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A Time-Domain Methodology for Rotordynamics: Analysis and Force Identification

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Abstract

In this paper, three time-domain force identification methods, known by the acronyms SWAT, ISF and DMISF, are for the first time (to the authors knowledge) applied to rotordynamics. These time-domain identifications have the advantage of allowing the determination of the amplitude and evolution of the forces acting on a given structure practically in real-time. Here, a short description on the dynamic behaviour of rotors is given, after which the mentioned methods are briefly presented. These methods are then adapted to be numerically applied to a simple rotor configuration rotating at constant speed with a given mass unbalance. A comparison of the results from the three methods is presented and some conclusions are drawn.

Keywords: rotordynamics, mass unbalance, force identification, time-domain, SWAT, ISF, DMISF.

1 Introduction

As in any engineering application, reliable and efficient operation is intended on machines with rotating components, and to achieve this, practical diagnosis methods are needed to record and display accurately the performance that is actually being obtained, so that further improvements can be implemented. A method of achieving a real-time depiction of the forces acting on a rotor is a valuable tool for such a task, in which the rotor structural performance enhancement is concerned.
Efforts in the last few decades have been made in the sense of achieving a practical method capable of solving in a satisfactory way the inverse problem of dynamic systems [1]. The concept of inverse problem is very simple and it is just based on the inversion of the conventional determination of response from a given loading (the forward or direct problem). The problem is that the responses are neither always easy to measure nor the accuracy of the system data is sufficient to obtain a

satisfactory result. In fact the inverse problem is more sensitive to inaccuracies in the data than the direct problem [2].
The Frequency Domain method (FD) is, at the present time, one of the most known techniques in the force identification field. A very good review in the subject can be found in [3]. It makes use of the measured response in several points of the system and of the frequency response function to compute an estimation of the applied forces.
The force identification in time domain has been less studied than its frequency domain equivalent, therefore there are not that many force identification studies in the literature.
The inverse problem is posed differently in the time domain and in the frequency domain. This means that the faced challenges come from other sources previously unknown, which require solution, but can also lead to more robust and accurate force identification if those difficulties are overcome.
The Sum of the Weighted Accelerations Technique (SWAT) is one of the time domain methods presented in the literature. It was first presented by Carne et al. [4], despite the fact that it was previously developed by Priddy and Smallwood [5]. An extension of the SWAT method allowing the simultaneous determination of multiple forces acting on a structure has been developed by Genaro and Rade [6], but this was not included in this work.
Another technique presented in the literature is based on the inversion of the equations of motion of the system. Kammer and Steltzner [7] have presented a method that uses this principle, named the Inverse Structural Filter (ISF). In this method the discrete-time equations of motion are inverted; this approach is more advantageous than the one involving the inversion of the continuous-time equations of motion, since it avoids the integration and differentiation of the measured responses.
Unfortunately, in both the discrete and the continuous cases the inverse system is contaminated with unstable poles that lead to erroneous solutions in the estimated forces.
Recently Allen and Carne [2] presented an extension to the ISF that makes use of the non-causal concept, the delayed multi-step inverse structural filter (DMISF); the objective is to improve the state space representation of the original formulation of the ISF, using multiple time future response samples to reconstruct the current force input and has shown a good performance in increasing the stability of the solution.
To the best knowledge of the authors, no direct application of the SWAT, ISF and DMISF methods to rotating systems has been reported so far. The purpose of this work consists in developing and testing such an application. Despite this, there are already some publications on the subject of force identification in rotordynamics. Zutavern and Childs [8] developed a force identification algorithm using magnetic bearings; Verhoeven published a work in this field making use of analytical synthetized transfer functions [9]. Spirig and Staubli [10] presented a method that determines the forces acting on the seals of the rotor integrating the circumferential pressure distribution measured in the seal coordinate.
Here, some examples are presented to illustrate the potential of the developed methods when applied to monorotors. These methods are also successful when

applied to the situation where the rotor is in transient motion, i.e. accelerating/decelerating.

2 Brief introduction to rotordynamics

Rotors are usually divided into several components, each of them obeying to certain assumptions, such as: rigid disk(s), elastic linear deformable shaft(s) and rigid and/or elastic bearings. In the finite element models used in this work, the disks are modeled as punctual masses of equivalent inertia and the shafts as beam elements. The simplest rotor configuration consists in a simple supported shaft with an accompanying disk. This configuration is known as Monorotor or Jeffcott rotor [11]. An example of this kind of rotor is displayed in Figure 1. When a rotor has more than one coaxial shaft, it is named Multirotor. Lalanne et al. [12] include a great revision on the dynamics of such rotors and, most relevant to this article, it includes a deduction of simple finite element models to study this kind of structures.

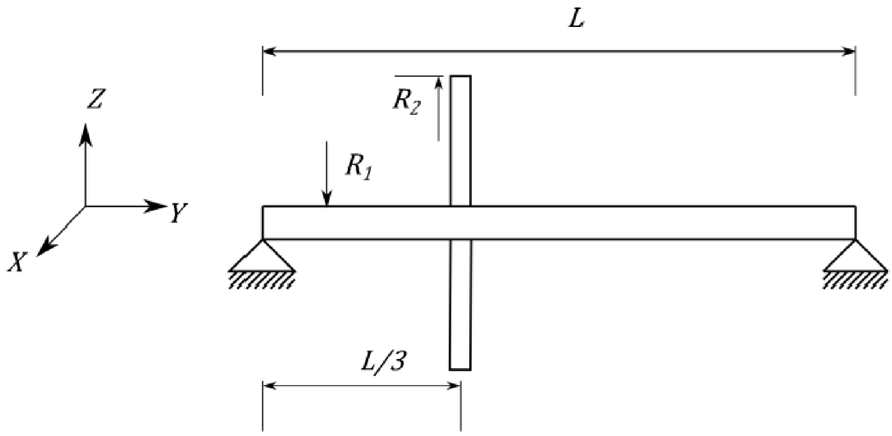

Figure 1: The Monorotor model.

So, for the most general case of two coaxial rotors with a viscously damped bearing, rigid disk, symmetric shaft, with external transversal applied forces F(t), Coriolis acceleration effect and assuming no motion in the axial direction, the resulting motion equations are given by:

$$[M]\ddot{\delta}+\underbrace{\left([C_{br}]+\Omega\begin{bmatrix}0 & -(a_1+na_2)\\(a_1+na_2) & 0\end{bmatrix}\right)}_{[C]}\dot{\delta}+[K]\delta=\{F(t)\} \tag{1}$$

In the equation above, $[M]$ is the mass matrix, a_1 and a_2 the gyroscopic effect parameters on rotor 1 and 2 respectively, Ω is the rotating speed, n the inner-outer spool rotating speed ratio, $[C_{br}]$ includes the damping coefficients in the bearings, $[K]$ the total stiffness matrix (bearings plus strain stiffness) and $[C]$ is the total damping matrix. The displacement vector δ is:

$$\delta = [u, w, \theta, \psi]^T \quad (2)$$

in which *u, w, θ and ψ*, are respectively the translations and rotations, along and about the *X* and *Z* axis (see the considered inertial frame in Figure 1).
The system matrices are displayed in detail in chapter 3.1.2 of [13]. If the rotor is in transient motion then some corrections should be included in the system matrices, whose details are explained in the Appendix 6 of [13].
A common force excitation in rotordynamics is caused by the presence of a mass unbalance in a disk of the rotor. The force acting in the node of the disk, in permanent motion can be modeled as follows:

$$\begin{bmatrix} F_u \\ F_w \\ F_\theta \\ F_\psi \end{bmatrix} = -m_u d\Omega^2 \begin{bmatrix} \sin\Omega t \\ \cos\Omega t \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

where m_u is the mass unbalance value and d the distance offset between the mass unbalance position and the rotating axis.

3 Force identification methods

The three time domain force identification methods studied in this work, SWAT, ISF and DMISF, are briefly introduced next.

3.1 SWAT – sum of weighted accelerations technique

The SWAT method makes use of the concept of modal filter. The whole method is based on the computation of a weighting matrix that when applied to the response of the structure, isolates the rigid body accelerations of the structure from the influence of the flexible modes response. The weighting matrix is the modal filter, since it is detaching the rigid body accelerations from a response consisting of both rigid body and flexible modes. Following the computation of the rigid body accelerations, the loads acting at the center of gravity of a structure can be estimated from the multiplication between the rigid body accelerations and its mass properties.

In this text the approach followed by Carne et al. [14] to derive the SWAT method is presented and implemented. As previously mentioned the weighting matrix is applied to the measures to obtain the rigid body accelerations. Following this statement it can be established that:

$$\{a_{RB}\} = [W]^T \{a\}_{measured} \quad (4)$$

where the sought $N_o \times N_{RB}$ weighting matrix $[W]$ is used to extract the $N_{RB} \times 1$ vector $\{a_{RB}\}$ of the rigid body accelerations, N_{RB} being the number of rigid body modes and N_0 the number of measurement points.

The method is completed after multiplying the rigid body accelerations by the rigid body mass properties to obtain the N_{RB} load vector $\{F(t)\}_{CG}$.

$$m_{RB}\{a_{RB}\} = \{F(t)\}_{CG} \tag{5}$$

If this method is systematically applied to the measurements at each time instant, a time domain force reconstruction at the center of gravity of the structure is obtained. Let us now introduce how the weighting matrix $[W]$ is computed. Firstly it begins by establishing a relationship between the measured accelerations and the modal shapes using a sum of modal contributions representation.

$$\{a\}_{measured} = [\Phi]\{\ddot{\eta}\} \tag{6}$$

where $\{a\}_{measured}$ is a $N_0 \times 1$ vector that contains the measured accelerations at the measurement points, $\{\ddot{\eta}\}$ is the $N \times 1$ vector of modal accelerations, $[\Phi]$ is a $N_0 \times N$ matrix containing the mode shapes, N is the number of modes (both rigid body and flexible). It should be noted that any continuous system has an infinite number of modes, meaning that equation (6) is just an approximation. Matrix $[\Phi]$ can be computed solving an eigenproblem, if a finite element model of the structure is available.
Equation (6) can be rewritten in a more convenient way if the rigid body mode vectors are considered as mass normalized:

$$\{a\}_{measured} = [[\Phi_{RB}] \quad [\Phi_e]]\begin{Bmatrix}\{a_{RB}\}\\ \{\ddot{\eta}_e\}\end{Bmatrix} \tag{7}$$

where the RB and e indices correspond to the rigid body and elastic modes respectively and $\{\ddot{\eta}_e\}$ represent the second derivative of the elastic modal displacements which will multiply by the $[\Phi_e]$ matrix columns. If Equation (4) is substituted in Equation (7), and taking into account once again that the weighting matrix must isolate the rigid body modes, it follows that:

$$\begin{gathered}[W]^T[[\Phi_{RB}] \quad [\phi_e]] = [[I]_{N_{RB}\times N_{RB}} \quad [0]_{N_e\times N_{RB}}]\\ [W] = ([[\Phi_{RB}] \quad [\phi_e]]^T)^+\begin{bmatrix}[I]_{N_{RB}\times N_{RB}}\\ [0]_{N_e\times N_{RB}}\end{bmatrix}\end{gathered} \tag{8}$$

It should be noted, that as long as the condition $N_0 \geq N_e + N_{RB}$ is verified, the N_{RB} columns of the weighting matrix are able to extract the rigid body accelerations from the modal data.
After $[W]$ is determined Equations (4-5) are ready to be computed, giving the load time-history of the resultant of the loads in the center of gravity of the structure, corresponding to the rigid body motions introduced in the $[\Phi_{RB}]$ matrix in Equation (8).

3.2 ISF – inverse structural filter

The aim of ISF is to invert the discrete time equations of motion of a given structural system, in order to estimate the forces using the dynamic response as input.

The first thing to do is to represent the dynamic system in its state-space continuous time representation in the form:

$$\begin{aligned}\{\dot{x}(t)\} &= [A_c]\{x(t)\} + [B_c]\{u(t)\} \\ \{y(t)\} &= [C_c]\{x(t)\} + [D_c]\{u(t)\}\end{aligned} \tag{9}$$

This was performed considering the following state vector:

$$\{x\}_{2N_{DOF}\times 1} = \begin{Bmatrix}\{\delta\} \\ \{\dot{\delta}\}\end{Bmatrix} \tag{10}$$

where N_{DOF} represents the number of dofs of the structural system. Substituting the state vector of Equation (10) into (1), and considering the responses of the structure as outputs, the following expression for the continuous state-space system is obtained:

$$\begin{aligned}\{\dot{x}\} &= \underbrace{\begin{bmatrix}[0]_{N_{DOF}\times N_{DOF}} & [I]_{N_{DOF}\times N_{DOF}} \\ -[K]/[M] & -[C]/[M]\end{bmatrix}}_{[A_c]}\{x\} + \underbrace{\begin{bmatrix}[0]_{N_{DOF}\times N_{DOF}} \\ [I]/[M]\end{bmatrix}[F_{in}]}_{[B_c]}\{u\} \\ \{y\}_{N_{DOF}\times 1} &= \underbrace{\left[[I]_{N_{DOF}\times N_{DOF}}[0]_{N_{DOF}\times N_{DOF}}\right]}_{[C_c]}\{x\}\end{aligned} \tag{11}$$

where $\{x\}$ is the state vector containing the N states of the system, the input vector $\{u\}$ contains the N_i load inputs of the system, and the N_o outputs of the state-space system are condensed in vector $\{y\}$. The matrix $[F_{in}]$ of ones and zeros establishes the relationship between the N_i applied loads and the N_{DOF} dofs of the structure. Keep in mind that matrix $[C]$ depends on the rotating speed of the rotor. $[D_c]$ is not definable for this system.

The continuous state-space system involves integration of the state vector. A much simpler representation can be achieved if the discrete state-space system is applied. This means that the time will be sampled in k time-steps at a given T_s sampling rate.

Before this, an adjustment in the continuous state-space system should be performed. In experimental applications accelerations are more easily measured than displacements. Therefore the second derivative of the state and output equations will be computed in order to obtain a state space representation of the accelerations. These steps are detailed in chapter 2.2.4 of [13], leading to the following representation of the discrete state-space system:

$$\{x_{k+1}\} = [A]\{\dot{x}_k\} + [B]\{\dot{u}_k\}$$
$$\{\ddot{y}_k\} = [C]\{\dot{x}_k\} + [D]\{\dot{u}_k\} \tag{12}$$

The changes in the matrices involved are detailed in the appendix of (Allen & Carne, 2008).The ZOH (Zero Order Hold) assumption is used to ensure that the output of the discrete-time state-space system reproduces the one of the continuous time state-space system.

Steltzner and Kammer have shown (Kammer & Steltzner, 1999) that the discrete time state-space system in Equation (12) can be inverted as follows:

$$\{x_{k+1}\} = [\hat{A}]\{\dot{x}_k\} + [\hat{B}]\{\ddot{y}_k\},$$
$$\{\dot{u}_k\} = [\hat{C}]\{\dot{x}_k\} + [\hat{D}]\{\ddot{y}_k\} \tag{13}$$

where

$$\hat{A} = A - BD^{+}C,$$
$$\hat{B} = BD^{+},$$
$$\hat{C} = -D^{+}C,$$
$$\hat{D} = D^{+}$$

are the inverse state space system matrices.

The inverse system uses as input the structural responses and gives an estimation of the loads acting in the structure as output. Steltzner and Kammer did not use the derived ISF in equation (10) but have opted instead for an algorithm that computes the load vector directly from response data using the Markov parameters of the inverse system (Kammer & Steltzner, 1999).

An important setback related with the ISF method, is that it leads to diverging force estimations when the eigenvalues of the ISF system are unstable.

Also in (Kammer & Steltzner, 1999), Steltzner and Kammer pointed out that if a non-causal ISF was used then more stable and/or accurate results could be obtained. When future values of the response are used to estimate the forces at the present time instant the ISF is named non-causal ISF. In this spirit they have enhanced their Markov parameter based algorithm, with the introduction of a time offset or delay term l, to introduce this non-causal effect,yielding:

$$\dot{u}_k = \sum_{i=0}^{\infty} \hat{h}_i \ddot{y}_{k+l-i}$$
$$\hat{h}_0 = \hat{D}, \hat{h}_i = \hat{C}\hat{A}^{i-1}\hat{B}, i = 1,2,\dots,\infty \tag{14}$$

where $\hat{h}_i$ are the Markov parameters of the inverse system. In the Matlab® code where ISF was implemented, the user is able to choose the number of Markov parameters to use as well as the value of the l parameter.

3.3 DMISF – delayed multi-step inverse structural filter

Allen and Carne (Allen & Carne, 2008) found additionally that when the output of the discrete-time state-space system in Equation (12) was stepped forward one sample and the resulting direct transmission matrix neglected (since it tends to be small in lightly damped structures), the ISF of the resulting discrete time state-space system (see Equation (15)) led to an improvement on the force estimation performance.

$$
\begin{gathered}
\{x_{k+1}\} = [A]\{\dot{x}_k\} + [B]\{\dot{u}_k\}, \\
\{\ddot{y}_{k+1}\} = [C]([A]\{\dot{x}_k\} + [B]\{\dot{u}_k\}) + \underbrace{[D]\{\dot{u}_{k+1}\}}_{=0\ (neglected)}, \\
\{\ddot{y}_{k+1}\} = \underbrace{[C][A]}_{[C_d]}\{\dot{x}_k\} + \underbrace{[C][B]}_{[D_d]}\{\dot{u}_k\}
\end{gathered}
\tag{15}
$$

In their work Allen and Carne have brought together the non-causal ISF concept with the previous modification and named the new method *Delayed Multistep ISF*. They suggested the stacking of p time instants in the load input and response output vectors of the discrete time state-space system, obtaining the following modified discrete-time state-space system:

$$
\begin{gathered}
\underbrace{\begin{Bmatrix} \{\ddot{y}_{k+1}\} \\ \{\ddot{y}_{k+2}\} \\ \vdots \\ \{\ddot{y}_{k+p}\} \end{Bmatrix}}_{\{\ddot{y}^S_{k+1}\}} = \\
\underbrace{\begin{bmatrix} [C_d] \\ [C_d][A] \\ \vdots \\ [C_d][A]^{p-1} \end{bmatrix}}_{[C_{dm}]}\{\dot{x}_k\} + \underbrace{\begin{bmatrix} [D_d] & [0] & \cdots & [0] \\ [C_d][B] & [D_d] & \ddots & [0] \\ \vdots & \ddots & \ddots & [0] \\ [C_d][A]^{p-2}[B] & [C_d][A]^{p-3}[B] & \cdots & [D_d] \end{bmatrix}}_{[D_{dm}]} \underbrace{\begin{Bmatrix} \{\dot{u}_k\} \\ \{\dot{u}_{k+1}\} \\ \vdots \\ \{\dot{u}_{k+p-1}\} \end{Bmatrix}}_{\{\dot{u}^S_k\}}
\end{gathered}
\tag{16}
$$

$$
\begin{gathered}
\{x_{k+1}\} = [A]\{\dot{x}_k\} + [B][P]\{\dot{u}^S_k\} \\
[P] = \left[[I]^{N_i \times N_i} \quad [0] \quad \dots \quad [0] \right]
\end{gathered}
\tag{17}
$$

which after inversion will generate the estimation of the forces between t_k and t_{k+p-1} from the responses of the structure between t_{k+1} and t_{k+p}. This method is applied to the response from t_k to t_{end-p} resulting in p force estimations per time step. Since this method allows the reposition of the poles of the ISF system it can enhance the stability of the force estimations, all the user of the method needs to do is to modify p until stable results are found.

4 Results and discussion

4.1 Source data

Before the time domain force identification methods are applied, it is necessary to extract the response data. The response data were extracted from a full transient analysis to each of the studied finite element models.
The full transient analysis uses the Newmark method. Bearing in mind that a constant time step should be chosen to drive the data acquisition of the variables from the analysis. This time discretization is especially important in ISF and DMISF methods where discrete state-space systems are employed. After the full transient analysis is performed, the accelerations at the desired dofs are extracted from the time history.
These response data are then loaded in Matlab® where the response vector is built. The way the measurements are organized inside the structure response vector will depend on the sequence of the dofs of the numerical method used to build the inverse system of the structure.
After this is accomplished the Matlab® code proceeds with the specific force identification algorithms for each of the implemented time-domain force identification methods. In the first application example, the system matrices required by each method were obtained using a programmed finite element model assembler. Another procedure was followed in the second application example.

4.2 Application to a monorotor

The considered rotor is displayed in Figure 1 and it will be subjected to a harmonic force excitation caused by a mass unbalance in the disk. The rotor is considered to be rotating at constant speed and it is simple supported on two rigid bearings. All relevant data are shown in Table 1.

Material Constants	***Dimensions***	***Mass Unbalance***
$\rho = 7800\ {}^{Kg}/_{m^3}$ *(Density)*	$R_1 = 0.01\ m$	$m_u = 10^{-4} Kg$
$E = 210\ GPa$ *(Young Modulus)*	$R_2 = 0.15\ m$	$d = R_2$ *(mass unbalance position)*
$\nu = 0.3$ *(Poisson Ratio)*	$L = 0.4\ m$	
	$h = 0.03\ m$	

Table 1 – Numeric data of the Monorotor

The system matrices were obtained using the finite element method. The shaft was divided in 9 equal size beam elements (10 nodes) and the disk was modeled as a punctual mass element (where the ANSYS® elements BEAM188 and MASS21 [15] are the correspondent elements to this model). The employed element matrices in the Matlab® codes were the ones presented in chapter 3.1.2 of [13]. The translational displacements at the extremity of the shaft were constrained. In this example one uses the calculated response at every dof of the Monorotor in the force identification methods.

The data of transient analysis performed to the rotor are displayed in table 2.

Recorded Time	$0.1\ s$
Rotating speed	$5000\ rpm$
Sampling Time Interval	$0.0001\ s$

Table 2 – Transient analysis data

SWAT (Figure 2), ISF (Figure 3) and DMISF (Figure 4) were successfully implemented to the rotor model in the described conditions. Notice that since the rotating speed is constant, matrix $[C]$ remains constant between time samples. All the relevant parameters associated with each force identification method are indicated above the graphs.

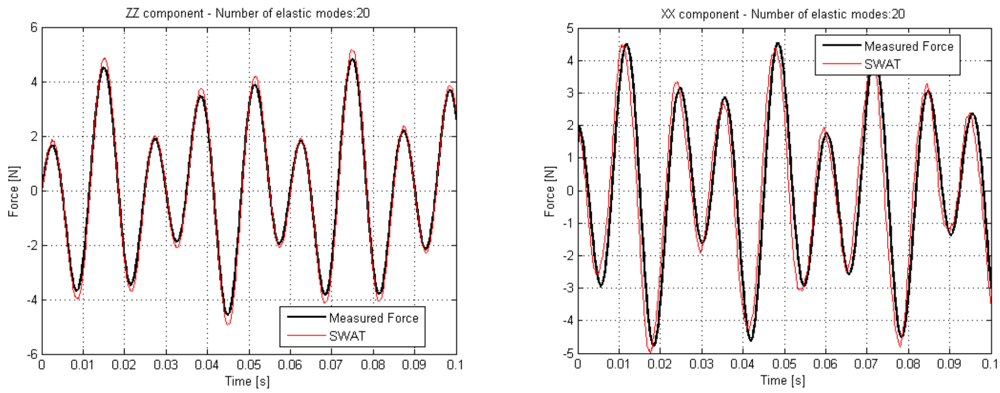

Figure 2: SWAT applied to the Monorotor.

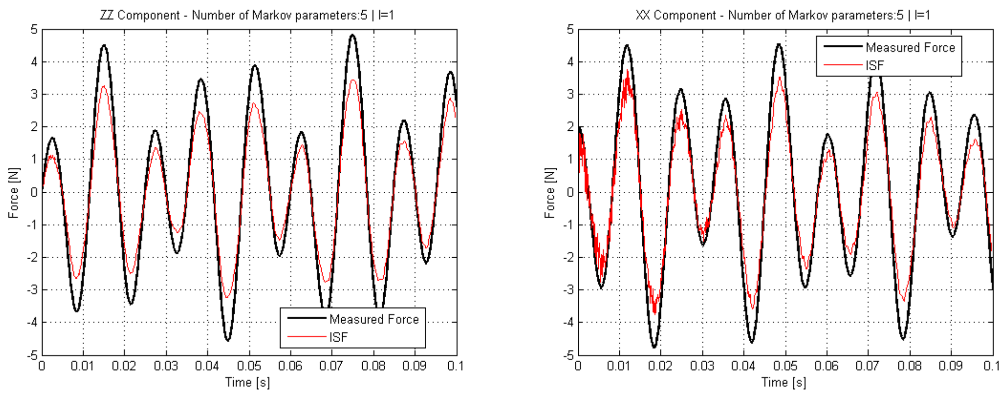

Figure 3: ISF applied to the Monorotor.

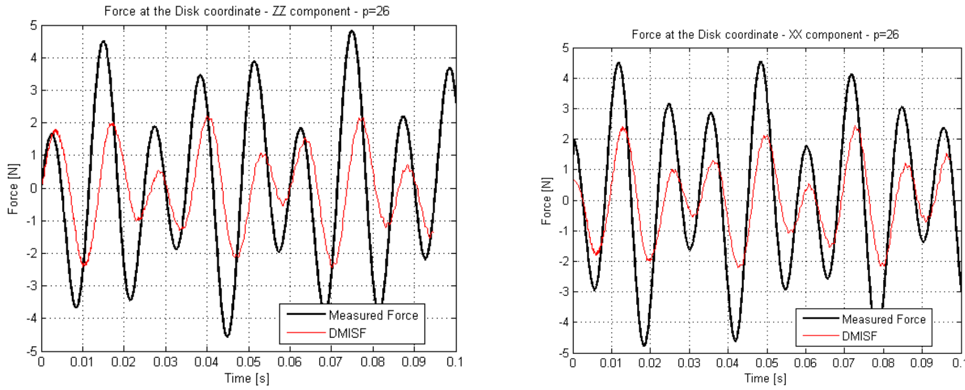

Figure 4: DMISF applied to the Monorotor.

Globally it can be concluded that these time-domain force identification methods present good results when applied to simple examples of rotordynamics. SWAT presents good results qualitatively speaking. The ISF method, despite showing some ringing has proven to be highly reliable in force amplitude prediction, while the DMISF method has shown difficulties in determining the force amplitude at the higher force amplitude peaks, but diminishes the ringing of the force curve. Further studies are now being done to understand how to improve it.

5 Conclusions

The application to rotordynamics of the three time-domain force identification methods, known by the acronyms SWAT, ISF and DMISF, was developed and tested. These methods were adapted to be numerically applied to a simple rotor configuration rotating at constant speed with a given mass unbalance.

A comparison of the results from the three methods was presented. In the rotor application example the SWAT method presented some values of the force estimation curve higher than the measured ones ones while the opposite was verified in the ISF and DMISF results.

The authors found difficulties in the application of the DMISF method in this specific example in comparison with the ISF example. Stable results were easily obtained in every method applied to the rotor, making time-domain force reconstruction methods a valid option for constant spin speed rotors.

Despite all the reconstructed forces in this paper originate in a mass unbalance, the authors do not see any limitation on the application of the studied time-domain force reconstruction methods to other load configurations such as punctual synchronous and asynchronous forces.

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