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The Eigenpairs of a Coupled Rectangular Cavity

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Abstract

This paper deals with the eigenvalue problem of a coupled rectangular cavity comprising five rigid walls and one flexible panel frequently employed in the literature. The purpose of this paper is to derive explicitly the eigenpairs of the coupled cavity, which are yet to be found. First, the coupling orthogonality conditions the eigenpairs need to satisfy are derived, thereby enabling the verification of the eigenpairs newly sought or already existent. Using the coupling orthogonality conditions, the modal equation of the coupled cavity system is then obtained, permitting one to deal with a forced response of the coupled cavity. It is shown that the eigenfunctions governing the dynamics of the sound field are expressed as the infinite sum of degenerate eigenfunctions. The characteristic matrix equation is then derived, specifying the eigenpairs of the coupled cavity. In order to investigate the fundamental properties of the eigenpairs derived, a numerical analysis is conducted, revealing the presence of evanescent modes in addition to the conventional standing wave modes.

Keywords: eigenpairs, rectangular cavity, coupling, evanescent modes, standing wave modes, modal orthogonality.

1 Introduction

Noise inside an acoustic-structural coupled cavity is a common problem in numerous practical situations including those involving aircrafts, automobiles, and trains. The term “coupling” indicates interference between a structural and an acoustic field of a cavity, resulting in the alternation of the eigenpairs of uncoupled system dynamics. Depending on the degree of coupling, cavity systems can be classified into two categories: a modally coupled cavity system and a coupled cavity system.

A modally coupled cavity system often introduced in sound transmission control problems [1-8] is based on the modal coupling theorem [9,10] established under the assumption that the fluid medium is non-dense and the cavity walls not "thin." The characteristic of this system is that the eigenfunctions of a coupled system remain the same as those of an acoustically rigid walled cavity, while only the eigenfrequencies of the cavity change.

When cavity walls become thin and the cavity gap shallow, the assumption of a modal coupling is no longer valid; thus, such a case falls into the second category, i.e., a coupled cavity system. Considerable efforts have been made in literature to derive the exact solution of coupled rectangular cavity system that comprises five rigid walls and a flexible panel. Lyon [11] examined the possibility of estimating noise reduction in various frequency ranges of the cavity; Dowell and Voss [12] expressed the sound pressure acting on a cavity-backed panel as a linearized form of Bernoulli's equation; Pretlove [13,14], in an attempt to spatially match the structural and acoustic mode shapes, introduced a cosine series expansion for simulating the cavity-backed panel deflection which is originally expressed as a sine-sine function, however, convergence of the method was not shown; Battacharya and Crocker [15] employed Fourier and Laplace transforms to derive a general solution of the wave equation under inhomogeneous boundary conditions; Guy and Bhattacharya [16] aimed to further improve the accuracy of the sound transmission of a cavity-backed panel. The main focus of these studies was to understand sound transmission through a cavity-backed panel, and thus their concern was directed toward deriving the exact solution of a forced vibration of the cavity-backed panel subject to external sound pressure. As such, the exact eigenpairs of the coupled cavity, which are intrinsic parameters governing the system dynamics and independent of any extraneous forces, are yet to be found.

With a view to suppressing cavity noise generated by external noise or disturbance forces acting on the panel, and/or noise sources inside the cavity, it is common practice to introduce active control. To establish a control strategy, the eigenpairs which are the governing parameters of the system dynamics are essential because the control law for minimizing some performance index is expressed using system parameters. System parameters such as sound pressure inside the cavity, acoustic potential energy, surface velocity of the panel, or structural kinetic energy may be described using the expansion theorem in terms of the eigenpairs. Hence, the control variables may be methodically manipulated if the eigenpairs are available.

It is the purpose of this paper to derive explicitly the eigenpairs of a coupled rectangular cavity comprising five rigid walls and a flexible panel. First, the coupling orthogonality conditions the eigenpairs need to satisfy are derived, thereby verifying the validity of the eigenpairs newly found or already existent. Using the coupling orthogonality conditions, the modal equation of the coupled cavity system is then obtained, permitting one to deal with a forced response of the coupled cavity. Furthermore, eigenfunctions governing the dynamics of both the sound field and the vibration field are expressed as the infinite sum of cluster eigenfunctions that possess the same attribute in common. The characteristic matrix equation is then derived, enabling one to specify the eigenpairs of the coupled cavity. In order to investigate the fundamental properties of the eigenpairs derived, a numerical analysis is conducted, revealing the presence of evanescent modes together with the conventional standing wave modes. It is shown that the evanescent modes emerge when the eigen wavenumber of the cluster eigenfunction becomes pure imaginary and the associated coefficient large.

2 Fundamental Properties of a Coupled Rectangular Cavity

2.1 Coupling Modal Orthogonal Conditions

Consider a rectangular cavity as shown in Fig. 1 with the dimensions of $L_x \times L_y \times L_z$, consisting of five rigid walls and one simply supported flexible panel S placed on top. Using the velocity potential ϕ, the wave equation of sound in the cavity may be written as

$$c^2\nabla^2\phi(x,y,z,t) - \ddot{\phi}(x,y,z,t) = 0 \quad (1)$$

where c, ∇^2, t and $\dot{}$ denote the sound velocity in air, the Laplacian operator, time and temporal differentiation, respectively. Furthermore, the equation of motion of a flexible panel is given by

$$D\nabla^4 w(x,y,t) + \rho h\ddot{w}(x,y,t) = \Delta p\,(x,y,z,t)\big|_{z=L_z} \quad (2)$$

where D is the flexural rigidity, w the displacement of the panel, ρ the density, h the thickness, Δp the sound pressure in the cavity, respectively. Differentiating Eq.(1) with respect to χ $(\chi = x,y,z)$ then gives

$$c^2\nabla^2 u_\chi(x,y,z,t) - \ddot{u}_\chi(x,y,z,t) = 0 \quad (3)$$

where u_χ denotes the particle velocity in the χ direction. Moreover, differentiating Eq.(2) with respect to t yields

$$D\nabla^4 v(x,y,t) + \rho h\ddot{v}(x,y,t) = \Delta\,\dot{p}(x,y,z,t)\big|_{z=L_z} \quad (4)$$

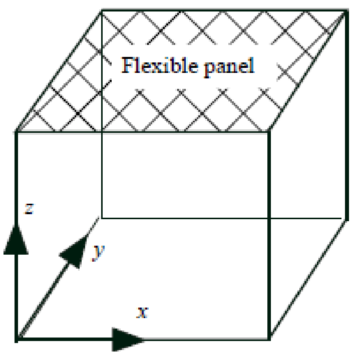

Fig1: Rectangular cavity model with a flexible panel on the top

where v denotes the surface velocity of the panel. Because the sound pressure is described as $\Delta p = -\rho_a \dot{\phi}$ where ρ_a denotes the air density, the right-hand side of Eq.(4) may be expanded to

$$\begin{aligned}\Delta\dot{p}(x,y,z,t)\big|_{z=L_z} &= -\rho_a\,\ddot{\phi}(x,y,z,t)\big|_{z=L_z} \\ &= -\rho_a c^2 \nabla^2 \phi(x,y,z,t)\big|_{z=L_z} \\ &= -\rho_a c^2 \sum_{\chi=x,y,z} \left.\frac{\partial u_\chi(x,y,z,t)}{\partial \chi}\right|_{z=L_z}\end{aligned} \quad . \qquad (5)$$

The ith eigenfunction and the associated eigenfrequency of the coupled cavity satisfy Eqs.(3) and (4), and hence the following equations are obtained,

$$c^2\nabla^2\overline{\phi}_{\chi,i}(x,y,z) + \overline{\omega}_i^2\overline{\phi}_{\chi,i}(x,y,z) = 0 \quad (\chi = x,y,z)\,, \qquad (6)$$

$$D\nabla^4\overline{\varphi}_i(x,y) - \overline{\omega}_i^2\rho h\overline{\varphi}_i(x,y) + \rho_a c^2 \sum_{\chi=x,y,z} \left.\frac{\partial\overline{\phi}_{\chi,i}(x,y,z)}{\partial\chi}\right|_{z=L_z} = 0 \qquad (7)$$

where $\overline{\phi}_{\chi,i}$, $\overline{\varphi}_i$ and $\overline{\omega}_i$ are the ith acoustic eigenfunction of the coupled cavity in terms of particle velocity in the χ direction, the ith structural eigenfunction and the associated eigenfrequency of the coupled cavity, respectively. Similarly, the jth eigen system may also be written as

$$c^2\nabla^2\overline{\phi}_{\chi,j}(x,y,z) + \overline{\omega}_j^2\overline{\phi}_{\chi,j}(x,y,z) = 0 \quad (\chi = x,y,z)\,, \qquad (8)$$

$$D\nabla^4\overline{\varphi}_j(x,y) - \overline{\omega}_j^2\rho h\overline{\varphi}_j(x,y) + \rho_a c^2 \sum_{\chi=x,y,z} \left.\frac{\partial\overline{\phi}_{\chi,j}(x,y,z)}{\partial\chi}\right|_{z=L_z} = 0\,. \qquad (9)$$

Using Eqs.(6)~(9), and with some manipulations of equations, the coupling modal orthogonality condition with respect to mass may be given by

$$\int_0^{L_y}\int_0^{L_x} \overline{\varphi}_i(x,y)\rho h\overline{\varphi}_j(x,y)dxdy + \int_0^{L_z}\int_0^{L_y}\int_0^{L_x} \sum_{\chi=x,y,z} \overline{\phi}_{\chi,i}(x,y,z)\rho_a\overline{\phi}_{\chi,j}(x,y,z)dxdydz = \delta_{ij} \qquad (10)$$

where δ_{ij} denotes the Kronecker delta. Likewise, the coupling modal orthogonality condition with respect to compliance may be written as

$$\begin{aligned}&\int_0^{L_y}\int_0^{L_x} \overline{\varphi}_i(x,y)D\nabla^4\overline{\varphi}_j(x,y)dxdy \\ &+\int_0^{L_y}\int_0^{L_x} \sum_{\chi_1=x,y}\left(\overline{\varphi}_i(x,y)\rho_a c^2\frac{\partial\overline{\phi}_{\chi_1,j}(x,y,L_z)}{\partial\chi_1} + \overline{\varphi}_j(x,y)\rho_a c^2\frac{\partial\overline{\phi}_{\chi_1,i}(x,y,L_z)}{\partial\chi_1}\right)dxdy \\ &+\int_0^{L_z}\int_0^{L_z}\int_0^{L_z} \sum_{\chi_2=x,y,z}\sum_{\chi_1=x,y,z} \frac{\partial\overline{\phi}_{\chi_2,i}(x,y,z)}{\partial\chi_1}\rho_a c^2\frac{\partial\overline{\phi}_{\chi_2,j}(x,y,z)}{\partial\chi_1}dxdydz = \overline{\omega}_i^2\delta_{ij}\end{aligned} \quad . \qquad (11)$$

Note that the coupling modal orthogonality conditions in Eqs.(10) and (11) signify that the eigenfunctions of a coupled cavity are no longer orthogonal to each other in the sole sound field or in the sole vibration field, but are orthogonal in a manner that a set of eigenfunctions are orthogonal with another set of eigenfunctions as will be discussed in the following subsection.

2.2 Eigenpairs of a Coupled Rectangular Cavity and Degenerate Eigenfunctions

Suppose that a coupled cavity system is oscillating at frequency ω. Equations (1) and (2) are then written as

$$\nabla^2\phi(x,y,z)+k^2\phi(x,y,z)=0\,, \tag{12}$$

where $k=\dfrac{\omega}{c}$ denotes the wavenumber of air. Assume that the velocity potential is described as a separable function of the three space variables

$$\phi(x,y,z)=X(x)Y(y)Z(z)\,. \tag{13}$$

Substituting Eq.(13) into Eq.(12) yields

$$\frac{X''(x)}{X(x)}+\frac{Y''(y)}{Y(y)}+\frac{Z''(z)}{Z(z)}=-k^2 \tag{14}$$

where X'', for instance, expresses the second derivative of X with respect to x. Now that the wavenumber k is independent of space, Eq.(14) may then be expressed as

$${k_x}^2+{k_y}^2+{k_z}^2=k^2 \tag{15}$$

where

$${k_x}^2=-\frac{X''}{X}\,,\ {k_y}^2=-\frac{Y''}{Y}\,,\ {k_z}^2=-\frac{Z''}{Z}\,. \tag{16}$$

The solution of Eq.(16) may produces the fundamental form of the velocity potential as

$$\phi_\kappa(x,y,z)=\cos\frac{l_\kappa\pi}{L_x}x\cos\frac{m_\kappa\pi}{L_y}y\cos k_z z \quad (\kappa=1,2,3\cdots)\,. \tag{17}$$

Substituting Eq.(17) into (14) yields

$$\left(\frac{l_\kappa\pi}{L_x}\right)^2+\left(\frac{m_\kappa\pi}{L_y}\right)^2+{k_z}^2=\bar{k}^{\,2} \tag{18}$$

where $\bar{k}^{\,2}$ (or $\bar{\omega}$) implies the ith eigen wavenumber (or eigenfrequency) of the coupled cavity. It should be noted that the eigen wavenumber $\bar{k}$ is dependent on modal indices; l_κ and m_κ $(\kappa=1,2,3\cdots)$.

Given that $\bar{k}$ is specified using the characteristic matrix equation of the coupled cavity as will be shown later, depending on the modal indices l_κ and m_κ, k_z is determined such that Eq.(18) holds. In order to emphasize this fact, let k_z be written as $k_{l_\kappa m_\kappa}$. Also, in order to distinguish the corresponding eigenfunction ϕ_κ in Eq.(17) from ordinary eigenfunctions, let ϕ_κ be expressed as $\tilde{\phi}_\kappa$. Hence

$$\tilde{\phi}_\kappa(x,y,z) = \cos\frac{l_\kappa \pi}{L_x} x \cos\frac{m_\kappa \pi}{L_y} y \cos k_{l_\kappa m_\kappa} z \quad (\kappa = 1,2,3\cdots). \tag{19}$$

Note that depending on the modal indices l_κ and m_κ, the wavenumber component $k_{l_\kappa m_\kappa}$ becomes real or pure imaginary as will be shown in Section III. Moreover, because of the constraint in Eq.(18), a set of $\tilde{\phi}_\kappa$ $(\kappa = 1,2,3\cdots)$ has the same eigenfrequency $\bar{\omega}$ in common and thus $\tilde{\phi}_\kappa$ $(\kappa = 1,2,3\cdots)$ is termed the degenerate eigenfunctions in the article.

As such, the acoustic eigenfunction $\bar{\phi}$ of the coupled cavity satisfies the following sound equation

$$\nabla^2 \bar{\phi}(x,y,z) + \bar{k}^2 \bar{\phi}(x,y,z) = 0 \tag{20}$$

where

$$\bar{\phi}(x,y,z) = \sum_{\kappa=1}^{\infty} a_\kappa \tilde{\phi}_\kappa(x,y,z) \approx \mathbf{a}^T \tilde{\boldsymbol{\phi}}(x,y,z) \tag{21}$$

where

$$\mathbf{a} = \begin{pmatrix} a_1 & a_2 & a_3 & \cdots \end{pmatrix}^T, \tag{22}$$

$$\tilde{\boldsymbol{\phi}} = \begin{pmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 & \tilde{\phi}_3 & \cdots \end{pmatrix}^T \tag{23}$$

and where a_κ is the κth amplitude of the degenerated eigenfunction of the coupled cavity. Note that the acoustic eigenfunction $\bar{\phi}$ of the coupled cavity is expressed as the infinite sum of the degenerate eigenfunctions $\tilde{\phi}_\kappa$, whereas the eigenfunction of a rigid walled cavity ϕ_i is described using a single term as

$$\phi_i(x,y,z) = \cos\frac{l_i \pi}{L_x} x \cos\frac{m_i \pi}{L_y} y \cos\frac{n_i \pi}{L_z} z. \tag{24}$$

More important, the sum of normal eigenfunctions does not satisfy the homogeneous Helmholtz wave equation in Eq.(12), while the sum of the degenerate eigenfunctions in Eq.(21) does because of the attribute the degenerate eigenfunctions possess as shown in Eq.(18).

Suppose that the coupled cavity is oscillating with the eigenfrequency $\bar{\omega}$. Then the equation of motion of a flexible panel is given by

$$D\nabla^4 \bar{\varphi}(x,y) - \bar{\omega}^2 \rho h \bar{\varphi}(x,y) = \bar{\omega}^2 \rho_a \bar{\phi}(x,y,z)\big|_{z=L_z} \tag{25}$$

where $\bar{\varphi}$ denotes the structural eigenfunction of the coupled cavity, which may be expressed as the infinite sum of the *in vacuo* structural eigenfunctions of the panel φ_κ $(\kappa = 1,2,3\cdots)$ satisfying the following homogeneous equation of motion of a panel,

$$D\nabla^4 \varphi_\kappa(x,y) - \omega_\kappa^2 \rho h \varphi_\kappa(x,y) = 0. \tag{26}$$

Hence,

$$\bar{\varphi}(x,y) = \sum_{\kappa=1}^{\infty} b_\kappa \varphi_\kappa(x,y) \approx \mathbf{b}^T \boldsymbol{\varphi}(x,y), \tag{27}$$

$$\mathbf{b} = \begin{pmatrix} b_1 & b_2 & b_3 & \cdots \end{pmatrix}^T, \tag{28}$$

$$\boldsymbol{\varphi} = \begin{pmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \cdots \end{pmatrix}^T \tag{29}$$

where b_κ is the κth modal coefficient of the structural eigenfunction of the coupled cavity. It should be noted that the sum of the *in vacuo* structural eigenfunctions does not satisfy the homogeneous equation of motion in Eq.(26), however satisfies the expression in Eq.(25) because of the non-homogeneous equation. Because of the boundary condition of a simply supported panel, the structural eigenfunction φ_κ is then given by

$$\varphi_\kappa(x,y) = \sin\frac{l'_\kappa \pi}{L_x}x \sin\frac{m'_\kappa \pi}{L_y}y \tag{30}$$

where l'_κ and m'_κ are modal indices. Substituting Eqs.(21) and (27) in Eq.(25) results in

$$\sum_{\kappa=1}^{\infty} \left(D\nabla^4 - \varpi^2 \rho h\right) b_\kappa \varphi_\kappa(x,y) = \varpi^2 \rho_a \sum_{\kappa=1}^{\infty} a_\kappa \tilde{\phi}_\kappa(x,y,z)\big|_{z=L_z}. \tag{31}$$

Moreover, substituting Eqs.(19) and (30) in Eq.(31) leads to

$$\begin{aligned} &\sum_{\kappa=1}^{\infty} \left(\omega_\kappa^2 - \varpi^2\right) \rho h b_\kappa \sin\frac{l'_\kappa \pi}{L_x}x \sin\frac{m'_\kappa \pi}{L_y}y \\ &= \varpi^2 \rho_a \sum_{\kappa=1}^{\infty} a_\kappa \cos\frac{l_\kappa \pi}{L_x}x \cos\frac{m_\kappa \pi}{L_y}y \cos k_{l_\kappa m_\kappa} L_z \end{aligned}. \tag{32}$$

Multiplying Eq.(32) by $\varphi_s(x,y) = \sin\frac{l'_s \pi}{L_x}x \sin\frac{m'_s \pi}{L_y}y$ and integrating over the panel S produces

$$\left(\omega_s^2 - \varpi^2\right) \rho h \frac{L_x L_y}{4} b_s = \varpi^2 \rho_a \sum_{\kappa=1}^{\infty} a_\kappa \beta_{s\kappa} \cos k_{l_\kappa m_\kappa} L_z \tag{33}$$

where $\beta_{s\kappa}$ denotes the coupling coefficient[14] defined as

$$\beta_{s\kappa} = \int_0^{L_y}\int_0^{L_x} \sin\frac{l'_s \pi}{L_x}x \sin\frac{m'_s \pi}{L_y}y \cos\frac{l_\kappa \pi}{L_x}x \cos\frac{m_\kappa \pi}{L_y}y\,dxdy. \tag{34}$$

Equation (33) may be expressed in a matrix form as

$$\mathbf{B}_{couple} \boldsymbol{\Lambda}_c \mathbf{a} = \boldsymbol{\Lambda}_\omega \mathbf{b} \tag{35}$$

where

$$\boldsymbol{\Lambda}_c = \varpi^2 \rho_a \begin{pmatrix} \cos k_{l_1 m_1} L_z & & \mathbf{0} \\ & \cos k_{l_2 m_2} L_z & \\ \mathbf{0} & & \ddots \end{pmatrix}, \tag{36}$$

$$\mathbf{\Lambda}_{\omega} = \frac{\rho h L_x L_y}{4}\begin{pmatrix} \omega_1^2 - \bar{\omega}^2 & & \mathbf{0} \\ & \omega_2^2 - \bar{\omega}^2 & \\ \mathbf{0} & & \ddots \end{pmatrix}, \tag{37}$$

$$\mathbf{B}_{couple} = \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots \\ \beta_{21} & \beta_{22} & \cdots \\ \vdots & \vdots & \end{pmatrix}. \tag{38}$$

Next, the boundary condition in which the surface velocity of the panel spatially meets the particle velocity is expressed as

$$\bar{\varphi}(x,y) = \frac{\partial}{\partial z}\bar{\phi}(x,y,z)\Big|_{z=L_z}. \tag{39}$$

Substituting Eqs.(21) and (27) into Eq.(39) leads to

$$\sum_{\kappa=1}^{\infty} b_{\kappa} \sin\frac{l'_{\kappa}\pi}{L_x}x \sin\frac{m'_{\kappa}\pi}{L_y}y = -\sum_{\kappa=1}^{\infty} k_{l_{\kappa}m_{\kappa}} a_{\kappa} \cos\frac{l_{\kappa}\pi}{L_x}x \cos\frac{m_{\kappa}\pi}{L_y}y \sin k_{l_{\kappa}m_{\kappa}} L_z. \tag{40}$$

Furthermore, multiplying Eq.(40) by $\varphi_s(x,y) = \sin\frac{l'_s\pi}{L_x}x \sin\frac{m'_s\pi}{L_y}y$ and integrating over the panel S produces

$$b_s \frac{L_x L_y}{4} = -\sum_{\kappa=1}^{\infty} k_{l_{\kappa}m_{\kappa}} a_{\kappa} \beta_{s\kappa} \sin k_{l_{\kappa}m_{\kappa}} L_z. \tag{41}$$

Equation (41) may be expressed in a form of matrix as

$$\frac{L_x L_y}{4}\mathbf{b} = -\mathbf{B}_{couple}\mathbf{\Lambda}_s \mathbf{a} \tag{42}$$

where

$$\mathbf{\Lambda}_s = \begin{pmatrix} k_{l_1m_1}\sin k_{l_1m_1}L_z & & \mathbf{0} \\ & k_{l_2m_2}\sin k_{l_2m_2}L_z & \\ \mathbf{0} & & \ddots \end{pmatrix}. \tag{43}$$

Using Eqs.(35) and (42), the characteristic matrix equation of the coupled cavity is then given by

$$\bar{\mathbf{A}}\bar{\mathbf{x}} = \mathbf{0} \tag{44}$$

where

$$\bar{\mathbf{A}} = \begin{pmatrix} \mathbf{B}_{couple}\mathbf{\Lambda}_c & -\mathbf{\Lambda}_{\omega} \\ \mathbf{B}_{couple}\mathbf{\Lambda}_s & \frac{L_x L_y}{4}\mathbf{I} \end{pmatrix}, \tag{45}$$

$$\bar{\mathbf{x}} = \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} \tag{46}$$

and where $\mathbf{I}$ denotes the identity matrix. Note that the frequency nullifying the determinant of the matrix $\overline{\mathbf{A}}$ in Eq.(45) yields the eigenfrequencies $\bar{\omega}_\kappa$ $(\kappa = 1,2,3\cdots)$ of the coupled rectangular cavity system. When the eigenfrequency $\bar{\omega}_\kappa$ is specified, the associated eigenvector $\overline{\mathbf{x}}_\kappa$ is then obtained. Consequently, the eigenpairs of the coupled rectangular cavity are acquired: the eigenfunctions in terms of the sound field $\bar{\phi}_\kappa(x,y,z)$ in Eq.(21), those in terms of the vibration field $\bar{\varphi}_\kappa$ in Eq.(27).

3 Numerical Analysis

Using the specifications of a rectangular cavity comprising a simply supported flexible panel and five rigid walls, a numerical analysis is conducted. For this purpose, 15 acoustic modes and 15 structural modes of the panel are taken into consideration, the frequency of interest being set up to 250 Hz. The frequency of the 15th acoustic mode; (0,2,2) mode, and the structural mode; (1,7) mode, are, respectively, 977 Hz and 716 Hz, thereby covering enough the frequency of interest. In accordance with the procedure shown in Section 2, the eigenpairs of the coupled rectangular cavity are acquired, the results of which are shown in Table 1. After coupling, the acoustic eigenfunction may not be expressed as an (l,m,n) mode, so that the corresponding column is left blank. Observe that the (0,0,0) and (0,0,1) acoustic modes in the cavity before coupling couples with only the (odd, odd) structural mode[9], thereby affecting the (1,1) and (1,3) structural mode by shifting the resonant frequency. As with the (1,2) mode, it couples with the (0,1,0) acoustic mode and its eigenfrequency is 447Hz, which is not enlisted in the table though.

	in vacuo		*coupled*	
Mode number	Modal indices (l,m,n)	Frequency Hz	Modal indices (l,m,n)	Frequency Hz
1	(0,0,0)	0	-	77
2	(1,1)	73	-	113
3	(1,2)	113	-	179
4	(1,3)	180	-	201
5	(0,0,1)	196	-	251
6	(2,1)	252	-	273
7	(1,4)	274	-	291

Table 1: Modal parameters before and after coupling

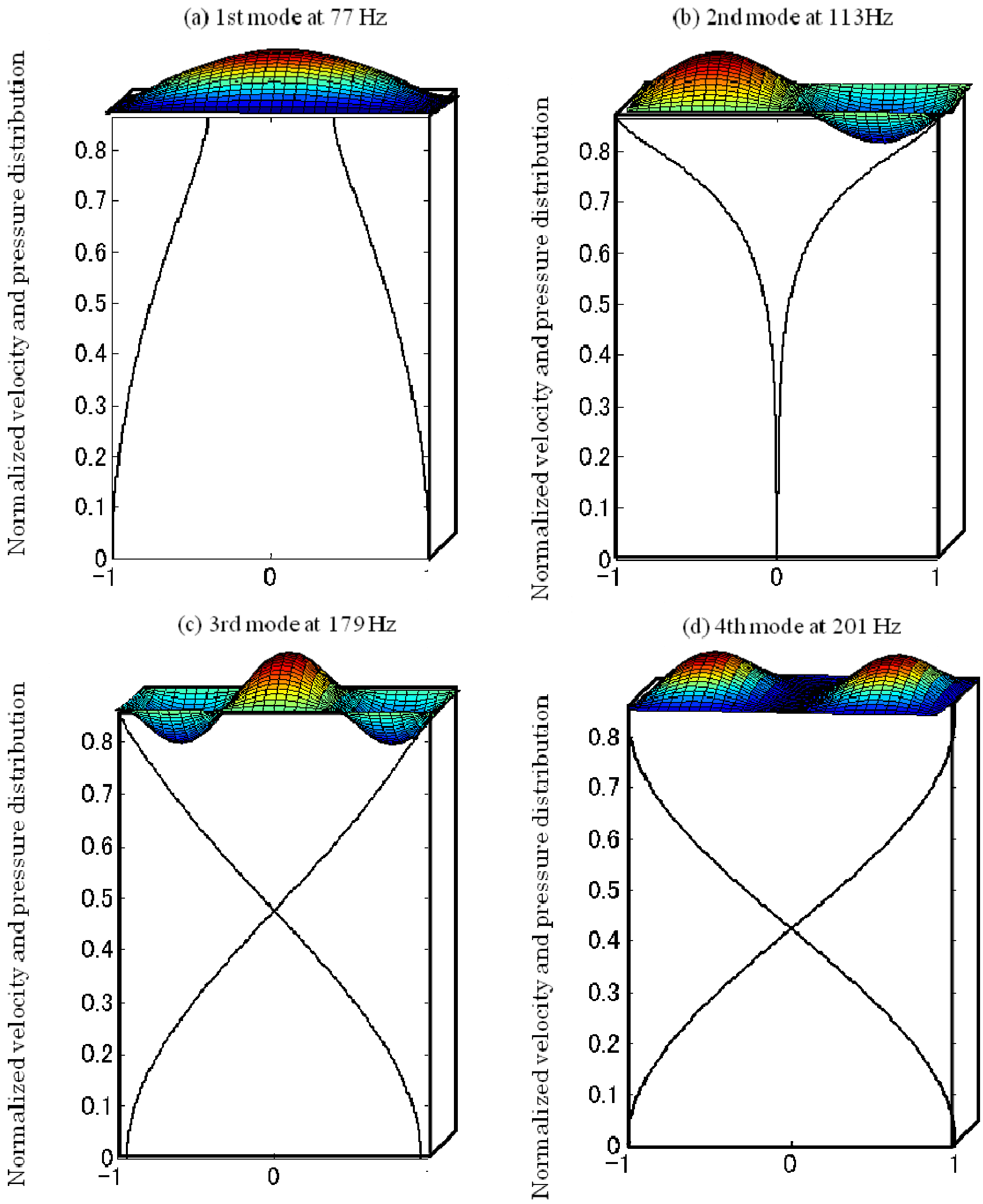

Figure 2: Normalized velocity mode shape of a flexible panel and the acoustic mode shapes in the cavity at the 1st ~ 4th coupled mode

Illustrated in Fig.2 are the normalized velocity mode shapes of a flexible panel of the coupled cavity, which are depicted using the expression: $\bar{\varphi}_i(x,y) \approx \mathbf{b}_i^T \boldsymbol{\varphi}(x,y)$ $(i = 1 \sim 4)$ in Eq.(27). Observe that the structural modal behaviors from the 1st through 3rd mode are dominated by the *in vacuo* mode shape; (1,1), (1,2) and (1,3) mode, respectively. As for the 4th mode, however, the original *in vacuo* structural (2,1) mode is replaced by the deformed (1,3) mode because of the coupling effect. Figure 2 also shows the corresponding acoustic mode shapes along the z direction depicted using the expression: $\bar{\phi}_i(x,y,z) \approx \mathbf{a}_i^T \tilde{\boldsymbol{\phi}}(x,y,z)$ in Eq.(21) in the vicinity of $x = L_x/2$ and $y = L_y/2$. Unlike the acoustic mode shapes observed in a rigid wall cavity in Eq.(24), the mode shapes in the z direction appear different. Regarding the first mode at 77 Hz in Fig.2(a), although the rigid wall mode shape is depicted by two parallel straight lines along the z axis, the acoustic mode shape after coupling shrinks as z increases, leading to that of an open ended cavity at $z = L_z$ as an extreme case. Acoustic mode shapes in Figs.2(c) and (d) are similar to each other, albeit the 4th mode is considerably affected by the coupling effect. Note that among the four in Fig.2, the 2nd acoustic mode shape is different from the ordinary mode shapes. The acoustic mode shape in the z direction is described as $\cos k_z z$, implying that the amplitude at $z = 0$ reaches the maximum; however, the maximum appears at $z = L_z$ and minimum at $z = 0$, and therefore the fundamental properties of such an unusual mode needs investigating.

Recall that the ith acoustic eigenfunction $\bar{\phi}_i(x,y,z)$ of a coupled cavity may be expressed in Eq.(21); the infinite sum of the degenerate eigenfunctions $\tilde{\phi}_\kappa^i(x,y,z)$ in Eq.(19), the associated eigen wavenumber component $k_{l_\kappa m_\kappa}^i$ being given by Eq.(18). Now that the 2nd coupled frequency is 113Hz, the term $k_{l_\kappa m_\kappa}^{(2)}$ is determined when the modal indices l_κ and m_κ $(\kappa = 1,2,3\cdots)$ are designated. As a result of the numerical analysis, it is found that a couple of amplitudes with respect to $k_{l_\kappa m_\kappa}^{(2)}$ are dominant: that is, $k_{00}^{(2)}$ and $k_{01}^{(2)}$, the associated coefficients being $a_{00}^{(2)} = -1.6\times10^{-4}$ and $a_{01}^{(2)} = 1.3\times10^{-4}$, respectively, while the other coefficients are negligibly small.

Therefore, the acoustic eigenfunction of the coupled cavity at 113 Hz may approximately be expressed as

$$\bar{\phi}_2(x,y,z) \approx -1.6\times10^{-4}\cos k_{00}^{(2)} z + 1.3\times10^{-4}\cos\frac{\pi}{L_y} y \cos k_{01}^{(2)} z \ . \tag{47}$$

As with the first term in the right-hand side of Eq.(47), the wavenumber component $k_{00}^{(2)}$ is 1.45 due to $(l_1 = m_1 = 0)$; hence a harmonic function. On the other hand, the second term, $k_{01}^{(2)}$ is found to be $j7.99$; pure imaginary. Therefore, the term $\cos k_{01}^{(2)} z$ is expressed as $\cos k_{01}^{(2)} z = \dfrac{e^{7.99z} + e^{-7.99z}}{2}$; thus, not harmonic but a hyperbolic function. When this term becomes dominant, the mode shape along the z direction is approximately described as $\dfrac{e^{7.9z}}{2}$; therefore, the mode shape as shown in Fig. 2(b) emerges. If this is the case, the sound field at the resonant frequency 113 Hz is governed by the evanescent mode that decays exponentially from the vibrating panel without propagating in the sound field. As the distribution of the sound pressure is tantamount to that of the acoustic mode shape, the acoustic potential energy inside the cavity is low. As such, the evanescent modes verified [20] experimentally emerge as a result of the dynamical interference between the sound field and the vibration field. When the cavity gap L_z becomes deeper, the evanescent modes become more noticeable. In other words, the shallower the cavity gap, the less influential the evanescent modes.

4 Conclusions

The eigenpairs of a coupled rectangular cavity comprising a flexible panel and five rigid walls were derived. First, the coupling modal orthogonality conditions the eigenpairs of the coupled cavity need to satisfy were derived, which do not require the eigenfunctions expressed in an explicit form. The coupling modal orthogonality conditions therefore enable one to verify the validity of the eigenfunctions newly sought or already existent. Taking into consideration the coupling effect between the sound field and the vibration filed, the eigenfunctions of a coupled cavity were expressed as the infinite sum of degenerate eigenfunctions possessing the same eigenfrequency in common. Using the coupling modal orthogonality conditions obtained, the modal equation of the coupled cavity was derived, hence allowing one to deal with the forced response of the coupled cavity. Combining the dominant equations of both the sound field and the vibration field, the characteristic matrix equation was then produced, thereby enabling the derivation of the eigenpairs of the coupled cavity. The acoustic potential energy inside the cavity was expressed using the expansion theorem in terms of the eigenpairs. Moreover, in order to investigate the fundamental properties of the eigenpairs derived, a numerical analysis was conducted. It is found that in addition to conventional standing wave modes, evanescent modes emerge in the coupled cavity when the eigen wavenumber is purely imaginary and the associated coefficient large.

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