Paper 276

©Civil-Comp Press, 2012
Proceedings of the Eleventh International Conference
on Computational Structures Technology,
B.H.V. Topping, (Editor),
Civil-Comp Press, Stirlingshire, Scotland

The Influence of Various Fractional Models of Viscoelastic Dampers on the Dynamic Behaviour of Structures

Z. Pawlak and R. Lewandowski
Institute of Structural Engineering
Poznan University of Technology, Poland

Abstract

The focus of this paper is the determination of dynamic parameters for structural systems with fractional viscoelastic (VE) dampers installed on them. The structures are treated as linearly elastic systems, subjected to earthquake-induced ground motion. Fractional Kelvin and Maxwell models with three or four parameters are used. These models, apart from stiffness and damping coefficients, are defined by the number representing the order of fractional derivatives. It is the aim of the study presented in this paper to establish a criterion, enabling the comparison of various damper models so as to obtain same or similar dynamic parameters of systems with different damper models.

Keywords: viscoelastic dampers, rheological models, fractional derivative.

1 Introduction

Passive damping systems are mounted on structures in order to reduce excessive vibrations caused by winds and earthquakes. Various kinds of mechanical devices, including viscoelastic (VE) dampers, are used in the passive systems. A number of applications of VE dampers in civil engineering are listed in [1, 2, and 3]. In the past, several rheological models were proposed for describing the dynamic behaviour of VE dampers [4÷10]. Both classic and so-called fractional-derivative models of dampers are available. In the classic approach, mechanical models consisting of springs and dashpots are used to describe the rheological properties of VE dampers (see [5, 6, and 7]). In this approach, the rheological properties of VE dampers are described using the fractional calculus [8, 9, and 10].

The fractional models have an ability to correctly describe the behaviour of VE materials and dampers using a small number of model parameters. Three parameters sufficiently describe the VE damper dynamics, which is an important advantage of the model discussed. However, in this case, the VE damper equation of motion is the fractional differential equation.

In this paper, planar frame structures with VE dampers mounted on them are considered. The VE dampers are modelled using the fractional Kelvin and Maxwell models. The nonlinear eigenvalue problem is formulated from which the dynamic parameters of the system can be determined. The continuation method is used to solve the above mentioned nonlinear eigenvalue problem. Similarly to the method described in [10], the dynamic behaviour of a frame with viscoelastic dampers is characterized by the natural frequencies and the non-dimensional damping factors. The above-mentioned properties are defined on the basis of eigenvalues, obtained from the nonlinear eigenproblem.

It is the aim of the study to establish a criterion, enabling the comparison of various damper models so as to obtain same or similar dynamic responses of systems with different damper models. The natural frequencies and the non-dimensional damping factors were evaluated for the whole system, i.e., the frame with VE dampers. In addition, the frequency response function of displacements and interstorey drifts were calculated.

The studies were carried out by adopting four different rheological models with fractional derivatives to describe VE dampers. In this work, the authors proposed a criterion which enables the comparison of the considered models. The equivalent models are those for which the dynamic parameters of structure are similar or identical.

2 Rheological model of damper

The rheological properties of VE dampers were described using four different fractional models, i.e., three- and four-parameter Kelvin and Maxwell models. The three-parameter Kelvin and Maxwell models consist of a fractional dashpot with the constants: c_d, α ($0 < \alpha \leq 1$) connected in parallel or in series with a spring of the stiffness k_i (see Figure 1). In the four-parameter models, there is an additional element of the stiffness k_0 (see Figure 2).

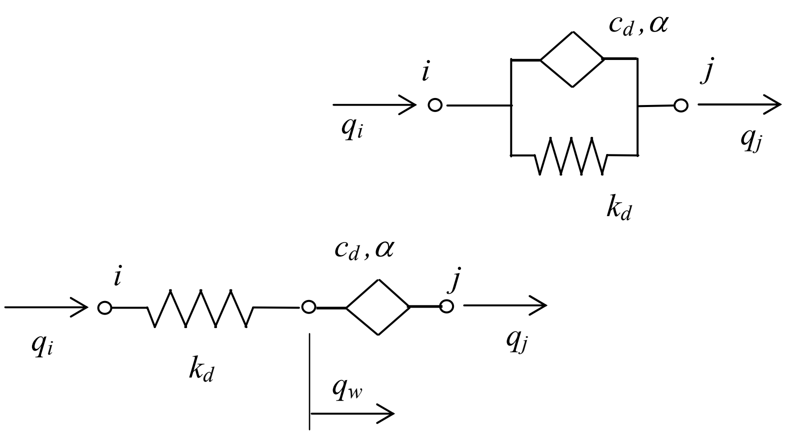

Figure 1: A three-parameter Kelvin (k3) and Maxwell (m3) models of damper.

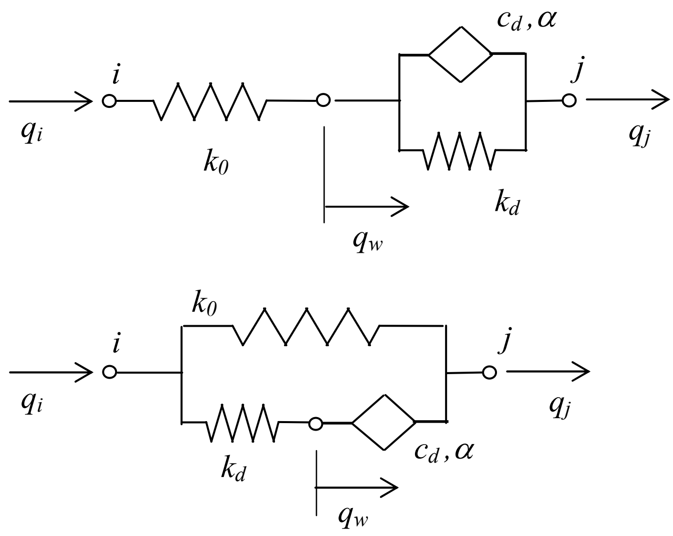

Figure 2: A four-parameter Kelvin (k4) and Maxwell (m4) models of damper.

The equations of motion describing the force in damper versus displacements could be written as follows:

$$u_i = k_d (q_j - q_i) + c_d D_t^{\alpha} (q_j - q_i) , \tag{1}$$

for a three-parameter Kelvin model,

$$\begin{aligned} u_i &= k_d (q_w - q_i) , \\ u_j &= c_d D_t^{\alpha} (q_j - q_w) , \end{aligned} \tag{2}$$

for a three-parameter Maxwell model,

$$\begin{aligned} u_i &= k_0 (q_w - q_i) , \\ u_j &= k_d (q_j - q_w) + c_d D_t^{\alpha} (q_j - q_w) , \end{aligned} \tag{3}$$

for a four-parameter Kelvin model,

$$\begin{aligned} u_j &= k_0 (q_j - q_i) + c_d D_t^{\alpha} (q_j - q_w) , \\ u_i &= k_0 (q_j - q_i) + k_d (q_w - q_i) , \end{aligned} \tag{4}$$

for a four-parameter Maxwell model, where u_i, u_j are the external forces at node *i* and *j*, respectively, q_i, q_j and q_w denote appropriate nodal displacements of the considered model. Moreover, $D_t^{\alpha}(\bullet)$ denotes the Riemann-Liouville fractional derivative of the order α with respect to time, *t*. More information concerning the fractional rheological models can be found in [11]. The equation of motion of the classic Kelvin or Maxwell model could be obtained after introducing into Equations (1) ÷ (4) $\alpha = 1$.

3 The damper parameters

The influence of fractional VE dampers on the dynamic behaviour of a structure could be analyzed on the basis of the dynamic parameters of the structure with and without dampers. The response of the structure depends on the type of the damper model used and on the value of coefficients c_d, α, k_d and k_0. When the stiffness and damping coefficients are identical for each model, then the value of forces generated by the dampers and the dynamic behaviour of the structure will be different for each case of the model used [12,13]. In order to obtain comparable values of force we must choose the stiffness and damping coefficients is such a way that the energy dissipated by damper is similar for each model. For dampers described by fractional rheological models vibrating harmonically in time $x(t) = q_j(t) - q_i(t) = x_0 \sin(\lambda t)$ the amount of dissipated energy can be expressed by the formulas:

- for the three-parameter Kelvin model (k3):

$$E_d = \pi \cdot c_d \cdot \lambda^\alpha x_0^2 \sin(\alpha \pi/2), \tag{5}$$

- for the three-parameter Maxwell model (m3):

$$E_d = \pi \cdot c_d \cdot \lambda^\alpha x_0^2 \frac{\sin(\alpha \pi/2)}{1 + \tau^\alpha \lambda^\alpha \cos(\alpha \pi/2) + (\tau\lambda)^{2\alpha}}, \tag{6}$$

- for the four-parameter Kelvin model (k4):

$$E_d = \pi \cdot x_0^2 \frac{k_0 k_e (k_0 - k_e)(\tau\lambda)^\alpha \sin(\alpha \pi/2)}{k_0^2 + 2k_0 k_e \tau^\alpha \lambda^\alpha \cos(\alpha \pi/2) + k_e^2 (\tau\lambda)^{2\alpha}}, \tag{7}$$

- for the four-parameter Maxwell model (m4):

$$E_d = \pi \cdot c_d \cdot \lambda^\alpha x_0^2 \frac{\sin(\alpha \pi/2)}{1 + 2\tau^\alpha \lambda^\alpha \cos(\alpha \pi/2) + (\tau\lambda)^{2\alpha}}, \tag{8}$$

where λ is the frequency of vibrations, x_0 is the amplitude of displacements,

$$\tau^\alpha = \frac{c_d}{k_d}, \quad k_e = \frac{k_0 k_d}{k_0 + k_d}.$$

When the value of parameters are appropriately selected $c_d = c_{dm}$ and $k_d = k_{dm}$ in Formula (6) then the functions of energy $E_d(\lambda)$ dissipated by models (m3) and (k3) have similar values in a limited range of frequency λ. For the four-parameter Kelvin model (k4) it is possible to choose parameters $c_d = c_4$, $k_d = k_4$ and then calculate the coefficients of model (m4):

$$c_d = c_{4m} = \frac{c_4 \cdot k_0^2}{(k_0 + k_4)^2}, \qquad k_d = k_{4m} = \frac{k_0^2}{(k_0 + k_4)}, \tag{9}$$

which was derived from the condition that the energy dissipated by both models of damper executing harmonically varying vibrations is the same.

Another way to compare the rheological models mentioned above is to adjust the stiffness and damping coefficients in such a way that the loss factor for all of the considered damper models is the same or similar. The loss factor for the fractional models of damper is defined as follows:

- for the three-parameter Kelvin model (k3):

$$\eta(\lambda) = \frac{(\tau\lambda)^\alpha \sin(\alpha\pi/2)}{1 + (\tau\lambda)^\alpha \cos(\alpha\pi/2)}, \tag{10}$$

- for the three-parameter Maxwell model (m3):

$$\eta(\lambda) = \frac{\sin(\alpha\pi/2)}{(\tau\lambda)^\alpha + \cos(\alpha\pi/2)}, \tag{11}$$

- for the four-parameter Kelvin model (k4):

$$\eta(\lambda) = \frac{(k_0 - k_e)(\tau\lambda)^\alpha \sin(\alpha\pi/2)}{k_0 + 2k_e(\tau\lambda)^\alpha \cos(\alpha\pi/2) + k_e(\tau\lambda)^{2\alpha}}, \tag{12}$$

- for the four-parameter Maxwell model (m4):

$$\eta(\lambda) = \frac{k_d(\tau\lambda)^\alpha \sin(\alpha\pi/2)}{k_0\left[1 + (\tau\lambda)^\alpha \cos(\alpha\pi/2)\right] + (k_0 + k_d)(\tau\lambda)^\alpha\left[(\tau\lambda)^\alpha + \cos(\alpha\pi/2)\right]}. \tag{13}$$

For appropriately selected coefficients c_d and k_d in the models (k3), (k4) and (m4) one can obtain similar values of the loss factor for all values of λ. In this case, the function of loss factor increases for higher values of frequency λ. However, for the model (m3), function $\eta(\lambda)$ derived according to Formula (11) declines with frequency of vibration. Thus, the same value of loss factor for all models can be obtained only for one selected value of frequency.

4 The equations of motion

A structure with VE dampers is treated as an elastic linear system which could be modelled as the shear frame. The mass of the system is lumped at the level of storeys. The viscoelastic dampers are installed between two successive storeys. The equation of motion of the structure with dampers can be written as follows:

$$\mathbf{M}_s \ddot{\mathbf{q}}_s(t) + \mathbf{C}_s \dot{\mathbf{q}}_s(t) + \mathbf{K}_s \mathbf{q}_s(t) = \mathbf{f}(t) + \mathbf{p}(t), \tag{14}$$

where the symbols $\mathbf{M}_s$, $\mathbf{C}_s$ and $\mathbf{K}_s$ denote the mass, the damping, and the stiffness matrices, respectively. Moreover, $\mathbf{q}_s(t) = [q_{s,1},..., q_{s,k},..., q_{s,n}]^T$ denotes the vector of displacements of the structure and $\mathbf{p}(t) = [p_1,..., p_k,..., p_n]^T$ the vector of excitation forces. The components of vector $\mathbf{f}(t) = [f_1, f_2,..., f_n]^T$ are the interaction forces between the frame and the dampers (see Figure 3).

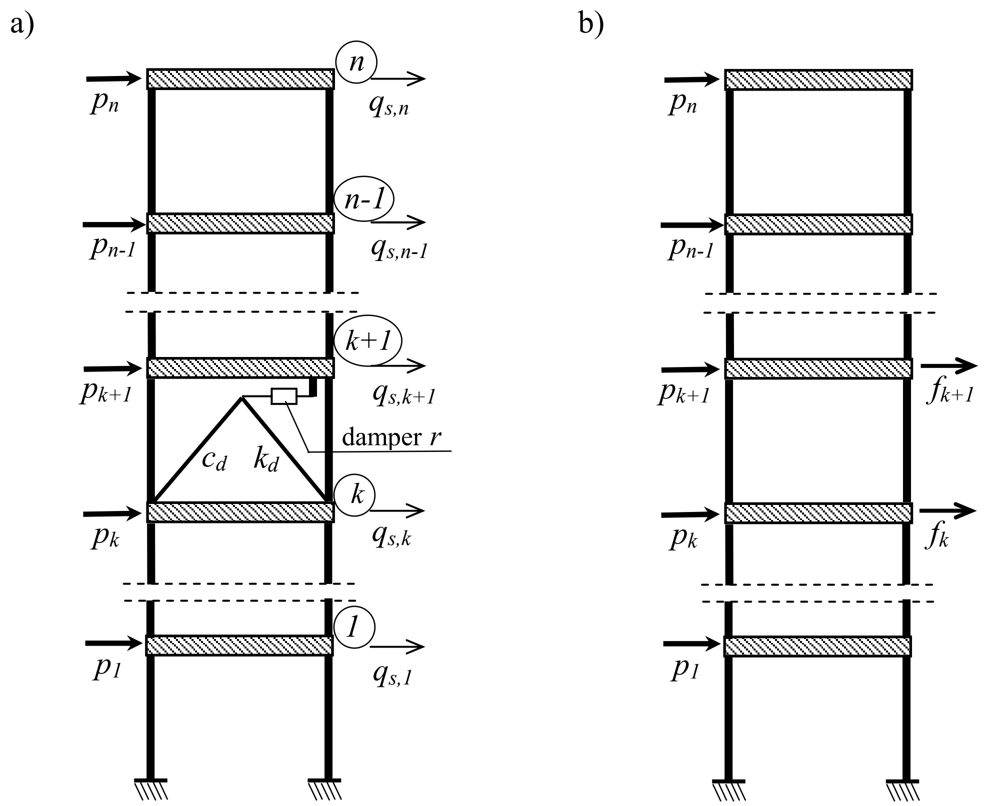

Figure 3: Diagram of frame with VE dampers.

If a structure with only one damper, denoted as the damper number *r*, mounted between two successive storeys, *k* and *k+1*, is considered then the vector of damper forces could be written in the following form:

$$\mathbf{f}_r(t) = [0,...,f_k = u_r, f_{k+1} = -u_r,.....,0]^T = \mathbf{e}_r u_r(t)\,, \tag{15}$$

where $\mathbf{e}_r = [0,...,e_k = 1, e_{k+1} = -1,...,0]^T$. For a structure with *m* dampers the vector of interaction forces is:

$$\mathbf{f}(t) = \sum_{r=1}^{m} \mathbf{f}_r(t)\ . \tag{16}$$

In this paper we investigate the dynamic parameters of a system with many viscoelastic dampers, but their rheological properties are represented by the same type of fractional model.

After some transformations, it is possible to eliminate the variable q_w from relationships (2), (3) and (4). Then the forces of interaction between the frame and the dampers can be written in the following form:

$$\begin{aligned} u(t) &= k_d \Delta q(t) + c_d D_t^{\alpha} \Delta q(t)\,, \\ D_t^{\alpha} u(t) + \nu\, u(t) &= k_d D_t^{\alpha} \Delta q(t)\,, \\ u(t) + \tau^{\alpha} D_t^{\alpha} u(t) &= \widetilde{k}_0 \Delta q(t) + k_0 \tau^{\alpha} D_t^{\alpha} \Delta q(t)\,, \\ u(t) + \tau_1^{\alpha} D_t^{\alpha} u(t) &= k_0 \Delta q(t) + k_{\infty} \tau_1^{\alpha} D_t^{\alpha} \Delta q(t)\,, \end{aligned} \tag{17}$$

where $\Delta q(t) = q_i - q_j$, $\nu = \dfrac{k_d}{c_d}$, $\tau^{\alpha} = \dfrac{c_d}{k_0 + k_d}$, $\tau_1^{\alpha} = \dfrac{c_d}{k_d}$, $\widetilde{k}_0 = \dfrac{k_0 k_d}{k_0 + k_d}$, $k_{\infty} = k_0 + k_d$.

After applying the Laplace transform, the equation of motion (14) can be written as:

$$\left(s^2 \mathbf{M}_s + s\ \mathbf{C}_s + \mathbf{K}_s\right) \overline{\mathbf{q}}_s(s) = \overline{\mathbf{f}}(s) + \overline{\mathbf{p}}(s) \tag{18}$$

The vectors $\overline{\mathbf{q}}_s(s)$, $\overline{\mathbf{f}}(t)$ and $\overline{\mathbf{p}}(s)$ denote the Laplace displacement and force transforms, respectively. For damper *r* the force transform is $\overline{\mathbf{f}}_r(s) = \mathbf{e}_r \overline{u}_r(s)$. The Laplace transform converts Equations (1) ÷ (4) into one relationship which is valid for each considered model of damper:

$$\overline{u}_i(s) = \left[k_v + s^{\alpha} G(s)\right] \Delta q(s)\,, \tag{19}$$

where k_v and $G(s)$ stands for, respectively:

- for the three-parameter Kelvin model (k3):

$$k_v = k_d\,, \quad G(s) = c_d\ , \tag{20}$$

- for the three-parameter Maxwell model (m3):

$$k_v = 0\ , \quad G(s) = \frac{k_d}{\nu + s^{\alpha}}\ , \tag{21}$$

- for the four-parameter Kelvin model (k4):

$$k_v = \frac{k_0 k_d}{k_0 + k_d}\ , \quad G(s) = \frac{b}{\nu + s^{\alpha}}\ , \quad b = \frac{k_0^2}{k_d + k_0}\ , \tag{22}$$

- for the four-parameter Maxwell model (m4):

$$k_v = k_0\,, \quad G(s) = \frac{k_d}{\nu + s^{\alpha}}\ . \tag{23}$$

Finally, for the structure with viscoelastic dampers we can write:

$$\left[s^2\mathbf{M}_s + s\ \mathbf{C}_s + \mathbf{K} + \sum_{i=1}^{m} s^{\alpha}\mathbf{G}(s) \right] \overline{\mathbf{q}}_s(s) = \overline{\mathbf{p}}(s)\,, \tag{24}$$

where $\mathbf{K} = \mathbf{K}_s + \mathbf{K}_v$.

5 The dynamic characteristics of structures

For $\overline{\mathbf{p}}(s) = \mathbf{0}$ Equation (24) constitutes a nonlinear eigenproblem from which one can obtain eigenvalus and eigenvectors. A nonlinear eigenproblem can be solved using the continuation method which is similar to the one described in the paper [10]. In this way the complex and conjugate eigenvalues s_i and the corresponding vector $\overline{\mathbf{q}}_i$ are determined.

The dynamic behaviour of a frame with viscoelastic dampers is characterized by the natural frequency ω_i and the non-dimensional damping parameter γ_i. Similarly to viscous damping, the above-mentioned properties are defined as follows:

$$\omega_i^2 = \mu_i^2 + \eta_i^2\,, \quad \gamma_i = -\mu_i / \omega_i\,, \tag{25}$$

where $\mu_i = \mathrm{Re}(s_i)$, $\eta_i = \mathrm{Im}(s_i)$.

When the structure is subjected to base acceleration $\ddot{u}_g(t)$, the excitation vector is written as:

$$\mathbf{p}(t) = -\mathbf{M}\ \mathbf{r}\ \ddot{u}_g(t), \tag{26}$$

where $\mathbf{r} = [1, 1, ...,1]^T$. For the harmonic external forces ($\ddot{u}_g(t) = \ddot{U}_g \exp(i\lambda t)$), the displacement response of the structure is given by relationship $\mathbf{q}_s(t) = \overline{\mathbf{q}}_s \exp(i\lambda t)$, where $\overline{\mathbf{q}}_s(\lambda)$ is determined from:

$$\overline{\mathbf{q}}_s(\lambda) = \mathbf{H}(\lambda)\ \ddot{U}_g. \tag{27}$$

The vector $\mathbf{H}(\lambda)$ derived from relationship (24):

$$\mathbf{H}(\lambda) = -\left[s^2 \mathbf{M}_s + s\ \mathbf{C}_s + \mathbf{K} + \sum_{k=1}^{m} s^{\alpha} \mathbf{G}_k(s) \right]^{-1} \mathbf{M}\ \mathbf{r}, \tag{28}$$

will be called the vector of response transfer functions of displacements. The elements of this vector are functions which are the Laplace transforms of displacements for $\ddot{U}_g = 1$.

Other quantities, which characterise the dynamic behaviour of frame with viscoelastic dampers, are the Laplace transforms of interstorey drifts. The response transfer function of drift is the difference between the response transfer functions of displacements of two successive storeys ($H_k(\lambda)$ and $H_{k-1}(\lambda)$), i.e.:

$$\Delta H_k = H_k - H_{k-1}. \tag{29}$$

The Laplace transforms of bending moments M_r in columns could be written in the following form:

$$\overline{M}_r(\lambda) = \frac{6EI_r}{h_r^2} \Delta H_r(\lambda), \tag{30}$$

where h_r is the height of the storey, EI_r is the rigidity of the column *r*.

6 The results of calculation

In the numerical example, a ten-storey building structure modelled as a shear plane frame with VE dampers mounted on it is considered. Bending rigidity varies in sequence for every two storeys: $k_1 = k_2 = 8710 kN/m$, $k_3 = k_4 = 54010 kN/m$, $k_5 = k_6 = 42170 kN/m$, $k_7 = k_8 = 28660 kN/m$, $k_9 = k_{10} = 16450 kN/m$. The mass of every floor is the same: $m_s = 2.07Mg$. The structure's damping ratio corresponding to its stiffness is $\mathbf{C}_s = 0.0000693\ \mathbf{K}_s$ (data taken from [3]).

Viscoelastic dampers of which the rheological properties are represented by the same type of fractional model are installed on each storey. The calculations were carried out for several sets of data, i.e., the values of parameters chosen for the considered rheological models of dampers. The dynamic behaviour of structure with dampers was compared on the basis of dynamic characteristics derived for the considered system.

In the first case, we assume the same value of damping coefficient $c_d = 500kNs^{\alpha} / m$, the same value of a number which expresses the order of fractional derivative $\alpha = 0.6$ and the same value of stiffness coefficient $k_d = 25000kN / m$ for each model. Moreover, for each four-parameter model the stiffness of the additional element is $k_0 = 25000kN/m$.

The natural frequencies ω_i and the values of non-dimensional damping factors γ_i obtained for a frame with various kinds of damper models are presented in Table 1 and 2.

Modal number	Rheological model with			
	3 parameters		4 parameters	
	Kelvin (k3)	Maxwell (m3)	Kelvin (k4)	Maxwell (m4)
1	28.964	23.275	26.105	28.983
2	79.852	60.462	69.585	79.955
3	131.719	100.211	113.974	131.823
4	181.544	140.265	156.953	181.515
5	230.526	175.287	197.554	230.008
6	265.033	201.790	224.577	264.159
7	303.686	233.849	258.368	302.245
8	335.232	269.430	291.229	333.842
9	370.238	306.240	326.586	368.456
10	410.647	348.948	367.669	408.437

Table 1: Natural frequencies ω_i [rad/sec]

The results of calculations are different for every case. The most notable differences are seen between the three-parameter Kelvin and Maxwell models.

Modal number	Rheological model with			
	3 parameters		4 parameters	
	Kelvin (k3)	Maxwell (m3)	Kelvin (k4)	Maxwell (m4)
1	0.0215	0.0279	0.0112	0.0180
2	0.0473	0.0609	0.0229	0.0341
3	0.0639	0.0720	0.0283	0.0412
4	0.0760	0.0737	0.0308	0.0450
5	0.0858	0.0969	0.0356	0.0474
6	0.1015	0.0791	0.0364	0.0528
7	0.1048	0.0829	0.0370	0.0523
8	0.1011	0.0691	0.0336	0.0492
9	0.0961	0.0620	0.0317	0.0459
10	0.0908	0.0554	0.0301	0.0428

Table 2: Non-dimensional damping factors γ_i

Natural frequencies are higher whereas the values of non-dimensional damping factors are usually lower for a frame with dampers described by model (k3), in comparison with the frame with dampers (m3). When the four-parameter models are applied, both the natural frequencies ω_i and factors γ_i are higher for a frame with dampers (m4). The value of non-dimensional damping factors obtained for a frame with dampers (k4) are significantly lower, compared with frames with the dampers described using other models.

Next, we derived displacements of structure subjected to base acceleration. The maximal amplitude of interstorey drifts were determined for $s = \mathrm{i}\omega_j$ where $\mathrm{i} = \sqrt{-1}$, ω_j is the natural frequency of structure with dampers. On the basis of the results compiled in Table 3, it can be observed that the main influence on displacements is observed for just a few lower modes of vibration. Therefore, in our next analyses we will take into account the results obtained for the first three natural frequencies (resonance picks).

The displacements given in Table 3 correspond to the values of non-dimensional damping factors shown in Table 2. The smallest displacements occur in the frame with the highest values of factors γ_i. Moreover, the highest values of interstorey drifts were obtained for the frame with dampers described by model (k4), which generates the lowest values of non-dimensional damping factors.

Modal number	Rheological model with			
	3 parameters		4 parameters	
	Kelvin	Maxwell	Kelvin	Maxwell
	(k3)	(m3)	(k4)	(m4)
1	4.5750	5.9512	10.8149	5.4556
2	0.4187	0.8121	1.3335	0.5816
3	0.1173	0.2150	0.3888	0.1840
4	0.0443	0.0993	0.1784	0.0777
5	0.0237	0.0424	0.0987	0.0453
6	0.0171	0.0374	0.0619	0.0229
7	0.0126	0.0216	0.0357	0.0142
8	0.0114	0.0180	0.0281	0.0132
9	0.0104	0.0163	0.0253	0.0125
10	0.0094	0.0172	0.0242	0.0131

Table 3: Maximal interstorey drift $q_{\max}$ [mm]

Figure 4 shows the function of energy dissipated by the considered models of dampers derived from formula (5) ÷ (8): red colour for model (k3), green for (m3), blue for (k4) and yellow for (m4).

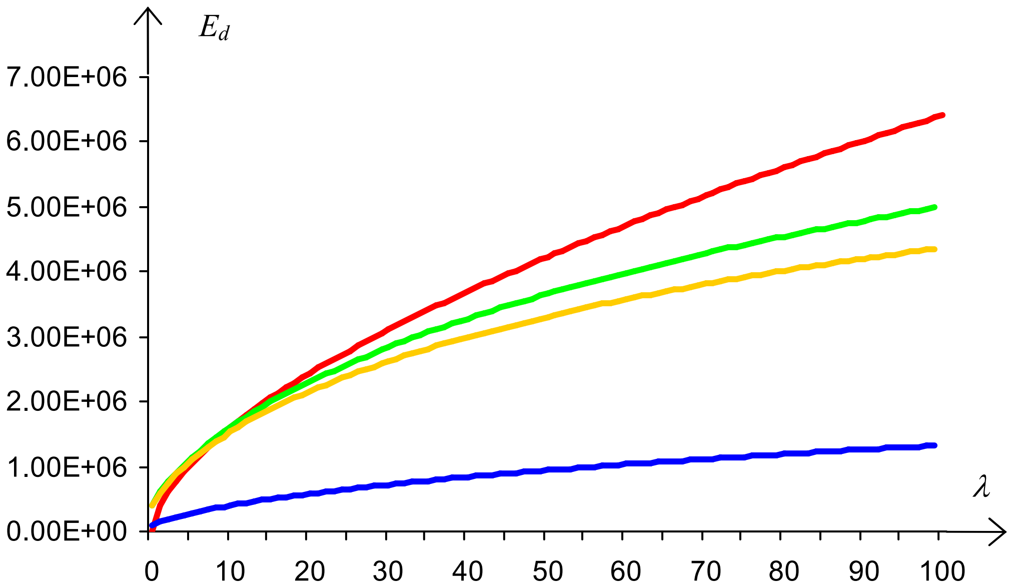

Figure 4: The function of energy dissipated by VE dampers – uniform data.

In each model we assumed the same value of damping and stiffness coefficients, so uniform data were taken as: $c_d = 500kNs^{\alpha}/m$, $\alpha = 0.6$, $k_d = 25000kN/m$, $k_0 = 25000\ kN/m$. The damper described by model (k4) has the smallest ability to dissipate energy, which corresponds with the results given in Table 3. For the structure with dampers modelled by model (k4) the highest values of displacements were obtained. The application of model (k3) yields the lowest displacements because the damper with this model has the highest ability to dissipate energy.

Because of the large differences in the functions shown in Figure 4 the values of stiffness and damping coefficients were modified in such a way that the amount of energy dissipated by each model of damper was similar. The coefficients for the three-parameter Kelvin model remain unchanged. After a few trials, new values of coefficients were established for the three-parameter Maxwell model (m3a): $c_{dm} = 600kNs^{\alpha}/m$, $k_{dm} = 15000kN/m$ and for the four-parameter Kelvin model (k4a): $c_4 = 700kNs^{\alpha}/m$, $k_4 = 1050kN/m$, so now they dissipate a similar amount of energy. Next, the parameters for model (m4a) were derived using Formula (9): $c_{4m} = 645\ kNs^{\alpha}/m$, $k_{4m} = 24000kN/m$. If the modified parameters are applied in the considered models, then the function of energy dissipation has similar values for excitation frequency in the range from 0 to $30.0\ \text{rad/sec}$ (see Figure 5). Moreover, for dampers modelled by model (k4a) and (m4a) functions of dissipated energy overlap (blue colour).

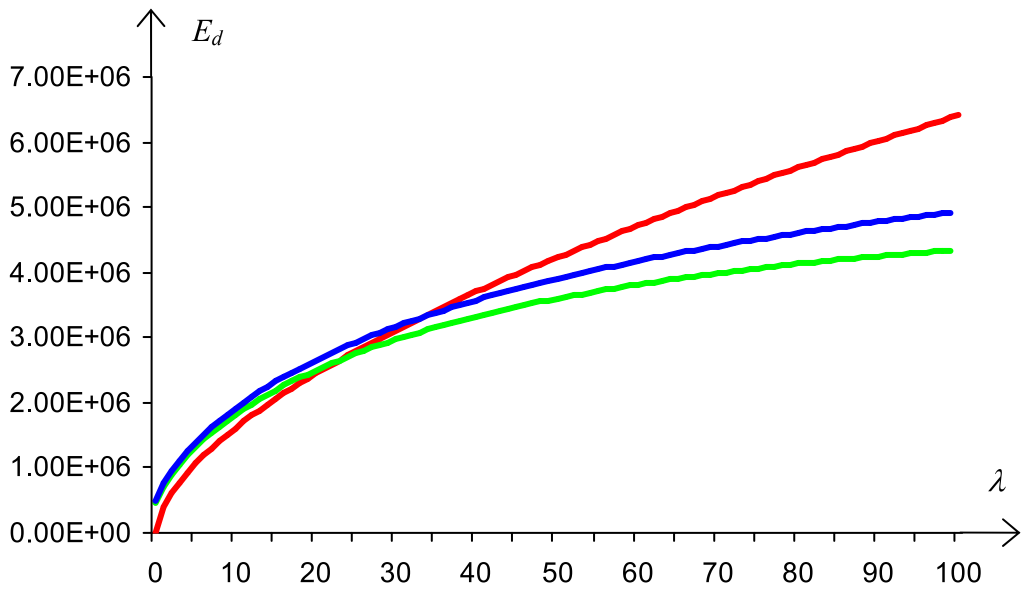

Figure 5: The function of energy dissipated by VE dampers – modified data.

The dynamic characteristics of structure with dampers modelled using the modified parameters are presented in Table (4). The bending moments and sums of amplitudes of drifts (interstorey displacements) were calculated for a structure which vibrates with natural frequency ω_1.

	Rheological model with			
	3 parameters		4 parameters	
	Kelvin	Maxwell	Kelvin	Maxwell
	(k3)	(m3a)	(k4a)	(m4a)
First natural frequency [rad/sec]	28.964	23.393	27.931	29.122
First non-dimensional damping ratio	0.0215	0.0281	0.0236	0.0215
Maximal interstorey drift [mm]	4.58	5.83	4.49	4.53
Sum of drifts [mm]	37.39	46.05	36.83	37.01
Maximal bending moment [kNm]	279.4	277.4	267.5	277.6
Sum of bending moments [MNm]	1.476	1.684	1.440	1.463

Table 4: The dynamic parameters of structures with modified dampers

The modification presented above of parameters of the considered models of dampers caused small changes in the values of first natural frequencies. In all cases, the amplitudes of displacements decreased, which is associated with the growth of the values of non-dimensional damping factors γ_1. It can be observed that the modified dampers provide more similar responses of structure in comparison with dampers modelled with identical coefficients c_d and k_d. This is because the energy dissipated by each model of damper has a similar value for frequency, equal to ω_1.

Using the same value of stiffness and damping parameters for all models of dampers, the present authors derived the loss factor functions from formulas (10) ÷ (13). The diagram of loss factor functions for all of the considered fractional models of dampers is presented in Figure 6: red colour for model (k3), green for model (m3), blue for (k4) and yellow for (m4). It is easy to observe that the value of loss factor differs widely for each damper model.

The aim of another modification of coefficients c_d and k_d is to adjust the value of loss factor performed by each model of damper. For the damper modeled by model (m3) the value of loss factor decreases with frequency while for other models it increases. Consequently, it is possibly to find the value of coefficients c_d and k_d in such a way that the value of loss factor is similar for each model of damper only for the selected frequency λ.

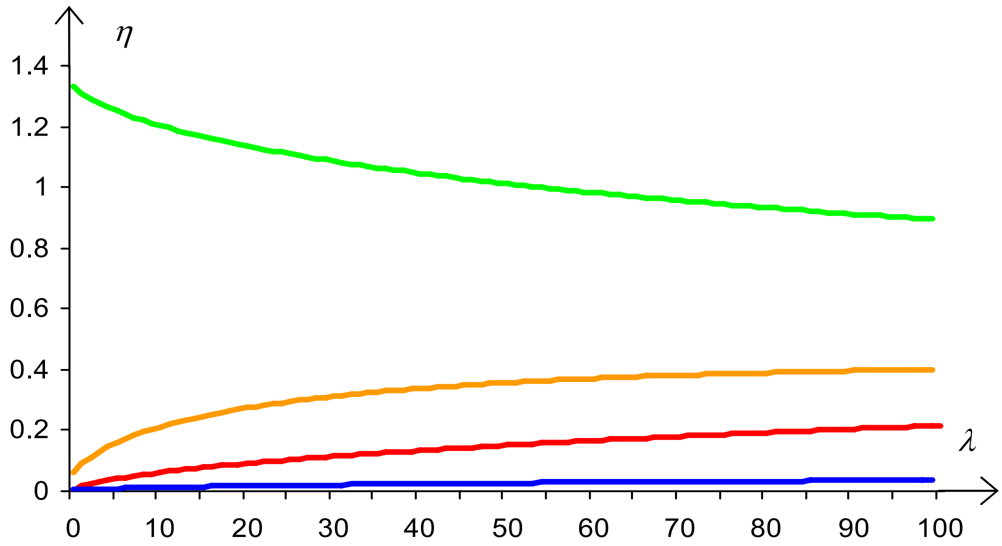

Figure 6: The function of loss factor for VE dampers – uniform data.

The displacements of structure have maximal amplitudes at the first resonance peak, so we adopted the values of parameters c_d , k_d in such way that the loss factor has similar values at the first natural frequency for each damper (see Figure 7).

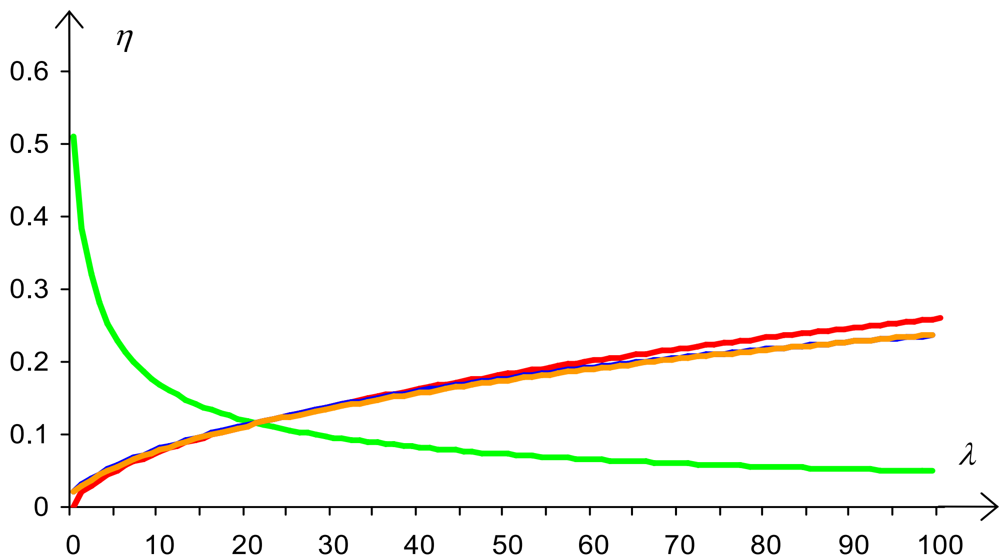

Figure 7: The function of loss factor for VE dampers – modified data.

Next calculations were carried out for new values of stiffness and damping coefficients: for the three-parameter Maxwell model (m3b): $c_{dm}' = 5000 kNs^{\alpha} / m$, $k_{dm}' = 5000 kN / m$, for the four-parameter Kelvin model (k4b): $c_4' = 500 kNs^{\alpha} / m$, $k_4' = 13000 kN / m$ and for the four-parameter Maxwell model (m4b):

$c_{4m}' = 650kNs^{\alpha} / m$, $k_{4m}' = 100000kN / m$. For those parameters the loss factor equals approximately 0.15 when the frequency of vibration is in the range between 23 and 29 rad/sec (see Figure 7).

As shown in Figure 7 for model (k4b) and (m4b), an identical diagram of loss factor was obtained (yellow colour). The dynamic characteristics of structure with dampers calculated for a new value of dampers coefficients are presented in Table 5.

	Rheological model with			
	3 parameters		4 parameters	
	Kelvin	Maxwell	Kelvin	Maxwell
	(k3)	(m3b)	(k4b)	(m4b)
First natural frequency [rad/sec]	28.964	23.928	25.727	29.091
First non-dimensional damping ratio	0.0215	0.0062	0.0247	0.0256
Maximal interstorey drift [mm]	4.58	24.47	4.96	3.80
Sum of drifts [mm]	37.39	195.22	40.70	31.08
Maximal bending moment [kNm]	279.4	1208.5	260.6	233.1
Sum of bending moments [MNm]	1.476	7.214	1.538	1.228

Table 5: The dynamic characteristics of structures with dampers modelled using coefficients c_d' and k_d'

The dynamic behaviour of structures with VE dampers described by model (k4b) is comparable to the response of system with model (m4b). The damper modelled by model (m3b) has the smallest ability to reduce frame vibrations. The reason why model (m3b) can not be adjusted to other models is because of the small values of the loss factor obtained by this model for higher frequencies λ.

7 Concluding remarks

In this paper the authors have studied the effectiveness of viscoelastic dampers described by three- and four-parameter fractional rheological models. The dynamic behaviour of a structure with dampers mounted on it was expressed by dynamic characteristics: natural frequencies, non-dimensional damping ratios, the amplitudes of interstorey drifts, and the amplitudes of bending moments in columns. The calculations were carried out for a selected shear frame with dampers modelled by four various rheological models. The influence of dampers was compared by adopting various criteria of equivalence for the considered models.

When the same values of damping and stiffness coefficients were assumed for each model, the smallest amplitudes of displacements were caused by dampers with the largest energy dissipation ability. Moreover, there are significant differences between the dynamic characteristics of a structure with various dampers, including the amplitudes of displacements (interstorey drifts) because the energy dissipated by each considered model was different.

In the second approach, the value of stiffness and damping coefficients were modified in such a way that the amount of energy dissipated by each model of damper was similar. It was possible to derive an identical function of dissipated energy in the frequency domain only for four-parameter Kelvin and Maxwell models. On the basis of fixed values of damping and stiffness coefficients for model (k3), the parameters for models (m3) and (k4) were established in order to align the ability of energy dissipation for each damper model. In this case, the dynamic characteristics of a structure with dampers modelled by various fractional models were more alike.

The third criterion enabling comparison was the loss factor derived for each considered damper model. The values of stiffness and damping coefficients for models (m3), (k4) and (m4) were investigated in order to obtain a similar value of loss factor derived for first natural frequency. The results for a structure with dampers modelled by the three-parameter Maxwell model were significantly different from those obtained for a structure with other models.

The results of numerical analysis show that the parameters of fractional VE dampers could be chosen in such a way that the effect of their interaction with the structure led to a similar behaviour of the system. Rheological models with such properties could be treated as equivalent models of dampers. The best criterion which enables various models of viscoelastic dampers to be compared is the function of dissipation of energy. The amount of dissipated energy depends on many factors, primarily it is a function of frequency.

Acknowledgments

The authors wish to acknowledge the financial support received from the Poznan University of Technology (Grant No. DS 11-088/12) in connection with this work.

References

[1] T.T. Soong, B.F. Spencer, "Supplemental energy dissipation: state-of-the-art and state-of-the-practice", Engineering Structures, 24, 243-259, 2002.

[2] C. Christopoulos, A. Filiatrault, Principles of Passive Supplemental Damping and Seismic Isolation, IUSS Press, Pavia, Italy, 2006.

[3] R.H. Zhang, T. T. Soong, "Seismic design of viscoelastic dampers for structural applications, J. Structural Engineering, Vol. 118, 1375 – 1392, 1992.

[4] S.W. Park, “Analytical modelling of viscoelastic dampers for structural and vibration control”, International Journal of Solids and Structures, 38, 8065 – 8092, 2001.
[5] I. Takewaki, “ Building control with passive dampers, Optimal performance-based design for earthquakes”, Wiley and Sons (Asia), Singapore, 2009.
[6] M.P. Singh, T.S. Chang, “Seismic analysis of structures with viscoelastic dampers”, J. Eng. Mech., 135, 571-580, 2009.
[7] T. Hatada, T. Kobori, M.A. Ishida and N. Niwa, “Dynamic analysis of structures with Maxwell model”, Earthq. Eng. Struct. Dyn. Earthq., 29, 159-176, 2000.
[8] R.L. Bagley, P.J. Torvik, ”Fractional calculus – a different approach to the analysis of viscoelastically damped structures”, AIAA Journal, 27, 1412–1417, 1989.
[9] T. S. Chang, M. P. Singh, “Seismic analysis of structures with a fractional derivative model of viscoelastic dampers”, Earthq. Eng. Eng. Vib., 1, 251-260, 2002.
[10] R. Lewandowski, Z. Pawlak, “Dynamic Analysis of Frames with Viscoelastic Dampers Modelled by Rheological Models with Fractional Derivatives”, Journal of Sound and Vibration, 330, 923-936, 2011.
[11] I. Podlubny, “Fractional Differential Equations”, Academic Press, 1999.
[12] E. Barkanov, W. Hufenbach, L. Kroll, “Transient response analysis of systems with different damping models”, Computer Meth. Appl. Mech. Eng., 192, 33-46, 2003.
[13] R. Lewandowski, B. Chorążyczewski, “Identification of the parameters of the Kelvin-Voigt and the Maxwell fractional models, used to the modelling of viscoelastic dampers”, Computers and Structures, 88, 1-17, 2010.