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Nonlinear Finite Element Modelling of Welded Pinned Connections with and without Web Angles

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Abstract

Nonlinear finite element analyses of various welded beam-to-column seated connections were performed in the current study. Effects of various components such as size, length and position of web framing angles, top or seat angles, bottom stiffeners and side weld of top angle were investigated. Analyses results were utilized to derive the moment-rotation curves required for determination of the degrees of rigidity of the joints. Stress distribution and yield propagation in connecting element were also investigated. In many cases, presumably simple connections exhibited substantial amount of rigidity. Results showed that the joint rigidity increase as elements such as web angles or bottom stiffeners are added to the joints.

Keywords: finite element, nonlinear, web angles, seated connections, rigidity, weld, moment-rotation curves.

1 Introduction and Objective

No accurate analysis of steel structures can be achieved without a precise approximation of the rigidity of the beam to column connections. However it is a common practice to idealize the moment-rotation characteristic of the connections as either simple or fixed, but actual structures do not behave in such a perfectly rigid or a fully hinged manner. The idealization that a beam to column joints was either fixed or simple makes the tasks of analysis and design process greatly simplified hence enabling design engineers to utilize typical details widely used as a hinged or fixed connection.
Design codes recommend that the effects of the behaviour of the joints on the distribution of internal forces and moments within a structure and on the overall

deformations of the frame should generally be taken into account but where these effects are sufficiently small they may be neglected. To identify whether the effects of joint behaviour on the analysis need to be taken into account, three types of connections are recognized in standard codes of practice, e.g. simple, fully restrained (FR) and partially restrained (PR) moment connection [1].

The most common types of simple connection which is designed to transfer only the vertical shear reaction use either web framing angles or seated connection with a top angle to transmit negligible bending moment and preserve beam stability. The combination of web angles and seated connection are not normally practised for design and construction of the simple joints, hence little information is existed regarding the rigidity of such connections. The current research is aimed to examine the moment-rotation characteristic and nonlinear behaviour of such basically "simple" connections with added component which makes them susceptible of being "semi rigid". Such understanding is particularly useful when seismic performance of an existing building with simple connections is being evaluated and upgrade to improve their seismic resistance is required. In addition to simultaneously using web angles and seated connections, variation of other elements like thickness of top angle and length of top angle side weld may also contribute to the rigidity of initially pinned joints. It may be mentioned that some of these elements can be unintentionally existed in the joints due to construction errors; therefore a thorough investigation of the effects of the components which may possibly change the behaviour of a "simple" connection to "semi-rigid" is undertaken in the present study.

In 1937 Lyse & Gibson [2] performed experimental tests on full scale specimens of beam-to-column top and seat welded angle connections. In this study they investigated the effect of various parameters such as the size and length of angles or welds and also the failure modes of different configurations.

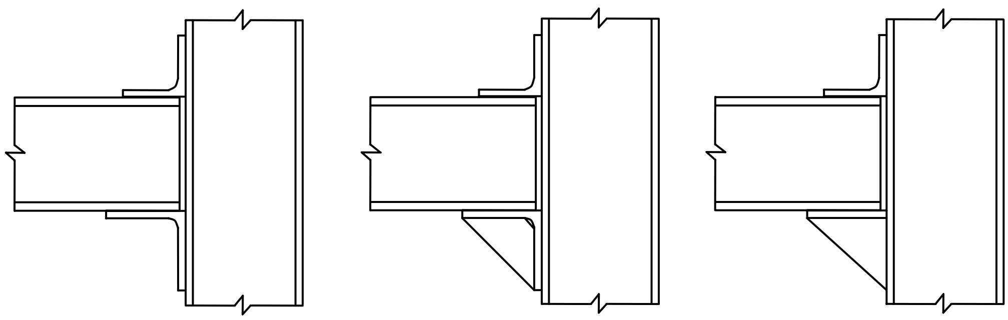
In 1999 Abolmaali [3] studied the nonlinear dynamic behaviour of 5 types of semi-rigid joints to derive the moment-rotation curves of the joints under cyclic loads. Effect of various components including angles and connecting welds or bolts on the joint's flexural behaviour was studied and the best matching numerical models were identified to illustrate the moment-rotation characteristics of each joint type. The results from this study were used for the preparation of a computer program to model the nonlinear dynamic behaviour of frames with semi-rigid joints.

Other researchers including Calado & Mele [4], Degertekin [5] and Akbas [6] also conducted studies on the behaviour of various semi-rigid joints. These studies were used to compare the different response of frames with semi-rigid and rigid joints.

2 Configurations of connections

The current study deals with the behaviour and flexibility of common beam-to-column welded pinned joints subject to static loads. For this purpose 3-dimensional models of top and seat angle connections were analysed and their moment-rotation curves were formed.

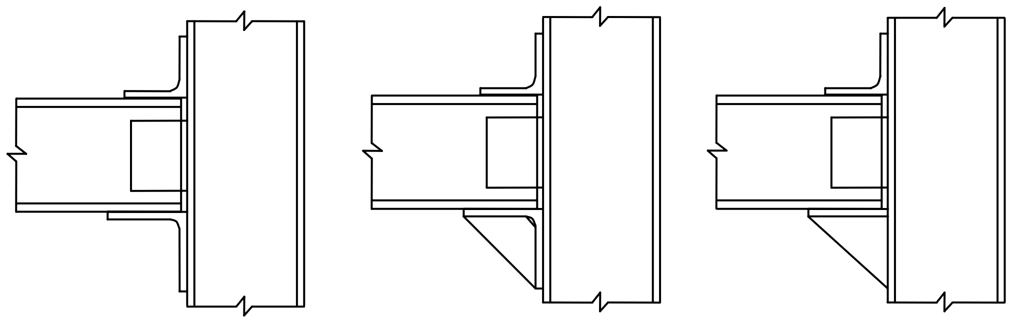
Two connection types were chosen for analysis in this study: Flexible angle connections and stiffened angle connections. These connections were classified into 6 main groups which are illustrated in figure 1.
Each of these groups represent variations of the basic simple joint which are created applying some changes in their components such as the addition of web angles or bottom stiffeners, the changes in the length and size of side welds or the size of top angles, the length and position of web angles, the replacement of bottom angles with seat plates, etc.



a. flexible seated connection-Type A

c. stiffened seated connection-Type C

e. stiffened bottom plate connection-Type G



b. flexible seated connection with web angles-Type B

d. stiffened seated connection with web angles -Type D

f. stiffened bottom plate connection with web angles -Type H

Figure 1. Connection types

Type A represents the basic flexible seated connections which consist of top and bottom angles (Fig 1.a). Type C models as illustrated in figure 1.b are a set of stiffened seated connections. In this connection type, a triangular plate is added to stiffen the seat angle and finally, type G is a variation of type C where the bottom angle is replaced with a horizontal plate stiffened by a triangular stiffener. (Fig 1.c)
Connection types B, D and H are variations of groups A, C and G respectively with double web angles added to each system. (Figures 1.b to 1.f)
One of the other important parameters studied here is the length of the side welds connecting top angles to the beam and column flanges. In an ideal pinned

connection the top angle must allow the free rotation of the connected beam. In the models analysed, the effect of the length of these weld lines on the flexibility of the joints is also investigated.
The details of the connections modelled for analysis are listed in table 1.

Connection	Top A.	Bot A.	Web A.	Web A. L.	Bot Pl.	Bot Stf.	Side Weld L.
A1a	100*10	120*12	-	-	-	-	2t=16
A1b	100*10	120*12	-	-	-	-	L/2=50
A1c	100*10	120*12	-	-	-	-	L=90
A51	80*8	120*12	-	-	-	-	40
A52	120*12	120*12	-	-	-	-	60
B1b	100*10	120*12	60*6	120	-	-	50
B1c	100*10	120*12	60*6	120	-	-	90
B3b	100*10	120*12	60*6	60-mid	-	-	50
B3b-1	100*10	120*12	60*6	60-top	-	-	50
B51	80*8	120*12	60*6	120	-	-	40
B52	120*12	120*12	60*6	120	-	-	60
C1a	100*10	120*12	-	-	-	100*10	16
C1b	100*10	120*12	-	-	-	100*10	50
C1c	100*10	120*12	-	-	-	100*10	90
C4a	100*10	120*12	-	-	-	100*10@2	90
D1a	100*10	120*12	60*6	120	-	100*10	16
D1b	100*10	120*12	60*6	120	-	100*10	50
D1c	100*10	120*12	60*6	120	-	100*10	90
D3a	100*10	120*12	60*6	60-mid	-	100*10	16
D3a-1	100*10	120*12	60*6	60-top	-	100*10	16
G1a	100*10	-	-	-	130*120*12	120*110*10	16
G1b	100*10	-	-	-	130*120*12	120*110*10	50
G1c	100*10	-	-	-	130*120*12	120*110*10	90
G1d	100*10	-	-	-	130*120*12	-	16
H1a	100*10	-	60*6	120	130*120*12	120*110*10	16
H1b	100*10	-	60*6	120	130*120*12	120*110*10	50
H4b	100*10	-	60*6	120	130*120*12	120*110*10@2	50
H1c	100*10	-	60*6	120	130*120*12	120*110*10	90

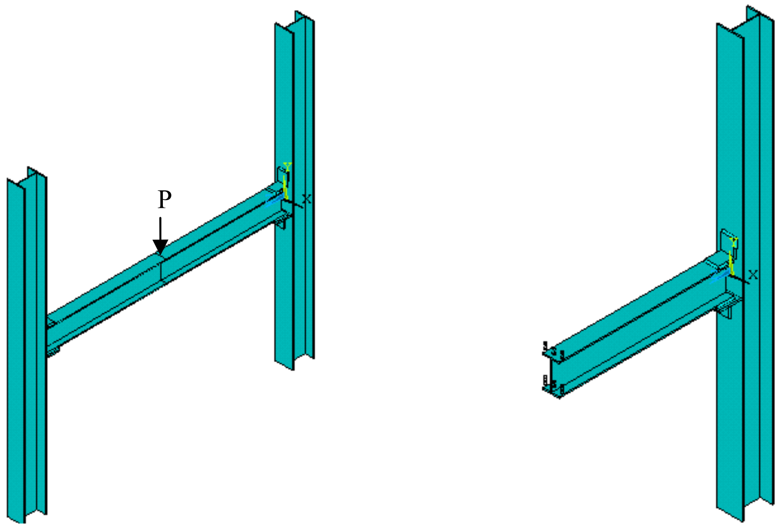
* Top A: Top angle, Bot A: Bottom angle, Web A.: Web angle, Web A. L.: Length of web angles, Bot Pl: Bottom plate, Bot Stf: Bottom stiffener, Side Weld L: length of side welds in top angles

Table 1. Connections' Details

3 Modelling for FE analysis

Models created in the FE software consist of IPE180 beams of 2 meters length that are connected to two 2.4-meter-long columns at two ends. The translational degrees of freedom in the two ends of the columns are restrained in all 3 directions but the

method used in restraining the end nodes in the column webs allows for the end rotation. Beams are connected to the columns using the joints described in section 2. (Fig. 2)
Three-dimensional nonlinear FE models for the simple connections displayed in figure 2 are generated using commercial FE software [7]. Due to the presence of a plane of symmetry along the beam mid span, only half of the structure has been considered for modelling and analysis (Fig 2.b)



a. Complete model b. Symmetrically halved model

Figure 2. Beam and column setting in FEM models

3.1 Elements used in modelling

Beam, column and joint components including angles, stiffeners and welds were modelled using 3D, 8-node solid elements with 3 degrees of freedom at each node.
In order to simulate the interaction of the beam and column flanges and angles, 3D point-to-point contact elements were used. These elements represent two surfaces which may maintain or break physical contact and may slide relative to each other. They are also capable of supporting compression in the direction normal to the surfaces and shear in the tangential direction. The element has three degrees of freedom at each node.
Fillet weld was used for connecting angles to beams and columns. These weld lines which were also modelled using solid elements were meshed with tetrahedral and quadratic elements. Mesh sizes were adjusted at different sections of the areas to be joined to each other in order to simulate the bonds created along the weld lines. The presence of adjacent elements and neighbouring nodes in the weld, beam, column and angle elements allowed for the merging of common nodes and simulation of weld lines.
Figure 3 shows an example stiffened seated angle connection model assembled in the FEM analysis using the aforementioned elements. Since the accuracy of analyses

and the duration of time for each solution procedure are both important in modelling, different mesh sizes were used in models. For instance in the areas near the joints finer mesh was used providing the derivation of results more precisely while in less significant areas bigger elements were employed creating coarse meshed areas.

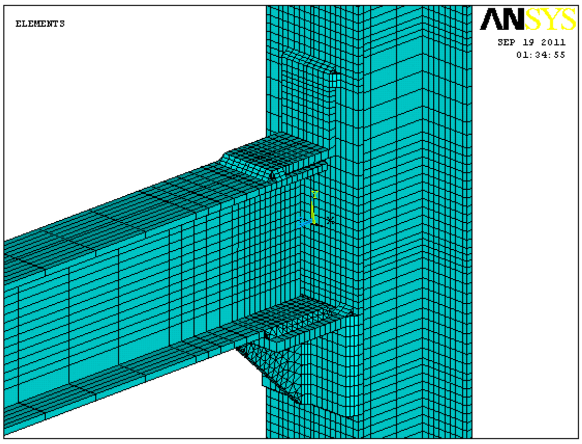


Figure 3. Meshing of model C1a

3.2 Material properties

Material properties of both steel and weld elements were defined using bilinear stress-strain diagram with the initial slope of the diagram taken as the module of elasticity $E=2100\ kg/mm^2$. This slope is decreased after the yielding stress point equal to $24\ kg/mm^2$ to 0.01 of the initial stiffness in the second line representing the post-yield stiffness.
Geometrical nonlinearity was considered in the modelling to account for large displacements. Both linear isotropic and bilinear kinematic hardening material properties were defined for the elements used.

3.3 Loading and solution

Models were analysed in the FE program under static load which was applied incrementally in a number of steps defined to satisfy the post processing demands and prevent divergence. The concentrated Force, P was applied in the mid span of the beam in Y direction. The same solution assumptions were used for all the models and loading continued until the point where excessive deformation or rotation was observed or the results showed signs of divergence. After reaching this point the loading was stopped as further steps required more complex solution procedures which were irrelevant to the objectives of the current study.
Newton-Raphson method was used in the solution procedure which uses line search technique to achieve rapid convergence utilizing both force and displacement norms.

In order to evaluate the accuracy of the FE models, results from an experimental study of welded connections performed by Mazroee et. al. [8], were used as the basis of a verification test. The model was produced with boundary and loading conditions similar to the experimental specimen and the results were compared. The connection modelled in the software exhibited a higher degree of rigidity which was expected considering the differences in presumptions about initial boundary conditions and material and geometric properties. The amount of agreement between the verification and experimental models served the purposes of the current study. Further details are presented in [9].

4 Derivation of moment-rotation curves

Following the FE analysis, stresses and the vertical displacements observed in the joint area were used to draw moment-rotation curves needed for the determination of the degree of rigidity in each joint. In order to collect this information, values of the beams' deformations and stress in the joint area were extracted from the program's post processing results at a section selected close enough to the column face to make sure the values of the beam's flexural deformation remain negligible. The chosen point must also be distanced from the stress concentration zones so that the results remain reliable and unexaggerated. For this purpose values of stress at different beam sections were evaluated at a distance between h and h/2 from the column face (h being the beam height) in order to choose the best section meeting the aforementioned demands.

The relative vertical deformations in the beam's top flange at the same area were then converted to rotations having their specific distance from the column face. The average value of the stresses imposed on the top and bottom flanges at the same section were also obtained and used for calculating moments. The Moment-Rotation curves were drawn along with the beam line which was plotted relative to the geometric properties of the 2-meter long IPE180 beam subjected to concentrated force (Represented as BLine in diagrams).

5 Degrees of rigidity

The main objective of this study has been to evaluate the degrees of rigidity in various connections and compare the effect of various components on this factor. The connections primarily classified as pinned were tested applying FEM and the moment-rotation curves elicited from their analysis were evaluated in relation with the beam-line diagram appropriate for the modelled beam. The resulting rigidity ratios are compared with the rigidity boundaries of 20% and 90% for pinned and rigid connections respectively. According to the well-known criteria defined by many design codes and many researchers [1 & 10], the connections with degrees of rigidity falling between 20% and 90% are classified as semi-rigid.

The models and their degrees of rigidity are listed in Table 2 the models rigidity varies from as low as almost 6% in the most flexible joint i.e. A1a to 60% in the stiffest configuration e.g. D1c.

Connection	Rigidity%	Connection	Rigidity%	Connection	Rigidity%
A1a	5.7	B51	16.8	D3a	19.9
A1b	16.3	B52	23.8	D3a-1	23.5
A1c	49.2	C1a	12.2	G1a	11.7
A51	12.1	C1b	30.3	G1b	29
A52	20	C1c	60.6	G1c	59.5
B1b	20.8	C4a	13.2	H1a	28.4
B1c	50	D1a	26.4	H1b	39
B3b	17.9	D1b	38.8	H4b	39
B3b-1	19.7	D1c	60.6	H1c	59.7

Table 2. Connections and degrees of rigidity

According to this table and the boundaries mentioned earlier the following models were classified as pinned:
A1a, A1b, A51, A52: Top and seat angle connections without web angles and with side welds of length 16mm and 50mm. In these models top angles were changed to both smaller and larger sizes as well.
B51, B3b, B3b-1: Top and seat angle connections with web angles and side welds of length 50mm. In B51 smaller top angle was used and in the other two models the length of web angles was decreased to half.
C1a, C4a, D3a and G1a all have side welds of length 16mm. C1a and C4a are top and seat angle connection with 1 and 2 bottom stiffeners respectively. D3a is a top and seat angle connection with bottom stiffener and web angles of half length. G1a is similar to C1a in all components except for the seat plate which is used instead of seat angle.
The rest of the models meet the requirements of the semi-rigid group.

6 Models and parameters to compare

6.1 Effect of web angles

In order to study the effect of web angles on the behaviour of the connections and their rigidity, models which are similar in all configurations except having web angles were compared. These models are listed in Table 3 in pairs.

A1b&B1b	A1c&B1c	A51&B51	A52&B52
C1a&D1a	C1b&D1b	C1c&D1c	
G1a&H1a	G1b&H1b	G1c&H1c	

Table 3. Models to compare for the effect of web angles

Figures 4 to 6 show the Moment-Rotation curves and degrees of rigidity derived for these models.
As it can be observed in the diagrams, all 3 pairs follow quite an analogous pattern, with the slope of the curves rising as the web angles are added. However the growth is attenuated as the overall stiffness of the joint increases. For instance in figure 5 the difference between the slope of curves for connection C1a and D1a is significant and the degree of rigidity in this pair has risen more than twice (12.2% to 26.4%). Also in the case of connections C1b and D1b, the growth is obvious though not as large as the previous pair (30.3% to 38.8%). In comparison with these two groups, the curves for joints C1c and D1c which belong to the stiffest configurations of this type, remain very close in the linear section. As the loading continues and the curves grow nonlinear, the difference gets easier to distinguish.

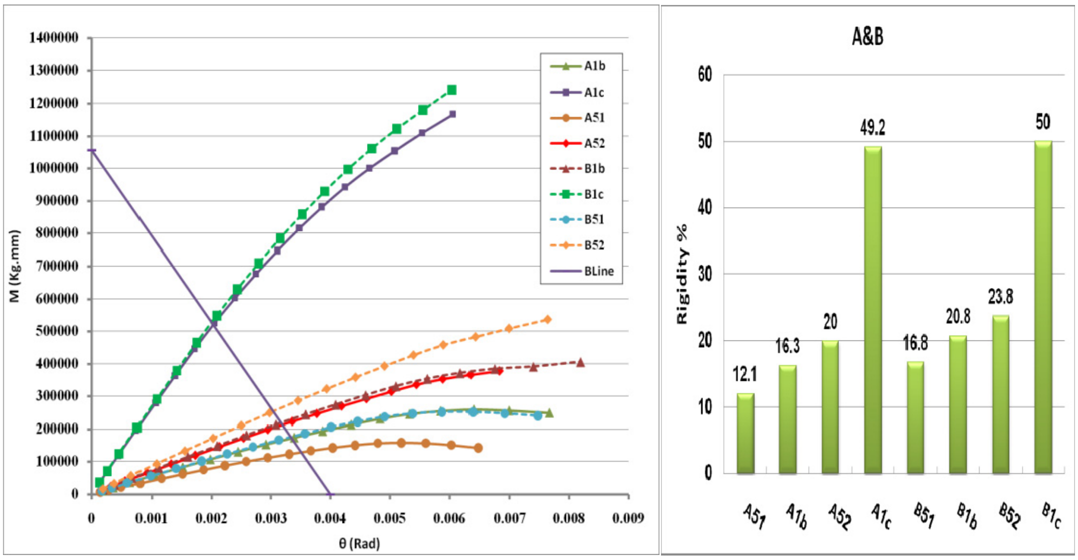


Figure 4. Connection types A&B varying in web angles

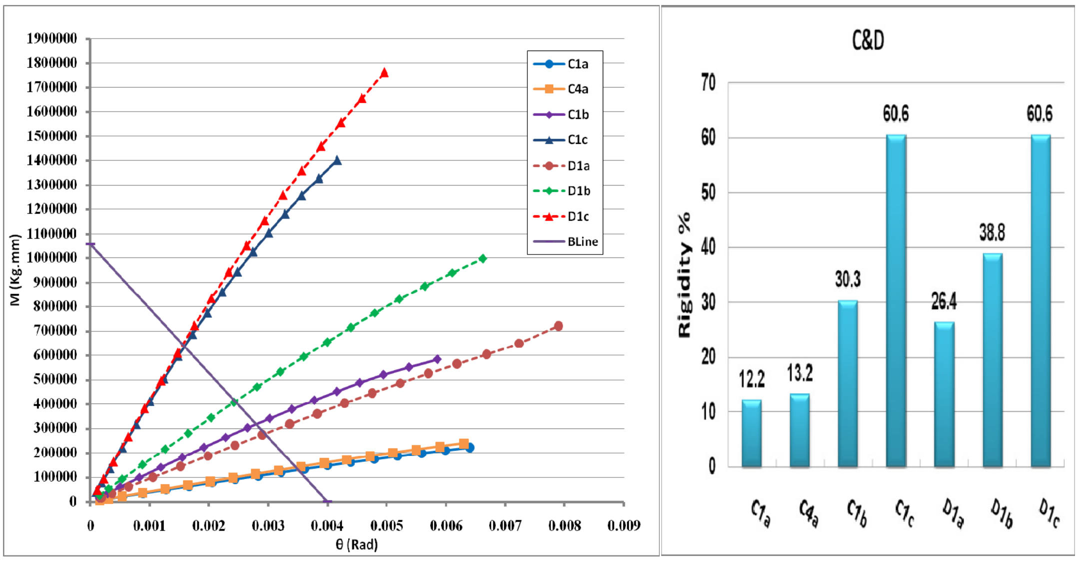


Figure 5. Connection types C&D varying in web angles

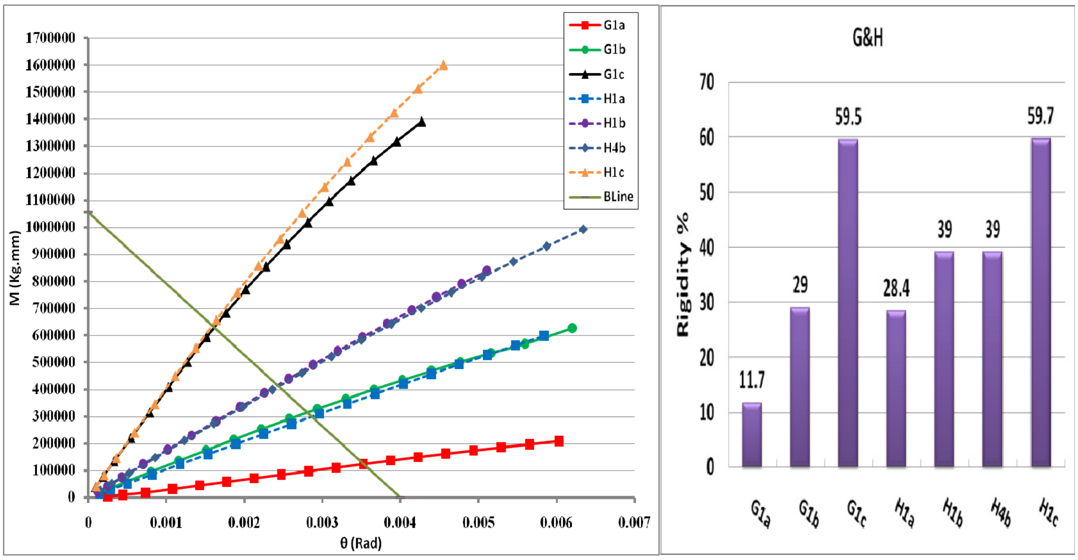


Figure 6. Connection types G&H varying in web angles

Comparing the curves, it is clear that certain connections, despite their varying configurations, display similar degrees of rigidity. For instance in figures 4 to 6 all the connections with full-length side welds posed quite the same degrees of rigidity in pairs e.g. A1c&B1c or C1c&D1c which means in full length side welds the addition of web angles did not change the overall rigidity of the connection significantly. Another example of such similarities was the case of two more flexible connections, G1b and H1a in which increasing the length of side welds to half was as effective as the addition of double web angles. Similar comparisons about the efficacy of other elements will be discussed in the following sections.

6.2 Effect of web angles length & position

In the analysis and design of welded web framing angle connections [10] special attention is directed toward the choice of leg size of connecting welds on the supporting and beam webs. Since the connection is assumed to act as pinned, the load determining the design criteria is the vertical reaction or shear. There are also considerations about the length of vertical weld lines connecting the web angles to the webs. It is suggested that this length should be long enough to prevent overstressing of beam web or support plate.
The objective in this section of the study is to find out whether changing the length of the web angles affects the connection's behaviour.
In this section, all the connections have web angles but they vary in length and position. In models B3b and D3a the length of the web angles is decreased to half and they are positioned in the middle of the web's height. In models B3b-1 and D3a-1 the halved web angles are situated in the upper half of the beam web.
In type B, connections remained quite similar in behaviour although when the web angles length was reduced to half and located in the upper half of the beam web (B3b-1), the degree of rigidity was very close to the original condition (B1b). When

the web angles were located in the middle position (B3b), the rigidity was less than the other two specimens.
In type D the differences between the curves are more noticeable but still a similar trend is maintained.
According to figure 7 it can also be concluded that in this condition, connections B1b, B3b-1 and D3a with their different configurations, seem to have a very analogous linear behaviour which means up to a certain point, changing some components in the joint such as the length of web angles doesn't have a remarkable impact on the joint's rigidity. At the same time applying some other changes can compensate for undesirable rigidity. For instance in connection D decreasing the length of web angles and positioning them in the middle of the beam web along with shorter side welds can put the curve close to that of joint B1b which is not stiffened in the bottom angle.

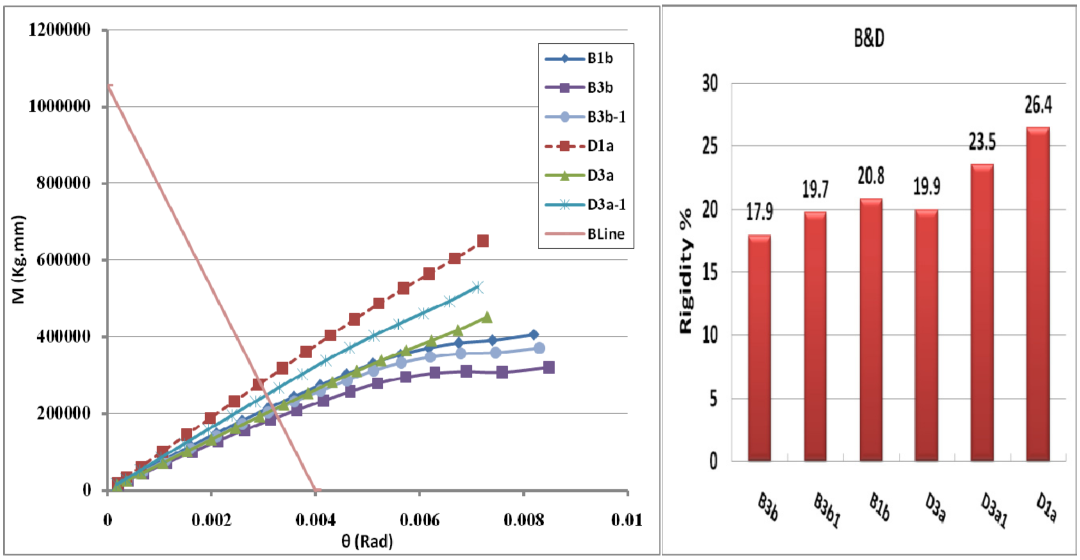


Figure 7. Connection varying in web angles' length and position

6.3 Effect of the size of top angle

Although common design codes state that top angles in pinned connections only is used for lateral stability, their effect on the joint's behaviour is quite significant. Basically, this angle must deform easily allowing the free rotation of the connected members and preventing the transmission of flexural moment. In this study three sizes of top angles (80×8, 100×10 and 120×12) were used in connection types A and B and the rigidity of connections in each setting was measured.
The models studied in the section are A1b, A51, A52 and B1b, B51, B52. The details of the components used in each model can be found in table 1.

Looking back at figure 4 which shows the moment-rotation curves derived from the analysis of models in groups A and B, it can be concluded that changing the top angle can affect the rigidity of the joint quite significantly. Apart from the conforming results in each group, the comparison of curves shows how the connections in groups behave similarly to the others when the components are altered. For instance when the 100×10 top angle in connection A1b (with no web angles) is replaced by a 120×12 angle of the same length in connection A52, it behaves like joint B1b which is a variation of A1b with double web angles. Likewise, when the top angle in a double web angle connection like B1b is weakened in B51, it can project the flexibility of an unstiffened seated connection without web angles like A1b.

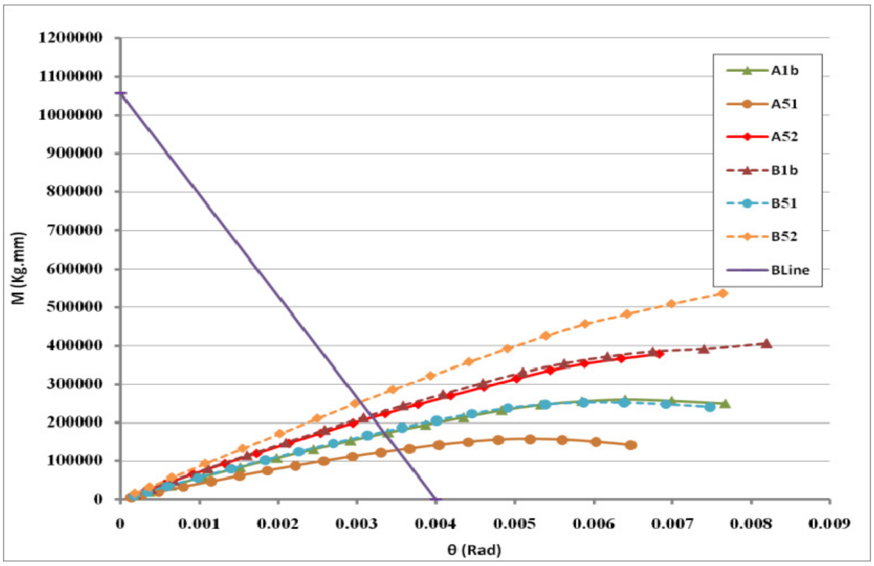


Figure 8. Models varying in top angle size

6.4 Effect of seat plates

The bottom angle in top and seat connections can be replaced by a seat plate and one or more stiffeners especially when heavy reaction force is being transmitted to the column.
In this part of the study the effect of this replacement on the rigidity of the connections is examined.
In both groups the substitution of bottom plates for bottom angles leads into negligible changes in connections' rigidity. As it can be observed in figures 9&10, in more flexible joints this alteration either increased the rigidity very insignificantly (D1a&H1a) or had no effect (C1b&G1b).

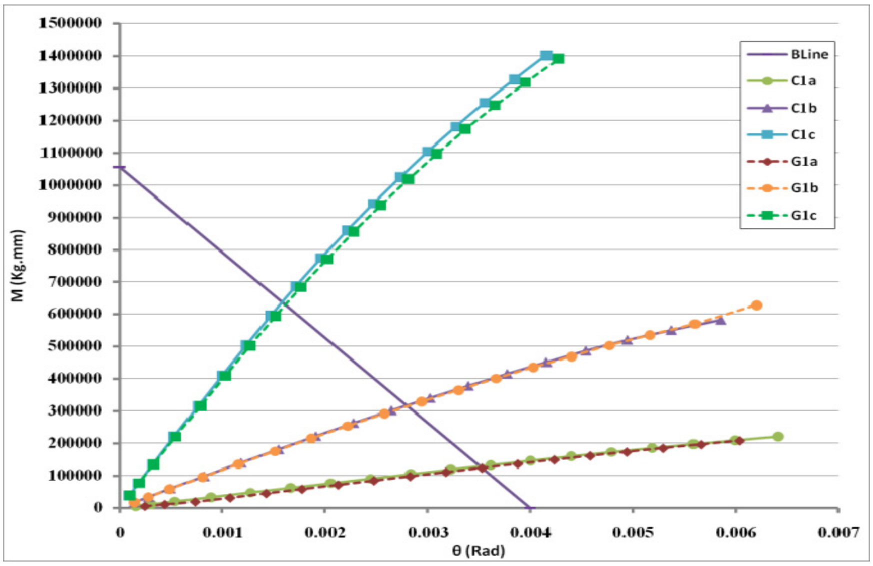


Figure 9. Models C&G with bottom plates

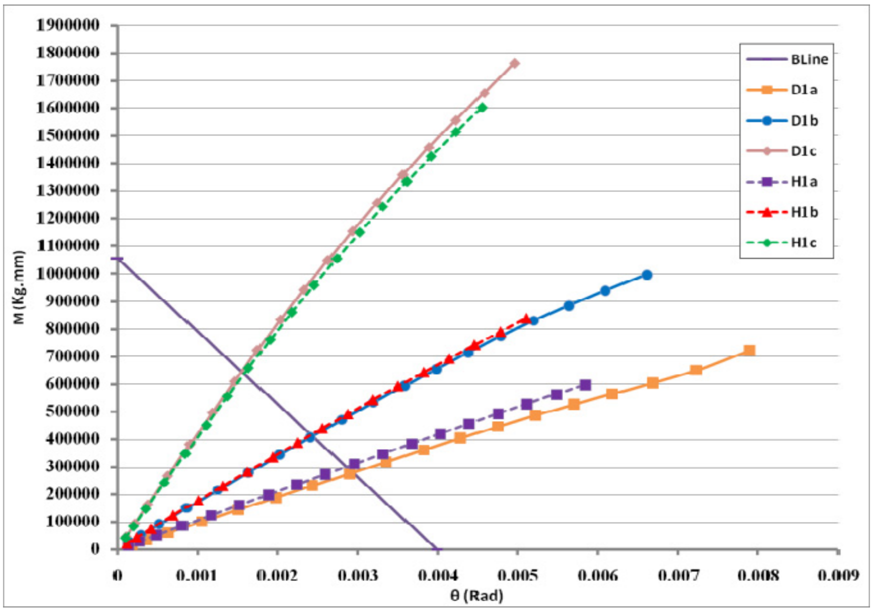


Figure 10, Models D&H with bottom plates

6.5 Effect of bottom stiffeners

One of the elements commonly added to the seated connections to enhance their stiffness is the triangular stiffener which is welded to the bottom angle. In order to study the influence of this element on the rigidity of seated connections, two pairs of connections with and without web angles are compared.

As expected, the addition of bottom stiffeners has improved the rigidity of joints in all cases. This alteration can even change a pinned joint to a semi-rigid one. (e.g. A1b&C1b)

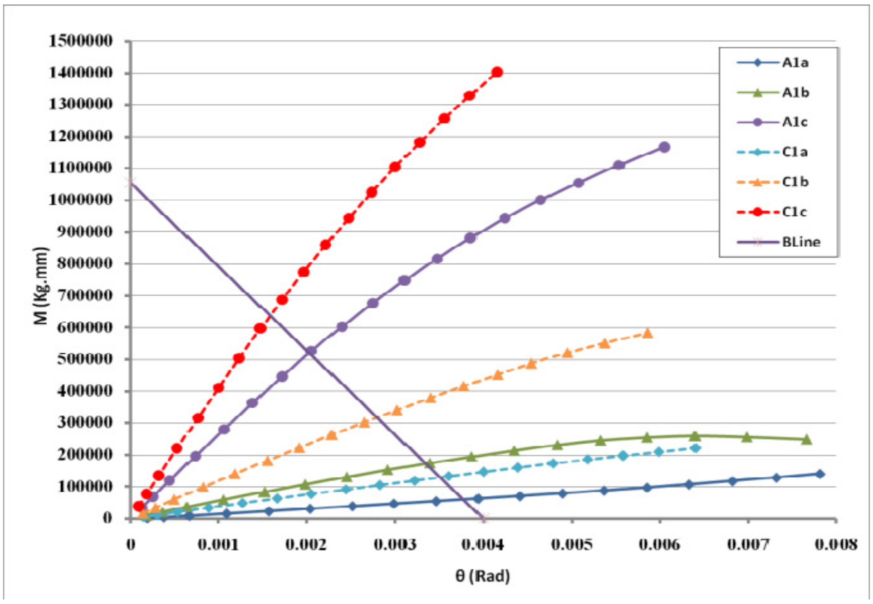


Figure 11. Effect of bottom stiffeners

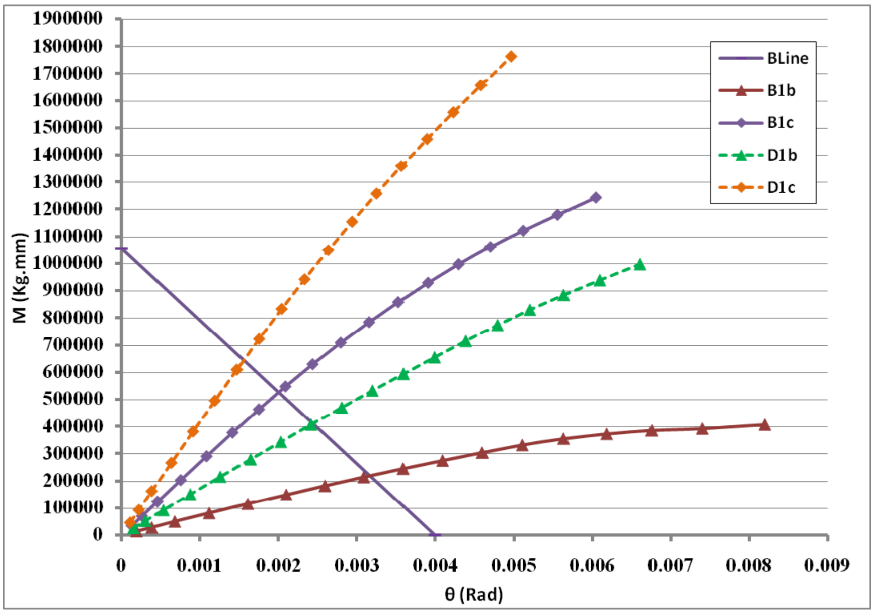


Figure 12. Effect of bottom stiffeners

In the case of connections C4a, H4b and H4c the seat angle or plate are stiffened with two stiffeners instead of one. This increase supposedly enhances the capacity of the joint in bearing vertical loads but it might also change the overall flexibility of the joint. The effect of using two bottom stiffeners on the rigidity of stiffened connections has been investigated. According to the results from this study, addition of bottom stiffeners has insignificant influence on the overall rigidity of the connection. The moment-rotation curves for these connections are illustrated in figure 13.

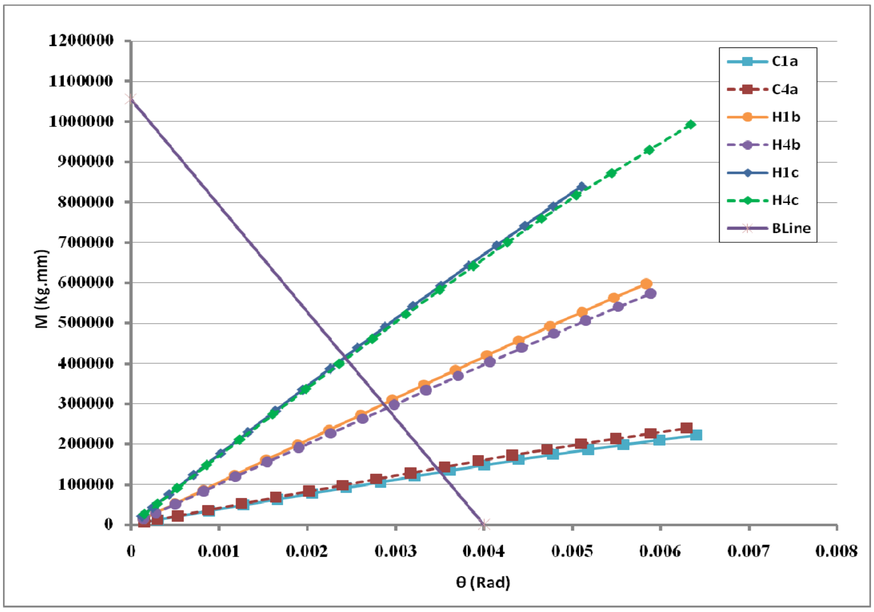


Figure 13. Models C&H varying in number of bottom stiffeners

7 Stress distribution and yield propagation

The other parameter studied is the yield stress and the way it is spread among elements. In this section the pattern in which elements reached yield point is compared in models in order to find out how different configurations contribute to load bearing capacity.

Generally in all models the first elements reaching yield stress are located at the beginning of the weld lines connecting top angles to columns. This can be due to the concentration of stress in this area. When the yielding spreads to the elements near the corner of the top angles and this area starts to deform remarkably, the elements located in this area undergo significant stresses. In figure 14 these deformations are illustrated for model A1a. In order to be visually noticeable, the deformation diagrams have been exaggerated 10 times in display.

In connections with the shortest side-welds e.g. A1a and D1a since these two stress concentration points are relatively distanced from each other, the distribution of stress in the critical yield area, i.e. angle corner, is uniform. In models with full-length side welds on the other hand, first the outer elements yield and then stress is distributed over the middle elements. The yield pattern in these areas is shown in figure 15.

The next point reaching yield stress after the top angle is the section in the middle of the beam span. At this section, elements in the beam flanges reach yield point uniformly after at least 70% of the final load is applied to the system.

In almost all models without web angles, the bottom angles start to yield nearly at the end of the loading process which is apparently due to limited rotations in that area.

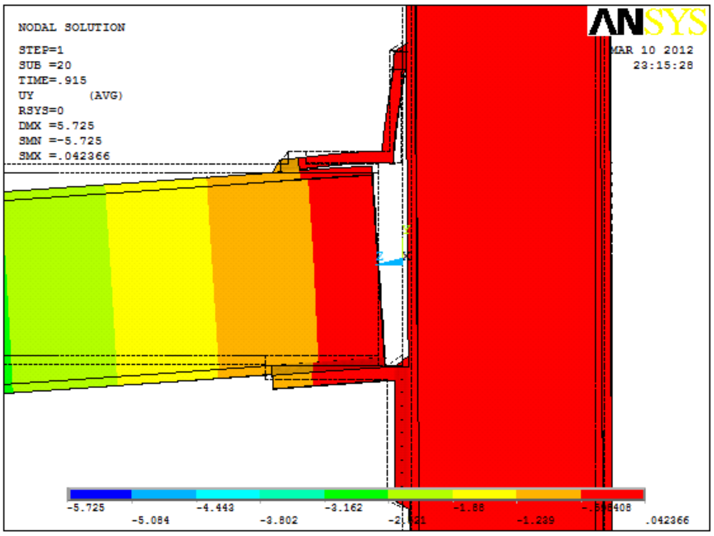


Figure 14. Deformation (Uy) in model A1a

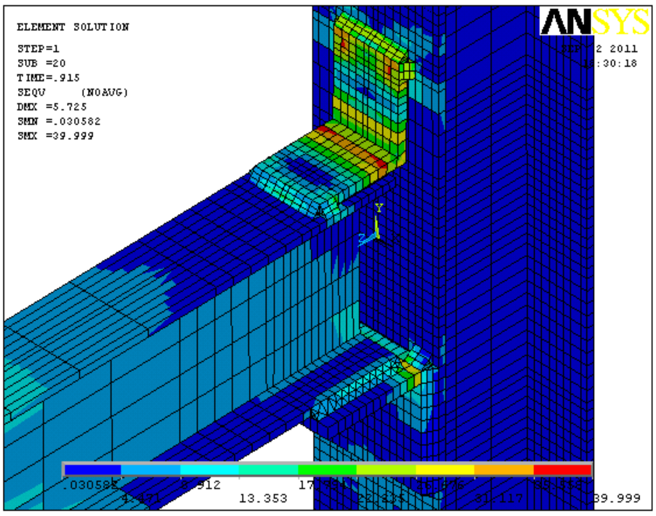


Figure 15. Von Mises Stress in elements in A1a

In the case of connections with web angles, when the length of side welds in top angles allows for deformations in the upper area of the joint, elements in the web angles start yielding sooner than the bottom angles. But in the case of more rigid models with full-length side-welds in top angles, the stress values in web angles remain negligible. When the length of web angles was shortened, very little changes were observed in the stress distribution and yield progress in various elements.

In stiffened seat connections as shown in Figure 16, stress in the lower part of the connection is mainly concentrated along the free edge of the stiffener and the beam's flange near that area and the values of stress in the seat angle elements remains small. In these models due to the high rigidity of the lower half of the joint, complete yielding only happens in the top angle and beam's mid span sections.

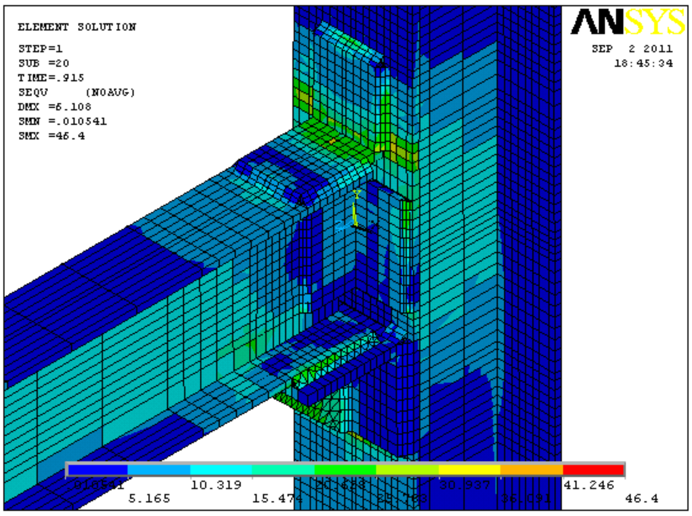


Figure 16. Von Mises Stress in elements in D1c

8 Conclusions

Nonlinear finite element models of various welded top and seat connections which are initially classified as pinned joints were analysed under static loads. The results from these analyses were used for deriving moment-rotation curves to evaluate the degrees of rigidity of the models. The effect of the changes applied to various components of the joints such as side welds, web angles, stiffeners and bottom plates on the flexural behaviour of the connections was investigated. According to the results obtained, many of these connections, when altered in details, do not behave as pinned joints. Instead, the degrees of rigidity they exhibit can place them in the semi-rigid joints category.

1. Addition of web angles to the simple or stiffened top and seat connections in all cases resulted in the increase of stiffness. However this increase was less significant in connections with higher degrees of rigidity.
2. Decreasing the length of web angles to half do not change the joint's stiffness significantly. When these halved web angles were located in the upper half of the beam web, the degree of rigidity remained very close to the original condition. When the web angles were located in the middle position the rigidity generally decreased.
3. Changing the size of top angles in the models can affect their rigidity quite significantly. In some cases using a larger top angle had similar effect on the connection's degree of rigidity as the addition of double web angles. Likewise, when the top angle in a top and seat joint with double web angles was weakened, the model projected the flexibility of an unstiffened seated connection without web angles.
4. The addition of bottom stiffeners improved the rigidity of joints in all cases. This alteration can even change a pinned joint into a semi-rigid one. On the other hand increasing the number of bottom stiffeners to two had insignificant influence on the overall rigidity of the connection.

5. Substitution of bottom plates for bottom angles leads into negligible changes in connections' rigidity.
6. Due to the concentration of stresses and the deformations that occur in the models, yielding started at the weld lines connecting top angles to columns. When the yielding spreads to the elements near the corner of the top angles and the angle starts to deform remarkably, elements in this are undergo significant stresses.
7. The length of side welds in the top angle affected the yield pattern and the uniformity of yield stress distribution in the critical areas of the angle.
8. In the models stiffened in the bottom area due to the high rigidity of the lower half of the joint, complete yielding only happened in the top angle and beam's mid span sections.
It is necessary to notice that the results listed here are obtained from a number of numerical models. Thus more adequate and definitive judgments can only be made after more precise examinations both experimentally and theoretically.

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