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The Coupling of Robust Metamodel and Heuristic Methods in Reliability Based Design Optimization

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Abstract

The context of this paper is the establishment of help tools to aid in the decisions for robust design of mechanical structures in an uncertain environment. The problem considered is how to determine an optimized structure with regard to the security level criteria. This problem is called reliability optimization; that means the optimization is with regard to the constraint reliability. With a falling probability level, the engineer can decide if the structure has sufficient reliability.
This paper presents a coupling between the methods of reliability approximation (FORM, SORM) and a set of modern techniques from the domain of the artificial intelligence which are characterized by their importance in the process of collection and analysis of the uncertainties.
Among these techniques, the particular swarm, the genetic algorithms and the ant colony techniques should be mentioned. In this paper, a coupling of the reliability method and dynamics with metamodels (condensed models) to guide reliability optimization of such systems is proposed. Two numerical examples are presented to illustrate the performance of the proposed approach.

Keywords: FORM, SORM, robust design, optimization, heuristic methods, reduced model.

1 Introduction

In order to find the optimal design of structure submitting to certain constraints, the optimization methods are widely used in the field of the conception in particular that of the reliability of dynamic structures. The objective about it is often to minimize the total weight of a structure under some constraint such as the deformation, the maximum constraint, the eigenfrequency etc. Meanwhile, the parameters of conception contain systematically few uncertainties. The reliability optimization permits to find an optimum solution of structures by simultaneously satisfying a

certain level of reliability. Besides, the strategy to solve the reliability optimization is by effecting an inverted calculation for which we realize the optimization by taking into account the accepted or desired level of reliability.

The first works which treat the reliability optimization problem are based on few analyses of sensibility [1], where one considers the failure probability as a constraint. Other methods are based on Kühn Tucker's theorem [2, 3]. These methods are effective, but are not easier because generally it is obligatory to know the random variables as to analytically calculate the derivatives of metrical systems in relation with their variables. Monte Carlo simulation [4] is the easiest and exactest of stochastic methods. However, this method has a disadvantage as the calculation time is too long.

In this context, a strategy of reliability optimization which combines the classic approximation methods [5, 6], condensed models [7] and the heuristic optimization algorithms (Particule Swarm Optimization PSO [8, 9]) is proposed. As a result, one defines an objective function (cost function or profit function), which one seeks to optimize (minimize or maximize) in relation to all parameters (or degrees of freedom) concerned keeping in mind the reliability indication among the constraint functions of the problem. This procedure constitutes the objective of the work done in this article which is in fact a coupling between the optimization and the reliability analysis. The idea is how to make a design with a level of acceptable reliability. Within the aim of showing the performance and efficiency of this strategy, the applications for the minimization of mass are presented by requiring a target reliability indication.

2 Optimization by Particle Swarm method

The optimization by particle swarm is a relatively recent and competitive optimization method for the continuous functions, introduced by Russel Eberhart and James Kennedy [8, 9] in 1995 in the USA under Particle Swarm Optimization (PSO) label.

2.1 Principle and formalization

The essential ethics in PSO are very easy. A group of particles in movement (the swarm) is spread initially inside the space of the research. Each particle has the following characteristics:

• Each particle is able to evaluate the quality of its position and of keeping in memory its best performance.

• Each particle is able communicate with several of its neighbours (exchange of inform) and obtaining from each among them its best proper performance.

• At every pace in time, each particle chooses the best of the best performances with which it has knowledge, modifies its speed due to this information and its proper premises, and displaces as a consequence.

Once the best informant is detected, the behaviour of a given particle is a compromise between three possible choices (Fig 1):

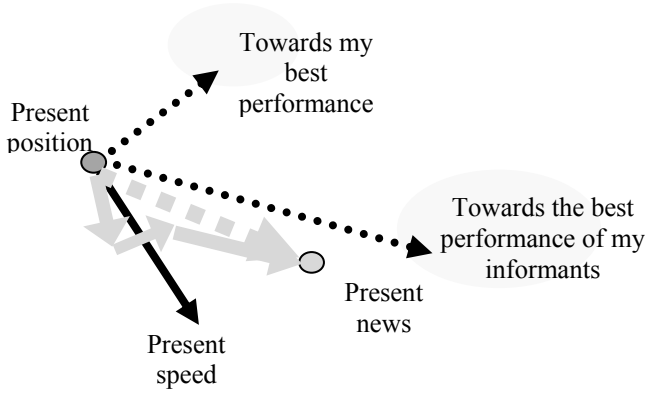

Fig 1: Principle of displacement of a particle to realize its following movement

• follow its proper track
• go towards its best position already found
• approximately orienting towards the best position corresponds to the best information

This compromise is formalized by the following equations:

$$\begin{cases} v_{t+1} = c_1 v_t + c_2 \left(p_{i,t} - x_t \right) + c_3 \left(p_{g,t} - x_t \right) \\ x_{t+1} = x_t + v_{t+1} \end{cases} \tag{1}$$

with:
v_t : speed in pace of time t,
x_t : position in pace of time t,
$p_{i,t}$: best anterior position in pace of time temps t,
$p_{g,t}$: the best of the best attained positions in proximity to pace of time t,
c_1, c_2, c_3 : the confidence coefficients balancing the three possible directions.

In almost applications, the confidence coefficients are hazardously chosen in each pace of time in a known interval.

2.2 The proximity

The proximity describes between which particles it exist an exchange of information. There are two great methods: whether a geographic proximity (the most

nearby particles) which is rarely used and which require a re-calculation at each pace in time or a « social » proximity which is the most used and which is defined at once.

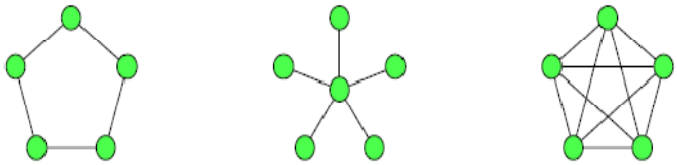

Fig 2: Example of proximity.

- Topology in star: each particle is adjoining each particle. The optimum of proximity is therefore the optimum of each swarm
- Topology in chain: each particle communicates with n (habitually 3) other particles
- Topology in rays: all the particles communicate only with a unique central particle

3 Reliability optimization based on the heuristic methods

Due to the development of the FORM/SORM approximation methods, we can calculate the failure probability without going through Monte Carlo method [4]. In the FORM/SORM method, we must first of all change all the random variable in normalized space. Moreover, by means of the limit-state information of the failing scenario, we can look for the point of conception which is defined by the point the most nearby to the origin. The hypothesis of the FORM method comes back to replace the surface of real limit state with a hyper plan in point of conception [10 - 12].

Finally, the approximation of the failure probability is defined by:

$$P_f = \Phi(-\beta) \tag{2}$$

where P_f is the failure probability $\Phi(\cdots)$ is the distribution function of the normal central small scale law. β is Hasofer Lind's reliability index, defined by the minimum distance of the limit state to the origin of the central small scale space.
Indeed, the reliability analysis allows us to calculate the failure probability of the structure by taking into consideration the uncertainties of the design parameters. In addition, in the field of engineering, it will be interesting to realize the conception of the structure respecting the level of acceptable reliability [13 - 15]. We arise, in the frame of this work, the question of minimizing the mass of a structure with a respected level of reliability. We present, here, the reliability optimization problem in which we realize the calculation of optimization taking into account the constraint upon the failure probability [16].

3.1 Position of the problem

We show interest in optimizing the structure by assuring the maximum of failure probability. The problem considered can then be written.

$$\begin{gathered} min \sum_i \rho V_i \\ under \\ p_f \leq \hat{p}_f \quad or \quad \beta \geq \hat{\beta} \end{gathered} \tag{3}$$

where V_i, $\hat{p}_f$ and $\hat{\beta}$ are respectively the volume of the structure, the failure probability and the reliability index to be respected. Reliability analysis shows that the failure probability is implicitly determined by $\hat{d}$, the displacement of the structure is due to excitation force. Indeed, to accomplish the optimization calculation of equation 3, it is necessary to find the criterion of displacement so that the structure satisfies the reliability criteria. We take into consideration the uncertainties about the design variables.

Under the hypothesis of the FORM method (equation 2), the displacement criterion $\hat{d}$ which satisfies the reliability criterion $\hat{\beta}$ can be calculated starting from the information of the limit state.

$$\begin{gathered} Maximised \\ \hat{d} \\ such\ as \\ min \sqrt{\sum_i u_i^2} = \hat{\beta} \quad with\ G(u_i) = 0 \end{gathered} \tag{4}$$

where u_i are the random variables in the normalized space.

Furthermore, after having calculated the maximum displacement criterion by the application of heuristic algorithms of Particle Swarm Optimisation (PSO) which corresponds to the reliability constraint, we realize the modification of the optimisation problem (equation 3). It is to be noted here, that the criteria of displacement $\hat{d}$ which satisfy the reliability criterion $\hat{\beta}$ and later on the failure probability to be respected, we can realize the new optimization problem of the structure:

$$\begin{gathered} min \sum_i \rho V_i \\ under \\ d \leq \hat{d} \quad and \quad w \geq \hat{w} \end{gathered} \tag{5}$$

Finally, our problem becomes a classic optimization problem under the moving and frequency constraints. So, we use the heuristic methods for this step of calculation and for the determination of the optimum weight which translate the required reliability [16].

4 Numerical simulations

4.1 Example 1

The studied structure is a plane plate represented by the figure 3. Its finite element model contains 3978 degrees of freedom (dofs).

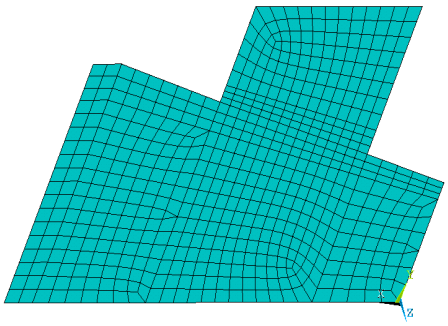

Fig 3: Plate FEM.

The dynamic analysis is realized in the frequency band [0–300 Hz] already fixed and containing the 9 first eignmodes. The observation point of the response is chosen at the same dof of the localized excitation.

For this problem, we realize the optimization of the mass under the failure constraint such as the failure probability is chosen $\hat{P}_f = 10^{-3}$, which corresponds to the reliability index $\hat{\beta} = 3.090$.

The deterministic, random, geometric and mechanic characteristics are grouped in the tables 1 and 2:

Parameter /variable		Value
Force	F	1 N
area of the plate	A	0.344 m^2
Poisson's ratio	υ	0.3

Table 1. Deterministic variables

Component		mean value	Standard deviation
Young's modulus	E	3.6e09 N/ m^2	3.6e08 N/ m^2
Density	ρ	1650 kg/ m^3	165 kg/m^3
Thickness	e	20 mm	2 mm

Table 2. Uncertain variables

The optimum of this problem can therefore be calculated in two steps, for each step of algorithm used is that of PSO: for the first step, according to the information of the failure scenario:

$$G = \hat{d} - \left(-\omega^2 M + (1 + j\eta) K\right)^{-1} \times F \tag{6}$$

where K, M, and F are respectively the stiffness matrix, the mass matrix and the excitation force.

The failure probability depends on the displacement constraint for the first eigenfrequency f = 50 Hz. We must therefore find the values of displacement which satisfy the reliability constraint by applying the maximization presented by the following equation.

$$\begin{gathered} Maximised \\ \hat{d} \\ such\ as \\ min\sqrt{u_e^2 + u_E^2 + u_\rho^2} = \hat{\beta} \quad with \\ G = \hat{d} - \left(-\omega^2 \left(1 + \frac{\sigma_e}{\mu_e} u_e\right)\left(1 + \frac{\sigma_\rho}{\mu_\rho} u_\rho\right) M + (1 + j\eta) K \left(1 + \frac{\sigma_e}{\mu_e} u_e\right)^3 \left(1 + \frac{\sigma_E}{\mu_E} u_E\right)\right)^{-1} \times F \end{gathered} \tag{7}$$

where $\mu_\rho, \sigma_\rho, \mu_E, \sigma_E$ and μ_e, σ_e are respectively the mean and the standard deviation of the density, of Young's modulus and of the thickness. u_P, u_E and u_e are respectively the random variables of uncertain parameters in normalized space.
The resolution of the maximization problem gives: $\hat{d} = 2.6mm$ as displacement constraint which corresponds to the reliability constraint $\hat{P}_f = 10^{-3}$.

In the second step, we realize the optimization problem under a displacement constraint $\hat{d} = 2.6mm$ and a frequency constraint f = 50 Hz, presented by the following equation:

$$
\begin{gathered}
min \sum_i \rho e_i A_i \\
under \\
d \leq \hat{d} \quad and \quad w \geq \hat{w}
\end{gathered}
\tag{8}
$$

Parameters		Initial value	Optimal value	
Young's modulus	E	3.6e09 N/ m^2	μ	σ
			3.96e08 N/ m^2	8.01e7 N/ m^2
Density	ρ	1650 kg/ m^3	1485 kg/m^3	80 kg/m^3
Thickness	e	20 mm	16.6 mm	1mm
The mass of the plates optimized from 11.352 Kg to 8.480 Kg				

Table 3. Results of optimization

The presented results in table 3 show that the proposed method allows us to find the optimal uncertain design parameters which produce the optimum weight of the structure with respect to a fixed reliability index.
This procedure is interesting as the design of the structure requires the optimum position which satisfies the reliability constraint in order to find the optimum design.

4.2 Example 2

The following example is extracted from [17]. It is about a GARTEUR excited by forces. The observation points of the response are chosen in same dof of excitation. Its finite element model comprises 10584 dofs. This structure is devised into five components (figure 4). The model of components is presented in table 4.

	MEF	Junction dof
SS_1	756	36
SS_2	756	36
SS_3	4104	72
SS_4	3636	108
SS_5	1332	36

Table 4. Finite element model of the components

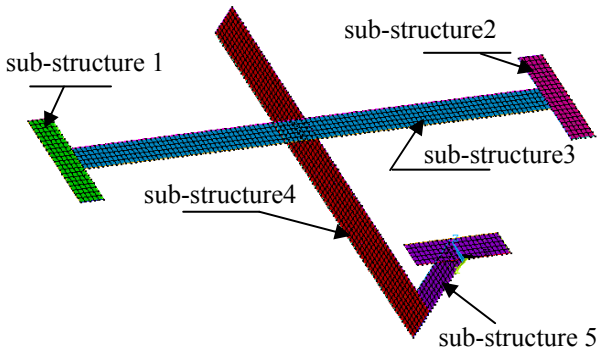

Fig 4: GARTEUR.

The deterministic geometric and mechanic characteristics are grouped in table 5. The variation coefficient of random variables is equal to 10%.

In a direct calculation of variability of responses or in the optimization procedure, we propose in this example, to use the method based on a stochastic metamodel [7]. It has the advantage of reducing the analysis cost of uncertainties classically obtained by the MC method. The integration of this method in the robust optimization procedure with respect to uncertainties leads to a drastic reduction of time calculation all by conserving a level of sufficient precision.

	Deterministic variables			Random variables		
				mean value		
	F (N)	A (m^2)	Y	E (N/m^2)	ρ (kg/ m^3)	e (mm)
SS_1	1	0.4	0.3	70e9	2700	10
SS_2	1	0.4	0.3	70e9	2700	10
SS_3	1	0.225	0.3	70e9	2700	10
SS_4	1	0.2	0.3	70e9	2700	10
SS_5	1	0.07	0.3	70e9	2700	10

Table 5. Random and deterministic variables

Through component modal synthesis method of Craig-Bampton's (CB), the initial model in 10584 dofs is brought back in a condensed model in 412 dofs (41 dofs of SS_1, 41 dofs of SS_2, 72 dofs of SS_3, 63 dofs of SS_4, 51 dofs of SS_5, 144 dofs of junctions). After the enrichment of Craig-Bampton's transformation basis by 50 random static residues, one obtains a new condensed robust model in 462 dofs (CBE).

In this example, we look for optimize the mass of the structure by assuring the maximum of failure probability $\hat{P}_f = 10^{-6}$, which corresponds to the reliability index $\hat{\beta} = 4.753$. The problem considered can therefore be written:

$$\begin{gathered} min \sum_i \rho A_i e_i \\ under \\ p_f \leq \hat{p}_f \quad or \quad \beta \geq \hat{\beta} \end{gathered} \tag{9}$$

where $\hat{P}_f$ and $\hat{\beta}$ are respectively the failure probability and the reliability indication to be respected. Besides by applying the FORM method, we can transfer the reliability criterion to the displacement value of the structure. Indeed, this displacement corresponds to the value of the failure probability desired. One considers the uncertainties, under the random variables (Table 5).

Like previous problem, we study two steps of calculation. For the first step, according to the information of the failure scenario:

$$G = \hat{d} - y(\omega) \tag{10}$$

with $y(\omega)$ is the forced reply of the structure due to the excitation force:

$$y(\omega) = Y\left(\left(1 + j\eta\right)\Lambda - \omega^2 I\right)^{-1} Y^T F \tag{11}$$

where Λ is the spectral matrix and Y is the modal matrix.
By replacing $y(\omega)$ with its expression in the equation (10) one obtains:

$$G = \hat{d} - Y\left(\left(1 + j\eta\right)\Lambda - \omega^2 I\right)^{-1} Y^T F \tag{12}$$

To find the displacement values that satisfies the reliability constraint by applying the maximisation below:

$$\begin{gathered} Maximised \\ \hat{d} \\ such\ as \\ min\sqrt{u_e^2 + u_E^2 + u_\rho^2} = \hat{\beta} \quad with \\ G = \hat{d} - Y\left(-\omega^2\left(1 + \frac{\sigma_e}{\mu_e}u_e\right)\left(1 + \frac{\sigma_\rho}{\mu_\rho}u_\rho\right)I + \left(1 + j\eta\right)\Lambda\left(1 + \frac{\sigma_e}{\mu_e}u_e\right)^3\left(1 + \frac{\sigma_E}{\mu_E}u_E\right)\right)^{-1} Y^T.F \end{gathered} \tag{13}$$

The maximisation problem solution gives: $\hat{d} = 1.6\text{mm}$ moving constraint which corresponds to the reliability constraint $\hat{P}_f = 10^{-6}$.

Then, we carry out the optimization problem under the moving constraint $\hat{d} = 1.6\text{mm}$ and a frequency constraint $f_1 = 7.29Hz$.

$$\begin{gathered} min \sum_i \rho A_i e_i \\ under \\ d \le \hat{d} \quad and \quad w \ge \hat{w} \end{gathered} \tag{14}$$

The optimization results are shown in the table 6.

With the results, the optimization method can therefore optimize the studied structure under the reliability constraint.

Parameters		Initial value	Optimal value	
Young's modulus	E	70.3e9 N/m^2	μ	σ
			70e8 N/m^2	5.4e7 N/m^2
Density	ρ	2690 kg/m^3	2700 kg/m^3	155 kg/m^3
Thickness	E	10 mm	8.5 mm	1mm
The weight of GARTEUR is optimized from 15.467 Kg to 13.19 Kg				

Table 6. Optimization results

5 Summary

Our purpose was to study the structures optimization considering reliability constraint. We adopt in this study the heuristic algorithms, based on biological phenomena. These algorithms allow the reduction of the mass of the structure under static constraints matching the maximum location and a dynamic constraint corresponding to the minimum eigenfrequency. Besides, this algorithm is used to solve the reliability optimisation problem in which the reliability constraints are replaced with its static equivalent.. The simulation results show the convergence to solve the optimization subject to the reliability constraints. We demonstrate in this

study, using two classic optimisation problems the displacement constraints of the eigenfrequency, that the decomposition of an optimization problem subject to the reliability constraints can be used to correctly solve the reliability optimisation problem. These works contribute to the structure robust concepts by considering the reliability problem.

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