



An Evolutionary Structural Optimization applied to Fluid-Structure Problems

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Abstract

This work presents a topology optimization formulation based on the frequency response of a multiphysics problem involving fluid-structure interaction. A mixed formulation (\mathbf{u}/p) is used, in which the pressure and displacement are governed by the Helmholtz equation and the elasticity equation, respectively. The optimization method used in this paper is the bi-directional evolutionary structural optimization (BESO), this process consist of a successive elimination and replacement of elements in the design domain. The feasible space of solution is defined initially and through a sensitivity analysis of the frequency response functions using finite element analysis the evolutionary algorithm removes or adds solid elements. A sensitive analysis is described for the dynamic problems and the sensitive number is evaluated for several conditions. The formulation implemented in FORTRAN and the ANSYS Parametric Design Language (APDL) seeks the optimum topology, considering the mean pressure minimization in specific nodes of the system for a certain range of frequency excitation and subject to volume constraints. A number of final topologies for fluid-structure problems are shown, as well as their intermediary topologies and evolutionary history. The results demonstrate that this methodology can be applied to this type of problem with good efficiency.

Keywords: topology optimization, fluid-structure interaction, finite element analysis, mixed formulation, frequency response function, ESO/BESO method.

1 Introduction

In order to improve the vibration or pressure characteristics of a system, topology optimization might be applied [1]. The basic idea is to find an optimal distribution of material in a structural design domain, considering an objective function. In this

work, we investigated how the vibration or pressure characteristics of given examples can be improved on the basis of the structural frequency response function (FRF) with topology optimization.

Between researches considering dynamic responses, we can cite the work of Diaz and Kikuchi [2], they applied topology optimization considering the eigenvalues of a structure for optimal plate reinforcement using the homogenization method. Ma et al. [3] extended the same method to problems of vibrating structures. The topology optimization considering dynamic responses was also investigated with the classical material interpolation method 'SIMP' (*Solid Isotropic Material with Penalization*) [1]. Many efforts have been made to develop and optimize dynamic systems. Frequency optimization is of great importance in many engineering fields and dynamic responses must be taken into account in a wide range of problems. One of these problems is the class of multiphysics systems. The applications of topology optimization methods have also been extended to multiphysics problems [4, 5, 6]. Although these procedures have reached a satisfactory level of maturity, there are still many topology optimization features open to research or less-than-satisfactorily resolved issues [6]. Commercial FEM packages often contain solvers for multiphysics problems, however, they do not enable optimization. A particular class of multiphysics problems involves fluid-structure interaction (FSI) is the main topic of this work.

Among of current optimization methods, the Bi-directional Evolutionary Structural Optimization (BESO) was chosen in this work for FSI problems analysis, since the method is almost not explored in this area. The technique, so called Evolutionary Structural Optimization (ESO), was first introduced in the 90's by Xie and Steven [7]. Associated with the finite element analysis, the ESO method was initially proposed as a gradual removal of inefficient material from the design domain until the remaining structure converges to the optimum topology. Material elimination is carried out after a sensitivity analysis. However, it is questionable that eliminated elements can never return to the design domain, since the sensitivity analysis is carried out only for the solid elements in the structure. Thus, a later development of this method was called Bi-directional ESO (BESO), in this new methodology elements are also added in void positions near to the elements with the highest sensitivity numbers [8]. In this case, the sensitivity number is a local index and represents the sensitivity of each element with respect to the objective function. Studies with the BESO method have recently been published presenting convergent and mesh independent solutions for stiffness maximization [9], for natural frequency maximization [10] among others. Critical analysis of ESO-based methods are found in [11], while a later review about the ESO/BESO methods is found in [12, 13]. In multiphysics problems, the ESO/BESO methods lack of methodologies and publications, especially in fluid-structure coupled systems.

The combination of evolutionary topology optimization techniques and multiphysics problems involving fluid-structure interaction is proposed. The scope of this work focuses on the mean pressure minimization in coupled systems. Frequency response optimization is of great importance in many engineering fields like acoustics systems and fluid dynamics. The possibility of removing and adding material systematically

with the evolutionary method could be a helpful procedure to explicitly define fluid-structure interfaces. However, only cases with immovable interfaces will be considered in this paper. The paper is organized as follows: Section 2 presents the governing equations and the finite element model for the fluid-structure interaction coupled system. In Section 3, the topology optimization problem for pressure minimization is formulated and the sensitivity number is presented. Section 4 shows numerical examples of the optimization methodology, BESO. Finally, conclusions are shown in Section 5.

2 Fluid-Structure Interaction: Governing Equations and the Finite Element Model

Herein, the analyzed systems are limited to a flexible structure in contact with an enclosed acoustic fluid, where the responses of the structure are significantly affected by the fluid. For this fluid-structure system, the structure can be described by the differential equation of motion for a continuum body assuming small deformations and the fluid by the acoustic wave equation. For the standard approach, the governing equations for the fluid and structural domains as well as the coupling boundary conditions are defined as follows.

2.1 Helmholtz equation

In this paper, the fluid is considered inviscid, irrotational and only under small translations conditions. The governing equation for the pressure in a nonhomogeneous acoustic medium is described by the Helmholtz equation

$$\nabla^2 p_f + \frac{\omega^2}{c_f^2} \frac{\partial^2 p_f}{\partial t^2} = 0 \quad \left(k = \frac{\omega}{c_f} \right) \quad \Omega_f \quad (1)$$

where p_f is the pressure in the analysis domain Ω_f , c_f is the local speed of sound, ω is the angular frequency and k the wave number. The pressure field is obtained by solving the Helmholtz equation imposing proper boundary conditions. In this paper, it is considered the following boundary conditions:

$$p_f = p_0 \quad (2)$$

$$\mathbf{n} \nabla p_f = 0 \quad (3)$$

representing the pressure boundary condition, Equation (2), and the hard wall condition, Equation (3), where p_0 is the pressure input and \mathbf{n} is the outward unit normal to the fluid.

2.2 Linear elasticity

The linear structural analysis can be described by the elasticity equation

$$\tilde{\nabla}^T \sigma_s + \mathbf{b}_s = \rho_s \frac{\partial^2 \mathbf{u}_s}{\partial t^2} \quad \Omega_s \quad (4)$$

where σ_s is the stress tensor, \mathbf{b}_s are the body forces and \mathbf{u}_s is the displacements field. Boundary conditions are applied as follows:

$$\mathbf{n}_s \sigma = \mathbf{f}^{S_b} \quad S_b \quad (5)$$

$$\mathbf{u}_s = \mathbf{u}_s^{S_u} \quad S_u \quad (6)$$

Equation (5) and Equation (6) represent the Neumann and Dirichlet boundary conditions, respectively, where \mathbf{f}^{S_b} is the surface traction on S_b , $\mathbf{u}_s^{S_u}$ is the prescribed displacement on S_u and \mathbf{n}_s is the outward unit normal to the solid medium.

2.3 The coupled fluid-structure system

At the interface $\partial\Omega_{sf}$ between the structural and fluid domains, the fluid and the structure move together in the normal direction of the boundary. The normal vector $\mathbf{n} = \mathbf{n}_f = -\mathbf{n}_s$ can be used in order to obtain the displacement boundary condition

$$\mathbf{u}_s \mathbf{n}|_{\partial\Omega_{sf}} = \mathbf{u}_f \mathbf{n}|_{\partial\Omega_{sf}} \quad (7)$$

and the continuity in pressure

$$\sigma_s|_n = -p \quad (8)$$

With relations derived from the governing equations and the previous coupling conditions, the interface forces may be obtained. The force acting on the structure provided by the fluid pressure is

$$\mathbf{f}_f = \int_{\Omega_{sf}} \mathbf{N}_s^T \mathbf{n} \mathbf{N}_f ds \mathbf{p}_f \quad (9)$$

and the force acting on the fluid domain can be expressed in structural acceleration

$$\mathbf{f}_s = -\rho_f \int_{\Omega_{sf}} \mathbf{N}_f^T \mathbf{n} \mathbf{N}_s ds \ddot{\mathbf{u}}_s \quad (10)$$

where ρ_f is the static density of the fluid and \mathbf{N} contains the finite element shape functions for the interface.

The introduction of a spatial coupling matrix

$$\mathbf{L}_{sf} = \int_{\Omega_{sf}} \mathbf{N}_s^T \mathbf{n} \mathbf{N}_f ds \quad (11)$$

allows the coupling force to be written as

$$\mathbf{f}_f = \mathbf{L}_{sf} \mathbf{p}_f \quad (12)$$

and

$$\mathbf{f}_s = -\rho_f \mathbf{L}_{sf}^T \ddot{\mathbf{u}}_s \quad (13)$$

Thus, the fluid-structure problem can then be described by an unsymmetrical system of equations

$$\begin{bmatrix} \mathbf{M}_s & \mathbf{0} \\ \rho_f \mathbf{L}_{sf}^T & \mathbf{M}_f \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_s \\ \ddot{\mathbf{p}}_f \end{bmatrix} + \begin{bmatrix} \mathbf{K}_s & -\mathbf{L}_{sf} \\ \mathbf{0} & \mathbf{K}_f \end{bmatrix} \begin{bmatrix} \mathbf{u}_s \\ \mathbf{p}_f \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_f \end{bmatrix} \quad (14)$$

$$[\mathbf{M}_{fs}] [\ddot{\mathbf{u}}_{fs}] + [\mathbf{K}_{fs}] [\mathbf{u}_{fs}] = [\mathbf{f}_{fs}]$$

where \mathbf{f}_s and \mathbf{f}_f are the load vectors for both domains.

In the multiphysics coupling analysis, the fluid analysis provides pressure loads to the structural analysis, and the structural analysis provides accelerations to the fluid analysis. These finite element model has been used to calculate responses of acoustic-structure interaction problems. For details and derivations of the coupling integrals as well as the governing equations, see [14, 15].

3 Bi-directional Evolutionary Structural Optimization

3.1 Problem Statement

Considering volume constraint, the optimization problem of frequency response minimization can be stated as:

$$\begin{aligned} &\text{Minimize} && |\mathbf{u}_{fs}| \\ &\text{Subject to:} && V_f - \sum_{i=1}^n V_i x_i = 0 \end{aligned} \quad (15)$$

$$x_i = x_{min} \text{ or } 1$$

where \mathbf{u}_{fs} is the frequency response, V_i is the volume of an individual element, V_f the prescribed final structural volume and n is the total number of elements in the system. The binary design variable x_i declares the presence of a completely solid element (1) or the density of a void element with a small value of x_{min} (e.g. 10^{-4}).

3.2 Vibroacoustic sensitivity

The vibroacoustic sensitivity analysis involves the evaluation of the fluid-structure coupled system response under a structural change. The objective here is to evaluate the sensitivity of the vibroacoustic response $|\mathbf{u}_{fs}|$ with respect to the variables x_i .

The residual R_{fs} of the governing equation of the coupled dynamic problem is given by:

$$\begin{aligned} R_{fs} &= (\mathbf{K}_{fs} - \omega^2 \mathbf{M}_{fs}) |\mathbf{u}_{fs}| - |\mathbf{f}_{fs}| \\ R_{fs} &= \mathbf{Z} |\mathbf{u}_{fs}| - |\mathbf{f}_{fs}| \end{aligned} \quad (16)$$

With the analytical derivatives of Equation (16) and analyzing the partial derivatives of R_{fs} with respect to $|\mathbf{u}_{fs}|$ and x_i , the sensitivity of the coupled system response $|\mathbf{u}_{fs}|$ is:

$$\frac{d|\mathbf{u}_{fs}|}{dx_i} = -\mathbf{Z}^{-1} \frac{\partial \mathbf{Z}}{\partial x_i} |\mathbf{u}_{fs}| \quad (17)$$

It was observed that the sensitivity number from Equation (17) is equivalent to the following expression:

$$\frac{d|\mathbf{u}_{fs}|}{dx_i} = \mathbf{u}_{ij} \mathbf{Z} \mathbf{u}_i = \alpha_{ij} \quad (18)$$

where \mathbf{u}_{ij} is the solution of the dynamic equilibrium equation $\mathbf{Z} \mathbf{u}_j = \mathbf{F}_j$ with an unit load vector F_j at the j -th objective node. \mathbf{u}_i is the displacement vector containing the entries of \mathbf{u}_{fs} , which is related to the i -th element. This sensitivity number α_{ij} is equivalent to that proposed by [16] for stiffness maximization with displacement constraints. It indicates the change of the specified displacement or pressure component \mathbf{u}_j due to the removal of the i -th element. This number was initially proposed for static cases. Here, we combine it with the interpolation scheme proposed by [10] for the BESO method. In the proposed scheme, material is interpolated and not removed completely from the domain. It implies that the interpolation depends on x_{min} and an exponent penalty factor p , which indicate how soft is the void-condition elements. Thus, the sensitivity numbers proposed for frequency response minimization are the following:

$$\begin{aligned} \alpha_i = \frac{1}{p} \frac{d\mathbf{u}_{fs}}{dx_i} &= \frac{1}{2\omega} \mathbf{u}_j^T \left(\frac{1-x_{min}}{1-x_{min}^p} \mathbf{K}_i^1 - \frac{\omega^2}{p} \mathbf{M}_i^1 \right) \mathbf{u}_i & x_i &= 1 \\ &= \frac{1}{2\omega} \mathbf{u}_j^T \left(\frac{x_{min}^{p-1} - x_{min}^p}{1-x_{min}^p} \mathbf{K}_i^1 - \frac{\omega^2}{p} \mathbf{M}_i^1 \right) \mathbf{u}_i & x_i &= x_{min} \end{aligned} \quad (19)$$

or when x_{min} tends to 0

$$\begin{aligned} \alpha_i = \frac{1}{p} \frac{d\mathbf{u}_{fs}}{dx_i} &= \frac{1}{2\omega} \mathbf{u}_j^T \left(\mathbf{K}_i^1 - \frac{\omega^2}{p} \mathbf{M}_i^1 \right) \mathbf{u}_i & x_i &= 1 \\ &= -\frac{\omega}{p} \mathbf{u}_j^T \mathbf{M}_i^1 \mathbf{u}_i & x_i &= x_{min} \end{aligned} \quad (20)$$

3.3 Evolutionary Procedures for pressure minimization

The BESO method allows material to be removed and added through the initial domain covered by a fine finite element mesh until an optimum solution is found. A sensitivity analysis of the problem is needed for the adopted criterion. For dynamic problems, considering displacement and/or pressure minimization in cases of fluid-structure interaction, when a solid element is removed from the structure, the change on the frequency response is evaluated by the dynamic compliance. This change is defined as the elemental sensitivity number from Equation (20). In the BESO process, elements are organized according to their sensitivity number values, from the highest to the lowest. For pressure minimization is reasonable to remove elements with the lowest α .

A filter scheme is applied over the mesh in order to smooth the sensitivity numbers distribution. This filter scheme is similar to the mesh-independency filter used in [17]. This procedure consists, first of all, in averaging the elemental sensitivity number to the mesh nodes, according to the nodal connectivity, as follows:

$$\alpha_j^n = \frac{\sum_{i=1}^M V_i \alpha_i}{\sum_{i=1}^M V_i} \quad (21)$$

where M is the total number of elements connected to the node j .

The above nodal sensitivity numbers will then be converted into smoothed elemental sensitivity numbers by projecting a sub-domain Ω with length scale r_{min} and center in the i th element. All the nodes inside Ω will have their nodal sensitivity numbers averaged back to the elemental level, now as a smoothed projection. Thus, the sensitivity numbers of void elements are automatically obtained.

To stabilize the evolutionary process, an average of the sensitivity number with its historical information is carried out as:

$$\alpha_i = \frac{\alpha_i^k + \alpha_i^{k-1}}{2} \quad (22)$$

where k is the current iteration number. Thus, the updated sensitivity number includes the whole history of the sensitivity information in the previous iterations.

The removal/addition of elements is carried out by the target volume of the next iteration (V_{k+1}), defined as:

$$V_{k+1} = V_k(1 \pm ER) \quad (23)$$

where ER is the evolutionary ratio. Once the volume constraint is satisfied, the volume of the structure will be kept constant for remaining iterations as:

$$V_{k+1} = V_f \quad (24)$$

With the target volume evaluated, the threshold sensitivity numbers (α_{del}^{ath} and α_{add}^{ath}) are determined. For solid element (1), it will be removed (switched to 0) if:

$$\alpha_i \leq \alpha_{del}^{ath} \quad (25)$$

For void elements (0), it will be added (switched to 1) if:

$$\alpha_i \leq \alpha_{add}^{ath} \quad (26)$$

An admission volume ratio (AR) is introduced to ensure that not too many elements are added in a single iteration. AR is defined as the number of added elements divided by the total number of elements in the design domain. If AR is bigger than a prescribed value AR_{max} , then α_{del}^{ath} and α_{add}^{ath} need to be recalculated. More details are found in [9].

Besides the volume constraint, a convergence criterion is introduced. The cycle of finite element analysis and element removal and addition continues until the objective volume (V_f) is reached and the following convergence criterion defined in the variation of the objective function is satisfied:

$$error = \frac{|\sum_{i=1}^N \mathbf{u}_{k-i+1} - \sum_{i=1}^N \mathbf{u}_{k-N-i+1}|}{\sum_{i=1}^N \mathbf{u}_{k-i+1}} \leq \tau \quad (27)$$

where k is the current iteration number, τ is an allowable convergence error and N is an integral number, herewith selected as 5, which means a stable compliance at least in successive $2 \times N$ iterations, i.e., 10 iterations in this case.

In short, the evolutionary iteration procedure of the present BESO method is given as follow:

1. Discretize the design domain using a fine finite element mesh and declare the initial values of the elements (0 or 1) to construct an initial design.
2. Carry out the finite element analysis and calculate the elemental sensitivity numbers, according to Equation (20). Save the sensitivity number for the next iteration.
3. Determine the target volume of the next iteration, according to Equation (23).
4. Add and remove elements according to the procedure described previously.
5. Repeat steps 2 to 5 until the volume constraint (V_f) is reached and the convergence criterion, Equation (27), is satisfied.

As a consequence of the removing material the natural frequencies of the system are changed. Natural frequencies closer to the excitation frequency point in the FRF curve can increase or decrease depending on which side they are relatively to the excitation frequency point. This occurs as a consequence of the minimization process which tends to make the point of optimization to go down in the FRF curve.

4 Numerical Results

In this section, two examples considering FSI are solved using the BESO method. The meshes were generated in ANSYS, the optimization code was implemented in FORTRAN and the topologies were plotted in MATLAB. All the steps were automatically aggregated. In the first example, the intermediary topologies are presented to show the objective function and pressure distribution in several steps of the optimization. The second example presents the optimization process for six different excitation frequencies.

4.1 Example 1

This first example shows the FSI problem, where two fluid cavities are separated by a structure partition, Figure 1. The objective is to minimize the mean pressure in the two output points for a specific excitation frequency. Hard-wall condition and a prescribed pressure were imposed to the fluid domain and the structure domain has clamped ends. The incoming wave amplitude of the excitation is $p_{in} = 1$ Pa, in the left fluid domain. The excitation frequency for optimization is 5 Hz. The structure domain has length of 1 mm, height 10 mm and represents the design domain. For the optimization procedure the design domain is divided using a regular mesh of 64×320 four-node quadrilateral elements, totaling 20480 elements.

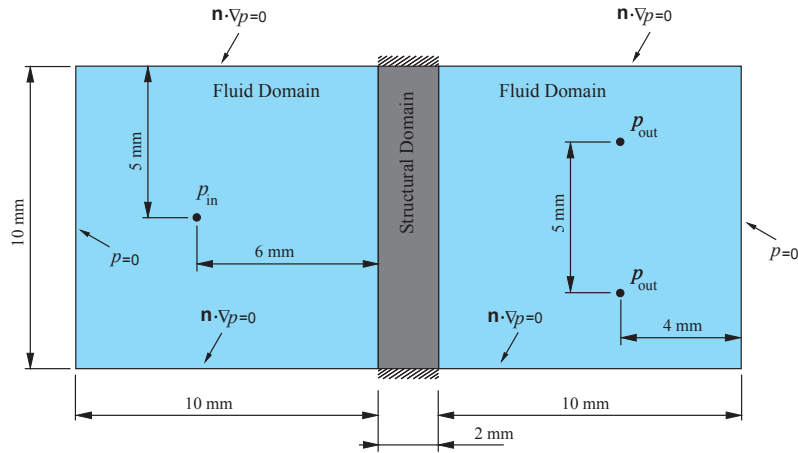


Figure 1: Model considered for optimization.

The parameters for the mesh in the finite elements analysis and the materials properties for fluid and structure domain are presented in Table 1. All constants related to the optimization process (BESO) are also listed in Table 1.

Figure 2 shows the FRF for the initial full design domain and the FRF after the optimization procedure for the mean pressure in the output point in the second fluid domain.

Variable	Description	Value
Mesh		
$nnos$	Number of nodes	60465
$nele$	Number of elements	60160
Structure		
E	Young's Modulus	100 GPa
ν	Poisson coefficient	0.3
ρ_s	Density	100 kg/m ³
Fluid		
ρ_f	Density	1.2 kg/m ³
c	Sound velocity	343 m/s
BESO		
V_i	Initial volume fraction	1.0
V_f	Final volume fraction	0.9
ER	Evolutionary ratio	0.02
AR_{max}	Maximum admission ratio	0.02
r_{min}	Filter radius	0.02 mm
τ	Convergence tolerance	0.001
N	Integral number	5
p	Penalty number	1.5

Table 1: Parameters for the numerical modelling.

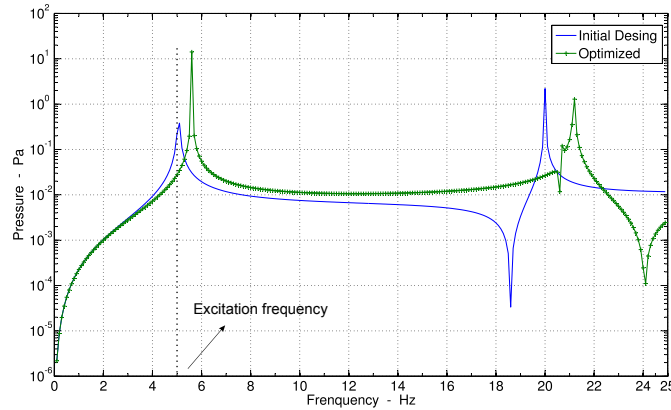


Figure 2: Frequency response for example 1.

The history of the mean pressure in the output points are shown in Figure 3. This figure shows that the optimization process starts from a full design domain (Structural Domain) and regularly removes material in a rate of 0.2% until it reaches the final volume. After 50 iterations the final volume is reached and the process is finalized when the convergence criterion is obtained, in this case in the 58th iteration. The total pressure reduction after the optimization process can be found in Table 2.

The intermediaries and final topologies are shown in Figure 4. The pressure distribution in both cavities and the shape of vibration of the structure domain can also be seen in Figure 4. The displacement of the structure is plotted $100\times$ bigger for a better visualization of the shape of vibration for this excitation frequency.

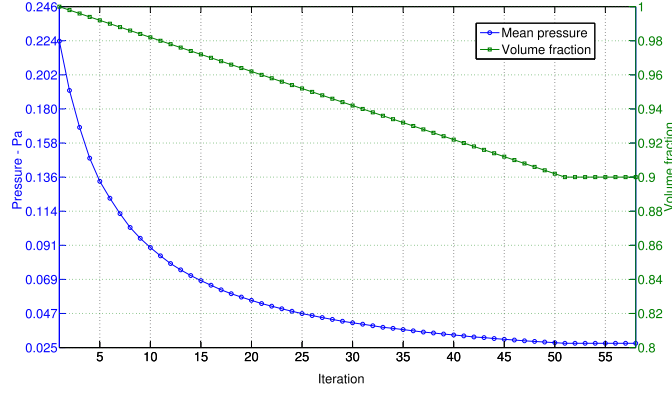


Figure 3: Mean pressure history for example 1.

Excitation Frequencies (Hz)	5.0
Initial pressure (Pa)	2.237E-01
Final pressure (Pa)	0.278E-01
Reduction	87.58%

Table 2: Mean pressure reduction at output points.

The Figure 5 shows the sensitivity numbers distribution in the design domain during the material removal material process. The sensitivity numbers are calculated taking in account the two output points, located in the right fluid domain, and indicates the area where material should be removed for the frequency response minimization.

It must be pointed out that there are no similar examples made for FSI problems with BESO method and the obtained result shows that this methodology can be effectively applied to this kind of problem, once the pattern topology found is very similar to that presented by Xie and Steven [18] for a cantilever beam, in which the boundary conditions are similar to this case. This confirms the physical meaning of the obtained topology.

4.2 Example 2

The system of the second example is shown in Figure 6. The materials properties are the same used in the first example. All the parameters for the BESO method are the same for the first example except the final volume which is different for each excitation frequency. The mesh in the design domain is divided in 32×320 four-node quadrilateral elements.

This example seeks to minimize the pressure in one output point located in the right fluid domain for six excitation frequencies in the left fluid domain. The incoming wave amplitude of the excitation on the input point is 1 Pa.

Based on the frequency response for this system six excitation frequencies were chosen. The first three frequencies are close of the first three natural frequencies of

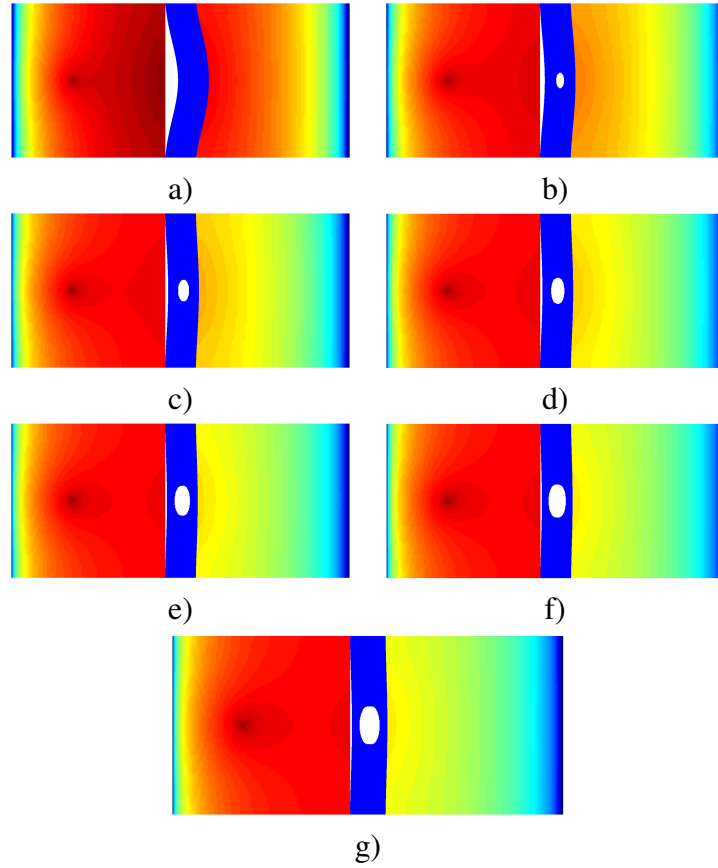


Figure 4: Evolutionary topologies for Example 1 with material volume of: a) 100% - Design Domain; b) 98% ; c) 96% ; d) 94% ; e) 92% ; f) 90% ; g) 90% after the convergence criteria - Final Topology.

the system, in the left side of the natural frequencies in the FRF curves, Figure 7. The second group of excitation frequencies is chosen to be in the right side of the natural frequencies, Figure 8. This division was done based on the different consequences of the optimization process for the FRF curves of each group, as explained in Section 3. In the first group the excitation frequencies are on the left side of the natural frequencies and in the second group they are on the right side of the natural frequencies.

Initial and final frequency responses for pressure in the six cases are shown in Figures 7 and 8. It can be observed in Figure 7 that in order to promote the minimization of the pressure, the optimization process results in an increase of the natural frequencies that are closer of the excitation frequencies, for this first group. However, Figure 8 shows the decrease of the natural frequencies near of the excitation frequencies, for these specific cases. It can be concluded that the optimization process can increase or decrease the natural frequencies of the system for the mean pressure minimization. It will depend of the relative position between the excitation frequency and natural frequencies in the FRF curve. The optimization process will always try to make the response to go down in the FRF curve.

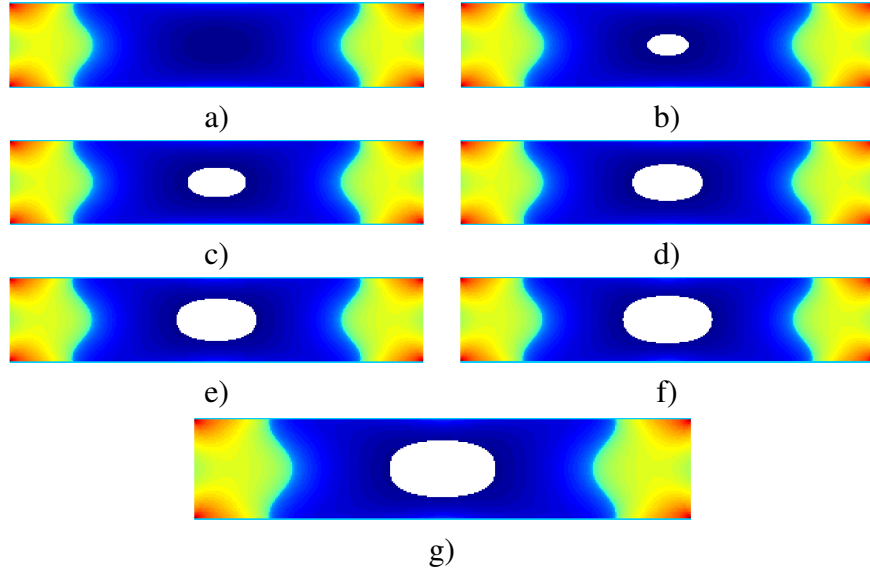


Figure 5: Objective Function for Example 1 with material volume of: a) 100% - Design Domain; b) 98% ; c) 96% ; d) 94% ; e) 92% ; f) 90% ; g) 90% after the convergence criteria - Final Topology.

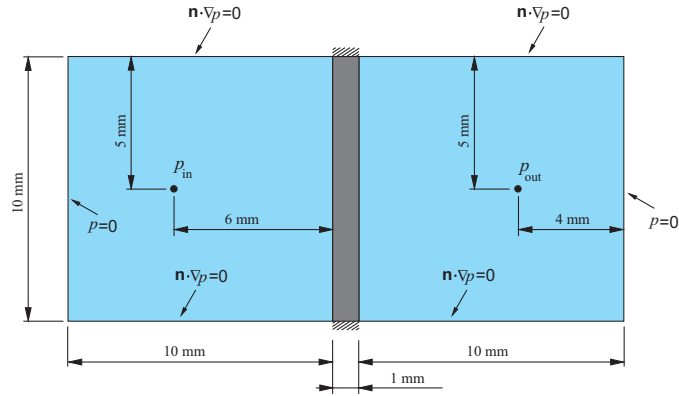


Figure 6: Model considered for optimization.

Figures 9-a, 9-c and 9-e presents the pressure distribution for the initial conditions and the shape of vibration for the three different frequencies and Figures 9-b, 9-d and 9-f present the final topology obtained for each case and the final pressure distribution after the optimization process for the first group of frequencies.

For the second group of frequencies the Figures 9-a, 9-c and 9-e show the pressure distribution in the initial conditions for the three different frequencies and Figures 9-b, 9-d and 9-f show the final topology obtained for each case and the final pressure distribution after the optimization process.

In each group of frequencies it can be noted that for lower frequencies, it is necessary to remove a greater volume of material than for higher frequencies in order to

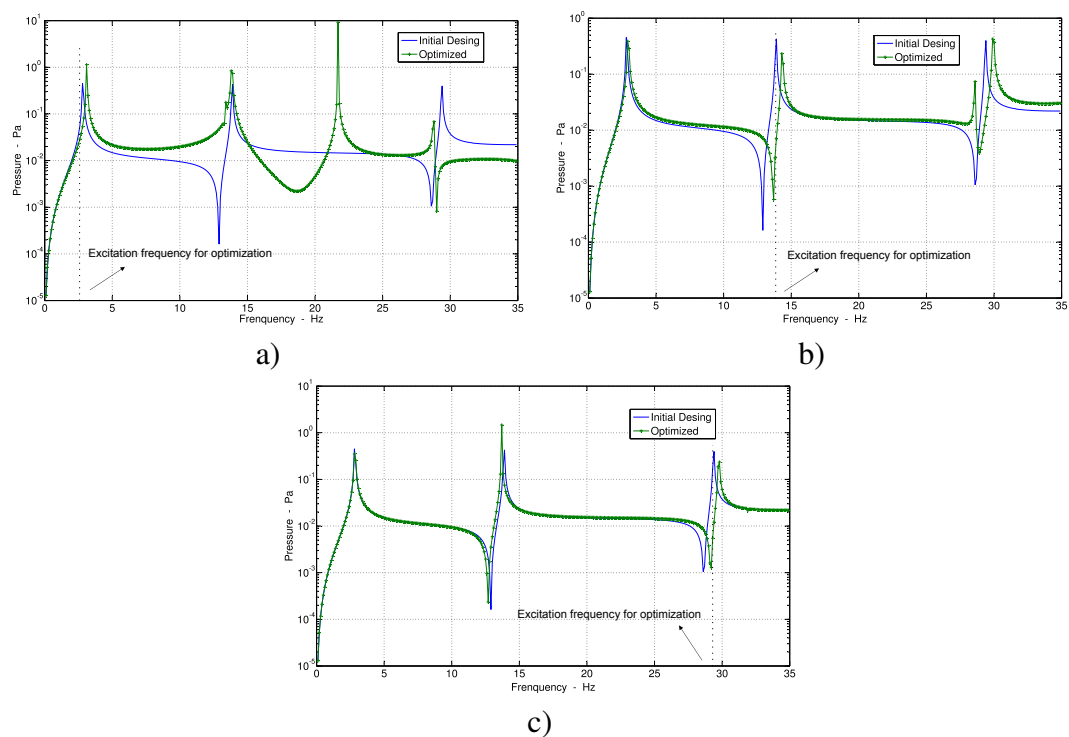


Figure 7: FRF for first group of excitation frequencies.

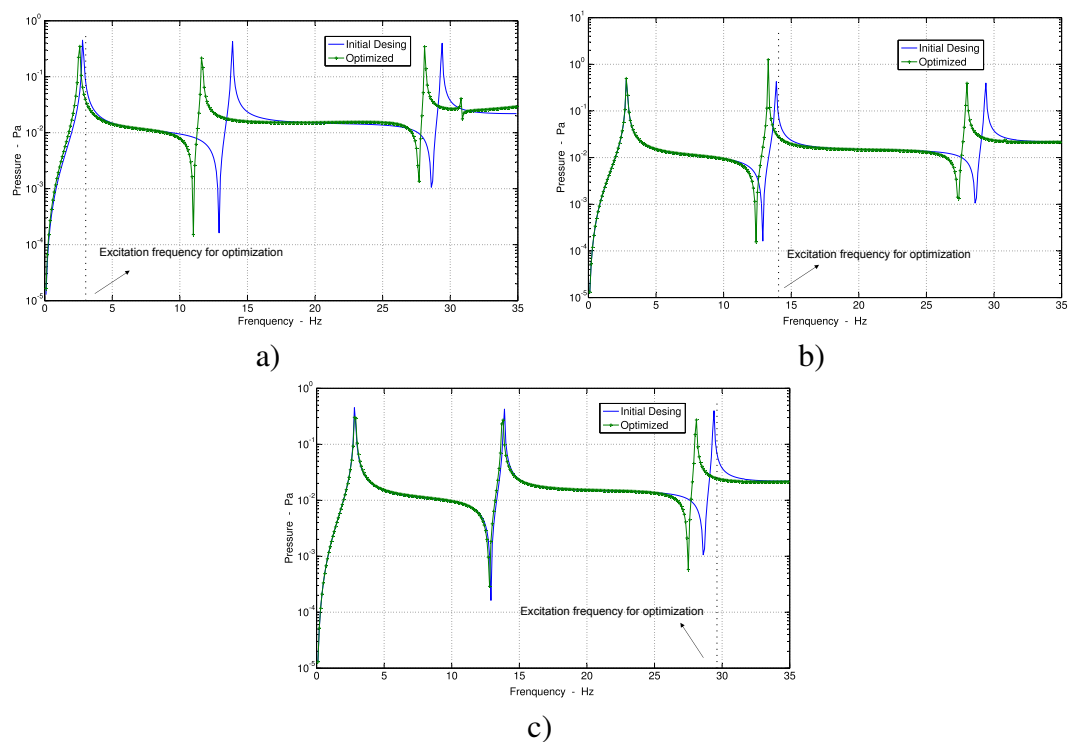


Figure 8: FRF for second group of excitation frequencies.

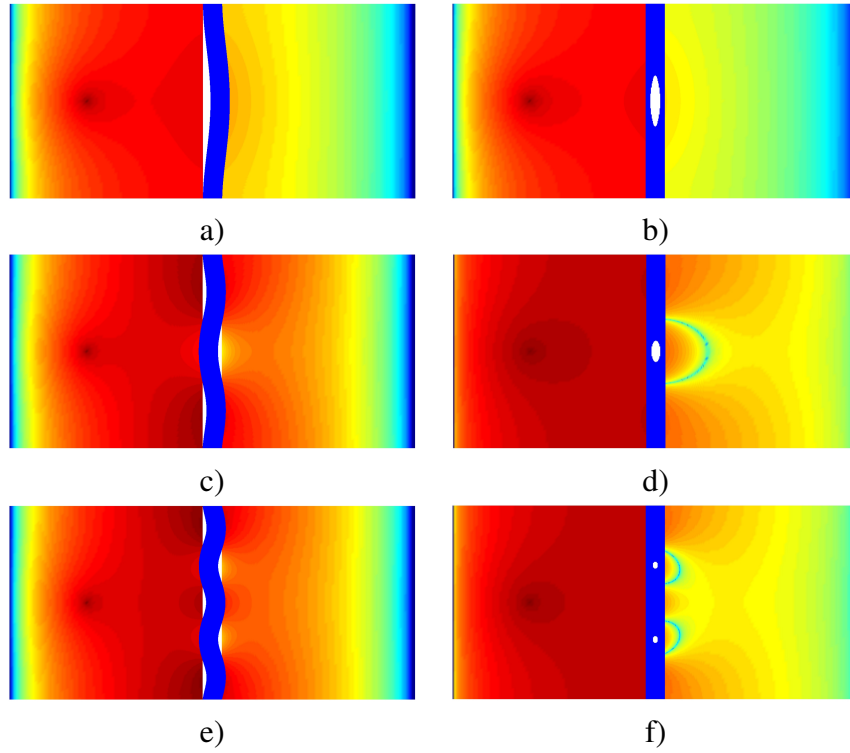


Figure 9: First set of topologies for different excitation frequencies: a) 2.6 Hz - 100%; b) 2.6 Hz - 90%; c) 13.8 Hz - 100%; d) 13.8 Hz - 96%; e) 29.3 Hz - 100%; f) 29.3 Hz - 98%;

obtain the same reduction of the pressure in the output point. The total reductions of the mean pressure in the output point for these six cases are shown in Table 3.

Excitation Frequencies (Hz)	1 st case			2 nd case		
	2.6	13.8	29.3	2.9	14	29.6
Initial pressure (Pa)	5.49E-02	1.64E-01	1.70E-01	2.19E-01	1.08E-01	7.15E-02
Final pressure (Pa)	2.89E-02	3.18E-03	5.49E-03	4.86E-02	2.92E-02	8.57E-03
Reduction	47.34%	98.06%	96.76%	77.79%	72.91%	88.01%

Table 3: Pressure reduction for different frequencies.

4.3 Example 3

This example presents a minimization of the pressure in an output point in a system with three fluid cavities and two structures partition, Figure 11. All the parameters for the BESO process are the same used for the first example. A total of 28,480 elements were used on the finite element mesh. The feasible design domain is represented by the two structural domains. The output point is located in the middle of the third fluid cavity. In this example it is explored the minimization in two different excitation

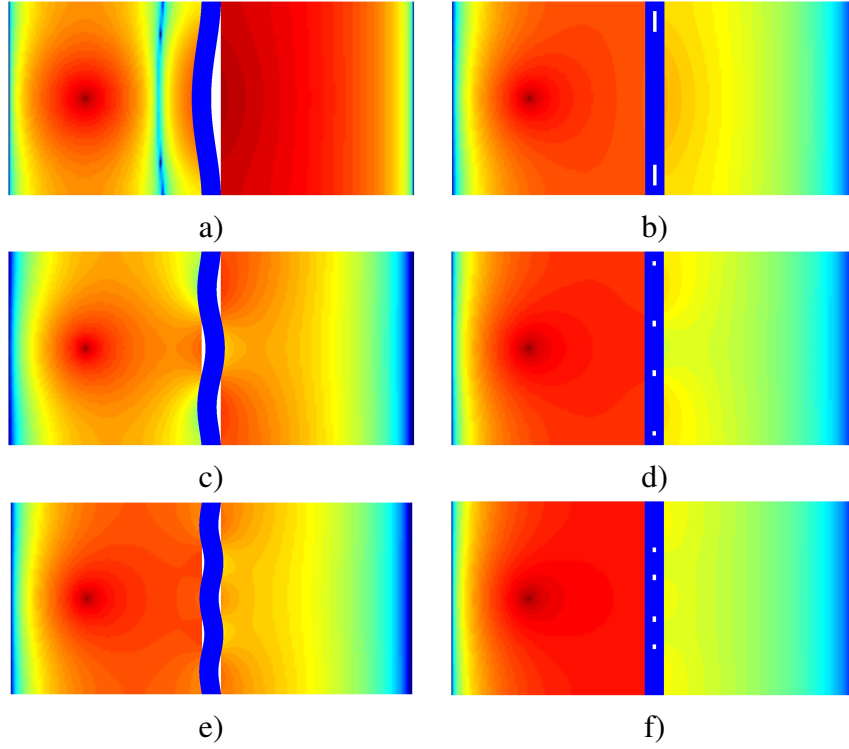


Figure 10: Second set of topologies for different excitation frequencies: a) 2.9 Hz - 100%; b) 2.9 Hz - 96%; c) 14.0 Hz - 100%; d) 14.0 Hz - 98%; e) 29.6 Hz - 100%; f) 29.6 Hz - 99%;

frequencies, the first one is close to the first natural frequency and the second one close to the second natural frequency of the system.

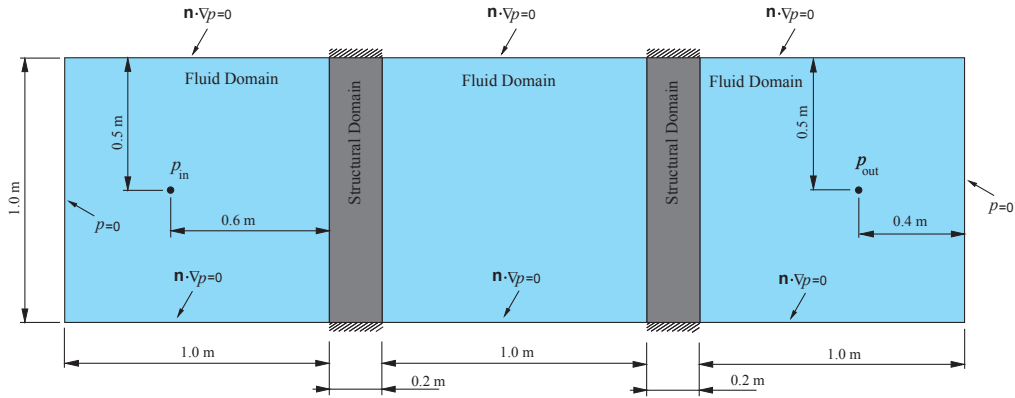


Figure 11: Model considered for optimization

For the excitation frequency of 5 Hz, both structural domains are moving in the same direction, Figure 12. Figure 12 also show the topology history for the optimization process starting from 100% of the design domain and removing material in a rate of 0.2%. The process stops when the prescribed volume of 80% is reached and the

error criterion is satisfied.

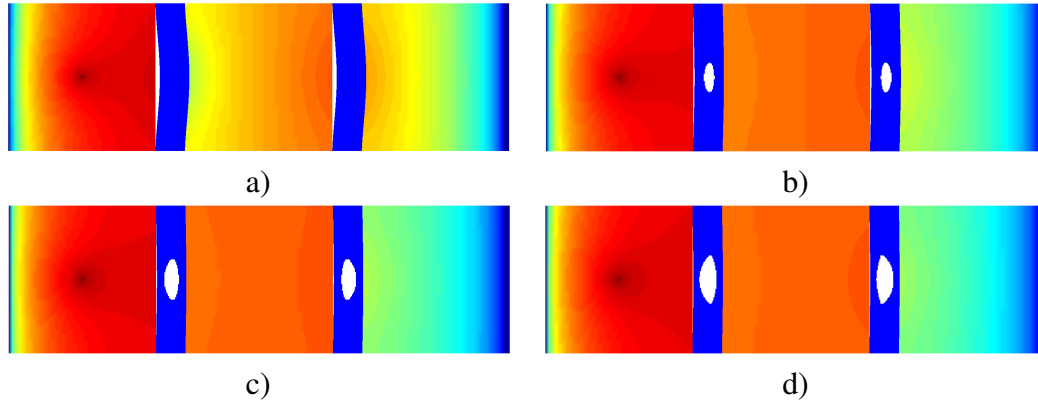


Figure 12: Evolutionary topologies for Example 3 with material volume of: a) 100% - Design Domain; b) 95%; c) 90%; d) 86% after the convergence criteria - Final Topology.

The final topology shows that the optimization process for this case results in structures with some symmetry. Figure 13 shows the FRF for the initial and final topology. The optimized topology produces a reduction of 85.4% in the mean pressure in the output location.

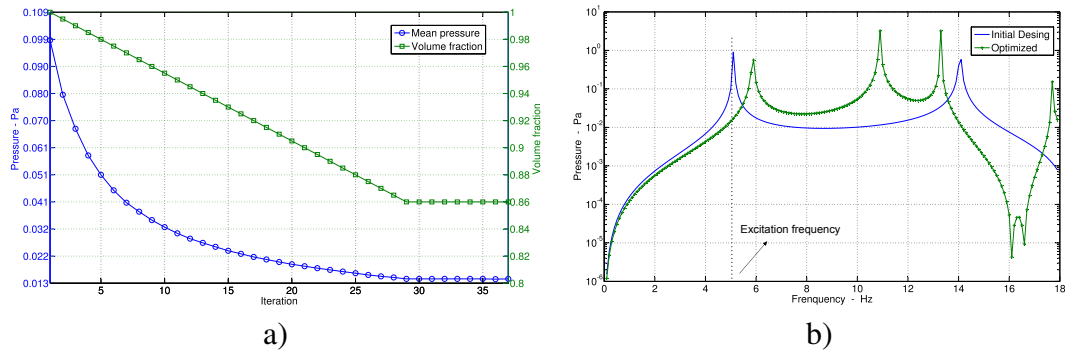


Figure 13: Response of the system for excitation frequency of 5 Hz: a) Evolution histories of mean pressure and volume fraction b) FRF for mean pressure.

The second excitation frequency is 14 Hz, which is closer to the second natural frequency of the system. For this frequency the structural domain vibrate in opposite directions, Figure 14. The optimization process starts in 100% of the design domain and removes material until the volume of 94% and finish when the error criterion is satisfied. Figure 14 also indicate that the final topologies of the first and second structural domain are quite different from each other, the optimization process has removed more material from the structural domain closer to the input source.

After the optimization process the total reduction of the mean pressure in the output point is 91.8%, this reduction can be seen in the Figure 15. In order to promote this minimization, the second natural frequency has been increased in 7.04%.

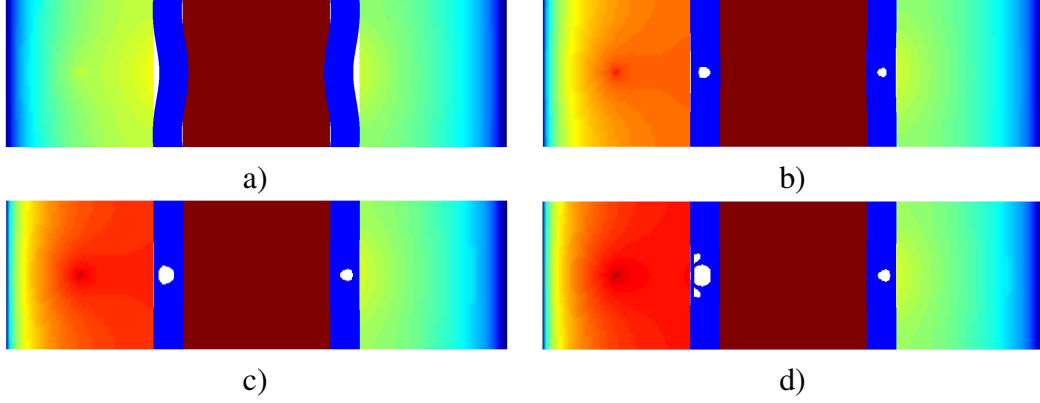


Figure 14: Evolutionary topologies for Example 3 with material volume of: a) 100% - Design Domain; b) 98% ; c) 96% ; d) 94% after the convergence criteria - Final Topology.

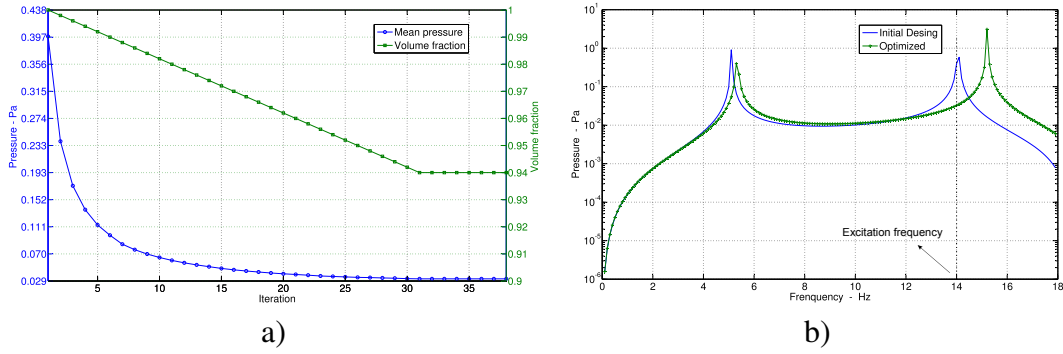


Figure 15: Response of the system for excitation frequency of 14 Hz: a) Evolution histories of mean pressure and volume fraction b) FRF for mean pressure.

5 Conclusions

In this paper we have extended the use of the BESO method to a class of fluid-structure systems under imposed pressure excitation. For this kind of problem, sensitivity numbers were presented. The objective function was the minimization of pressure in certain points of the fluid domain. For a number of excitation frequencies the methodology presented here was capable to minimize the pressure and to converge to optimized topologies. The efficiency of the method is demonstrated for a good range of frequencies. In particular for higher frequencies a great minimization of the pressure is achieved with a small percentage of material removed. For future works, movable interfaces between the fluid and structure domain will be considered as well as the extensions to three dimensions models.

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