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Paper 214

Firefly Metaheuristics for the Size, Shape and Topology Optimization of Truss Structures

L.F. Fadel Miguel[1], R.H. Lopez[1], A. El-Hami[2] and L.F.F. Miguel[3]
[1] Civil Engineering Department
Federal University of Santa Catarina, Florianopolis, Brazil
[2] Mechanical Engineering Department
National Institute of Applied Sciences (INSA), Rouen, France
[3] Mechanical Engineering Department
Federal University of Rio Grande do Sul, Porto Alegre, Brazil

Abstract

This paper presents an efficient single-stage firefly based algorithm to the simultaneous size, shape and topology optimization of truss structures. The optimization problem has as objective function the minimisation of the structural weight and it imposes displacement, stress and kinematically stability constraints. The effectiveness of the firefly algorithm (FA) to solve this kind of optimisation problem is demonstrated through a benchmark problem and the results obtained were better than the reported in the literature, with lower computational cost, emphasising the capacity of the proposed methodology.

Keywords: truss optimization, firefly algorithm, size optimization, shape optimization, topology optimization, mixed-variable optimization.

1 Introduction

Methods for the size optimization of truss structures, in which the member areas are taken as design variables, are well established and there is a rich literature on this subject [1]. However, it is well known that better results are achieved when size, shape, and topology optimization are carried out simultaneously [2]. In this case, the problem generally starts from a ground structure and the best topology of the elements must be determined. In addition, the truss geometry is also changed by taking nodal coordinates as design variables. This problem is nonconvex [3-5] and requires dealing simultaneously with discrete and continuous design variables, since the cross sectional areas are limited to some discrete values due to manufacturing constraints and the nodal coordinates are continuous variables.

Metaheuristic algorithms are well-suited to solve this kind of optimization problem. The genetic algorithms (GA), for instance, have been applied by several researches for truss optimization [6-9]. In the last decade, several researchers developed and improved robust search techniques that simulate the paradigm of a biological, chemical, or social system, the so-called metaheuristic algorithms [10]. In this context, the firefly algorithm (FA), recently developed by Yang [11], demonstrated to be more accurate and efficient than well-established heuristic algorithms such as the GA and the particle swarm optimization (PSO). Its implementation in structural optimization field is still new and precocious, requiring a substantial amount of additional studies [12]. Thus, the implementation of the FA in size, shape and topology optimization of truss structures have not been studied yet and seems to be promising.

In this paper, the FA is employed for the simultaneous optimization of size, shape, and topology of truss structures. Unstable and singular topologies are disregarded as possible solutions by checking the positive definiteness of the stiffness matrix. The effectiveness of the FA to solve truss optimization problems is demonstrated through a couple of benchmark problems. The paper is organized as follows. Section 2 presents the formulation of the optimization problem. The description of the FA is given in section 3. The numerical analysis is pursued in section 4, while section 5 presents the main conclusions of this paper.

2 Optimization problem statement

In the proposed methodology, the ground structure approach is followed. It starts from a universal truss containing all possible member connections (or almost all) among all the q nodes in the structure, i.e. it starts from the set of all possible members M_{all}, where $\dim\left(\mathrm{M}_{all}\right)=n$. Thereafter, the optimization procedure is employed to discard the unnecessary members, *i.e.* from M_{all}, the algorithm chooses the members that remain in the structures and these members form the design set M_{design}, where $\dim\left(\mathrm{M}_{design}\right)=m$. Simultaneously, the algorithm changes the cross-sectional dimensions ($\mathbf{A}\in\Re^{m}$) and nodal coordinates which are design variables ($\boldsymbol{\xi}\in\Re^{q'}$), looking for the minimum structural weigh and subject to stress, displacement and kinematically stability constraints. Thus, the optimization problem may be posed as:

$$
\begin{array}{ll}
\textit{Find} & \mathrm{M}_{design},\ \boldsymbol{\xi}\ \textit{and}\ \boldsymbol{A} \\
\textit{Minimize} & W(\boldsymbol{\xi},\mathbf{A})=\sum_{\mathrm{i}=1}^{\mathrm{m}}\rho_{\mathrm{i}}\ell(\xi_{\mathrm{i}})\mathrm{A}_{\mathrm{i}} \\
\textit{Subject to} & \mathrm{G1}\equiv \textit{truss is kinematically stable} \\
 & \mathrm{G2}\equiv \delta_k(\boldsymbol{\xi},\mathbf{A})-\delta_k^{\max}\le 0,\ k=1,\dots,q \\
 & \mathrm{G3}\equiv \left|\sigma_j(\boldsymbol{\xi},\mathbf{A})\right|-\sigma_j^{\max}\le 0,\ j=1,\dots,m \\
 & \mathrm{G4}\equiv A_j^{\min}\le A_j\le A_j^{\max},\ j=1,\dots,m \\
 & \mathrm{G5}\equiv \xi_i^{\min}\le \xi_i\le \xi_i^{\max},\ i=1,\dots,q'
\end{array}
\tag{1}
$$

where W is the structural weight, m is the number of members in the current design, ρ is the specific weight of the material of the bars, ℓ is the length of each bar and G are the set of constraints. δ_k and $\delta_k^{\max}$ are respectively the displacement and maximum allowed displacement at node k, σ_j and $\sigma_j^{\max}$ are respectively the stress and maximum allowed stress of the j^{th} bar (for simplicity, we consider the stresses in modulus, i.e. the same allowable stress for tension and compression), $A_j^{\min}$ and $A_j^{\max}$ are respectively the lower and upper bounds of the cross sectional area of the j^{th} bar, and finally, $\xi_i^{\min}$ and $\xi_i^{\max}$ are respectively the lower and upper bounds of the allowed move of the i^{th} design variable nodal coordinate.

Regarding the topology optimization, a specific member is eliminated of the ground structure following the criteria proposed by Deb and Gulati [9]. The cross-sectional area of a member is compared to a user defined small critical cross-sectional area ε. Then, if the member area is smaller than ε, this element is eliminated of the ground structure. This way defines how different topologies can be obtained for a continuous optimization procedure. Note that the value ε and the lower (A_{min}) and upper (A_{max}) bounds of the cross-sectional areas must be determined considering an adequate probability of a specific element to be absent of the final solution. For instance, if $A_{min} = -A_{max}$, and the critical cross-sectional area ε is almost zero, the probability of any member to be present in the final structure is practically 50 %. For discrete optimization, the user defines the number of zero-bar that are added to the available profiles list to generate a reasonable probability to eliminate an element of the ground structure.

3 Firefly algorithm

The firefly algorithm (FA) is a recent metaheuristic optimization algorithm, developed by Yang [11] and it is inspired by the flashing behavior of fireflies. According to Yang [11], there are three idealized rules in the FA optimization:

(a) all fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex;

(b) attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less brighter one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly, and,

(c) the brightness of a firefly is affected or determined by the landscape of the objective function.

Based on these three rules, the basic steps of the FA can be summarized as the pseudo code shown in Figure 1 [11].

There are two essential points in the FA: the variation of light intensity and the formulation of the attractiveness. The latter is assumed to be determined by the brightness of the firefly, which in turn is related to the objective function of the problem under study.

begin

Objective function f(x), $x = (x_1, ..., x_d)^T$

Generate initial population of fireflies x_i $(i = 1, 2, ..., n)$

Light intensity I_i *at* x_i *is determined by* $f(x_i)$

Define light absorption coefficient γ

while (t < *MaxGeneration*)

for $i = 1 : n$ *all n fireflies*

for $j = 1 : d$ *loop over all d dimensions*

if ($I_i < I_j$), *Move firefly i towards j*; end if

Vary attractiveness with distance r via exp$[-\gamma r]$

Evaluate new solutions and update light intensity

end for *j*

end for *i*

Rank the fireflies and find the current global best

end while

Post-process results and visualization

end

Figure 1: Pseudo code of firefly algorithm (adapted from [11]).

As light intensity and attractiveness decreases as the distance from the source increases, the variation of light intensity and attractiveness should be a monotonically decreasing functions, *e.g.* the light intensity may be:

$$I\left(r_{ij}\right) = I_0 e^{-\gamma r_{ij}^2} , \tag{2}$$

where the light absorption coefficient γ is a parameter of the FA and the r_{ij}, the distance between fireflies i and j at $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively, can be defined as the Cartesian distance $r_{ij} = \left\|\mathbf{x}_i - \mathbf{x}_j\right\|$. As a firefly's attractiveness is proportional to the light intensity seen by other fireflies, it can be defined by:

$$\beta\left(r_{ij}\right) = \beta_0 e^{-\gamma r_{ij}^2} \tag{3}$$

where β_0 is the attractiveness at $r = 0$. Finally, a firefly i is attracted at another more attractive (brighter) firefly j is determined by:

$$\Delta \mathbf{x}_i = \beta_0 e^{-\gamma r_{ij}^2} \left(\mathbf{x}_j^{\,t} - \mathbf{x}_i^{\,t}\right) + \alpha \boldsymbol{\varepsilon}_i , \quad \mathbf{x}_i^{\,t+1} = \mathbf{x}_i^{\,t} + \Delta \mathbf{x}_i , \tag{4}$$

where t is the generation number, $\boldsymbol{\varepsilon}_i$ is a random vector (*e.g.* standard Gaussian random vector: mean equals to 0 and standard deviation equals to 1) and α is the randomization parameter. The first term in the right hand side of Eq.(4) represents the attraction between the fireflies and the second term is the random move. In other words, Eq.(4) shows that a firefly will be attracted to brighter or more attractive fireflies, and at the same time they will move randomly.

4 Numerical analysis

Standard test problems are useful for the purpose of checking optimization algorithms. The benchmark examples given in this section have been widely used for this purpose. Moreover, the problems are presented in increasing order of complexity. In all the examples, we used the following set of parameters: ; $\beta_0 = 1$, $\gamma = 1$, $\alpha = 0.5$ and $\boldsymbol{\varepsilon}_i$ follows the uniform distribution between -0.5 and 0.5.

4.1 Eleven-bar truss example

This truss is often used as a benchmark problem and has been applied in most works on this field. The ground structure is shown in Figure 2 and the design parameters are given in Table 1. Two independent studies are carried out: (i) Sizing and topology optimization, and (ii) Sizing, shape and topology simultaneous optimization.

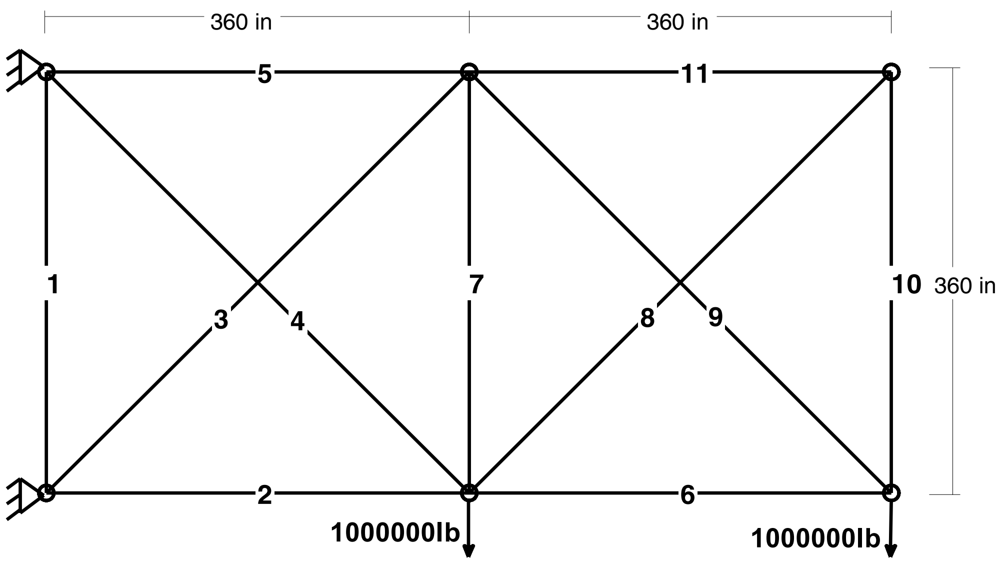

Figure 2: The 11-bar truss benchmark example: section 4.1.

4.1.1 Sizing and topology optimization

Several researches studied this problem using the GA. However, they applied two different sets of discrete design variables. Rajan [7] adopted cross-sectional areas from a set of 32 discrete values, whereas Hajela et al. [6] and Deb and Gulati [9] allowed the cross-sectional areas to vary in the range 0.0-30.0 in^2, with increments of 1.0 in^2. Since the best known solution in the literature is 4912.15 lb, found in [9], the present paper attempted to reproduce the same conditions.

Although Hajela et al. [6] and Deb and Gulati [9] studied the same problem. The latter used a population size P = 220 and a number of generations G = 225, resulting a total of 49500 objective function evaluations (OFE).

The present paper intends to achieve the same accuracy of Deb and Gulati [9]. In addition, we aim to achieve the optimal solution requiring fewer function evaluations than reference [9]. We employ the number of fireflies n = 10 and the number of searches S = 3000, resulting in 30000 OFE. Figure 3 shows the best structure found by the FA.

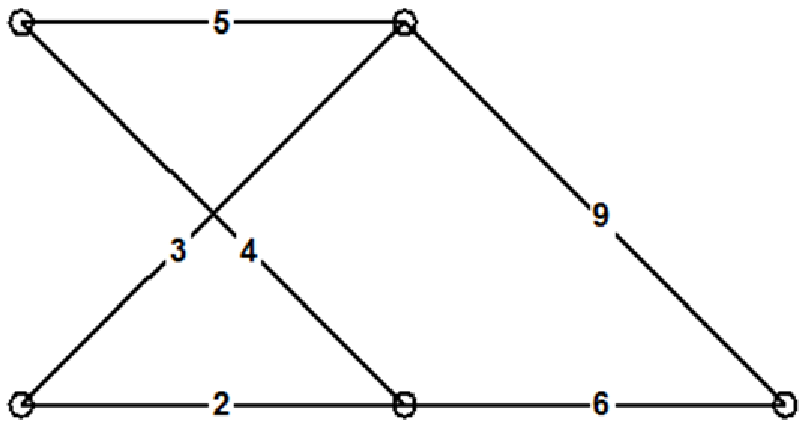

Figure 3: Optimal topology found in the size and topology 11-bar truss optimization problem.

Member Number	Member Areas (in^2)		
	Proposed	Deb and Gulati [8]	Hajela *et al.* [5]
2	24	24	24
3	20	20	21
4	6	6	6
5	30	30	28
6	16	16	16
9	21	21	22
Weight (lb)	4912.85	4912.85	4942.7

Table 2: Member areas of the optimized 11 bar truss benchmark example.

Figure 3 shows that the best topology was reduced to only six members and five nodes. The y-displacement approaches its limit for the nodes carrying loads, reaching 99.95 % (intersection of members 6 and 9) and 99.85 % (intersection of members 2, 4, and 6) of the allowable 2 in displacement. The cross-sectional areas and the corresponding truss weight, obtained in this paper and [6] and [9], are found in Table 2. As it can be seen, the proposed optimization scheme was able to reproduce the best results reported in literature.

4.1.2 Sizing, shape and topology optimization

This problem was studied by Rajan [7] and Balling *et al*. [13], which employed the GA, and Martini [14], which employed the Harmony Search (HS) optimization algorithm. Shape was optimized by allowing the vertical coordinates of the three superior nodes to moves between 180 in and 1000 in, considering the origin in the intersection of members 1, 2, and 3. Since the nodal coordinates are continuous and the cross-sectional areas are taken from a set of 32 discrete variables [7], the problem is a mixed variable optimization problem, *i.e.*, it deals simultaneously with integer and continuous design variables.

In this problem, Rajan [7] employed a population size P = 40 and number of generations G = 96, Balling *et al*. [13] carried out a multimodal analysis with P = 1000 and G = 500, resulting in a total of 500000 OFE. In addition, the latter did not eliminate the possibility of unstable topologies and as a consequence, the algorithm could find mechanisms. Martini [14] employed 75 evaluations on initialization of harmony memory plus 4000 design cycles, then, resulting 4075 OFE in a multimodal analysis.

Figure 4 shows that the best topology also was reduced to only six members and five nodes. The y-displacement approached its limit for the nodes carrying loads, reaching 99.99 % (intersection of members 6 and 9) and 99.85 % (intersection of members 2, 4, and 6) of allowable 2 in displacement.

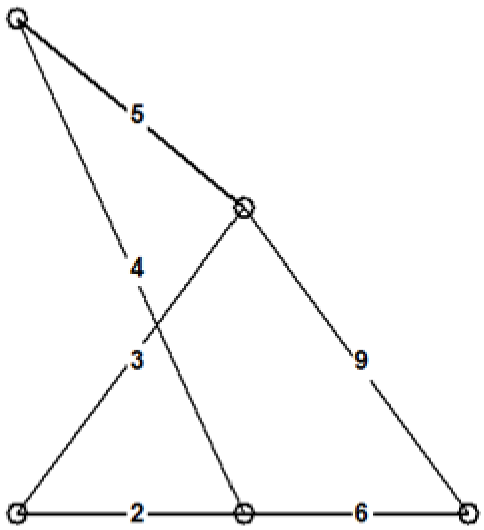

Figure 4: Optimal topology found in the size, shape and topology 11-bar truss optimization problem.

To achieve the optimal design, it was used a number of fireflies n = 10 and a number of searches S = 5000, which is much less than study of Balling *et al*. [13]. In despite of that, the best topology found here weights 2705 lb, which is slightly better than the optimum weight of Balling *et al*. [13] (2736 lb) and Rajan [7] (3254 lb). Martini [14] also presented a worse result, as it was expected, since the number of OFE was much lower. Thus, to the best of the authors' knowledge, the proposed scheme provided the best result found in the literature. Figure 4 shows the best design found by the FA. The cross-sectional areas, tension members and the corresponding weight of this optimal design are found in Table 3.

Member Number	Member Areas (in^2)	Stress (ksi)
2	11.50	10.3
3	2.88	10.0
4	5.74	19.1
5	11.50	10.4
6	7.22	10.2
9	13.50	9.2
Weight (lb)	2705.16	

Table 3: Cross-sectional areas and tension members for the size, shape, and topology 11 bar truss optimization problem.

From the results presented in this section, we may conclude that the proposed optimization scheme was able not only to reproduce the best results reported in literature, but also to find the best design for this problem.

5 Concluding remarks

In this paper, the firefly algorithm (FA) was employed for the simultaneous optimization of size, shape, and topology of truss structures in a single stage procedure. It was shown that the approach is especially suited for mixed variable optimization problems, which is a typical situation in this kind of problem.

The effectiveness of the FA to solve the simultaneous size, shape and topology optimization of trusses was demonstrated through a benchmark problem and the results obtained were better than those reported in the literature, with lower computational cost, emphasizing the capacity of the proposed methodology. The application of the FA to other examples of truss optimization is in progress.

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