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Dealing with Uncertainty in Advanced Frequency-Domain Operational Modal Analysis

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Abstract

This paper presents a new advanced frequency-domain operational modal analysis method that is able to deal with uncertainties. The idea is that uncertainties in the measurements (noise) should be taken into account when estimating the modal parameters of the tested structure, and that also confidence bounds on the modal parameters are estimated. The starting point for the new developments is the PolyMAX method that became quite popular in the field of experimental and operational modal analysis. Operational modal analysis, which is the domain in which this paper is situated, refers to the fact that the structure is tested in its operational conditions and that only response data are available (e.g. bridge acceleration response data due to unmeasurable wind or traffic excitation). The main advantages of PolyMAX are its computational efficiency and the very clear stabilisation diagrams it yields even in the case of highly-damped systems and noisy measurements. Moreover, the numerical stability of the algorithm allows for a large-bandwidth and high-model order analysis and makes it suitable both for lowly- and highly-damped structures. In this paper, an iterative maximum likelihood estimation method is introduced as an extension to PolyMAX of which the main benefits become clear in case of very noisy data and in cases where the estimation of confidence bounds on the modal parameters is of interest. This "PolyMAX Plus" approach will be illustrated using railway bridge vibration data that was acquired during the execution of the FADLESS research project.

Keywords: operational modal analysis, noise, uncertainty, confidence bounds, PolyMAX Plus.

1 Introduction

Over the last decades, a number of algorithms have been developed to estimate modal parameters from measured frequency or impulse response function data (or in case of operational modal analysis (OMA): … from measured cross spectra or cross

correlations.). A very popular implementation of the frequency-domain linear least squares estimator optimized for modal parameter estimation is called polyreference least-squares complex frequency-domain (pLSCF) estimator [1], also known as PolyMAX [2]. PolyMAX is a poly-reference version of the LSCF estimator [3] that was originally developed as a starting value generator for the iterative Maximum Likelihood Estimator (MLE) [4]. However, it was found that these "starting values" were already quite accurate. In addition, the main advantages of PolyMAX are its computational efficiency and the very clear stabilization diagrams.

Recently there has been an interest to apply stochastic estimators to modal analysis applications. These stochastic methods take into account the noise on the data and provide confidence bounds on the modal parameter estimates. The above-mentioned iterative frequency-domain MLE method is such a method [4][5]. The application to OMA is discussed in [6][7]. Also time-domain stochastic subspace identification methods can be used to estimate the confidence bounds on the OMA results [8].

In this paper, the PolyMAX Plus method will be discussed that keeps the advantages of PolyMAX and adds some maximum likelihood estimation (MLE) features such as the proper handling of uncertainty, estimation of confidence bounds and improved results in case of very noisy data. The method consists of 3 stages:

1. "Smoothing" of measurement functions such as frequency response functions (FRFs) or half spectra using a blind MLE fit.
2. Mode selection by applying PolyMAX to the smoothed data and constructing a stabilization diagram.
3. Estimation of the confidence bounds from an MLE modal model formulation.

The method was initially described in [9]. In this paper, also the application of the method to a numerical example originating from [10] was discussed. In this example, various levels of noise were added to the simulation results and it was shown that the deterministic methods underestimated the damping ratio of highly-damped and weakly-excited modes. It was proven in [9] that the PolyMAX Plus method was able to yield accurate modal parameter estimates even with very high noise levels. In [11], the 3rd stage of the PolyMAX Plus method, frequency-domain MLE applied to the modal model formulation, is explained in detail.

In Section 2, the PolyMAX Plus method for OMA will be discussed and in Section 3 a case study is presented: the application of the method to the Trezói Bridge, a steel railway bridge in Portugal.

2 The PolyMAX Plus method

In [12], the PolyMAX modal parameter estimation method for operational modal analysis was introduced. It was shown that the modal decomposition of half spectra (i.e. the Fourier transform of the positive time lags of the auto- and cross-correlation functions) has a similar structure as the modal decomposition of frequency response functions (FRFs):

$$S_{yy}^{+}(\omega)=\sum_{i=1}^{n}\frac{\{v_i\}<g_i>}{j\omega-\lambda_i}+\frac{\{v_i^*\}<g_i^*>}{j\omega-\lambda_i^*}+\frac{\mathrm{LR}}{j\omega}+j\omega\,\mathrm{UR} \tag{1}$$

in which $S_{yy}^{+}(\omega)\in\mathbb{C}^{N_o\times N_i}$ is the half spectrum matrix, where N_o is the number of outputs; N_i is the number of reference outputs; n is the number of complex conjugated mode pairs; $\bullet^*$ is the complex conjugate of a matrix; $\{v_i\}\in\mathbb{C}^{N_o}$ are the mode shapes; $<g_i>\in\mathbb{C}^{1\times N_i}$ are the so-called *operational reference factors*, which replace the modal participation factors in case of output-only data; and λ_i are the poles, which are related to the eigenfrequencies ω_i and damping ratios ξ_i as follows:

$$\lambda_i,\lambda_i^*=-\xi_i\omega_i\pm j\sqrt{1-\xi_i^2}\,\omega_i \tag{2}$$

The lower and upper residuals, $\mathrm{LR},\mathrm{UR}\in\mathbb{R}^{N_o\times N_i}$, have been introduced to model the influence of the out-of-band modes in the considered frequency band.

A poly-reference method such as PolyMAX yields a set of poles and corresponding operational reference factors from the interpretation of the stabilization diagram [12]. The mode shapes and the lower and upper residuals are then readily obtained by solving Equation (1) in a linear least-squares sense. This second step is commonly called least-squares frequency-domain (LSFD) method.

2.1 First stage: Maximum Likelihood Estimator

The method will be illustrated for FRFs, but as explained in previous paragraph, the FRF matrix can be replaced by the half spectrum matrix. Assuming the measured FRFs to be uncorrelated, the maximum likelihood cost function reduces to [4]:

$$\ell^{\mathrm{ML}}(\theta)=\sum_{o=1}^{N_o}\sum_{i=1}^{N_i}\sum_{k=1}^{N_f}\frac{\left|H_{oi}^{\mathrm{ML}}(\omega_k,\theta)-H_{oi}(\omega_k)\right|^2}{\sigma_{H_{oi}}^2(\omega_k)} \tag{3}$$

where $H_{oi}(\omega_k)$ and $\sigma_{H_{oi}}(\omega_k)$ are the measured FRF and its standard deviation at frequency ω_k. The summations run over the number of outputs N_o, the number of inputs N_i and the number of frequency lines N_f. The FRF model $H_{oi}^{\mathrm{ML}}(\omega_k,\theta)$ is a common-denominator model, where the numerator and denominator polynomials are expressed in the *z*-domain. The polynomial coefficients θ are estimated by minimizing equation (3). This can be done by means of a Gauss-Newton optimization algorithm, which takes advantage of the quadratic form of the cost function. The Levenberg-Marquardt algorithm is used to ensure a continuous decrease of the cost function during the iterations.

Alternatively, the MLE-like logarithmic estimator minimizes the following cost function:

$$\ell^{\mathrm{LOG-ML}}(\theta)=\sum_{o=1}^{N_o}\sum_{i=1}^{N_i}\sum_{k=1}^{N_f}\frac{\left|\log(H_{oi}^{\mathrm{ML}}(\omega_k,\theta)/H_{oi}(\omega_k))\right|^2}{\sigma_{H_{oi}}^2(\omega_k)/\left|H_{oi}(\omega_k)\right|^2} \tag{4}$$

This logarithmic estimator has nearly MLE properties and is more robust against the noise assumptions made as well as outliers, and can handle measurements with a large dynamic range. More details on the algorithm can be found in [9] and the references cited therein.

2.2 Second stage: PolyMAX applied to the MLE synthesized model

Despite the nice statistical properties of MLE, some hurdles still need to be taken to make it a practical method compatible with the "state-of-the-use" in modal parameter estimation:

- Often a stabilization diagram is used to distinguish real poles from numerical poles [13]. Due to the iterative character of the method, it is rather time-consuming to construct an MLE stabilization diagram. Also the resulting diagram is not so easy to interpret as in case of PolyMAX [9].
- The FRF model considered in MLE is a common-denominator model, which is not fully compatible to a poly-reference modal model.

Therefore, it is proposed to apply MLE using a fixed model order (i.e. without constructing a stabilization diagram) and post-process the synthesized FRFs using the PolyMAX method [1][2][12].

2.3 Third stage: Estimation of the confidence bounds from an MLE modal model formulation

After having identified a modal model (1), the confidence bounds on the modal parameter estimates still have to be computed. The basic principle is that the uncertainty on the measurement data $\sigma_{H_{oi}}$ is propagated to an uncertainty on the modal parameter estimates (standard deviation of eigenfrequency σ_{ω_i} and damping ratio σ_{ξ_i} :

$$\sigma_{H_{oi}} \rightarrow \sigma_{\omega_i}, \sigma_{\xi_i} \tag{5}$$

The (analytical) relation between the covariances of the FRFs and the covariances of the MLE model parameters is given by the Jacobian matrix evaluated during the last iteration step of the Gauss-Newton algorithm. The confidence bounds on frequency and damping estimates can then be calculated from the covariances of the MLE model parameters [7].

In the particular case of PolyMAX Plus, the MLE cost function like in equation (3) is formulated, but the FRF model is a modal model like in equation (1). Also, the 2nd stage modal parameters are retained and no iterative process is started, but only the Jacobian is calculated in order to calculate the covariances of the modal parameters. Details on the specific implementation for the MLE method applied to a modal model can be found in [11].

3 Case study: Operational Modal Analysis of a railway bridge

3.1 Trezói Bridge

The Trezói Birdge is a riveted metallic bridge (1956) and a part of an international railway line linking Portugal to Spain. It has a total length of 126 m, with three spans of 39 m, 48 m and 39 m. The two main beams have a height of 5.68 m. The distance between each vertical profile is 6.00 m in the first and third spans and 6.50 m in the central span. The bridge has a constant width of 4.40 m, throughout its length (Figure 1). The Laboratory of Vibrations and Structural Monitoring (ViBest) of FEUP (Faculty of Engineering at the University of Porto) was performing ambient vibration tests on this bridge as part of a larger effort to assess its the fatigue [14][15][16].

Figure 1: Trezói Bridge (Courtesy FEUP [14]).

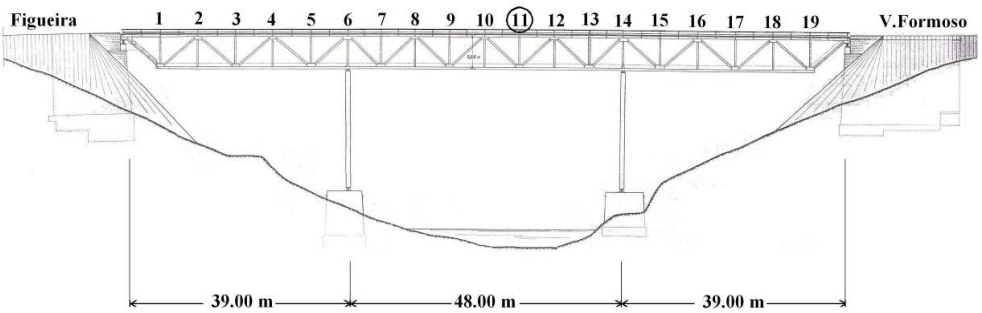

Figure 2: Location of the 19 instrumented sections (Courtesy FEUP).

Accelerations due to ambient vibrations were measured along three orthogonal directions at 19 sections at the top of one of the main beams Figure 2. In each "run" of 12 minutes, a seismograph was placed at point 11 (reference), and two others

were placed successively at the 18 remaining points of measurement. In a last run ("Run10"), the ambient responses at both sides of the reference section were measured, allowing distinguishing between vertical and torsional modes. A view on the data acquisition is given in Figure 3.

Figure 3: Placement of the seismographs (Courtesy FEUP [14]).

3.2 Data

The time recordings of a vertical reference sensor during the 10 different runs are represented in Figure 4. During most of the runs, ambient vibrations were captured, but during Run2, Run5 and Run8, a high-amplitude event occurred – most probably a train passage. The power spectra of these signals are shown in Figure 5. Whereas most of the spectra agree well on dominant resonance peaks, again Run2, Run5 and Run8 show a different spectrum where the resonances are less visible. Therefore, the high-amplitude events have been cut out of the data before further analysis. Figure 6 shows the power spectra of the reference sensor in transversal (Y) and vertical direction (Z) after discarding the high-amplitude events. Although these sensors remain at the same location during all 10 runs, the spectra differ from one run to another due to differences in ambient excitation (wind loading). Nevertheless, the resonance peaks agree well between the different runs.

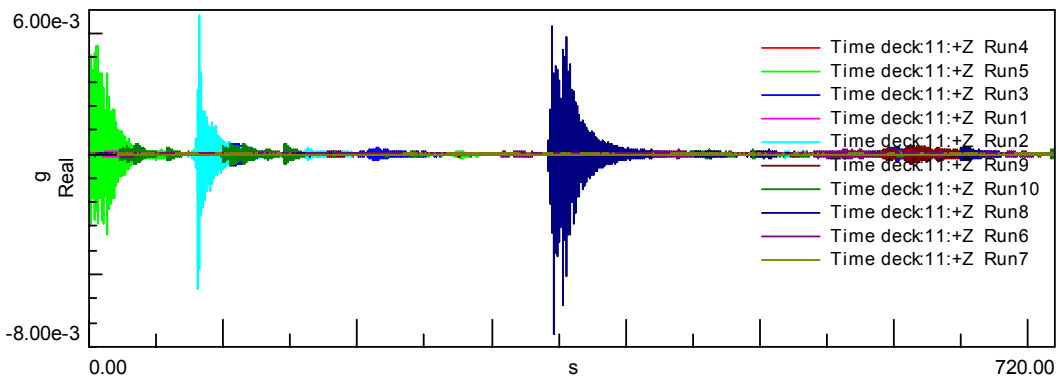

Figure 4: Time series from vertical (Z) reference sensor measured during all 10 runs.

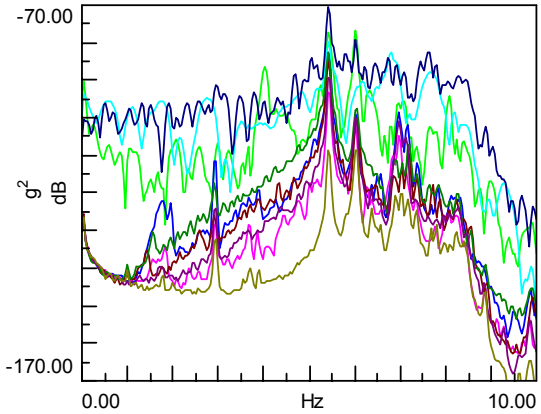

Figure 5: Power spectra from vertical (Z) reference sensor measured during all 10 runs.

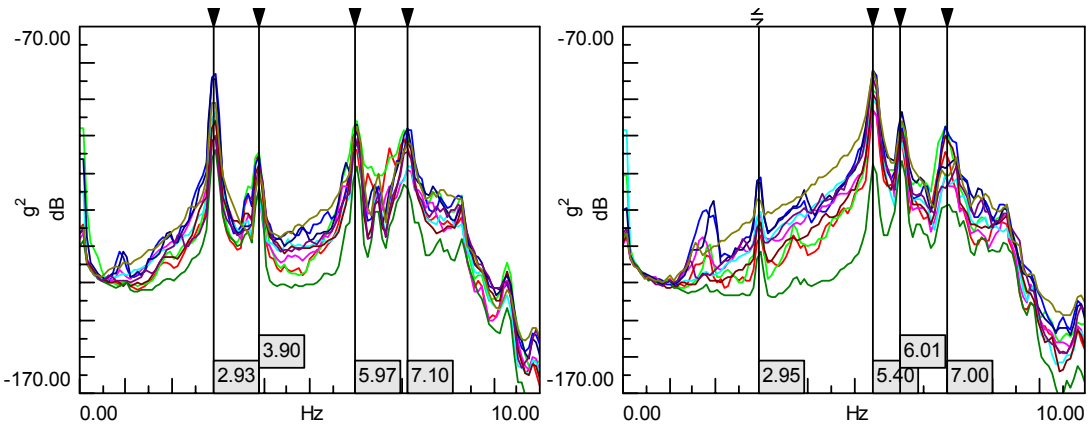

Figure 6: Power spectra from transversal (Y) and vertical (Z) reference sensor measured during all 10 runs. The resonance peaks are indicated with a cursor.

3.3 PolyMAX Plus results

Both the PolyMAX and the PolyMAX Plus method have been applied to the Trezói Bridge data. Figure 7 shows the stabilization diagrams from both methods. They can hardly be distinguished from each other, indicating that PolyMAX Plus is indeed able to retain the property of clear stabilization diagrams.

Figure 8 shows the measured half spectra, the PolyMAX Plus synthesized spectra and the "uncertainty" (i.e. standard deviation) on the data. In classical modal analysis, the data variance is related to the coherence between input and output. In operational (or output-only) modal analysis, the variances of the spectra that could be computed during the spectral averaging process have to be used with care. Indeed, it is not guaranteed that the excitation is stationary across the different data segments and the differences in spectra can be due to noise and differences in the excitation. Therefore, a smoothed residual error approach is adopted instead to

estimate the data variance. It consists of computing the difference between measured spectra and the MLE synthesis (from the 1st stage; see Section 2.1) and applying a smoothing across the frequency lines. These smoothed residual errors are represented by the dashed lines in Figure 8.

Some typical mode shapes from the bridge after merging the results from all 10 runs are presented in Figure 9.

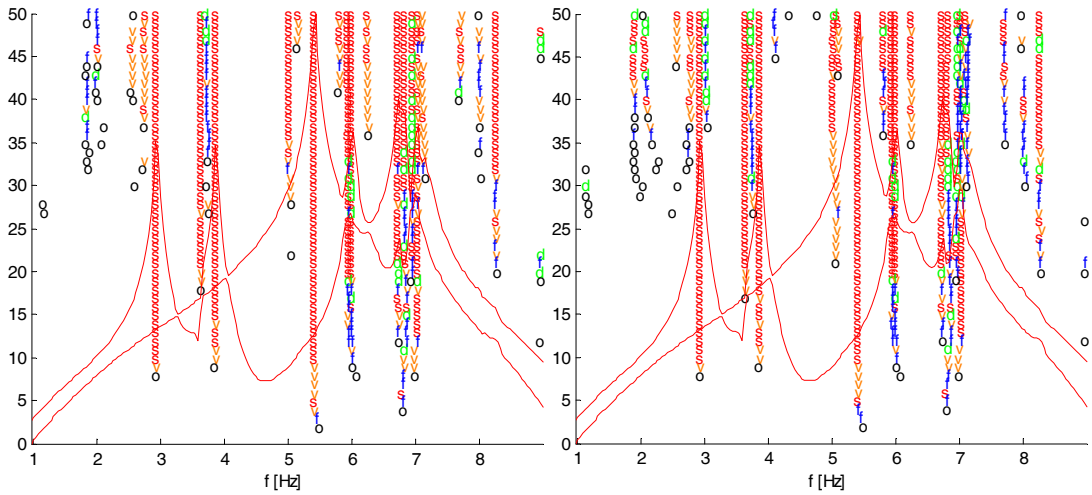

Figure 7: Run1 Stabilization diagrams: (Left) PolyMAX; (Right) PolyMAX Plus.

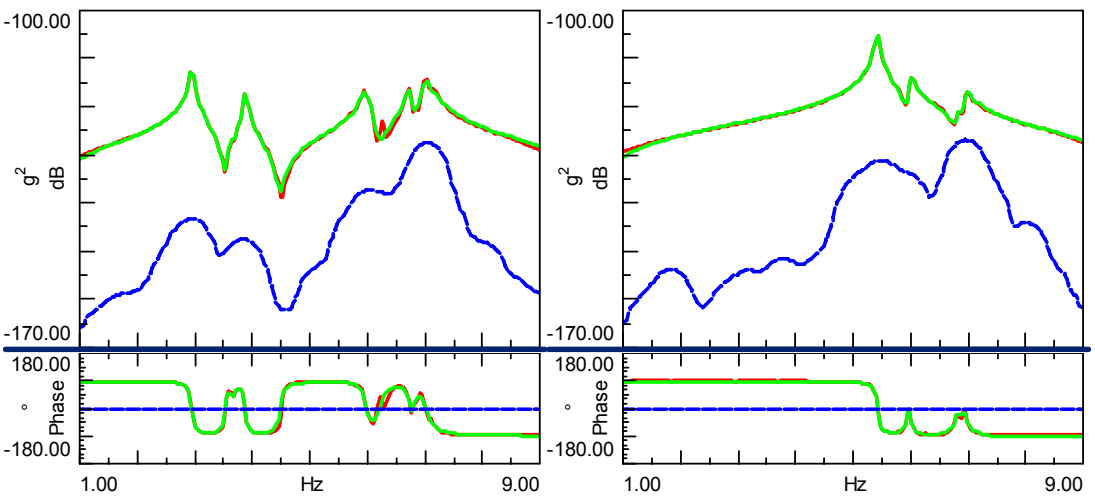

Figure 8: Run1 PolyMAX Plus results representing the measured spectra (red), synthesized spectra (green) and the "uncertainty" on the spectra (blue-dashed).

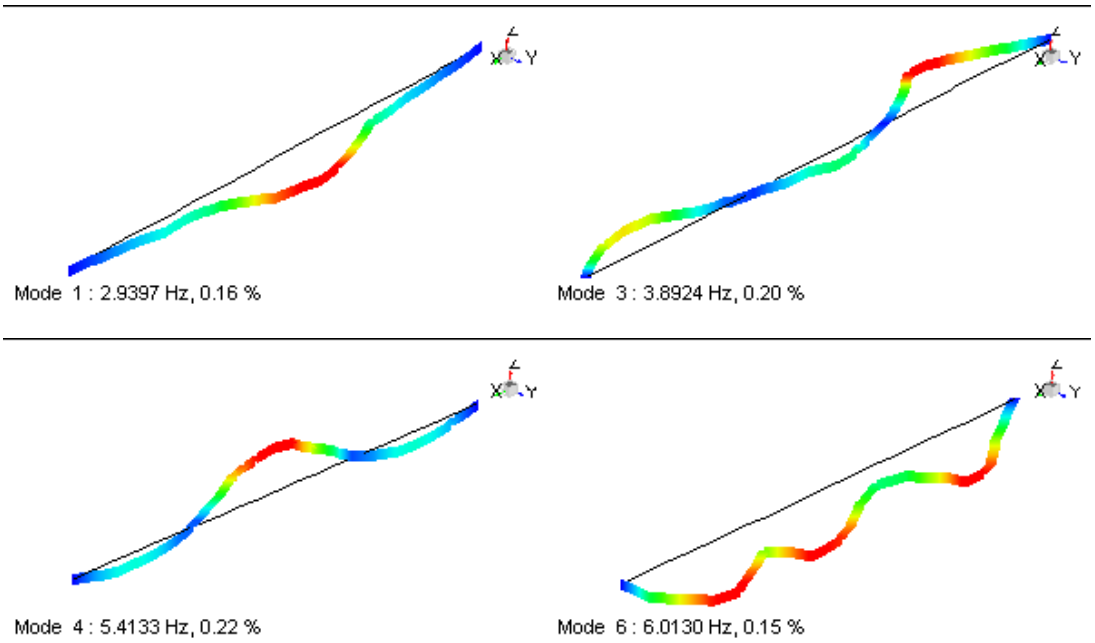

Figure 9: Some mode shapes merged together from 10 different runs. From left to right and top to bottom: 1st transversal bending, 2nd transversal bending, vertical bending, torsional mode.

From the data uncertainty, the confidence bounds on the estimated frequencies and damping ratios are computed as explained in Section 2.3. The results are represented in Table 1 for different data pre-processing choices (exponential window) in Run1 and in Table 2 for all 10 runs.

The following observations can be made from Table 1:

- Results for different values of the exponential window agree well. PolyMAX Plus yields results that are quite independent of the data pre-processing.
- In general, the uncertainties on the estimates become larger if a weaker exponential window was used (1% means very strong exponential window, 100% means no exponential window). This seems logical, as an exponential window reduces the noise on the data and hence the uncertainty on the estimates.
- As indicated by the colorbars, the relative uncertainty across the modes is very similar for all settings of the exponential window.
- The absolute uncertainties are perhaps unexpectedly low. However, this is in agreement with the very low data uncertainty within a single run (see dashed line in Figure 8).

	Exponential window = 1%				Exponential window = 10%				Exponential window = 100%			
Mode	f [Hz]	s_f [Hz]	xi [%]	s_xi [%]	f [Hz]	s_f [Hz]	xi [%]	s_xi [%]	f [Hz]	s_f [Hz]	xi [%]	s_xi [%]
1	2.9438	0.0000	0.1296	0.0005	2.9437	0.0000	0.1338	0.0008	2.9439	0.0000	0.1353	0.0006
2	3.6488	0.0001	0.3046	0.0035	3.6490	0.0001	0.3045	0.0040	3.6484	0.0002	0.2706	0.0060
3	3.8754	0.0000	0.0939	0.0004	3.8751	0.0000	0.1009	0.0003	3.8752	0.0000	0.0942	0.0004
4	5.4193	0.0000	0.1541	0.0007	5.4165	0.0001	0.1382	0.0010	5.4182	0.0000	0.1102	0.0008
5	5.9464	0.0002	0.2653	0.0033	5.9460	0.0003	0.2612	0.0052	5.9456	0.0004	0.1880	0.0069
6	5.9997	0.0001	0.1538	0.0023	5.9991	0.0002	0.1689	0.0034	5.9978	0.0003	0.1932	0.0051
7	6.7304	0.0004	0.1697	0.0061	6.7298	0.0004	0.1683	0.0066	6.7341	0.0007	0.1460	0.0105
8	6.8212	0.0007	0.2281	0.0103	6.8243	0.0007	0.2310	0.0107	6.8297	0.0010	0.1696	0.0144
9	6.9782	0.0012	0.3747	0.0170	6.9616	0.0007	0.2211	0.0110	6.9757	0.0015	0.1032	0.0221
10	7.0405	0.0008	0.7978	0.0111	7.0266	0.0004	0.1895	0.0066	7.0202	0.0012	0.3005	0.0176
11	8.2722	0.0004	0.4045	0.0044	8.2751	0.0004	0.3772	0.0052	8.2708	0.0006	0.3389	0.0076

Table 1: Run1 estimated modal parameters for different values of the exponential window (1%, 10%, 100%) used during pre-processing. Following columns are included: eigenfrequencies (f), standard deviation of eigenfrequencies (s_f), damping ratios (xi), standard deviation of damping ratios (s_xi).

The following observations can be made from Table 2:

- In this table, less modes are present than in Table 1. The reason is that here, only the modes have been retained that could be estimated in all runs.
- The difference between the sample standard deviations computed from the results of all runs and the PolyMAX Plus standard deviations is about one or two orders of magnitudes. This can be explained as follows. The standard deviations estimated by PolyMAX Plus represent the data variance of a single run. The sample standard deviations computed from all 10 run results also include changes that may occur between runs: the sensors are at different locations on the bridge (except for the reference sensors), different excitation levels (trains, ambient), temperature differences, …
- Despite these large differences in absolute sense, the relative standard deviations computed across the modes seem to agree quite well.

	sample mean	sample std	mean std	sample mean	sample std	mean std
Mode	f [Hz]	f [Hz]	f [Hz]	xi [%]	xi [%]	xi [%]
1	2.9391	0.0057	0.0000	0.1653	0.0506	0.0006
2	3.8901	0.0126	0.0001	0.2020	0.0891	0.0031
3	5.4136	0.0056	0.0000	0.2243	0.1126	0.0008
4	6.0137	0.0071	0.0001	0.1889	0.0749	0.0010
5	7.0904	0.0383	0.0003	0.3062	0.1921	0.0047
6	8.2810	0.0172	0.0003	0.4131	0.2499	0.0044

Table 2: Statistics on the modal parameters estimated from 10 different runs. The sample mean and sample standard deviation (std) are obtained by simply applying statistics to the estimates from the 10 runs. The mean of the standard deviations is obtained by averaging the PolyMAX Plus estimates of the standard deviation at each run.

4 Conclusions

In this paper, the PolyMAX Plus method was discussed that keeps the advantages of PolyMAX (computationally efficient, clear stabilization diagram) and adds some maximum likelihood estimation (MLE) features such as the proper handling of uncertainty, estimation of confidence bounds and improved results in case of very noisy data. The method consists of 3 stages: statistically optimal MLE data smoothing, PolyMAX mode selection, confidence bounds from MLE modal model formulation. The OMA variant of PolyMAX Plus was applied to data from the Trezói Bridge measured in 10 different runs. This data allowed getting experience with the confidence bounds as delivered by the method. It seems that these confidence bounds are useful to distinguish easily identifiable modes from weakly excited and/or noisy modes.

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