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Analysis of Steel-Concrete Beams: Influence of Time Dependent Effects, Cracking and Connection Flexibility

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Abstract

The influence of time dependent effects, namely the creep and shrinkage of the concrete flange, on the long-term behaviour of steel-concrete continuous beams with deformable connection is analysed. The stress and internal forces redistributions and evolution are evaluated. The non-linear behaviour of concrete is considered by taking into account the cracking of concrete flange in negative moments. The non-linear behaviour of the connection between the steel girder and the concrete flange is also considered in order to simulate experimental results. The analysis is performed using finite elements based on the internal forces approximation with functions that strictly satisfy equilibrium. The viscoelastic behaviour of the concrete is considered through an incremental process in time using a numerical method based on the creep approximation with a finite number of *Dirichlet* series terms. Finally, some numerical examples demonstrating the scope of this paper are presented.

Keywords: steel concrete composite beams, connection, time dependent effects.

1 Introduction

The time dependent analysis of composite beams with deformable connection taking in account the concrete cracking in hogging regions is presented in this paper. The study is focused on the stress and internal forces redistributions as a result of creep and shrinkage of the concrete slab simultaneously with the flexibility of the connection. In fact, the effect of the connection deformability in the behaviour of composite beam was already proven to be relevant through the seminal work of Newmark. Regarding this scope further works have been published, [1, 2, 3, 4]

The steel-concrete beam formulation corresponds to considers two Euler-Bernoulli beams interconnected at the interface through mechanical devices with an appropriated stiffness. The analysis is performed by a finite element model derived from the the corresponding beam differential equations admitting the connection to be distributed along the element.

The analysis of steel-concrete beam models have been performed either by modeling each of the beam components through adequate finite elements from the libraries of general purpose finite element software or through specific derived finite element models, [10]. This work considers a finite element derived by the authors for the analysis of composite beams with flexible connection that, since the analysis is focused on the redistributions of stresses and the respective internal forces, adopts the approximation of the internal forces instead of a more conventional displacement field approximation; as a result solutions that locally satisfy equilibrium being globally compatible are obtained, rather than local compatible solutions but equilibrated on a global sense, [7], [8].

Regarding the development of specific finite element for composite beams, several works can be found in the literature; the most of these formulations considers an approximation of the displacement field, which however have some limitations due to locking problems, [9]; finite element models based on a force method were developed by [5], which overcome the difficulties of locking but require a refined process of iteration for the physical non linear analysis, [12]. Mixed formulations that consider independently approximations for the displacement field and internal forces have also been developed and successfully applied, [11]. There are also other approaches, namely the finite difference implementation of [14] and a stiffness matrix derived by [13].

The influence of the limited strength of concrete in tension, the time dependent behaviour of concrete in the structural behaviour of the composite steel-concrete beam with flexible connection, in particularly regarding stresses and internal forces, is analyzed in this work through the analysis of a continuous beam.

2 Model formulation

The model adopted herein consists of two beam elements used to model the concrete slab and the steel girder, respectively, interconnected by an interface model. Although the shear connectors are discrete, a distributed stiffness of the shear connectors along the composite beam axis is adopted in order to simplify the model, [2], [3]. The shear deformations of the concrete and steel section are neglected and only the relative displacement along the beam axis "slip" is considered in the analysis.

2.1 Equilibrium equations

The differential equations of equilibrium are established for an infinitesimal element of the composite beam, where the geometric centre of each component is adopted for the corresponding reference point. The distance between the interface and the reference point of the concrete slab and steel girder is represented by h_c and h_s, respectively. The equilibrium equations in terms of internal forces are written as follows,

- concrete section

$$\frac{dN_c}{dx_c} + p_{x_c} + f_h = 0 \tag{1}$$

$$\frac{dV_c}{dx_c} + p_{z_c} + f_v = 0 \tag{2}$$

$$\frac{dM_c}{dx_c} - V_c + m_{y_c} + f_h\, h_c + f_\theta = 0 \tag{3}$$

- steel section

$$\frac{dN_s}{dx_s} + p_{x_s} - f_h = 0 \tag{4}$$

$$\frac{dV_s}{dx_s} + p_{z_s} - f_v = 0 \tag{5}$$

$$\frac{dM_s}{dx_s} - V_s + m_{y_s} + f_h\, h_s - f_\theta = 0 \tag{6}$$

in which N_c, V_c and M_c are, respectively, the axial force, shear force and bending moment in the reference point of the concrete section; and N_s, V_s and M_s are, respectively, the axial force, shear force and bending moment in the reference point of the steel section.

The shear forces need not to be considered explicitly in the equilibrium equations since the corresponding deformations are neglected. The elimination of the shear forces from the equilibrium equations is obtained by substituting equations (2) and (5) on the result obtained by taking the derivative of the sum between equations (3) and (6), leading to:

$$\frac{d^2 M}{dx_2} + H\,\frac{df}{dx} + p_z = 0 \tag{7}$$

in which $M = M_s + M_c$; $dx = dx_c = dx_s$; and $H = h_c + h_s$ represents the distance between the references axes adopted for the concrete and steel sections. Rewriting the equations (1), (4) and (7), the following equilibrium equations are obtained

$$\mathbb{D}\,\mathbf{s} + \mathbf{p} = 0 \tag{8}$$

being the internal forces, $\mathbf{s}$, and the span loads, $\mathbf{p}$, represented as follows,

$$\mathbf{s} = \begin{bmatrix} N_c & N_s & M & f_h \end{bmatrix}^t \quad \mathbf{p} = \begin{bmatrix} p_{x_c} & p_{x_s} & p_z \end{bmatrix}^t$$

The equilibrium differential operator is given by

$$\mathbb{D} = \begin{bmatrix} \frac{d}{dx} & \cdot & \cdot & 1 \\ \cdot & \frac{d}{dx} & \cdot & -1 \\ \cdot & \cdot & \frac{d^2}{dx^2} & H\,\frac{d}{dx} \end{bmatrix} \tag{9}$$

2.2 Compatibility conditions

Since the longitudinal deformability of the connection between the concrete slab and the steel girder is considered, the Bernoulli hypothesis is no longer valid for the composite beam section as a whole. Nevertheless, the above mentioned hypothesis is admitted to remain valid for the concrete and steel sections. The displacement field for each beam component is then given as follows,

$$u_{cz}(x,z) = \bar{u}_{cz}(x), \quad \theta_c(x,z) = \bar{\theta}_c(x) \quad u_{cx}(x,z) = \bar{u}_{cx}(x) + \bar{\theta}_c(x)\, z_c \tag{10}$$

$$u_{sz}(x,z) = \bar{u}_{sz}(x), \quad \theta_s(x,z) = \bar{\theta}_s(x) \quad u_{sx}(x,z) = \bar{u}_{sx}(x) + \bar{\theta}_s(x)\, z_s \tag{11}$$

Since that only the relative displacement along the beam axis, "slip", is considered and neglecting the shear deformation of the section, the following relations are obtained:

$$\bar{u}_{cz}(x) = \bar{u}_{sz}(x) = \bar{u}_z(x),\ \bar{\theta}_c(x) = \bar{\theta}_s(x) = \bar{\theta}(x) \text{ and } \bar{\theta}(x) = -\frac{d\bar{u}_z(x)}{dx} \tag{12}$$

Considering the small displacements hypothesis to be valid, the following compatibility conditions are obtained for the deformations of the composite beam element:

$$\mathbf{e} = \mathbb{D}^* \,\mathbf{u} \tag{13}$$

where $\mathbb{D}^*$ corresponds to the compatibility operator, which is self-adjointed to the equilibrium operator in equation (9) and

$$\mathbf{e} = \begin{bmatrix} \epsilon_c & \epsilon_s & \chi & s_h \end{bmatrix}^t \quad \text{and} \quad \mathbf{u} = \begin{bmatrix} \bar{u}_{cz} & \bar{u}_{sz} & \bar{u}_z \end{bmatrix}^t \tag{14}$$

where ϵ_c and ϵ_s represent the axial deformation at the concrete and steel reference point, respectively, χ corresponds to the section curvature and s_h represents the relative displacement along beam axis; $\bar{u}_{cz}$ and $\bar{u}_{sz}$ corresponds to the longitudinal displacement of the concrete and the steel part of the beam and $\bar{u}_z$ represent the vertical displacement of the beam.

2.3 Constitutive relations

The internal forces can be defined by weighting the stress field through the corresponding deformation modes as follows,

$$\bar{\mathbf{s}} = \int_A \mathbf{E}^t \boldsymbol{\sigma}\, dA \quad \text{with} \quad \bar{\mathbf{s}} = \begin{bmatrix} N_c & N_s & M \end{bmatrix}^t \quad \text{and} \quad \boldsymbol{\sigma} = \begin{bmatrix} \sigma_c & \sigma_s \end{bmatrix}^t \tag{15}$$

being σ_c and σ_s the axial stress for the concrete and steel section, respectively, and $\mathbf{E}$ the matrix that represents the deformation modes, which is written as follows,

$$\mathbf{E} = \begin{bmatrix} 1 & \cdot & z_c \\ \cdot & 1 & z_s \end{bmatrix} \tag{16}$$

The following constitutive relation at a cross section level is then obtained

$$\mathbf{s} = \mathbf{K}\,\mathbf{e} \tag{17}$$

being the siffness matrix $\mathbf{K}$ defined as follows,

$$\mathbf{k} = \begin{bmatrix} E_c\,dA_c & \cdot & E_c\,z_c\,dA_c & \cdot \\ \cdot & E_s\,dA_s & E_s\,z_s\,dA_s & \cdot \\ E_c\,z_c\,dA_c & E_s\,z_s\,dA_s & E_c\,z_c^2\,dA_c + E_s\,z_s^2\,dA_s & \cdot \\ \cdot & \cdot & \cdot & k_{ch} \end{bmatrix} \tag{18}$$

where E_c and E_s represent the concrete and steel elastic modulus and k_{ch} the connection stiffness.

3 Numerical model

3.1 Finite element formulation

The numerical model adopted in this work for the analysis of composite beams neglects the shear deformation of the beam section and considers the deformability of the connection along the beam axis. The set of equations previously presented (equilibrium, compatibility and constitutive), along with appropriate boundary conditions, define the behaviour of the composite beam finite element adopted.

Depending on the solution scheme adopted for those equations different formulations are suitable. In this paper, attention is focused on internal forces and stresses distributions in composite beams with flexible connection, and hence, a force field based finite element formulation is appropriate, [1], [5].

The equilibrium equation solution is considered to be of the following form,

$$\mathbf{s} = \mathbf{S}\,\mathbf{X} + \mathbf{s}_0 \tag{19}$$

in which $\mathbf{s}_0$ corresponds to a force field distribution in equilibrium with the applied loads, $\mathbf{S}$ collects the internal forces approximation functions that locally satisfy the equilibrium in the absence of applied loads, being $\mathbf{X}$ their associated weights.

The compatibility condition defined in (13) is enforced on average through the domain, integrated twice by parts in order to introduce the Dirichlet conditions, and introducing the constitutive relations (17) the following compatibility condition for the element is obtained:

$$\mathbf{A}\,\mathbf{q} = \mathbb{F}\,\mathbf{X} + \mathbf{u}_0 \tag{20}$$

with the flexibility matrix for the element, $\mathbb{F}$, and the generalised deformations associated with the beam loading, $\mathbf{u}_0$, given as follows:

$$\mathbb{F} = \int_A \mathbf{S}^t\,\mathbf{f}\,\mathbf{S}\,dx \quad \text{and} \quad \mathbf{u}_0 = \int_A \mathbf{S}^t\,\mathbf{f}\,\mathbf{s}_0\,dx \quad \text{with} \quad \mathbf{f} = \mathbf{k}^{-1} \tag{21}$$

where $\mathbf{A}$ denotes the compatibility matrix on the boundary; $\mathbf{q}$ represents the finite element nodal displacements; $\mathbb{F}$ represents the flexibility of the composite beam element and $\mathbf{u}_0$ represents the generalised strains due to the applied loads when $\mathbf{X}$ is null. The equilibrium on the boundary is satisfied imposing the following relation:

$$\mathbf{Q} \,=\, \mathbf{A}^t\,\mathbf{X} \,+\, \mathbf{A}_0^t \tag{22}$$

in which $\mathbf{A}_0^t$ correspond to the nodal forces in equilibrium with the applied loads and $\mathbf{Q}$ represents the nodal forces applied to the composite beam element.

The set of equations (20) and (22) describe the behaviour of the composite beam finite element and can be compactly written in terms of the nodal displacements $\mathbf{q}$, as follows

$$\mathbb{K}_b\,\mathbf{q} \,+\, \mathbf{Q}_0 \,=\, \mathbf{Q} \tag{23}$$

where $\mathbb{K}_b$ represents a stiffness matrix for the composite beam element and $\mathbf{Q}_0$ the corresponding fixed end forces, which are defined by:

$$\mathbb{K}_b \,=\, \mathbf{A}^t\,\mathbb{F}^{-1}\,\mathbf{A} \quad \text{and} \quad \mathbf{Q}_0 \,=\, -\,\mathbf{A}^t\,\mathbb{F}^{-1}\,\mathbf{u}_0 \tag{24}$$

3.2 Non-linear analysis

A numerically algorithm was developed in order to consider the non linear behaviour of the composite beam, particularly in what concerns the non linear constitutive relations of the concrete slab and the shear connectors. The algorithm consists in a load incremental analysis, determining iteratively for each increment the corresponding nodal displacements as follows:

$$\mathbf{q}^k \,=\, \mathbf{q}^{k-1} \,+\, \sum_{i=1}^{i_{NR}} \Delta\,\mathbf{q}_i^k \tag{25}$$

where k denotes the applied load step, i denotes the structural level iteration process, being i_{NR} the number of iterations needed to achieve convergence.

The nodal displacement increment are obtained from the solution of the following equations:

$$\begin{aligned} \mathbb{K}_{b,i}^k\,\Delta\,\mathbf{q}_i^k \,+\, \Delta\,\mathbf{Q}_0^k \,=\, \Delta\,\mathbf{Q}^k \qquad & \text{for} \qquad i \,=\, 1 \\ \mathbb{K}_{b,i}^k\,\Delta\,\mathbf{q}_i^k \,=\, \Delta\,\mathbf{Q}_{d,i}^k \qquad & \text{for} \qquad i \,>\, 1 \end{aligned}$$

The unbalanced nodal forces for each iteration are determined by

$$\mathbf{Q}_{d,i}^k \,=\, \mathbf{Q}^k \,-\, \mathbf{Q}_{E,i}^k \tag{26}$$

where $\mathbf{Q}^k$ corresponds to the applied loads and $\mathbf{Q}_{E,i}^k$ represent the element nodal forces associated to the non linear constitutive relations.

Since the finite element has been developed approximating internal forces, the element forces$\mathbf{Q}_{E,i}^k$ cannot be obtained through the internal forces integration along the element, as it would be in a displacement based finite element. In order to overcome this issue an iterative process based on the deformations associated to the unbalanced forces at each section is implemented, resulting for each step of iteration i (structural level) an iterative process at the element level, [4] and [5].

4 Time dependent analysis

The concrete strain for a generic instant t after the beam loading t_0 and in the absence of thermal variations strains can be defined as follows:

$$\epsilon_c(t, t_0) = \epsilon_{c\sigma}(t, t_0) + \epsilon_{cn}(t) \tag{27}$$

where $\epsilon_{c\sigma}(t, t_0)$ corresponds to the strain stress dependent and $\epsilon_{cn}(t)$ represents the shrinkage strain which is independent of the concrete stress. Creep deformation can be considered approximately linear in relation to stress whenever the stress values remain less than half of the concrete compression strength characteristic value, and as a result one obtains:

$$\epsilon_{c\sigma}(t, t_0) = \sigma_c(t_0)\,\phi(t, t_0) \quad \text{with} \quad \phi(t, t_0) = \frac{1}{E_c(t_0} + \frac{\varphi(t, t_0}{E_{cm}} \tag{28}$$

where $\phi(t, t_0)$ represents the creep function to adopt, in which E_{cm} is the concrete modulus at 28 days, and $\varphi(t, t_0)$ correspond to the creep coefficient defined as a ratio between the creep strain at time t and the initial elastic strain for a stress $\sigma_c(t_0)$ considered applied 28 days up to the casting.

However, the concrete stresses will be variable in time and hence the deformation dependent on the stress is rewritten as follows,

$$\epsilon_{c\sigma}(t, t_0) = \int_{\sigma_c(t_0)}^{\sigma_c(t)} \phi(t, \tau)\, d\,\sigma_c(\tau) = \phi(t, t_0)\,\sigma_c(t_0) + \int_{t_0} t\,\phi(t, \tau)\,\frac{\partial\,\sigma_c}{\partial\tau}\,d\tau \tag{29}$$

which corresponds to a *Volterra* integral equation. The implementation of this relation is considered by approximating the creep coefficient with a finite number of *Dirichlet* series terms. This allows a numerical solution of (28), obtaining a consitutive relation for each interval of time as follows,

$$\Delta\sigma_c^k = E_c^{k*}\left(\Delta\epsilon_{c\sigma}^k - \Delta\epsilon_{cc}^{k*} - \Delta\epsilon_{sh}^{k*}\right) \tag{30}$$

where E_c^{k*} represents the equivalent elastic modulus to be adopted in the k interval, being dependent of the creep function approximation; $\Delta\epsilon_{cc}^{k*}$ corresponds to the creep strain in the k interval due to the stress history of the previous intervals and $\Delta\epsilon_{sh}^{k*}$ represents the shrinkage strain.

Introducing the constitutive relation (30) in (17), the governing equation for an interval of time k, in the absence of applied loads, is written in terms of nodal displacements as follows:

$$\mathbb{K}_b^{k*}\,\Delta\mathbf{q}^k + \Delta\mathbf{Q}_{0,\varphi}^{k*} + \Delta\mathbf{Q}_{0,sh}^{k*} = \mathbf{0} \tag{31}$$

where $\mathbb{K}_b^{k*}$ represent beam element stiffness matrix for the the k time interval and $\Delta \mathbf{Q}_{0,\varphi}^{k*}$ and$\Delta \mathbf{Q}_{0,sh}^{k*}$ represent the fixed end forces corresponding to the creep and shrinkage strain, respectively.

5 Applications

The numerical example herein presented consists in the analysis of the time dependent behaviour of a continuous composite beam with three spans $(10\,m\, +\, 15\,m\, +\, 10\,m)$, regarding the stress and internal forces redistribution due to creep, shrinkage and cracking of concrete.

The numerical model presented was adopted for the analysis, considering 10 elements for the lateral spans and 15 for the central span. The period of time from 10 days to 10000 days is considered in the analysis, corresponding approximately to 30 years, being the period divided into 32 intervals for the analysis.

The concrete slab is connected to a steel section by rows of two studs ($\phi\, 19$) uniformly spaced along the beam axis. The concrete slab is $150\,mm$ thick with an effective flange of $1500\,mm$ and a concrete class $C25/30$. The steel section consists in a profile IPE500 grade S275JR.

In order to evaluate the influence of considering the connection flexibility and the concrete cracking on the composite beam long-term behaviour different kinds of analysis were performed. The considered analyses were: (i) a linear analysis neglecting the connection deformability, (ii) a linear analysis with deformable connection, (iii) a non linear analysis considering the connection non linear behaviour. For the last type of analysis two different situations were considered: (a) neglecting the concrete tensile strength (b) limiting the concrete tensile strength to $f_{ct} \,=\, 2\,MPa$.

The stress redistribution was evaluated considering the influence of i) the connection deformability, ii) the steel reinforcement area at hogging regions and iii) the flanges effective width. Towards this end, different spacings of the connectors were considered 150 mm, 300 mm, 600 mm and 750 mm; different reinforcement areas, $A_s \,=\, 10\,cm^2$ $A_s \,=\, 20\,cm^2$ $A_s \,=\, 30\,cm^2$ $A_s \,=\, 40\,cm^2$ and $A_s \,=\, 50\,cm^2$ and different effective flange widths.

The shear force obtained from a linear analysis and a non linear analysis is represented in figure (1) (given the problem symmetry, the beam is represented from one end to its mid-span). As it can be observed, the influence of the connection deformability is significant; in figure (2), several degrees of connection are considered by adopting different spacings between connectors. The influence of the reinforcement steel at the hogging region on the shear force distribution can be observed in figure (3), where the shear force distribution considering the connectors to be spaced longitudinally 300 mm is depicted for different reinforcement steel areas.

The stresses at the concrete slab in both top and bottom fibers are represented in figures (4) and (5), respectively, for the different analyses performed. The influence of the connection deformability is relevant as it can be verified, which together with

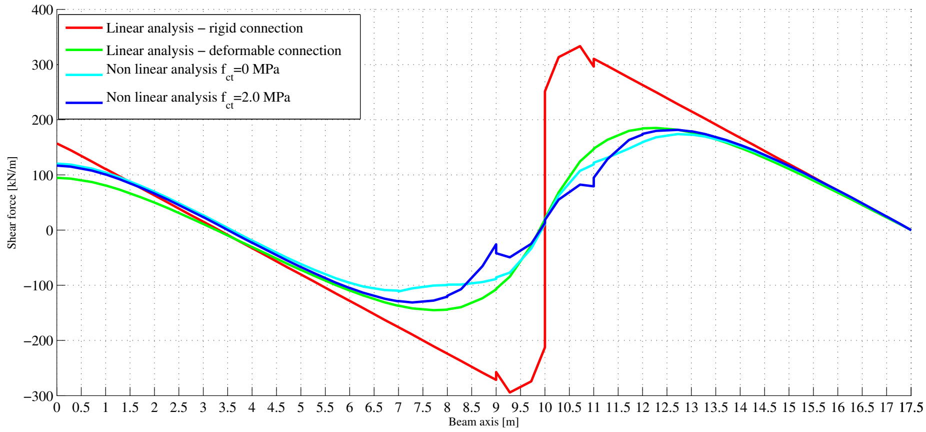

Figure 1: Distribution of shear force, comparison between analysis.

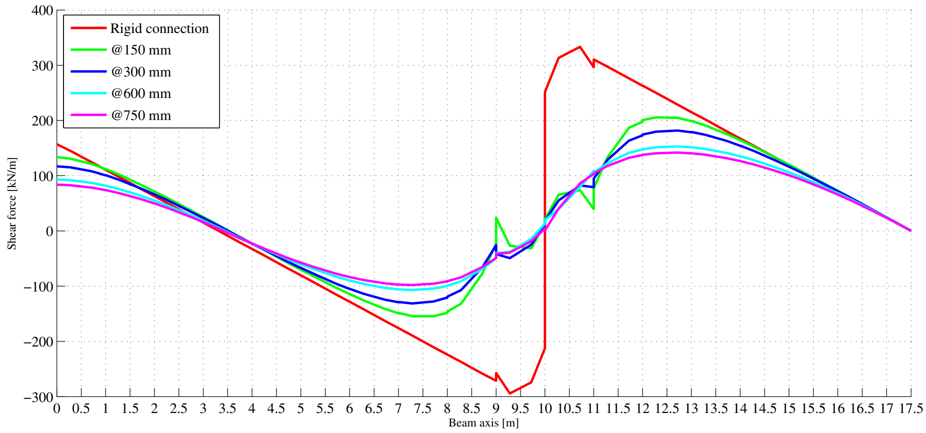

Figure 2: Distribution of shear force, connection deformability.

the concrete cracking leads to a significative redistribution of stresses. The evaluation of the redistribution of stresses considering the tensile strength limited to f_{ct} for different connections is represented in figures (6) and (7) for the top and bottom fibers respectively.

The concrete stress (bottom fiber) distribution along the beam axis is represented for different reinforcement degrees at the internal support in figure (8) and for different effective flange widths (considering a steel reinforcement of $20\,cm^2$) in figure (9); the connection degree is equivalent to connectors equally spaced by 300 mm in both conditions.

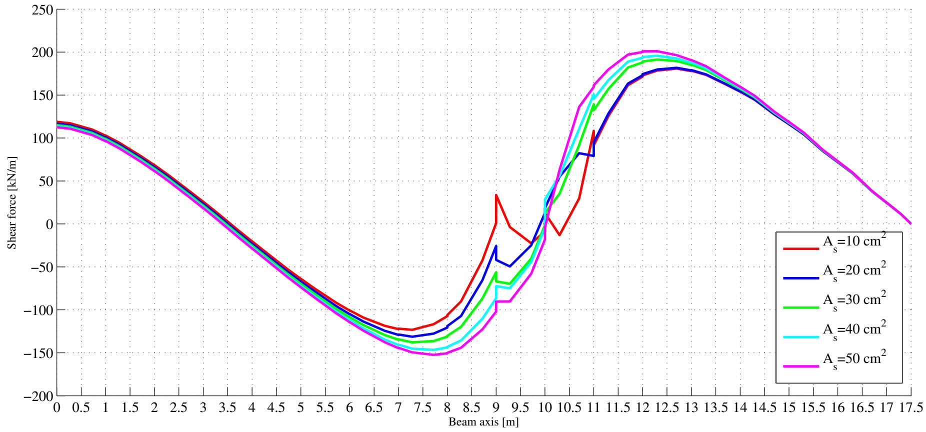

Figure 3: Distribution of shear force, reinforcement steel.

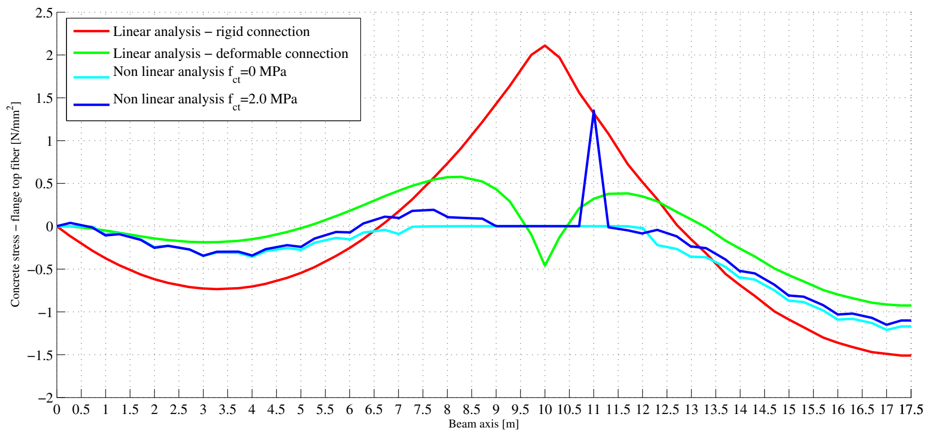

Figure 4: Concrete stress - top fiber.

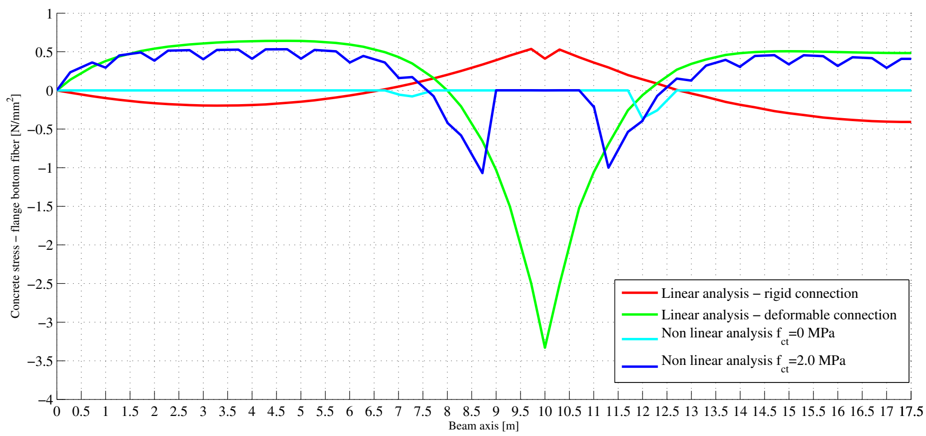

Figure 5: Concrete stress - bottom fiber.

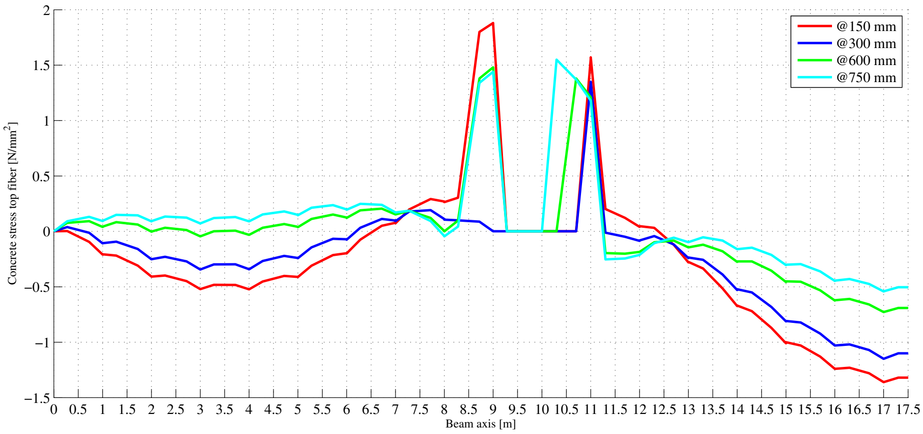

Figure 6: Concrete stress (different connection degrees) - top fiber.

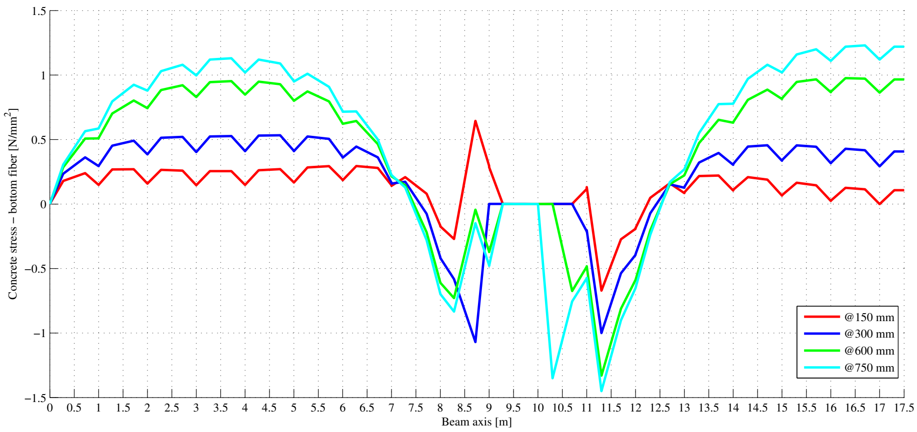

Figure 7: Concrete stress (different connection degrees) - bottom fiber.

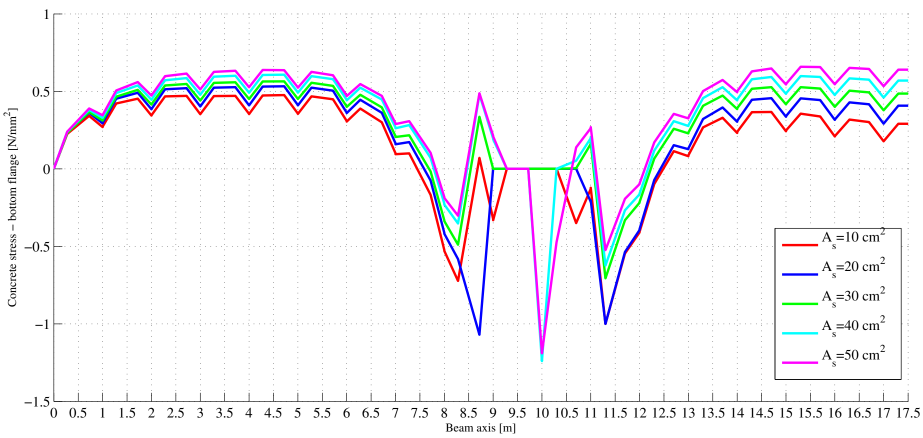

Figure 8: Concrete stress - bottom fiber, comparison between reinforcement steel.

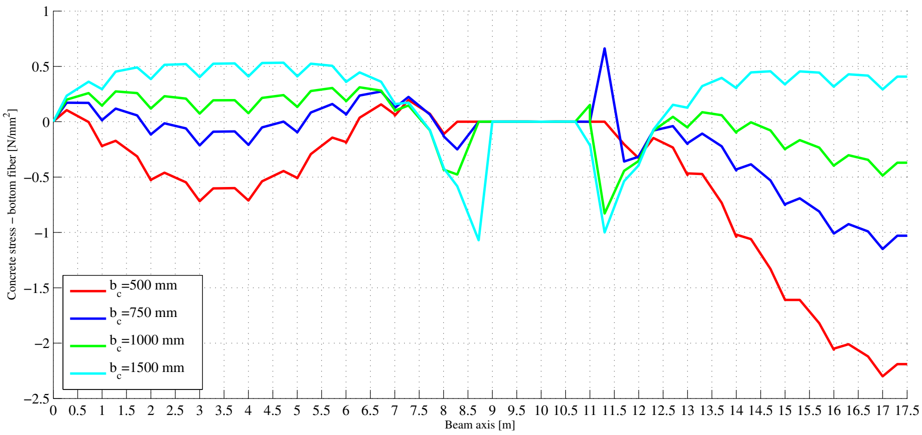

Figure 9: Concrete stress - bottom fiber, comparison between flange widths.

The long term redistribution of stresses and internal forces was also evaluated from 10 to 10000 days. The beam is considered to be loaded at 10 days, being the load constant over the period of time in analysis. Only the effect of the connection deformability together with the time dependent effects due to creep and shrinkage performed through a linear analysis is presented.

The distribution of the shear force for different connection degres is represented for 10000 days due only to creep in figure (10) and considering simultaneously the effect of creep and shrinkage in figure (11). The time evolution of the shear force at the beam end support is represent for different connections in figure (12) for both time dependent effects: creep only and creep simultaneously with shrinkage. The effect of shrinkage is far more significant than the effect of creep, being more expressive for more flexible connections.

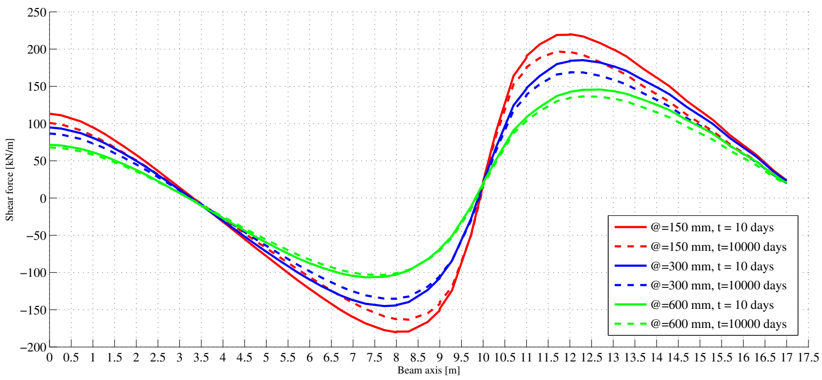

Figure 10: Distribution of shear force at 10 and 10000 days due to creep.

The concrete stress distributions along beam axis at 10 days and at 10000 days taking into account the creep effects and the creep and shrinkage effects in simultaneous

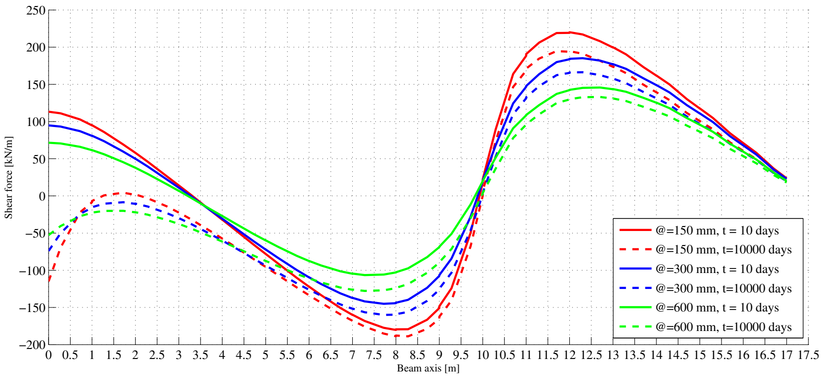

Figure 11: Distribution of shear force at 10 and 10000 days due to creep and shrinkage.

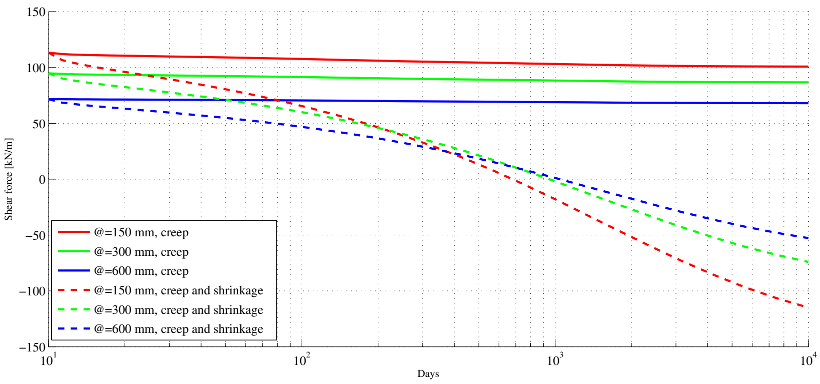

Figure 12: Time evolution of shear force at the beam en support.

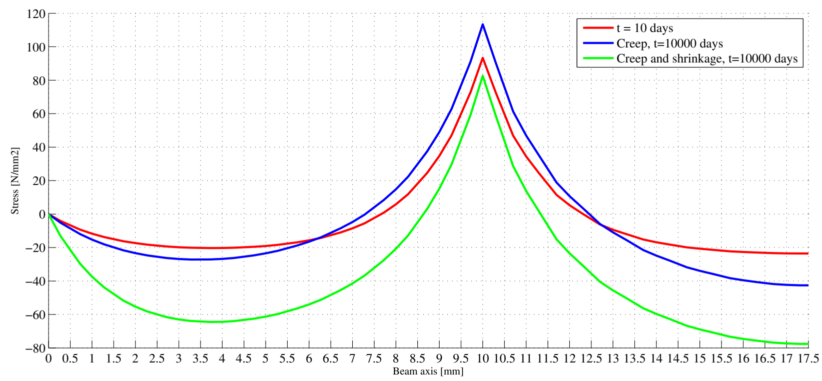

Figure 13: Distribution of stress at the upper flange of the steel section.

are represented for the upper and lower flange of the steel section in figures (13) and (14), respectively, for connectors spaced at 300 mm. The evolution in time for the compressive stress for the upper flange at the mid of the lateral span is represented in figure (15), being the stress at the lower flange at the steel section internal support represented in figure (16). The results were obtained for different connection degrees and considering creep and shrinkage effects.

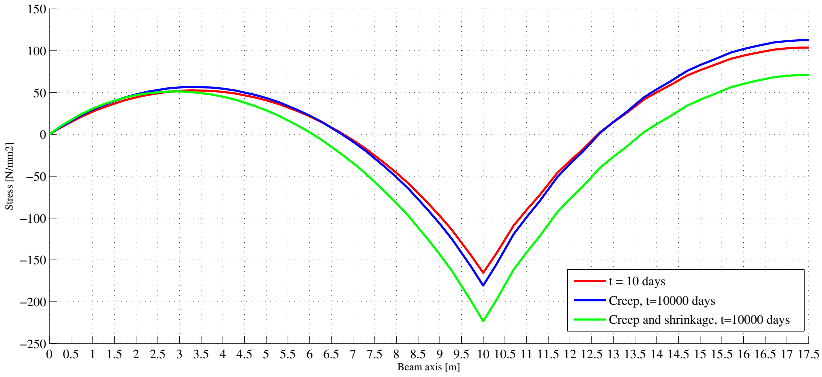

Figure 14: Distribution of stress at the lower flange of the steel section.

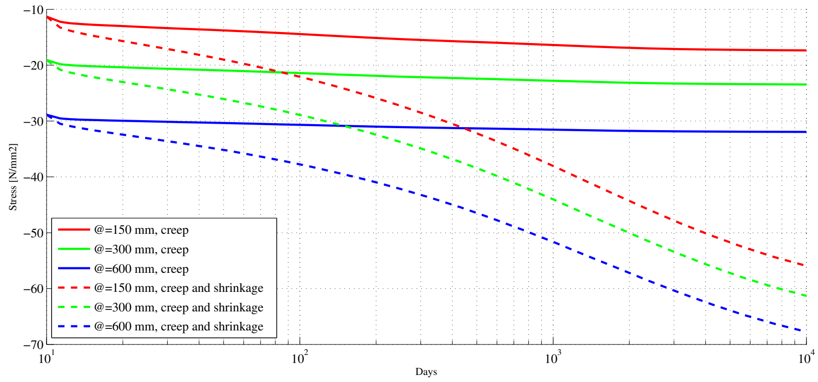

Figure 15: Distribution of shear force.

6 Conclusion

A finite element previously derived by the authors for the analysis of steel concrete composite beams with flexible connection was successful adopted for the study of the long-term behaviour of the referred beams. Moreover, the limited tensile strength of concrete was also considered in the analysis. The conclusion that as a result of the concrete slab creep and shrinkage a transfer of stresses from concrete to the steel

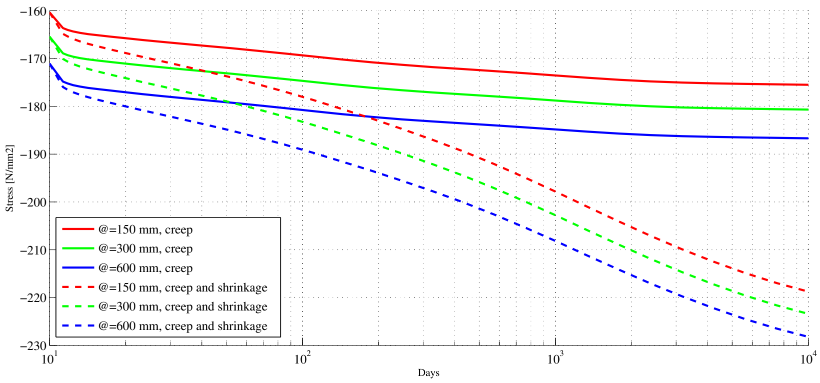

Figure 16: Distribution of shear force.

girder occurs can be drawn. Moreover, this stress redistribution is more meaningful as the connection system flexibility decreases.

The limited concrete tensile strength causes a redistribution of stresses, lowering the concrete stresses and increasing the steel girder stresses. Regarding the results obtained, the following can be concluded: a) Concrete stresses decreases with time as a result of the creep effects and are strongly dependent on (i) the concrete tensile strength and (ii) the connection flexibility; a significant part of the concrete slab is subject to tensile stresses arising from shrinkage effects; b) Steel stresses increase with time, being the increase more expressive when the shrinkage is simultaneously considered and where concrete cracking has occurred. c) The effect of creep in simultaneous with shrinkage is far more important than just the effect of creep. In any of those cases, the redistribution of stresses resulting from creep and shrinkage effects is as more significant as the connection the stiffness increases.

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