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Non-Linear Buckling of Conical Shells under External Hydrostatic Pressure

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Abstract

This paper presents a theoretical geometrical and material non-linear analyses of fifteen un-stiffened conical shells, using the finite element computer program ANSYS, to calculate their buckling pressures, for the first time. The cones buckled due to shell instability caused by the external hydrostatic pressure. Experimental tests were also carried to destruction of the fifteen vessels, under external hydrostatic pressure. The experimental results for the fifteen conical shells, were compared with multiple alternate theoretical methodologies, including ANSYS eigen-buckling, ANSYS nonlinear, together with other methodologies; which include PD5500 (BS5500), and the design charts of Ross. The ANSYS eigen-buckling results were poor; but the ANSYS nonlinear results were much more encouraging. PD5500 was too conservative, but Ross' design Charts were the best and the easiest to use. Whereas most of the conical shells failed by plastic shell instability, some of the failures were initiated by axisymmetric deformation; prior to plastic shell instability taking place. The PD5500 was hard to use and produced overly conservative results, which would lead to conservative pressure vessel designs, while the Ross Design Charts produced good results, and were very easy to use and understand and were not too conservative. The graphical displays from ANSYS were quite spectacular. For some of the ANSYS nonlinear analyses, it appeared that plastic axisymmetric yield initially took place at the larger ends of the cones; before triggering off, plastic shell instability.

Keywords: submarine pressure hulls, plastic shell instability, conical shells, ANSYS, PD5500.

1 Introduction

The world's economies depend largely on energy from fossil fuels and other sources, to continually expand and flourish our economies; and the form of energy of most demand is usually in the form of fossil fuels. While the western world is increasing its use of fossil fuels dramatically, emerging countries are becoming more and more interested in these fuels, to increase the size of their middle classes and to provide more of the population with privatised motor vehicles, etc., as private vehicles are more convenient than public transport [1]. As third and second world countries slowly become first world countries, by using fossil fuels, it is possible that Planet Earth will enter an energy resource crisis. With the supply of these fossil fuels decreasing above ground, research has been conducted into recovering the vast untouched supply of methane hydrates located at the very great depths of the oceans [2]. To access these untapped resources, submarine type pressure hulls will have to be designed to reach these great oceans depths, either to retrieve the fuels or to bury carbon dioxide, in an attempt to reduce the detrimental effects of global warming. These pressure hulls will be required to withstand the huge external pressures of the great depths of the oceans, the deeper that the vessels are submerged.

Now under uniform external pressure, a thin-walled circular conical shell can collapse due to shell instability as shown in Figure 1 [3].

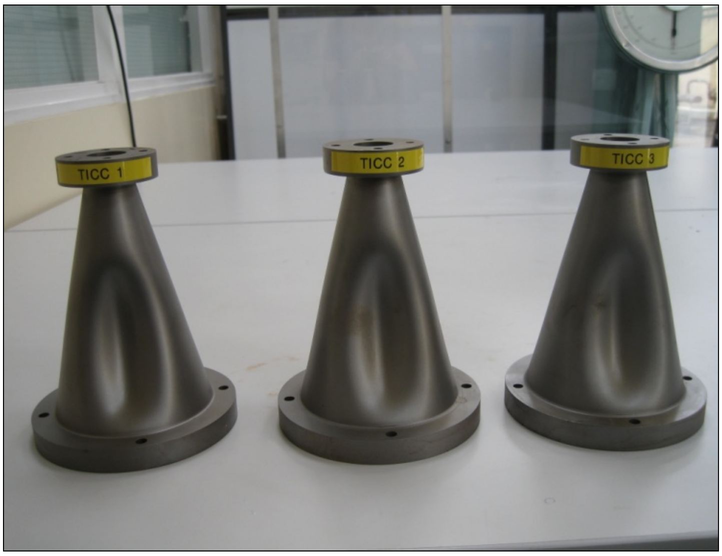

Figure 1: Shell instability.

Shell Instability [3 to 7], often occurs at a fraction of the pressure to cause axisymmetric buckling when a shell buckles into a number of circumferential lobes or waves as depicted in Figure 1.

This is a most undesirable mode of failure, as the collapse pressures can be extremely low and one way of improving their buckling resistance is to ring-stiffen the vessels in their flanks, as shown in Figure 2 [8].

If the ring-stiffeners are not strong enough, the entire ring-shell combination can buckle bodily in its flank, as shown in Figure 2. Another mode of failure of thin-walled conical shells, is through axisymmetric deformation, where the shell implodes inwards, keeping its circular form, as shown in Figure 3.

Figure 2: General instability of ring-stiffened circular conical shells.

Figure 3: Axisymmetric collapse.

In this paper, the finite element analysis program, namely ANSYS was used by the authors to model 15 unstiffened conical shells, together with other methods of analyses, including PD5500 and the Ross Design Charts [3, 4]. The said analyses, were also compared with experimentally obtained collapse pressures [3, 4]. This was done partly because the authors believed that PD5500 was overly conservative and may lead to an over-engineered design of a submarine pressure hull.

The ANSYS theoretical results for both elastic and plastic buckling were also compared with the experimental results, together the Ross Design Charts [3, 4]; which used a Plastic Knock Down factor (PKD) and a collapse pressure. These analyses were carried for the first time.

Previous work by Ross [3, 4] has developed a design chart listing a Plastic Knock down Factor. From this design chart a value of the appropriate PKD can be established, and then the actual experimental buckling pressure can be found by the following equations:

2 Theoretical Analyses

In these analyses the following notation was used.

a = mean radius of the cross section of the circular cylinder. R= a/cos (α)
l = length of unsupported shell in-between adjacent ring supports.
α = half cone angle. L= slant length = l/(cos(α))
λ = thinness ratio = $((L/2R)^2/(t/2R)^3)^{0.25} * (\sigma_{yp}/E)^{0.5}$
σ_{yp} = Yield stress = 250MPa E = Young's Modulus = 1.90e5MPa
ν = Poisson's Ratio = 0.3 (assumed)
P_{exp} = experimentally obtained buckling pressures. P_{cr} = von Mises buckling pressure.
t = thickness of shells wall. PKD = Plastic Knockdown Factor.

$$P_{exp} = \frac{P_{cr}}{PKD}$$

Then to find the final design pressure of the conical shell P_{des}, P_{exp} was divided by the relevant safety factor (SF):
$P_{des} = P_{exp} / SF$

To validate the results from the Ross Design Charts and PD5500, the commercial computer program ANSYS was used in this paper, as it was capable of carrying out both linear and nonlinear analyses; where the latter involved both material and geometrical nonlinear finite element analyses.

3 Experimental Work

The physical experiments were carried out by Ross [3, 4]; where a total of 15 thin-walled conical shells were tested to destruction. The conical shells were fixed to the top of a pressurised water tank and circumferential foil strain gauges were placed inside the circumference of each conical shell, at equal intervals, together with one strategically placed longitudinal strain gauge. The tank (Figure 4) was then filled with water and a hydraulic pump was used to slowly pressurise the tank. The pressure was increased in set increments and the strain was recorded at each increment. Eventually the conical shells collapsed and a collapse pressure could be read from the pressure gauge.

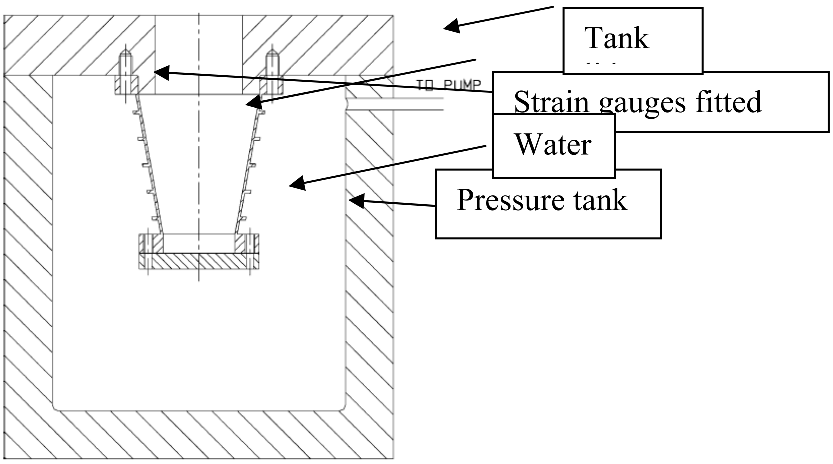

Figure 4: Experimental Set-up.

4 Computational Work

Each model was produced using ANSYS, as shown in Figure 5, and constrained at the large end, as shown in Figure 6.

After each cone was restrained, the external pressure had to be applied; where a value of -1MPa for the Eigen buckling analysis was used, as shown in Figure 7.

A typical Eigen buckling form is shown in Figure 8.

The deformed shape of each cone was displayed; where each cone buckled in a number of equally-spaced circumferentially lobes, in the shell instability mode of failure.

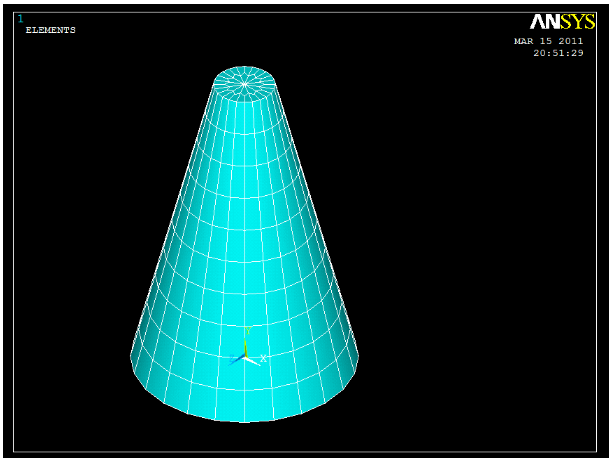

Figure 5: Mesh for cones, namely TICC series .

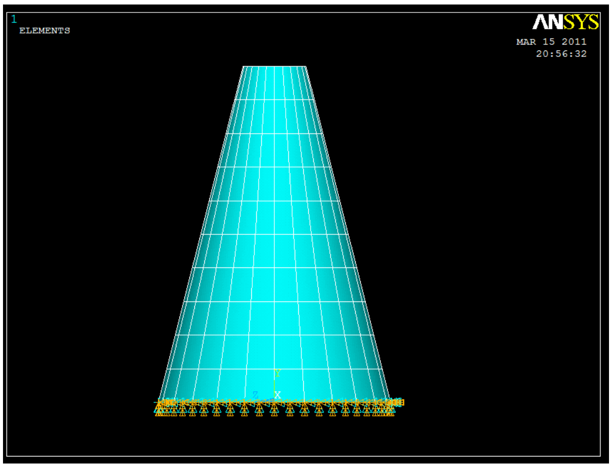

Figure 6: Constrained TICC cones

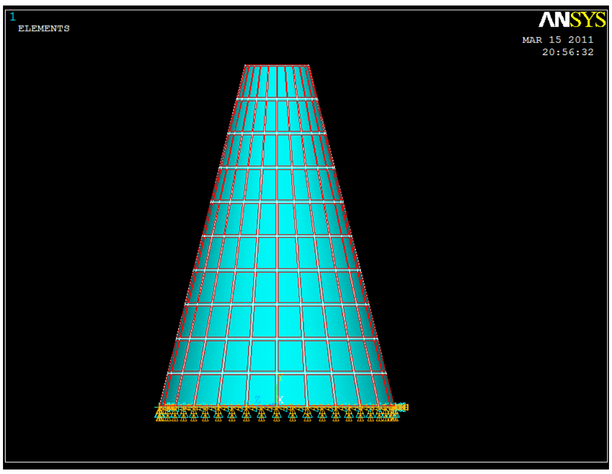

Figure 7: TICC 1; Constrained, with external pressure applied

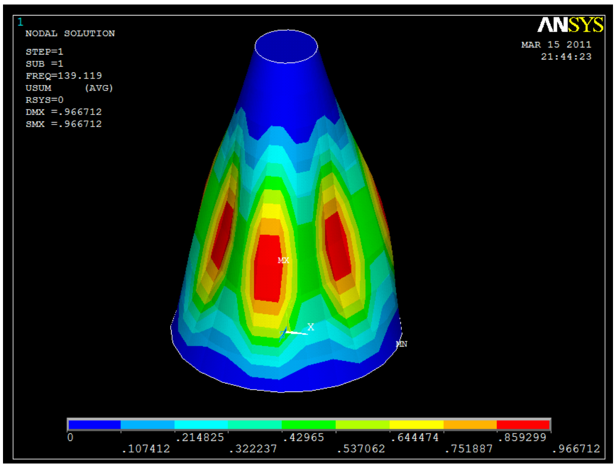

Figure 8: Eigen buckling results for TICC 1

4.1 Non-Linear Analyses

A deformed cone suffering geometrical and material nonlinearity is shown in Figure 9.

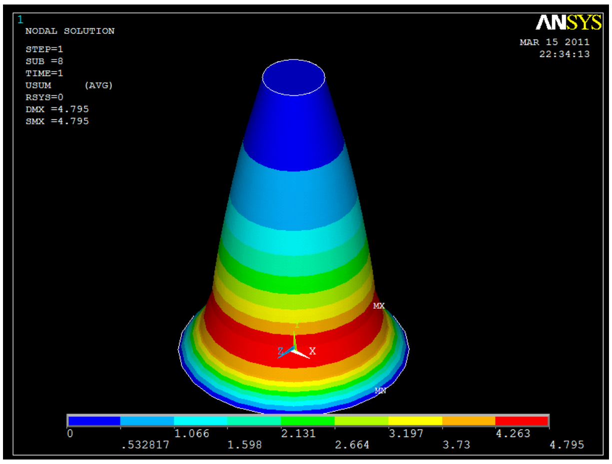

Figure 9: TICC 1, suffering non-linear deformation

Finally a graph was produced showing the historical displacement of a single node, in the region of greatest buckling, as shown in Figure 10.

The buckling pressure was read off of the graphs. The Y-Axis represented the Load Factor of the applied pressure, with 1 being 100% of the applied pressure (1 x 34MPa) and 0.1 representing 10% of the pressure (0.1 x 34 = 3.4MPa). The slope of the graph depended on which node was chosen to draw the graph and in which direction it displaced.

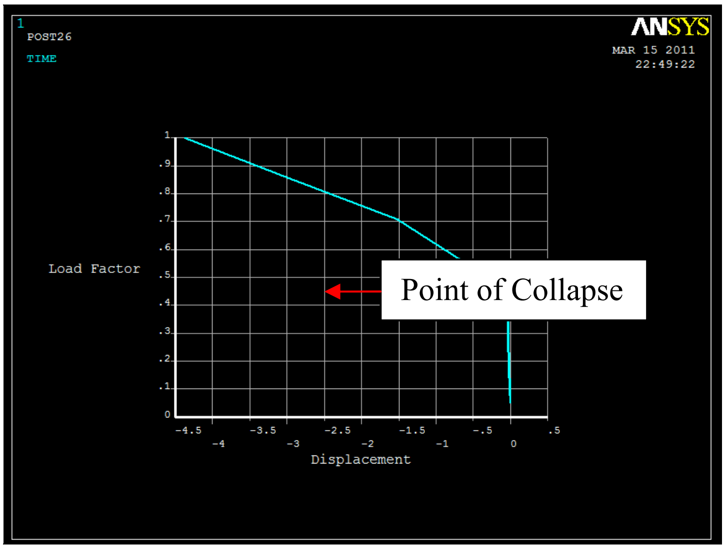

Figure 10: Load factor/displacement graph for TICC1

4.2 British Standards (PD5500:2000) (Figures 11 & 12)

PD5500 was used to calculate the collapse pressure for shell instability using its Section 3.6, namely, 'Vessels under external pressure'. The following calculations show the workings for calculating the collapse pressure for non-ring stiffened cones using PD5500. The following formulae are similar to the equations used for cylinders, but take into account the slant angle of the cone.

4.2.1 Notation for PD5500

P_{mc} = The von Mises [5] elastic instability pressure for collapse of conical section between stiffeners.
P_{yc} = Pressure at which mean circumferential stress in conical section between stiffners.
E = Young's Modulus of elasticity
e = Thickness of shell wall
ε = Mean elastic circumferential strain at
R_{mean} = The mean conical radius for a check on interstiffener collapse.
θ = The semi angle to the axis of a conical shell
n = An integer used in stiffener design calculations
$\grave{I}_c$ = Second moment of area of composite cross-section including stiffener and effective length of shell acting with it.
R_{max} = The maximum conical radius for a check on interstiffener collapse
R_{min} = The minimum conical radius for a check on interstiffener collapse
L = Effective unsupported length of shell.
L_s = Distance between heavy stiffeners
S = The factor relating *f* to effective yield point of material, which is taken as 1.4 for

carbon steels
f = Design stresses for shell; yield stress.
P = The required external design pressure.

4.2.2 Calculating P_{yc} for a typical vessel

P_{yc} was calculated, using the following equation:

$$P_{yc} = \frac{sfe\cos\theta}{R_{max}}$$

$$P_{yc} = \frac{1.4 \times 250 \times 10^6 \times 3 \times 10^{-3} \times cos14.1}{50.85 \times 10^{-3}}$$

$$P_{yc} = 20.027MPa$$

4.2.3 Calculating P_{mc}, for a typical vessel

P_{mc} was calculated using the following equation:

$$P_{mc} = \frac{E\varepsilon\varepsilon\cos^3\theta}{R_{mean}}$$

Where R_{mean} was the mean radius:

$$R_{mean} = \frac{R_{max} + R_{min}}{2} = \frac{50.85 + 12.67}{2} = 31.76mm$$

ε was determined using Figure 3.6-2 from the PD5500 using $\frac{L}{2R_{mean}Cos\theta}$ and $\frac{2R_{mean}Cos\theta}{e}$

For TICC 1:

$$\frac{L}{2R_{mean}Cos\theta} = \frac{156.7}{2 \times 31.763cos14.1} = 2.543$$

$$\frac{2R_{mean}Cos\theta}{e} = \frac{2 \times 31.763Cos14.1}{3} = 20.54$$

This gives a value for the Y-axis and lines for figure 3.6-2 in the PD5500, now values for ε can be read off:

$\varepsilon \approx 0.0055$

Once ε had been determined it was substituted back into the equation.

$$P_{mc} = \frac{190000 \times 3 \times 10^{-3} \times 0.0055 \times \cos^3 14.1}{31.763 \times 10^{-3}}$$

$$P_{mc} = 90.045MPa$$

4.2.4 Calculating P

Next a value for K was needed to calculate P.

$$K = \frac{P_{mc}}{P_{yc}}$$

$$K = \frac{90.045}{20.027} = 4.45$$

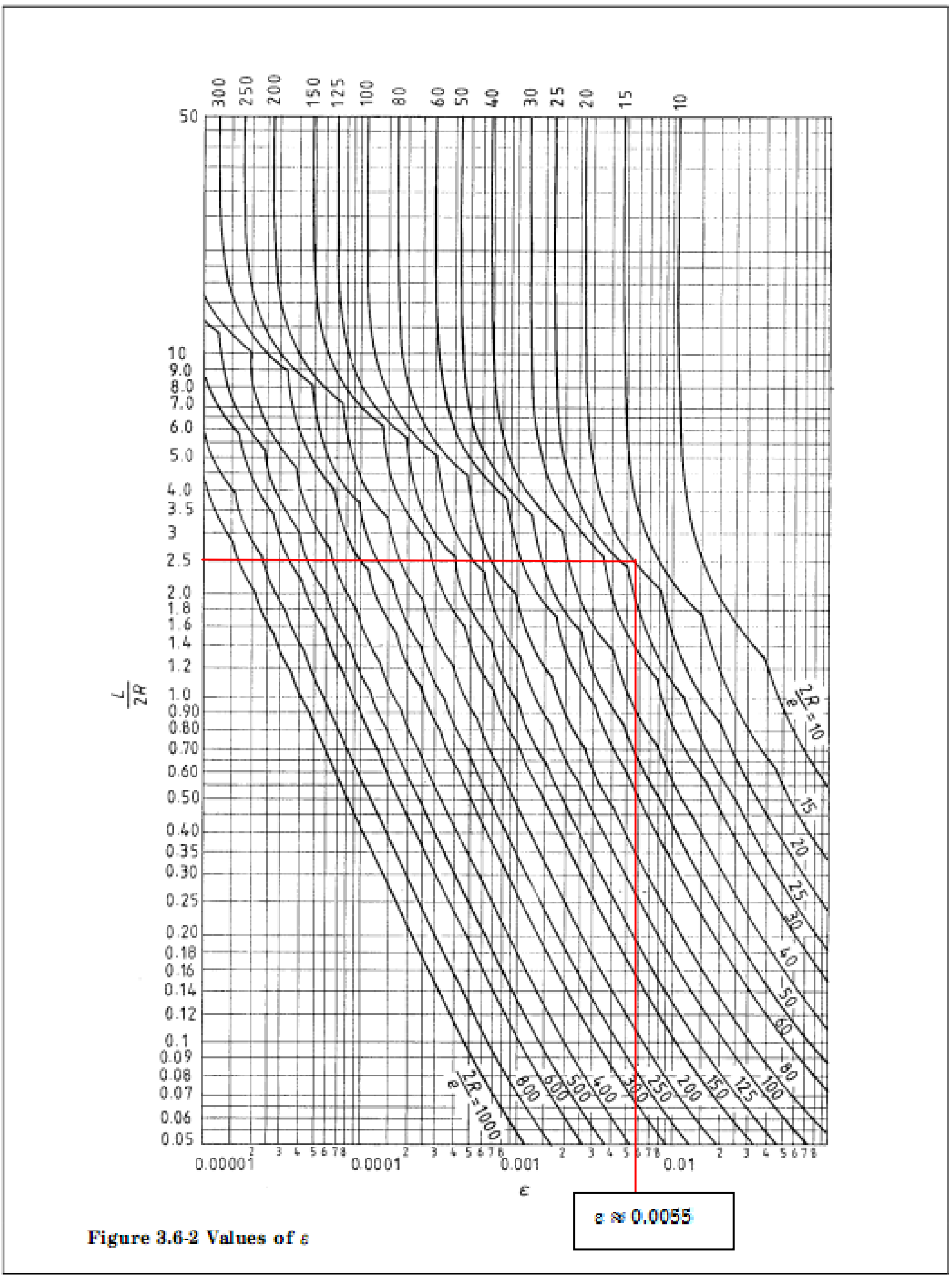

Figure 11: From PD5500's Figure 3.6-2.

Then using PD5500's Figure 3.6-4:

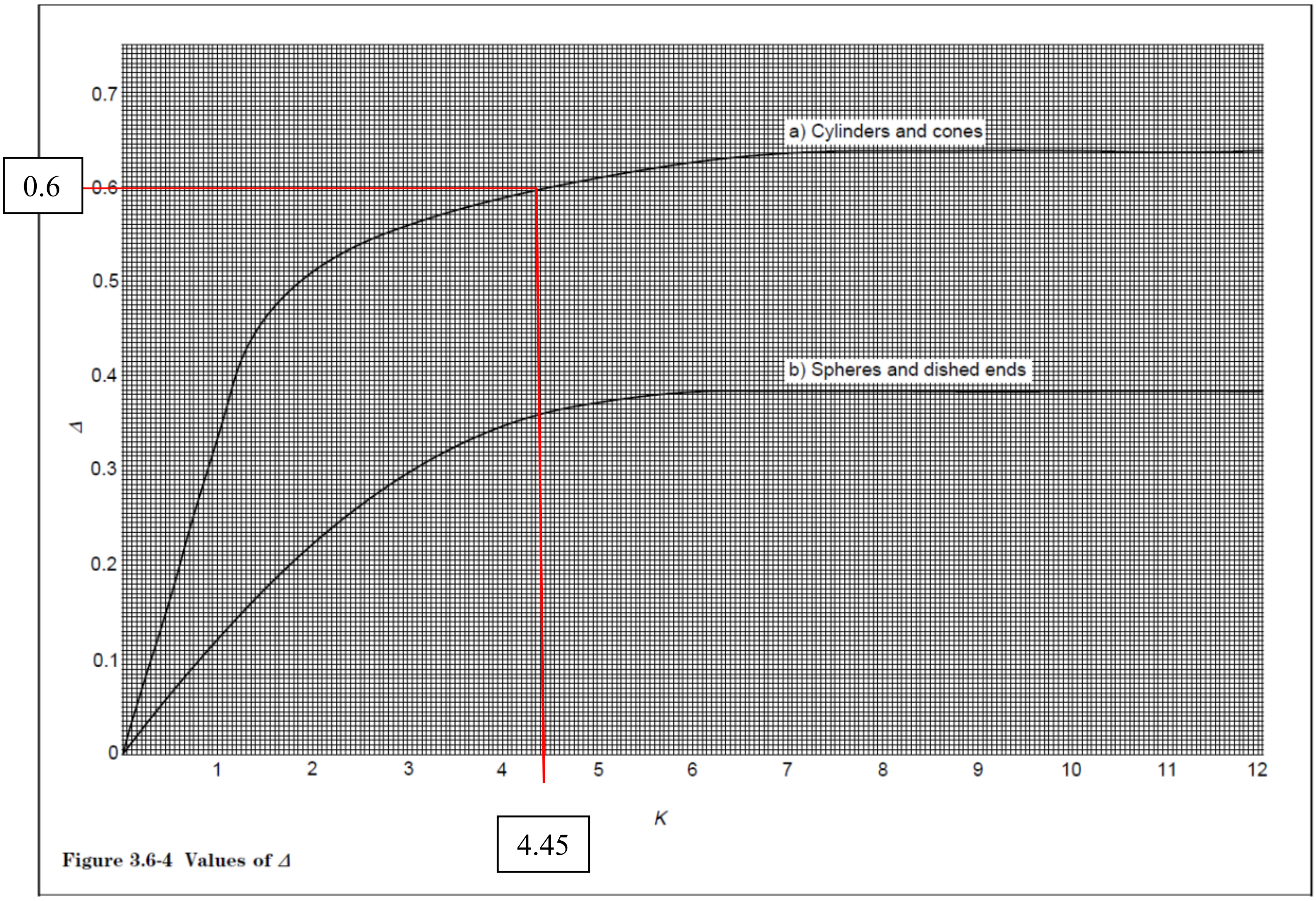

Figure 12: PD5500's Figure 3.6-4

$\Delta \approx 0.6$

$\Delta = \frac{P}{P_{yc}}$

$P = P_{yc} \times \Delta$

$P = 20.027 \times 0.6 = \mathbf{12.017}\boldsymbol{MPa}$

Calculations for TICC 2 and 3 can be found in appendices D and E respectively.

4.3 Ross' Design Charts [3, 4] (Figure 13 & 14)

4.3.1 The program MISESNP [3] (Figure 13).

MISESNP [3] is a program designed to calculate the buckling pressures of a cone using the von Mises[4] and DTMB[3] formulae. It also calculates the thinness ratio (λ) of a cone by inputting the above information.

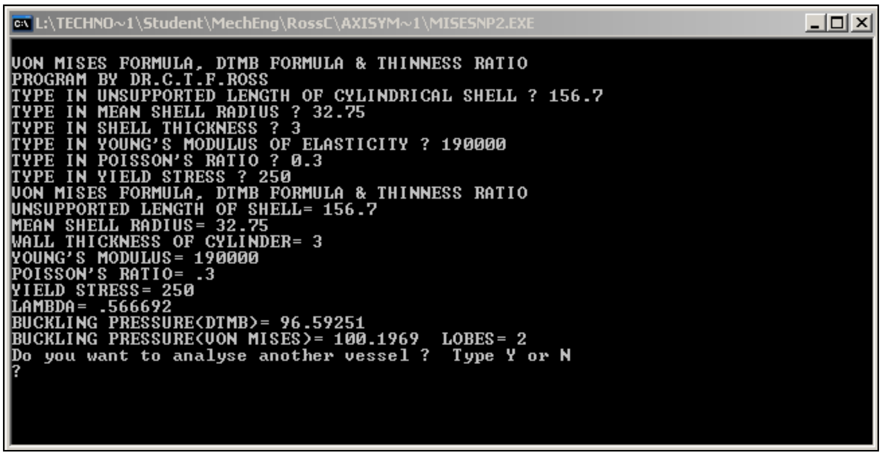

Figure 13 - Screenshot of MISESNP

The following results were obtained from MISESNP:
$P_n(von\ Mises) = 100.197$
P (DTMB) = 96.593
λ = 0.566692
Number of Lobes = 2

4.3.2 Calculating the Plastic Knock Down factor (PKD) (Figure 14)

$$PKD = \frac{P_{cr}}{P_{exp}}$$

Using the Ross Design Charts for conical shells [3, 4], the Plastic Knock Down Factor can be found from using the value of $\frac{1}{\lambda}$:

$$\frac{1}{\lambda} = \frac{1}{0.566692} = 1.7646$$

4.3.3 Calculating P_{exp}

$$PKD = \frac{P_{cr}}{P_{exp}} \approx 5.7$$

$$P_{exp} = \frac{P_{cr}}{PKD}$$

$$P_{exp} = \frac{P_{cr}}{5.7}$$

$$P_{exp} = \frac{100.1969}{5.7}$$

$$\boldsymbol{P_{exp} = 17.57MPa}$$

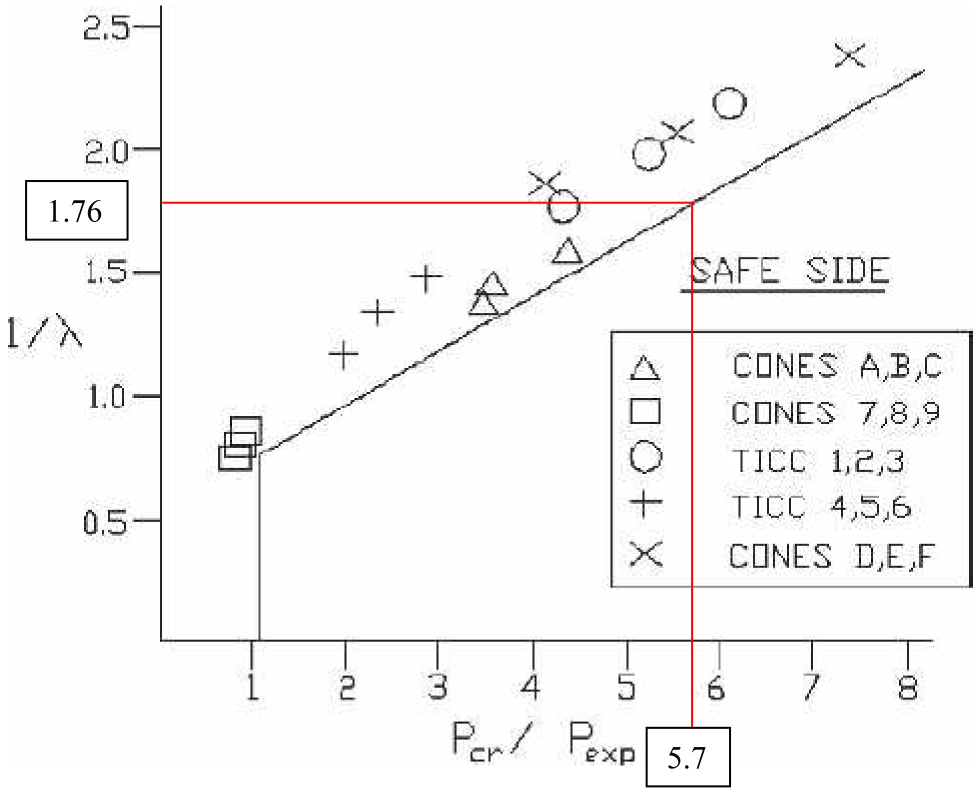

Figure 14: The Ross Design Chart for conical shells.

5 Comparison between various theories and experiment

Table 1 shows the theoretical predicted collapse pressures (P_{cr}) for Eigen and Non-Linear analysis compared to that calculated from the PD5500, Ross Design Charts and the experimentally determined pressures P_{exp}.

Cone	Lambda	ANSYS-Eigen	ANSYS-Nonlinear	PD5500 (BS5500)	Ross	Experimental
TICC1		139.12	27.6	12.027	17.544	22.41
TICC2		199.73	24.85	14.48	20.22	27.24
TICC3		274.13	21.3	16.821	25.607	32.76
TICC4		71.57	19.32	10.09	15.38	22.21
TICC5		108.65	22.34	12.57	17.43	27.59

TICC6		**155.94**	**25.17**	**15.65**	**22.28**	**31.66**
ConeA		**30.317**	**9.4284**	**5.180**	**6.49**	**6.48**
ConeB		**32.494**	**9.3288**	**5.134**	**5.65**	**6.76**
ConeC		**40.174**	**9.2349**	**5.404**	**6.33**	**6.62**
ConeD		**55.587**	**8.400**	**5.166**	**5.906**	**8.828**
ConeE		**77.874**	**9.450**	**5.397**	**6.605**	**8.414**
ConeF		**149.43**	**9.000**	**5.752**	**7.758**	**8.966**
Cone7		**5.96**	**5.67**	**1.31**	**3.22**	**4.97**
Cone8		**6.31**	**5.70**	**1.42**	**2.69**	**5.17**
Cone9		**6.56**	**5.52**	**1.50**	**2.47**	5.21

Table 1: Buckling Pressures (MPa)

6 Discussion

It is known that the ANSYS software can be difficult to use and can produce inaccurate results for eigen-buckling. Initially, the authors found ANSYS hard to use, but with practice they eventually found it was much easier to use. Other FEM software such as the ProEngineer application, Mechanica, ETC., were easier to use, but had limited capability; this was because the user did not require the same level of FEM knowledge that ANSYS required.

With regards to the results that ANSYS yielded, the authors were unsure on how accurate they were. The eigen-buckling results were grossly over-optimistic, but the non-linear results were much better, but were not entirely accurate. In general, they were relatively close to the experimental buckling pressures, but did not always follow the same pattern.

The authors found the Ross design charts the easiest to use and they were the most reliable, and not as conservative as PD5500.

7 Conclusions

The methodologies used by the authors in this report to obtain the buckling pressures for thin-walled conical shells, under external hydrostatic pressure, namely ANSYS Eigen buckling and nonlinear, together with PD5500:2000 and the use of the Ross Design Charts were all utilised successfully. ANSYS non-linear analysis produced some fairly accurate results, but ANSYS eigen-buckling was far too optimistic and dangerous, while the PD5500:2000 results were far too conservative. The Ross Design Charts proved to be very helpful; they were the easiest to use, and produced the most consistent results. Moreover, they were not as conservative as PD5500. Only fifteen cones were investigated and their behaviours were analysed, so it is not completely clear that the results obtained from these 15 cones were representative for all un-stiffened cones.

The British Standards PD5500 (BS5500) is widely acknowledged as being outdated and requires revision to produce less conservative results. Future work could include the use of other standards which aren't so conservative. For example, the European Standard EN13445 for 'Unfired Pressure Vessels', which claims to have European consensus; after more than 10 years of discussion by experts. The EN13445 is very competitive when compared to similar standards and is considered to be more comprehensive when compared to the PD5500, as it covers areas such as different methods for vessel supports such as ring and saddle supports for horizontal vessels and leg, ring and skirt supports for vertical vessels. The EN13445 is also logical and easy to use.

FEM software such as ANSYS provides great information and possibilities for the design of pressure vessels and similar designs. An alternate software package that could be used to further investigate the non-linear buckling of conical shells is called ABAQUS FEM. ABAQUS was originally designed for the investigation of non-linear analysis and hence provides a wide range of material models for more in-depth analysis and reliable results. ABAQUS FEM also has a more user friendly and logical interface. One great attraction of ANSYS, was that it produced quite spectacular and useful graphical displays; especially for the vessels undergoing material and geometrical non-linearity.

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