



## Statistical Multi-Objective Structural Damage Identification based on Dynamic Parameters

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### Abstract

Evolutionary algorithms are suitable to solve damage identification problems in a multi-objective context. However, the performance of these methods can deteriorate quickly with increasing noise intensities originating numerous uncertainties. In this paper, a statistic structural damage detection method formulated in a multi-objective context is proposed. The statistic analysis is implemented to take into account the uncertainties existing in the structural model and measured structural modal parameters. The presented method is verified by a number of simulated damage scenarios. The effects of noise and damage levels on damage detection are investigated.

**Keywords:** structural health monitoring, damage detection, multi-objective optimization, statistical analysis.

## 1 Introduction

The implementation of a damage detection strategy for aerospace, civil and mechanical engineering infrastructures is referred as structural health monitoring (SHM). Over the last years, there have been increasing demands to develop SHM systems over different kind of systems because of the huge economical and life-safety benefits that such technologies have the potential to provide.

The most usual approach for solving this sort of problem is the use of the finite element model updating method [1, 2, 3, 4]. To apply the method, one objective function measuring the fit between measured and model predicted data is chosen. Then, optimization techniques are used to find the optimal values of the model parameters that minimize the value of the objective function, i.e., those values best fitting the experimental data. Damage detection methods based on model updating method have usually been developed as single objective optimization problems. However, the lack of a clear objective function in the context of real-world damage

detection problems advises simultaneous optimizations of several objectives with the purpose of improving the performance of the procedure [5, 6]. Often, these objectives are conflicting. As opposed to single-objective optimization problems which accept one single optimum solution, multi-objective optimization problems do not have a single optimal solution, but rather a set of alternative solutions, named the Pareto front set, which are optimal in the sense that no other solutions in the search space are superior to them when all objectives are considered. Because evolutionary computation algorithms deal with a group of candidate solutions, it seems natural to use evolutionary computation algorithms in multi-objective optimization problems to find a group of Pareto optimal solutions simultaneously instead of traditional optimization techniques. Although, in the past, many evolutionary algorithms have been developed, most of them are based on genetic algorithms.

The efficiency of this kind of damage identification algorithms relies on accuracy of the analytical finite element model and the measured dynamic data. However, the presence of uncertainties or noise, affecting as the numerical model as the experimental results, is usual which might lead to inaccuracies in the detection procedure, giving erroneous estimations. In this sense, it would be interesting to analyze the influences from these uncertainties on the damage identification results in a multi-objective framework as well as proposing some improvements to decrease their effect.

In this paper, a statistic structural damage detection method formulated in a multi-objective context is proposed. The statistic analysis is implemented to take into account the uncertainties existing in the structural model and measured structural modal parameters. The presented method is verified by a number of simulated damage scenarios. The effects of noise and damage levels on damage detection are investigated.

## **2 FE updating procedure**

### **2.1 Damage parameterization**

Setting-up an objective function, selecting updating parameters and using robust optimization algorithms are three crucial steps in structural FE model updating. They require deep physical insight and usually trial-and error approaches are commonly used.

The multi-objective approach can provide more robust identification since it provides multiple solutions while single-objective optimization gives a single solution. Multiple solutions have a much better chance of capturing the true solution. Therefore, multiple solutions, combined with the demotion of incorrect solutions by a non-domination scheme, enable low false-negative damage detection, which is essential for robust damage detection.

In this work, all the numerical simulations have been carried out by using single beam elements to represent the structure. The updating parameters are the uncertain physical properties of the numerical model. It has been assumed that no alteration

occurs before and after damage related to the mass, which is acceptable in most real applications. Therefore, the parameterization of the damage has been represented by a reduction factor or damage index of the element bending stiffness. According to Damage Mechanics [7], this damage index represents the relative variation of the element bending stiffness,  $(EI)_d^e$ , to the initial value,  $(EI)^e$ :

$$d_e = 1 - \frac{(EI)_d^e}{(EI)^e} \quad (1)$$

This definition of a damage index  $d_e$  for each element allows estimating not only the damage severity but also the damage location since the damage identification is then carried out at the element level. It should also be pointed out that although the damage index would take values between 0 and 1 for a damage identification procedure, their values would be included between -1 and 1 if a reference stage of the undamaged structure were to be identified, and would be considered, for this particular case, as more a correction factor of the bending stiffness than a damage index strictly speaking.

## 2.2 Objective functions

The multi-objective approach can provide more robust identification since it provides multiple solutions while single-objective optimization gives a single solution. Multiple solutions have a much better chance of capturing the true solution. Therefore, multiple solutions, combined with the demotion of incorrect solutions by a non-domination scheme, enable low false-negative damage detection, which is essential for robust damage detection.

In formulating a damage detection problem based on model updating the choice of the objective functions represents one of the most important decisions to be performed. In fact, many different single objective functions, depending directly or indirectly on basic modal parameters, have been proposed in recent years [2, 6, 8]. However, there is not a clear criterion for choosing the suitable objective function. Due to this, a combined consideration of some of them can be a good solution.

Among the various choices available, in this work, based on [6], the following objective functions have been adopted to perform multiobjective damage identification:

$$F_1 = 1 - \prod_{j=1}^m MTMAC_j = 1 - \prod_{j=1}^m \frac{MAC(\{\phi_{numj}\}, \{\phi_{expj}\})}{1 + \frac{|\lambda_{expj} - \lambda_{numj}|}{|\lambda_{expj} + \lambda_{numj}|}} \quad (2)$$

$$F_2 = 1 - \prod_{j=1}^m MACFLEX_j = 1 - \prod_{j=1}^m \frac{\left| \{F_{numj}\}^T \{F_{expj}\} \right|^2}{\left( \{F_{numj}\}^T \{F_{numj}\} \right) \left( \{F_{expj}\}^T \{F_{expj}\} \right)} \quad (3)$$

where  $\{\phi_j\}$  is the  $j$ th mode shape,  $\lambda_j = (2\pi f_j)^2$  where  $f_j$  is the eigenfrequency corresponding to  $j$ -th mode and  $\{F_j\}$  is the flexibility vector, collecting the diagonal terms of the flexibility matrix corresponding to the  $j$ -th mode; MAC is the modal assurance criterion [9] and the subscript num and exp are referred to numerical and experimental values, respectively. Both functions take values between zero and one.

### 2.3 Uncertainty

The presence of noise or uncertainty in both the finite element model and the measured modal data will affect to the value of the objective functions and, therefore, to the performance of the identification procedure. Then, it is very important to study its influence in the identification results. Although the noise might be introduced separately in the numerical model parameters and in the measurement data, in this work noise will be considered affecting to the original objective functions with the purpose of including in this perturbation all the possible kind of noises. This perturbation has been implemented on the basis of Gaussian noise as an additive normal distributed variable with zero mean and a variance  $\sigma^2$  representing the level of noise present:

$$\bar{F}_1 = F_1 + Normal(0, \sigma^2) \quad (4)$$

$$\bar{F}_2 = F_2 + Normal(0, \sigma^2) \quad (5)$$

where  $\bar{F}$  denote the noisy objective functions.

## 3 Probability of damage existence (PDE)

The PDE can be estimated from the statistical distributions of the stiffness parameter of the undamaged and damaged state [[10]0]. The basic idea is to compute the probability of an elemental stiffness parameter at a confidence level, defining the interval of the healthy stiffness parameter,  $\Omega(\alpha_i, \mu)$ , so that the probability of  $\alpha_i$  contained within the interval is  $\mu$ :

$$\text{prob}(x_\alpha \in \Omega(\alpha_i, \mu)) = \text{prob}(L_\Omega \leq x_\alpha < \infty) = \mu \quad (6)$$

where  $L_\Omega$  is the lower bound of the interval  $\Omega(\alpha_i, \mu)$ , which depends on the required confidence level. In this study,  $\mu$  is set to 95%, thus  $L_\Omega = E(\alpha_i) - 1.645 \times \sigma(\alpha_i)$ , which means that there is a probability of 95% that the healthy stiffness parameter falls in the range of  $[E(\alpha_i) - 1.645 \times \sigma(\alpha_i), \infty]$ .

In the same way, an interval can be defined for the stiffness parameter of the damaged state ( $\hat{\alpha}_i$ ). Thus, the PDE is defined as that of  $\hat{\alpha}_i$  not within the 95% confidence healthy interval  $\Omega(\alpha_i, 0.95)$ . Thus, the PDE of an element  $i$  is:

$$p_d^i = 1 - \text{prob}(x_{\bar{\alpha}} \in \Omega(\alpha_i, 0.95)) = 1 - \text{prob}(L_{\Omega} \leq x_{\bar{\alpha}} < \infty) \quad (7)$$

PDE is a value between 0 and 1. It is apparent that if the PDE is close to 1, the damage of the element is most likely; otherwise, if the PDE is close to 0, the damage of the element is very unlikely.

## 4 Evolutionary multi-objective optimization

The formulation of the multiobjective problem can be defined as finding the values of the damage parameter set  $\{d\}$  that simultaneously minimizes the objectives

$$F(\{d\}) = (F_1(\{d\}), F_2(\{d\}), \dots, F_m(\{d\})) \quad (8)$$

where  $\{d\} = (d_1, d_2, \dots, d_{NE})$  is the parameter set reflecting the damage value for each one of the  $NE$  elements of the structure and  $m$  is the number of objective functions.

The presence of multiple objectives in a problem, in principle, gives rise to a set of optimal solutions, known as Pareto-optimal solutions, instead of a single optimal solution. In the absence of any further information, one of these Pareto-optimal solutions cannot be said to be better than the other. This demands a user to find as many Pareto-optimal solutions as possible.

Classical optimization methods suggest converting the multiobjective optimization problem into a single-objective optimization problem by emphasizing one particular Pareto-optimal solution at a time. When such a method is to be used for finding multiple solutions, it has to be applied many times, hopefully finding a different solution at each simulation run. This approach may not necessarily result in an even distribution of Pareto optimal points and an accurate, complete representation of the Pareto optimal set. Another problem with this method is that it is impossible to obtain points on non-convex portions of the Pareto optimal set in the criterion space.

Pareto-based approaches constitute the main motivation for using evolutionary algorithms, such as genetic algorithms, to solve multi-objective optimization problems since in a single run of the algorithm several points of the Pareto-optimal set are found.

### 4.1 Modified Non-dominated Sorting Genetic Algorithm II

Genetic algorithm is a widely applied and efficient method of random search and optimization, the development of which is based on the theory of evolution. Its main features are groups search strategy and information exchange between individuals and moreover, the search does not depend on gradient information. Its basic theory was introduced by Holland [11] and developed in the engineering area by Goldberg's work [12]. Due to its potential for solving multi-objective optimization problems, a wide variety of algorithms has been proposed in the literature [13].

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) [14] is one of the most prominent multi-objective evolutionary algorithms and has been widely used. This algorithm is a modified version of the NSGA [15] algorithm which can be considered as belonging to the second generation of multi-objective evolutionary algorithms. This revised version is more efficient (computationally speaking) and incorporates a better sorting algorithm that keeps diversity without specifying any additional parameters, and elitism and no sharing parameter needs to be chosen a priori.

NSGA-II is based on several layers of classifications of the individuals. Once the population is initialized and before selection is performed, the population is ranked on the basis of non-domination. The first front being completely non-dominated individuals in the current population and the second front being dominated by the individuals in the first front only and so on. Each individual in each front is assigned rank (fitness) values or is based on the front to which it belongs. Individuals in the first front are given a fitness value of 1 and individuals in the second are assigned a fitness value of 2 and so on.

In addition to a fitness value a new parameter called crowding distance is calculated for each individual. The crowding distance is a measure of how close an individual is to its neighbours. A large average crowding distance will result in better diversity in the population. Parents are selected from the population by using binary tournament selection based on the rank and crowding distance. An individual is selected if the rank is less than the other or if the crowding distance is greater than the other. The selected population generates offsprings from crossover and mutation operators.

The population with the current population and current offsprings is sorted again based on non-domination and only the best  $N$  individuals are selected, where  $N$  is the population size. The selection is based on rank and the on the crowding distance in the last front. Unlike NSGA, an external memory is not required and elitism is introduced by combining the best parents with the best offspring obtained.

Different approaches have been proposed for the design of robust MOEAs for noisy multi-objective optimization. With this objective, a modified version of NSGA-II, which will be called MNSA-II, has been proposed [16] to handle noise in evolutionary multi-objective optimization. Its main characteristics are the introduction of explicit averaging into NSGA-II and the modification of the non-dominated sorting procedure to allow seemingly dominated solutions into the first non-dominated front. To prevent the loss of potentially useful solutions, a clustering mechanism is incorporated to induce solutions from the inferior non-dominated fronts into the first layers. This mechanism works by comparing the distances between solutions from the first non-dominated front and the other fronts. A higher ranked solution is re-assigned to the first non-dominated front if it is located in close proximity to a perceived non-dominated solution by using a criterion based on the variance.

Furthermore, this approach incorporates a procedure to remove unreliable solutions from the final set of non-dominated solutions by applying the clustering mechanism once again.

## 5 Numerical example

In order to investigate the performance of the methodology proposed here, several numerical simulations on a simply supported beam have been made. In all the studies performed, a crossover probability of 0.8 and a mutation probability of 0.01 have been assumed for the GA. Using a binary encoding, damage indices are between 0 and 1.

For all cases, the exact solution is compared with the solutions obtained by using the NSGA-II and MNSGA-II approaches. In order to decrease the influence of random effects characteristics to the evolutionary algorithm, 10 independent runs were performed per test problem to produce the mean  $\pm$  one sample standard deviation plot.

The problem for a comparative investigation consists in identifying damage for a simply supported concrete beam of length  $L=6$  m and rectangular cross section  $b \times h = 0.25$  m  $\times$  0.2 m. For numerical analysis purposes the beam was divided into 10 two-dimensional beam elements. The beam was assumed to have a Young's modulus  $E$  of 30 GPa and a density  $\rho$  of 2500 kg/m<sup>3</sup>.

The beam was subjected to a multiple simulated damage scenario (Figure 1) of complex identification. The “measured” dynamic responses of the beam before and after damage were generated previously. The baseline finite element model of the beam was created using Euler-Bernoulli planar elements with two degrees of freedom per node.

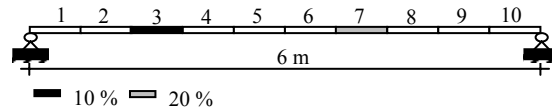


Figure 1. Multiple damage scenario for the numerical beam

To be more consistent with the field test conditions and to check the robustness of the proposed procedures, only the four lowest vibration modes were considered, and due to the limited number of sensors, the mode shape vector was only read at a limited number of locations coincident with the vertical degrees of freedom of the nodes in Figure 1. Furthermore, different levels of noise (1% and 5%) were included according to Equations(4) and (5). The presence of noise can affect the performance of the damage identification procedure especially with increasing noise intensities and, therefore, it is a way of evaluating the performance and robustness of the proposed methodology.

Figures 2 and 3 show the damage distribution for the beam problem when solved with the chosen criteria considering the different levels of noise. The results shown are the average of the 10 optimum solutions for the 10 runs carried out. In the same way, Figures 4 and 5 show the standard deviation for 1% and 5% noise, respectively. The standard deviation can be useful to measure the sensitivity of each damage parameter to the noise in the “measurements”. Sensitive parameters will generally show large standard deviations.

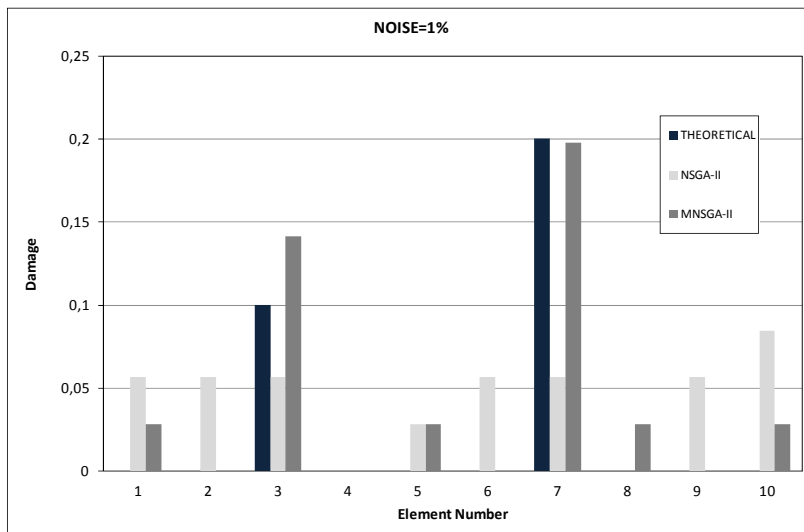


Figure 2. Damage distribution – 1% noise

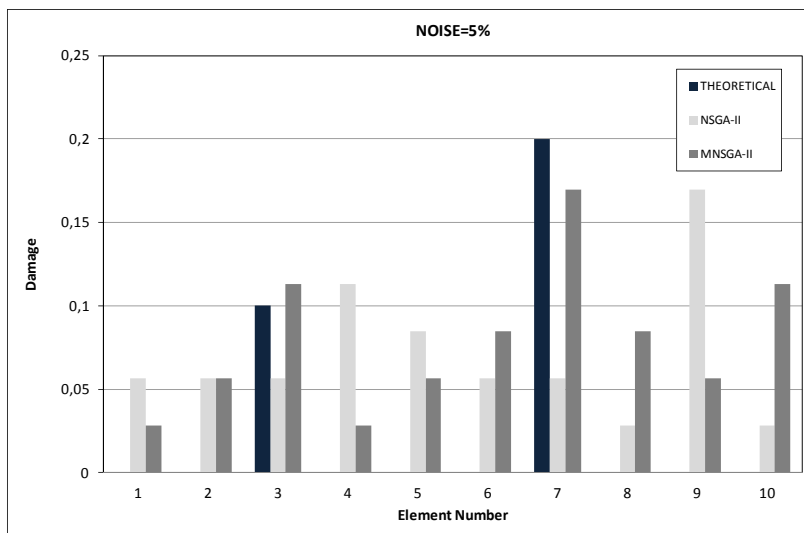


Figure 3. Damage distribution – 5% noise



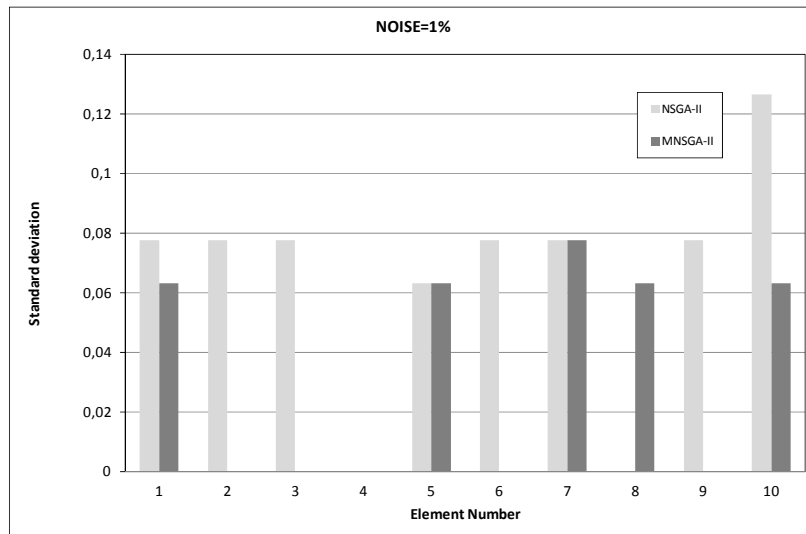


Figure 4. Standard deviation – 1% noise

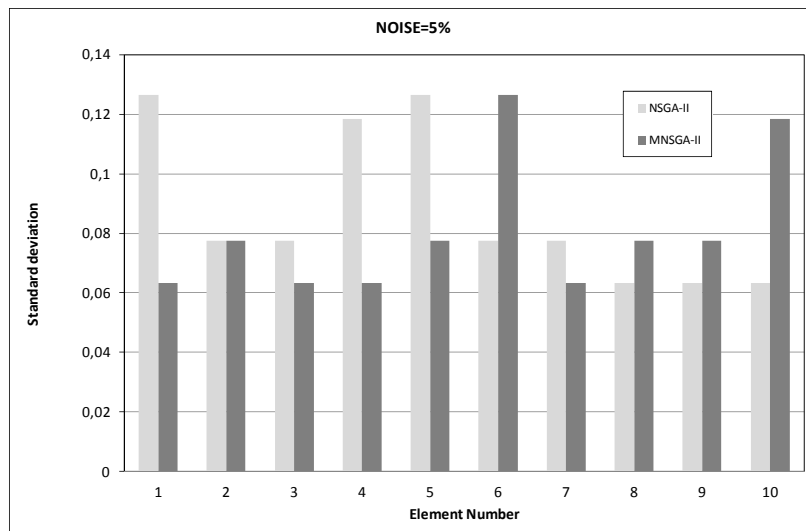


Figure 5. Standard deviation – 5% noise

Results demonstrate the improvement obtained when the MNSGA-II is used.

Figs 6 and 7 show the probability of damage existence (PDE) for the different elements and for levels of noise 1% and 15% by using the procedure shown in Section 3.

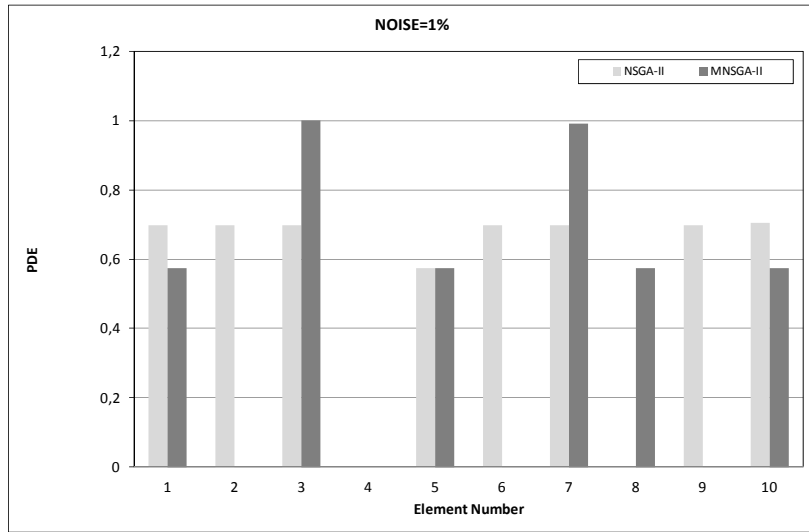


Figure 6. Probability of damage existence – 1% noise

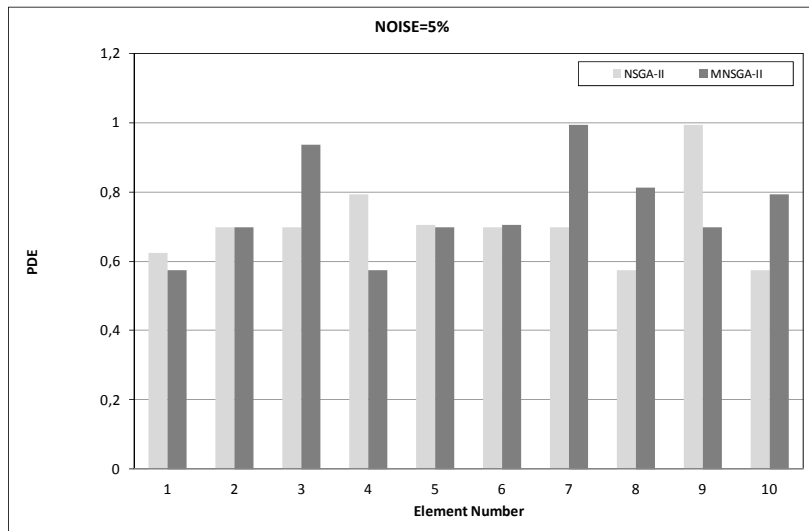


Figure 7. Probability of damage existence – 5% noise

## 6 Conclusions

A statistical multiobjective structural damage detection algorithm is developed in this paper. The uncertainties existing in the structural model and measured structural modal parameters have been taken into account. The probability of damage existence can also be obtained based on the probability density functions of structural stiffness parameters in the intact and damaged state. The method presented has been verified using a numerical study on a simple supported beam with different levels of noise.

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