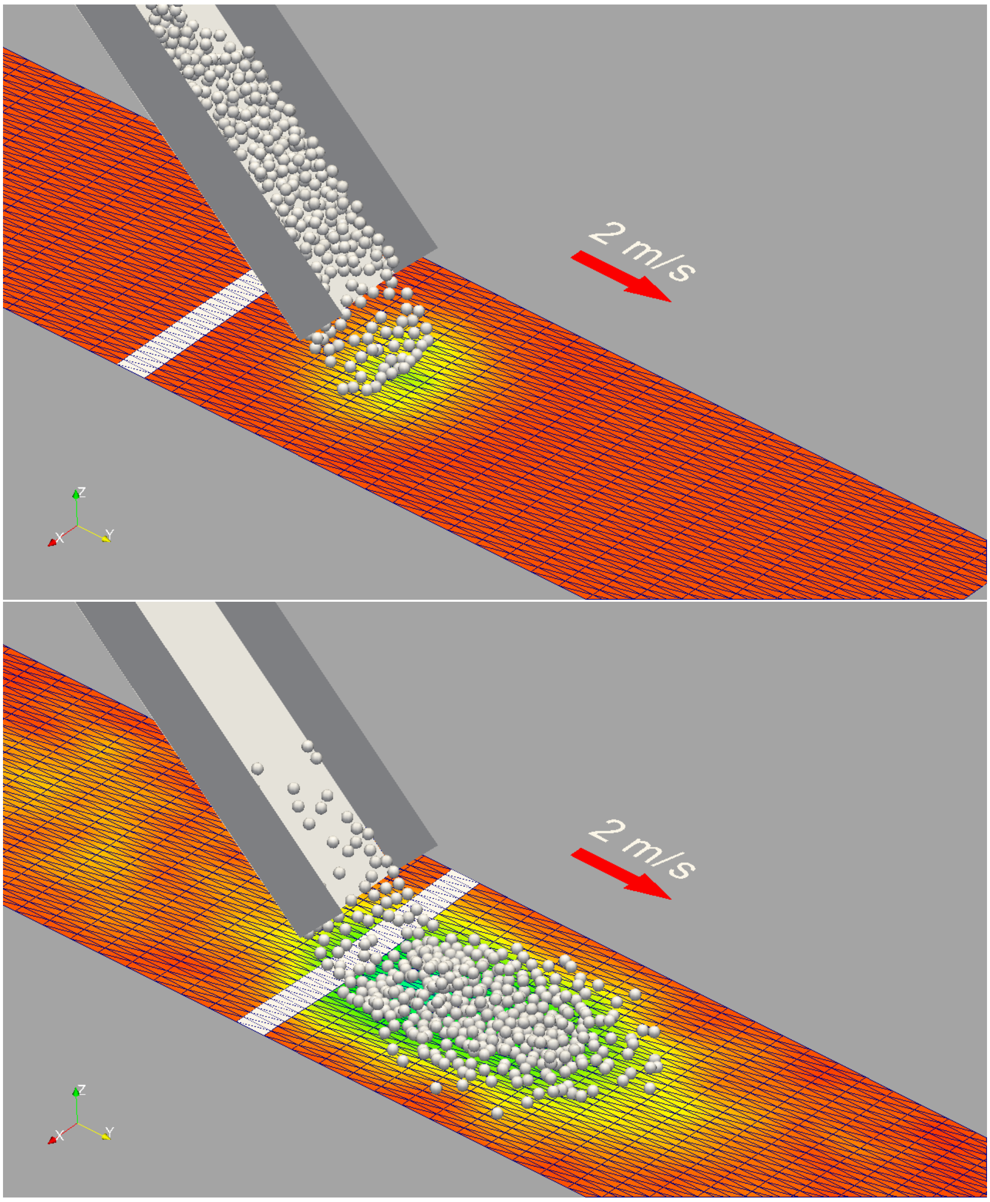

Figure 6: DE - FE Coupling Simulation of a Conveyer Belt Loading. The belt deformation is predicted by means of FEM

The coupling method not only resolves the different scales it further contributes to the efficiency of the computations. The method employs a fast contact detection algorithm, which spares valuable computation time by a fast separation of the important pairs of particles and surface element for the contact force prediction.

The DEM - FEM simulation technique is introduced with two engineering applications from entirely different fields. But both application inherit the similar physical problems of different length scales. In both cases individual particles are in contact with an widely used engineering devices that is in contact with granular material.

Thus, the DEM - FEM coupling is shown to resolve the different scale responses with in each domain separately.

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