Paper 160

Proceedings of the Eleventh International Conference
on Computational Structures Technology,
B.H.V. Topping, (Editor),
Civil-Comp Press, Stirlingshire, Scotland

Automatic Calculation of Optimum Reinforcement for Flexural and Axial Loading

A. Tomás, G. Sánchez and A. Alarcón
Department of Civil Engineering
Universidad Politécnica de Cartagena (UPCT), Spain

Abstract

The problem of a concrete cross section under flexural and axial loading is indeterminate. Among the infinite solutions, it is possible to find the optimum, which is that of minimum reinforcement that satisfies certain design constraints (section ductility, minimum reinforcement area, etc.). This paper proposes the automation of the optimum reinforcement calculation under any combination of flexural and axial loading. Conventional-strength or high-strength concrete may be chosen, minimum reinforcement area may be considered, and the neutral axis depth may be constrained in order to guarantee a certain sectional ductility. Some numerical examples are presented, drawing comparisons between the results obtained by ACI 318, EC 2, and the conventional method.

Keywords: cross section, flexural/axial loading, automated design, optimum reinforcement.

1 Introduction

The design of reinforced concrete members for combined flexure and axial load is a common case in structural engineering. In this type of design, the reinforcement of a cross section is obtained in order to resist some certain flexural moment and axial force. The section is usually rectangular. It is known that the solution to this kind of problems is indeterminate, since there are three unknown variables (the top and bottom reinforcements, and the neutral axis depth) and only two equilibrium equations. In conventional calculation procedures (see references [1, 2], among others), a further condition is required in order to solve the problem, so it is advisable to manage these procedures by programming them in computer codes.

Due to the infinite number of solutions, an optimization problem to obtain the optimum reinforcement in the section can be proposed as an ideal method of solving the equations system [3, 4, 5]. Optimization procedures involve, in most cases, complex calculations that require adequate computer methods. However, an easy-to-implement optimization problem is proposed in this paper, entailing a negligible computational cost (tenths of a second) in any currently available personal computer.

The proposed method allows:

i) considering high-strength concrete, i.e., specified compressive strength of concrete f_{ck} > 50 MPa in Eurocode 2 (EC 2) [6], and 41.37 MPa < f_c' < 82.74 MPa in ACI 318 [7],
ii) considering the minimum area of reinforcement,
iii) constraining the neutral axis depth in order to guarantee certain curvatures to comply with ductility criterion, and
iv) choosing between the standards ACI 318 or EC 2.

Four numerical examples are presented. In three of them, the results obtained by ACI 318 are compared with those obtained by EC 2 and by the conventional method, whilst in the fourth example, high-strength concrete is used.

This study aims not so much as to obtain the reinforcement, which has already been solved [5, 8], but to have an automated design procedure for calculating the optimum reinforcement by means of the implementation of a simple optimization method. Thus, the interest of the proposed procedure focuses on automating the calculation of reinforcement, in which the structural engineer may include certain design conditions, such as the minimum reinforcement, the high-strength concrete and/or a certain ductility criterion.

2 General principles and requirements

2.1 Design assumptions

In this study, the following fundamental assumptions have been considered:

- Homogeneous and isotropic material.
- Perfect bond between the compression reinforcement and concrete.
- Navier-Bernoulli hypothesis (plane cross section remains planar and normal to the rod axis after deformation).
- Negligible tensile strength of concrete.
- Failure of section caused by excessive plastic deformation.

2.2 Conditions of equilibrium and compatibility of strains

To obtain the equilibrium equations, a section with any shape, but which is symmetric about the bending plane is considered under any combination of flexural and axial loads (Figure 1). The equilibrium equations, in the limit state, can be written as

$$P_n = \int_0^h b_y \sigma_y dy + A_s \sigma_s + A_s' \sigma_s'$$
$$P_n e_1 = \int_0^h b_y \sigma_y (d_t - y) dy + A_s' \sigma_s' (d_t - d') \tag{1}$$

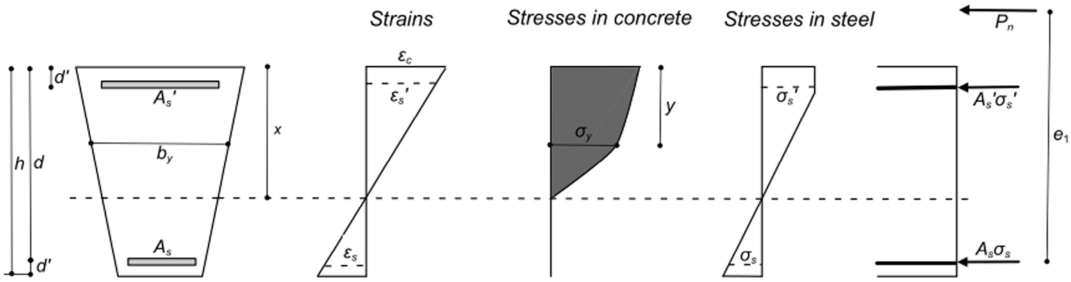

Figure 1: Sketch defining terms for conditions of equilibrium and compatibility of strains in a generic section of reinforced concrete.

The compatibility of strains between the most significant fibers of the section are expressed by the following equations

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_y}{x-y} = \frac{\varepsilon_s}{x-d} = \frac{\varepsilon_s'}{x-d'} \tag{2}$$

where

A_s	area of tension reinforcement
A'_s	area of compression reinforcement
b	width of rectangular cross section
b_y	width of cross section at generic depth y
d	distance from extreme compression fiber to centroid of tension reinforcement
d'	distance from extreme compression fiber to centroid of compression reinforcement
d_t	distance from extreme compression fiber to centroid of tension reinforcement
e_1	eccentricity of axial load from the centroid of tension reinforcement
h	height of cross section
P_n	nominal axial load normal to cross section
x	depth of neutral axis
y	generic depth of a fiber
ε_c	strain at extreme concrete compression fiber
ε_s	strain in tension reinforcement
ε_s'	strain in compression reinforcement
ε_y	reinforcement yield strain
σ_s	stress in tension reinforcement

σ_s' stress in compression reinforcement
σ_y stress at generic concrete compression fiber at depth y

2.3 Limit strains

Regarding the limit strains of materials, ACI 318 considers a strain of 3 ‰ in the extreme compression fiber, and a net tensile strain in tension reinforcement of 5 ‰ (a value that provides ductile behavior for most designs). EC 2 considers a strain of 10 ‰ for reinforcement, 2 ‰ for ε_{c0} (limit strain at extreme concrete compression fiber in sections under pure compression) and 3.5 ‰ for ε_{cu} (that strain in sections under bending), with f_{ck} less than or equal to 50 MPa.

In the case of high-strength concrete (f_{ck} > 50 MPa), the strain ε_{c0} for concrete under pure compression and ε_{cu} for concrete under bending is expressed in Equations (3) and (4), respectively

$$\varepsilon_{c0} = 2 + 0.085(f_{ck} - 50)^{0.53} \tag{3}$$

$$\varepsilon_{cu} = 2.6 + 35\left[\frac{(90 - f_{ck})}{100}\right]^4 \tag{4}$$

3 Overview of methods for calculating the reinforcement in concrete sections

Any method for calculating the reinforcement in a concrete section is based on the equilibrium conditions of moments and axial forces. Although different compressive stress distributions may be defined (see reference [9], among others), they are commonly idealized by a parabolic or rectangular stress block. The former shows a better behavior of concrete, but is more complex to use. The latter is more useful. The differences between the two are in practice often limited. Only two cases exist where pronounced differences arise, but with no practical implementation: (i) in the case of simple bending, for certain extreme values of the moment, resulting in an excessive reinforcement; and (ii) in some combined flexure and axial load cases, resulting in a reinforcement below the minimum area of reinforcement. For more information about these two cases, see Alarcón [10]. Some methods used to obtain the reinforcement in rectangular sections are presented below.

3.1 Standard method

This method may be found in classical literature of concrete structures, such as references [1, 2], among others. In this method, a formulation is applied to the calculation of reinforced concrete rectangular sections subjected to combined flexure and axial load. ACI 318 provides that the factored loads must be less than or equal to the design loads

$$P_u \leq \phi P_n \tag{5}$$

$$M_u \leq \phi M_n \tag{6}$$

where ϕ is the strength reduction factor (Figure 2), P_u the factored axial force normal to cross section, M_u the factored moment at section and M_n the nominal moment at section.

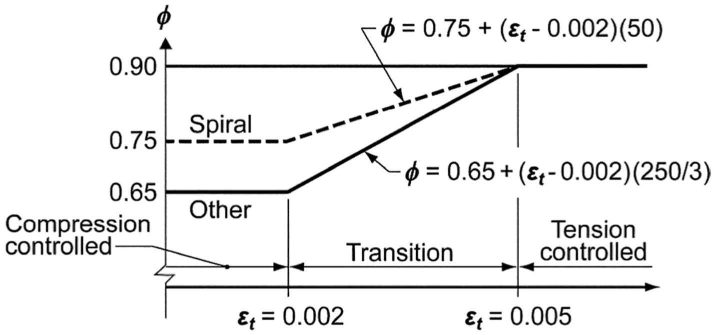

Figure 2: Variation of the strength reduction factor ϕ with net tensile strain ε_t [Source: ACI 318].

The parameter ε_t is the net tensile strain in the extreme tension steel at nominal strength, exclusive of strains due to prestress, creep, shrinkage and temperature.

This standard method states a rectangular concrete compressive stress block, with a compressive stress $0.85f_c'$ until a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain. The values that can be taken by β_1 have been developed in section 4.2, and c is the depth of neutral axis from extreme compression fiber.

Two cases are traditionally studied by the standard method. Firstly, only the tension reinforcement necessary, i.e. $A_s' = 0$, setting the balanced amount of steel ρ_b (ratio of A_s to bd producing balanced strain conditions) obtained considering a tensile strain $\varepsilon_t = \varepsilon_y$ and a strain of 0.003 in the concrete compression fiber. The expression for the ratio ρ_b is shown in Equation (7), considering a modulus of elasticity of reinforcement E_s = 200 GPa

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \frac{87000}{87000 + f_y} \tag{7}$$

with f_y the specified yield strength of reinforcement and f_c' the specified compressive strength of concrete, both in psi. ACI 318 recommends taking a reinforcement limit of $0.75\rho_b$. Unless unusual amounts of ductility are required, the $0.75\rho_b$ limitation will provide ductile behavior for most designs.

The second case is for double reinforcement (top and bottom). This happens when ρ (ratio of A_s to bd) is greater than ρ_b. In the case of cross sections under flexural and axial loading, the standard method recommends solving the problem using a computer, because of the complexity when operating with the conditions of equilibrium and compatibility of strains.

3.2 Reinforcement Sizing Diagrams

The Reinforcement Sizing Diagrams (RSD) method [8] is developed to calculate the reinforcement of rectangular concrete sections subjected to combined flexure and axial load. The reinforcement required to provide adequate strength is determined as a function of the neutral axis depth. Acceptable combinations of top and bottom reinforcement are plotted as a function of the neutral axis depth on a reinforcement sizing diagram.

From the conditions of equilibrium, using the rectangular concrete compressive stress block, it is possible to obtain the reinforcement sizing equations shown in Table 1.

	Tension reinforcement (A_s)	**Compression reinforcement (A'_s)**
If $c < 0$	$A_s = \dfrac{\frac{M_u}{\phi} - \frac{P_u}{\phi}\left(\frac{h}{2} - d'\right)}{-\sigma_s (d - d')}$	$A'_s = \dfrac{\frac{M_u}{\phi} + \frac{P_u}{\phi}\left(d - \frac{h}{2}\right)}{\sigma'_s (d - d')}$
If $0 \le \beta_1 c \le h$	$A_s = \dfrac{\frac{M_u}{\phi} - \frac{P_u}{\phi}\left(\frac{h}{2} - d'\right) - C_c\left(d' - \frac{\beta_1 c}{2}\right)}{-\sigma_s (d - d')}$	$A'_s = \dfrac{\frac{M_u}{\phi} + \frac{P_u}{\phi}\left(d - \frac{h}{2}\right) - C_c\left(d - \frac{\beta_1 c}{2}\right)}{\sigma'_s (d - d')}$
If $\beta_1 c > h$	$A_s = \dfrac{\frac{M_u}{\phi} - \left(\frac{P_u}{\phi} - C_c\right)\left(\frac{h}{2} - d'\right)}{-\sigma_s (d - d')}$	$A'_s = \dfrac{\frac{M_u}{\phi} + \left(\frac{P_u}{\phi} - C_c\right)\left(d - \frac{h}{2}\right)}{\sigma'_s (d - d')}$

Table 1. Reinforcement sizing equations of RSD method.

The most representative parameters in these equations are β_1 (rectangular stress block coefficient), C_c (resultant compressive force in the concrete), σ_s and σ'_s (stress in both tension and compression reinforcement, respectively), and d (distance from extreme compression fiber to centroid of tension reinforcement).

4 Reinforcement optimization of concrete rectangular cross sections

4.1 Optimum design problem

In the design of concrete sections, a question arises about whether the values obtained for the reinforcement are the most appropriate or not. This question is not only from the point of view of resistance, since these values are obtained from the equilibrium equations, but from the perspective of optimum reinforcement, which affects not only cost, but also the environmental aspects related to the reduction of resources consumed for the production of steel for reinforcement.

In the design of members under combined flexure and axial load it is common to use conventional methods to obtain the reinforcement with symmetrical distribution. This may be appropriate in some cases of flexural moments with different signs and similar values. However, in other cases this distribution may result in uneconomical constructive simplification and be environmentally inadequate, with it being more interesting not to use the symmetrical distribution, but to search for another distribution with optimum reinforcement. This is the case, for example, of retaining walls with a vertical load at the top (the soil pressure is causing single sign flexure in the wall), or circular piers for retaining walls, in which longitudinal reinforcement can be reduced by more than 50 % compared with traditional designs [11]. Admittedly, the probability of positioning error increases in this case of asymmetrical reinforcement, but it can be prevented with more careful control of this phase of the construction.

In this section of the paper, the problem of calculating the optimum reinforcement in a rectangular section subjected to combined flexural moment and axial force is studied. The resolution of this problem is based on research by Gil-Martín *et al.* [5] and Hernández-Montes *et al.* [8], where the RSD and the Theorem of Optimal Section Reinforcement are presented. A simple optimization method is implemented that allows considering high-strength concrete, with a minimum area of reinforcement according to ACI 318 or EC 2, and a ductility constraint on the neutral axis depth to guarantee certain curvature.

It should be highlighted that when solving the problem of optimum reinforcement what is mainly achieved is automating, with a negligible computational time, the reinforcement calculation of a cross section subjected to flexural and axial loading. Moreover, when observing the graphical results, particularly the depth of neutral axis, the physical sense of the problem can be visualized, since the stress-strain state of the section is known instantly.

4.2 Objective function

The objective function of the optimization problem is the total area of steel reinforcement A_{st}, which depends on the geometry of the section, the strength of materials, the depth of neutral axis, and the flexural moment and axial force

$$A_{st} = A_{st}\left(P_u, M_u, \phi, f_c', f_y, b, h, d, d', c\right) \tag{8}$$

This function is obtained from the equilibrium equations in the cross section; these equations have been developed in section 2.2, using the rectangular concrete compressive stress block (Figure 3). The reinforcement of the cross section depends on the depth of the neutral axis. Therefore, there is a design space that contains feasible reinforcement solutions among which there is one that provides the optimum reinforcement configuration. The so-called optimum depth of neutral axis corresponds with that optimum configuration.

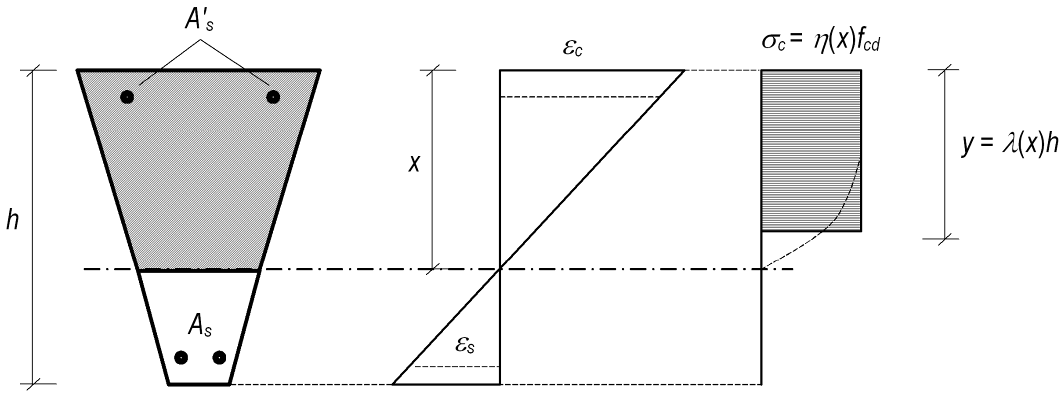

Figure 3: Equivalent rectangular compressive stress block in EC 2.

Equations (9) to (12) define the compressive block in concrete according to ACI 318

$$\sigma_c = 0.85 f_c^{'} \tag{9}$$

$$a = \beta_1 c \tag{10}$$

where σ_c is the stress at extreme concrete compression fiber, y the depth of equivalent rectangular stress block, and β_1 is defined by Equations (11) and (12)

$$\beta_1 = 0.85 - 0.008\left(f_c^{'} - 30\right) \tag{11}$$

$$0.65 \leq \beta_1 \leq 0.85 \tag{12}$$

with f_c' in MPa. The change of nomenclature from EC 2 to ACI 318 is $y = a$ and $x = c$.

Equations (13) to (16) define the compressive block in concrete according to EC 2

$$\sigma_c = \eta f_{cd} \tag{13}$$

$$y = \lambda x \tag{14}$$

where η is the effective strength factor and λ is the effective depth factor of the compressive stress block

$$\begin{matrix} \eta = 1 \\ \lambda = 0.8 \end{matrix} \qquad f_{ck} \leq 50 \text{ MPa} \tag{15}$$

$$\eta = 1 - \frac{f_{ck} - 50}{200}$$
$$\lambda = 0.8 - \frac{f_{ck} - 50}{400} \qquad f_{ck} > 50 \text{ MPa} \qquad (16)$$

The following equations for reinforcement and constraints are expressed according to ACI 318. Nevertheless, EC 2 has also been considered in the procedure implemented in the program code used in this research.

4.2.1 Area of tension reinforcement

Equations (17) to (19) are used to obtain the tension steel A_s depending on the neutral axis depth c or on the equivalent rectangular stress block depth a

$$A_s = \frac{\frac{M_u}{\phi} - \frac{P_u}{\phi}\left(\frac{h}{2} - d'\right)}{-\sigma_s(d - d')} \qquad \text{if} \quad c \le 0 \qquad (17)$$

$$A_s = \frac{\frac{M_u}{\varphi} - \frac{P_u}{\varphi}\left(\frac{h}{2} - d'\right) - 0.85 f_c^{'} ab\left(d' - \frac{a}{2}\right)}{-\sigma_s(d - d')} \qquad \text{if} \quad 0 < a \le h \qquad (18)$$

$$A_s = \frac{\frac{M_u}{\phi} - \left(\frac{P_u}{\phi} - 0.85 f_c^{'} hb\right)\left(\frac{h}{2} - d'\right)}{-\sigma_s(d - d')} \qquad \text{if} \quad a > h \qquad (19)$$

4.2.2 Area of compression reinforcement

Equations (20) to (22) are used to obtain the tension steel A'_s depending on the neutral axis depth c or on the equivalent rectangular stress block depth a

$$A'_s = \frac{\frac{M_u}{\phi} + \frac{P_u}{\phi}\left(d - \frac{h}{2}\right)}{\sigma'_s(d - d')} \qquad \text{if} \quad c \le 0 \qquad (20)$$

$$A'_s = \frac{\frac{M_u}{\phi} + \frac{P_u}{\phi}\left(d - \frac{h}{2}\right) - 0.85 f_c^{'} ab\left(d - \frac{a}{2}\right)}{\sigma'_s(d - d')} \qquad \text{if} \quad 0 < a \le h \qquad (21)$$

$$A'_s = \frac{\frac{M_u}{\phi} + \left(\frac{P_u}{\phi} - 0.85 f_c^{'} hb\right)\left(d - \frac{h}{2}\right)}{\sigma'_s(d - d')} \qquad \text{if} \quad a > h \qquad (22)$$

4.2.3 Total area of reinforcement

Equations (23) to (25) are used to calculate the total area of steel, the objective function is to minimize (addition of both tension and compression reinforcement).

$$A_{st} = \frac{\frac{M_u}{\phi} - \frac{P_u}{\phi}\left(\frac{h}{2} - d'\right)}{-\sigma_s (d - d')} + \frac{\frac{M_u}{\phi} + \frac{P_u}{\phi}\left(d - \frac{h}{2}\right)}{\sigma'_s (d - d')} \qquad \text{if} \quad c \le 0 \tag{23}$$

$$A_{st} = \frac{\frac{M_u}{\phi} - \frac{P_u}{\phi}\left(\frac{h}{2} - d'\right) - 0.85 f'_c ab \left(d - \frac{a}{2}\right)}{-\sigma_s (d - d')} + \dots$$

$$\dots + \frac{\frac{M_u}{\phi} + \frac{P_u}{\phi}\left(d - \frac{h}{2}\right) - 0.85 f'_c ab \left(d - \frac{a}{2}\right)}{\sigma'_s (d - d')} \qquad \text{if} \quad 0 < a \le h \tag{24}$$

$$A_{st} = \frac{\frac{M_u}{\phi} - \left(\frac{P_u}{\phi} - 0.85 f'_c hb\right)\left(\frac{h}{2} - d'\right)}{-\sigma_s (d - d')} + \dots$$

$$\dots + \frac{\frac{M_u}{\phi} + \left(\frac{P_u}{\phi} - 0.85 f'_c hb\right)\left(d - \frac{h}{2}\right)}{\sigma'_s (d - d')} \qquad \text{if} \quad a > h \tag{25}$$

4.3 Constraints

The constraints for the design variables A_s, A'_s and c are stated in the following sections.

4.3.1 Reinforcement constraints

As mentioned in section 4.2, the objective function is obtained from the RSD method [8]. In this method, the sign of stresses and forces in the materials is positive for compression and negative for tension. Since both the resultant force and its associate stress distribution have the same sign, the values of A_s and A'_s, which correspond to the relationship between them, must be positive.

Moreover, the option of considering minimum reinforcement according to ACI 318 or EC 2 may be activated before starting the calculation of the optimum reinforcement. According to ACI 318, the amount of steel in the tension reinforcement shall not be less than the amount

$$A_s \ge \frac{\sqrt{f'_c}}{4 f_y} bd \ge \frac{1.4 bd}{f_y} \tag{26}$$

However, this does not consider any minimum reinforcement for the compression zone.

According to EC 2, the minimum tension reinforcement is

$$A_s \geq 0.26 \frac{f_{ctm}}{f_{yk}} bd > 0.0013bd \tag{27}$$

where f_{ctm} is the average tensile strength of concrete and f_{yk} is the characteristic yield strength of reinforcement.

In the case of combined flexure and axial load, the minimum compression reinforcement is

$$A'_s \geq \frac{0.05N_d}{f_{yd}} \tag{28}$$

where N_d is the design axial force normal to cross section and f_{yd} is the design yield strength of reinforcement.

In the case of low eccentricity, the minimum total reinforcement is

$$A_{st} = A_s + A'_s \geq 0.1 \frac{N_d}{f_{yd}} > 0.002bh \tag{29}$$

Finally, in the case of flexural moment and tensile axial force, the total reinforcement must satisfy

$$A_{st} \geq A_c \frac{f_{ctm}}{f_{yd}} \tag{30}$$

where A_c is the area of concrete section.

4.3.2 Constraints on the depth of neutral axis

As mentioned in section 4.2, the objective function is evaluated for values of the depth of the neutral axis that are constrained within the range

$$\rho_i h \leq x \leq \rho_s h \tag{31}$$

where ρ_i is a bottom factor of proportion ($\rho_i \leq 0$) and ρ_s is an upper factor of proportion ($\rho_s > 0$).

In the case of combined flexure and tensile axial load (in which tensile axial load dominates the behavior of the cross section), the reinforcement (Equations (17), (20) and (23)) is constant for values of x (or c) between $-\infty$ and ($\rho_i h$) for which the strain in the tension reinforcement corresponds to a stress equal to the yield strength. In this case, it is only necessary to evaluate the objective function within the range $\rho_i h \leq x \leq 0$, with a value of ρ_i according to EC 2 as

$$\rho_i = \frac{\varepsilon_y d - 0.01d'}{(0.01 - \varepsilon_y)h} \tag{32}$$

It is necessary to use a maximum allowable strain in the steel to obtain ρ_i. The value 0.01 provided by EC 2 can be used in order to obtain a realistic value for ρ_i and evaluate the objective function within the mentioned interval.

The case of combined flexure and compressive axial load (in which compressive axial load dominates the behavior of the cross section) presents a similar situation. The reinforcement (Equations (17), (20) and (23)) is practically constant for values of x (or c) in a range from $\rho_s h$ to $+\infty$. For the value $\rho_s h$ the reinforcement reaches the maximum compressive stress of 400 MPa according to EC 2, or 420 MPa according to ACI 318. In this case, it is enough to evaluate the objective function in the range $h < x \leq \rho_s h$. The parameter ρ_s is not obtained analytically, but numerically, as described in section 4.4.

The extreme cases with only tensile or compressive axial load have been evaluated analytically, since the strain in the reinforcement is known, and consequently so is the stress, therefore the neutral axis depth is no longer an unknown.

Finally, if greater ductility in the section is required, the neutral axis depth should be constrained to a certain maximum value $\rho_s h$. For this purpose, ACI 318 recommends using a strain in the tension reinforcement of at least 0.0075, which means that the neutral axis depth must be less than the corresponding one to the strain of 0.005. This limitation may be used as ductility constraint by using a factor ρ_i obtained from a neutral axis depth c, corresponding to a strain in the reinforcement of 0.0075. For more detailed considerations about ductility criteria, see [12, 13].

4.4 Optimization methodology

Among the existing optimization methods, deterministic methods may be used in this kind of problems in order to find the global minimum by an exhaustive search over the design space. Due to the nature of the problem (with few design variables), it is appropriate to use the *covering methods* [14].

The basic idea of covering methods is the search for the global minimum "covering" the design space by evaluating the objective function at every point. This is, of course, an infinite calculation and is therefore impossible to implement and use. In the particular design problem of this research it is possible to apply a *brute force approach* as a simple way of evaluating the objective function in a finite number of points, with reasonable precision and with a low computational cost.

Until recently, these procedures would have been impossible to implement, since they employ complex optimization algorithms to solve the problem. However, this is possible today due to the development in the computing processors incorporated into any modern personal computer.

Thus, the methodology is quite simple, but effective. It consists of setting an interval of the neutral axis depth from an initial value of x ($\rho_i h$) to a final value ($\rho_s h$) according to section 4.3.2, and of using a value of this depth (x_i) that is increasing in a small increment (p) from the iteration i–1 to the i:

$$x_i = x_{i-1} + p \tag{33}$$

For cases with low eccentricity (neutral axis outside the section), the final value $\rho_s h$ is the point from which the reinforcement remains constant. This value can become very large because the behavior of the section tends towards the state of pure compression ($M_u = 0$), and therefore the neutral axis tends to $+\infty$.

This problem has been solved by making the increment p rise gradually while searching for the value of ρ_s for which the total reinforcement is minimum. This minimum is obtained when the difference in the reinforcement is less than or equal to a certain tolerance between consecutive iterations. Almost any tolerance may be chosen and be useful, with a low computational cost. In this case 10^{-6} mm^2 has been chosen as tolerance. Obviously this tolerance has no practical sense, but is purely mathematical, to find the global minimum.

The optimization process has been performed using a programming routine implemented in the program code used in this research. The code MATLAB [15] has been used.

5 Examples

Four numerical examples are presented to show the automation of the optimum reinforcement calculation under several combinations of flexural and axial loading. In the first three examples, conventional-strength concrete is used, and the results using ACI 318 and EC 2 are compared with those obtained by conventional methods. In the fourth example, high-strength concrete is used.

5.1 Section under flexure

A rectangular section of 400×500 mm under a factored flexural moment of 400 kN-m is studied. The strength of materials are f_c' = 30 MPa for concrete and f_y = 500 MPa for steel. The distance d' from centroid of upper reinforcement to upper fiber is 50 mm.

The variation of reinforcement depending on the neutral axis depth x using ACI 318 is shown in Figure 4. The optimum reinforcement is achieved at the point where compression reinforcement reaches the zero value.

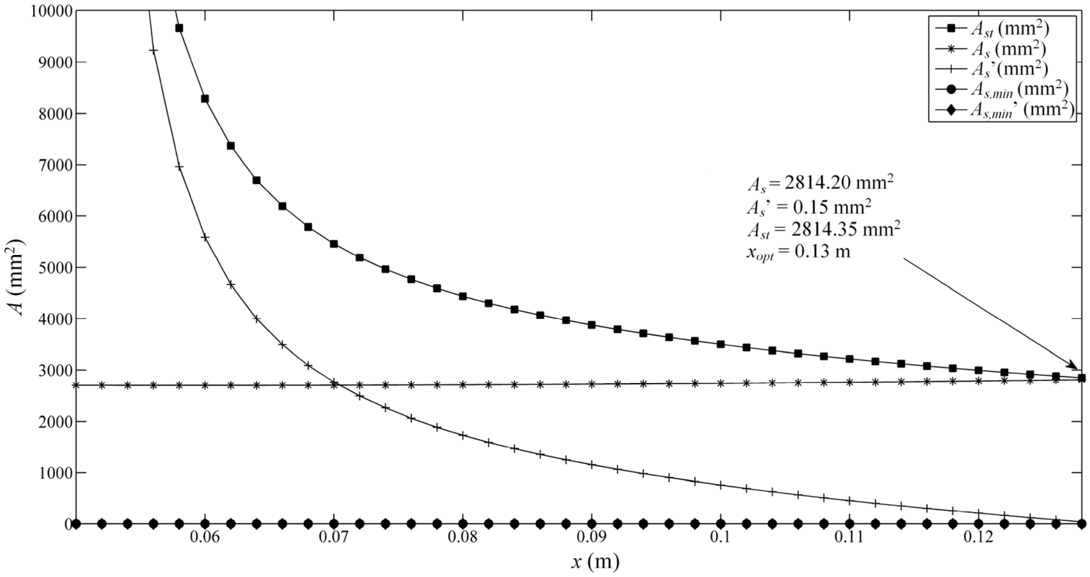

Figure 4: Reinforcement over the space design using ACI 318. Cross section of 400×500 mm under flexure.

The optimum results obtained using ACI 318, EC 2 and the conventional procedure, are shown in Table 2. The main difference between the standards is a 6.4 % increase in the total reinforcement using EC 2 as compared to ACI 318. The conventional method provides the same optimum results to those obtained using ACI 318, which always occurs when flexure acts alone.

	Optimization method (ACI 318)	**Optimization method (EC 2)**	**Conventional method**
A_s (mm^2)	2814.20	2389.11	2814.20
A'_s (mm^2)	0.15	0.05	0.15
A_{st} (mm^2)	2814.35	2389.16	2814.35
x (mm)	130	162	130

Table 2. Optimum results. Cross section of 400×500 mm under flexure.

5.2 Section under flexural and axial loading (high eccentricity)

A rectangular section of 300×400 mm is studied under a combined factored flexural moment of 400 kN-m and factored axial force of 1800 kN. The strength of materials are $f_c' = 30$ MPa for concrete and $f_y = 500$ MPa for steel. The distance d' from centroid of upper reinforcement to upper fiber is 50 mm.

The variation of reinforcement depending on the neutral axis depth *x* using ACI 318 is shown in Figure 5. In the case of EC 2, the optimum reinforcement is achieved for *x* equal to the theoretical limit depth (depth for which the tension reinforcement has a strain equal to the yield limit of steel). In the case of ACI 318, *x* is equal to a depth lower than that for balanced reinforcement. This depth is located in the “transition region”, as ACI 318 has named it.

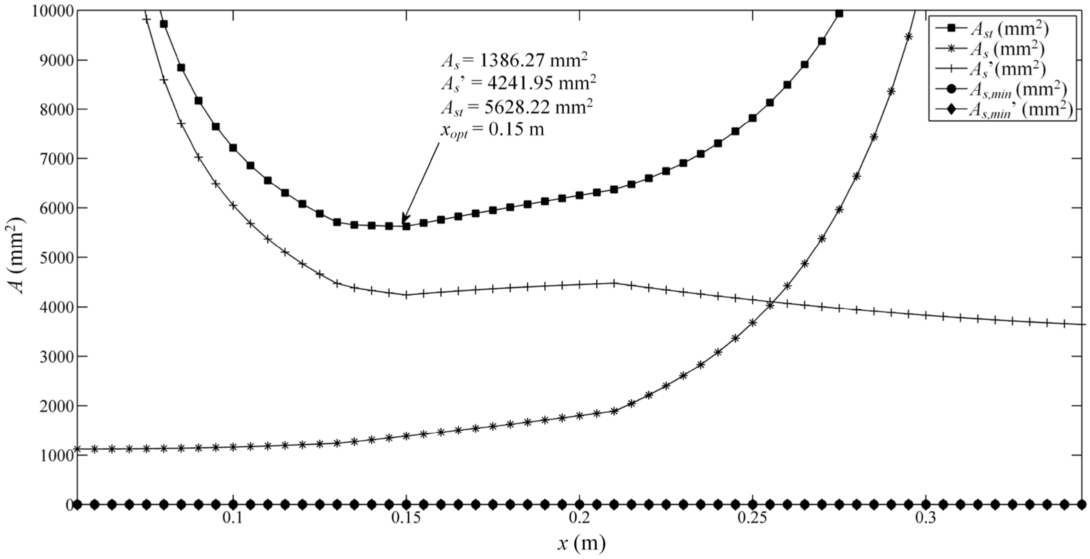

Figure 5. Reinforcement over the space design using ACI 318. Cross section of 300×400 mm under flexural and axial loading (high eccentricity).

In the resolution of the combined flexure and axial load case by the conventional procedure, it is necessary to use a computer or the interaction diagrams to obtain the value of reinforcement, including the compatibility of strains in both cases. The numerical optimization method proposed in this paper is perfectly adapted to the methodology of the conventional procedure. Since both methods are the same, the column of results for the conventional method has been removed in the tables of the following examples, and only the results of the optimization method are shown.

The results obtained by the proposed optimization method using both ACI 318 and EC 2 are shown in Table 3. The use of ACI 318 provides results 30 % higher than those obtained by EC 2.

5.3 Section under flexural and axial loading (low eccentricity)

A rectangular section of 400×400 mm is studied under a combined factored flexural moment of 80 kN-m and factored axial force of 4000 kN. The strength of materials are f_c' = 30 MPa for concrete and f_y = 500 MPa for steel. The distance d' from centroid of bottom reinforcement to upper fiber is 50 mm.

	Optimization Method (ACI 318)	Optimization Method (EC 2)
A_s (mm^2)	1386.27	1285.55
A'_s (mm^2)	4241.95	3042.01
A_{st} (mm^2)	5628.22	4327.56
x (mm)	150	216

Table 3. Optimum results. Cross section of 300×400 mm under flexural and axial loading (high eccentricity).

The relationship between the reinforcement and the neutral axis depth, using ACI 318, is shown in Figure 6. The optimum reinforcement is achieved for a neutral fiber depth where the reinforcement is nearly constant. That depth has not been shown in Figure 6 because of its high value (17,658,019,827 mm). To illustrate this, a stretch is shown in which the reinforcement is tending to a value practically constant and very close to the optimum.

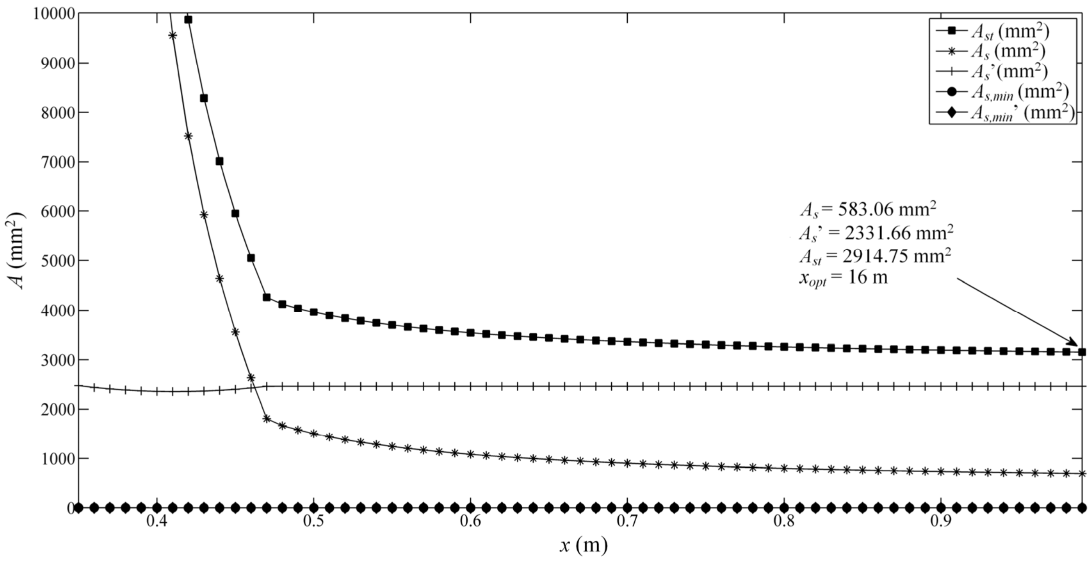

Figure 6. Reinforcement over the space design using ACI 318. Cross section of 400×400 mm under flexural and axial loading (low eccentricity).

The minimum reinforcement has not been shown in Figure 6 in order not to distort the results, and thus enable comparisons with the results obtained using ACI 318 and using EC 2 (Table 4). The reinforcement obtained with ACI 318 is 45.50 % higher than that obtained with EC 2.

	Optimization Method (ACI 318)	Optimization Method (EC 2)
A_s (mm^2)	583.06	333
A'_s (mm^2)	2331.69	1667
A_{st} (mm^2)	2914.75	2000
x (mm)	16000	17,658,019,840

Table 4. Optimum results. Cross section of 400×400 mm under flexural and axial loading (low eccentricity).

5.4 Section of high-strength concrete

The proposed optimization procedure is generalized to high-strength concrete. Then, the example in section 5.2 is used in this case with the only difference being in the strength of concrete (f_c' = 60 MPa). The relationship between the reinforcement and neutral fiber depth x is shown in Figure 7, and the values of the optimum reinforcement and its corresponding depth of neutral axis are indicated.

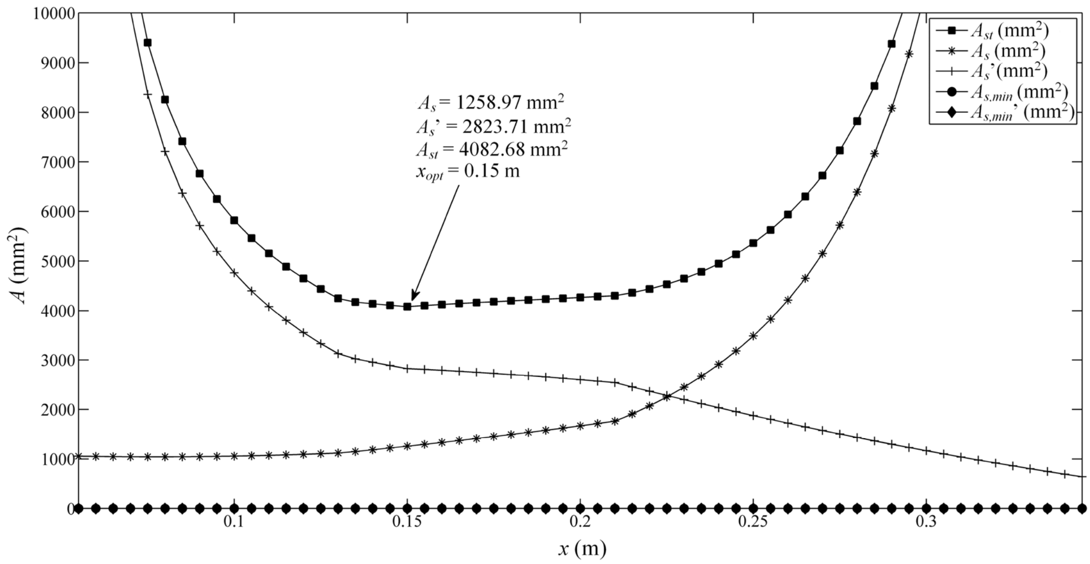

Figure 7. Reinforcement over the space design using ACI 318. Cross section of 300×400 mm under flexural and axial loading. High-strength concrete, f_c' = 60 MPa.

The results obtained with the proposed optimization procedure using both codes are shown in Table 5. The reinforcement obtained with ACI 318 is 45 % higher than that obtained with EC 2.

	Optimization Method (ACI 318)	Optimization Method (EC 2)
A_s (mm^2)	1258.97	1366.18
A'_s (mm^2)	2823.71	1450.92
A_{st} (mm^2)	4082.68	2817.10
x (mm)	150	200

Table 5. Optimum results. Cross section of 300×400 mm under flexural and axial loading. High-strength concrete, f_c' = 60 MPa.

6 Conclusions

Traditionally, computers have been used in the design process of concrete sections in order to obtain the reinforcement by conventional methods. These methods are usually subjected to a casuistry related to the values of the loads at the section and to the relationship between them. The application of optimization techniques to the design process widens the field of computer use, and allows the designer to obtain optimum designs for the design conditions that have been determined.

In this paper, an automated design procedure is proposed for calculating reinforced concrete sections under flexural and axial loading, being, moreover, the calculated reinforcement the optimum. The procedure, based on the equilibrium conditions of moments and forces at the section, includes the processing of high-strength concrete and several design constraints, such as minimum reinforcement and the possibility of limiting the depth of the neutral axis. The algorithm has been implemented in a simple program code. The results are achieved in a negligible calculation time (tenths of a second) by using any personal computer currently available.

References

[1] A.H. Nilson, D. Darwin, C.W. Dolan, “Design of concrete structures”, 14th Ed., McGraw-Hill, New York, 2010.
[2] J. Calavera, “Design and calculation of concrete structures” (in Spanish), Intemac, Madrid, 2008.
[3] E. Raue, S. Hahn, “Optimum reinforcement design of concrete cross-sections considering deformation constraints”, J. Civ. Eng. Manag., 11(1), 65-71, 2005.
[4] A. Tomás, P. Martí, “Design of reinforcement for concrete co-planar shell structures using optimization techniques”, Meccanica, 45(5), 657-669, 2010.
[5] L.M. Gil-Martín, M. Aschheim, E. Hernández-Montes, M. Pasadas-Fernández, “Recent developments in optimal reinforcement of RC beam and column sections”, Eng. Struct., 33(4), 1170-1180, 2011.

[6] Technical Committee CEN/TC250, "Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings", European Committee for Standardization, Brussels, 2004.
[7] ACI Committee 318, "Building code requirements for structural concrete (ACI 318-08) and commentary (318R-08)", ACI, Farmington Hills, MI, 2008.
[8] E. Hernández-Montes, L.M. Gil-Martín, M. Aschheim, "Design of concrete members subjected to uniaxial bending and compression using reinforcement sizing diagrams", ACI Struct. J., 102(1), 150-158, 2005.
[9] J.C.M. Ho, H.J. Pam, J. Peng, Y.L. Wong, "Maximum concrete stress developed in unconfined flexural RC members", Comput. Concrete, 8(2), 207-227, 2011.
[10] A. Alarcón, "Optimum reinforcement dimensioning in concrete rectangular sections under flexural and axial loading" (in Spanish), Universidad Politécnica de Cartagena (UPCT), Spain, 2010.
[11] L.M. Gil-Martín, E. Hernández-Montes, M. Aschheim, "Optimal reinforcement of RC columns for biaxial bending", Mater. Struct., 43(9), 1245-1256, 2010.
[12] M.Z. Cohn, P. Riva, "Flexural ductility of structural concrete sections", PCI J., 36(2), 72-87, 1991.
[13] F.T.K. Au, C.C.Y. Leung, A.K.H. Kwan, "Flexural ductility and deformability of reinforced and prestressed concrete sections", Comput. Concrete, 8(4), 473-489, 2011.
[14] J.S. Arora, "Introduction to optimum design", Elsevier Academic Press, London, 2004.
[15] Matlab (R2008a), The MathWorks, Inc., Natick, MA, 2008.