Paper 141

Proceedings of the Eleventh International Conference
on Computational Structures Technology,
B.H.V. Topping, (Editor),
Civil-Comp Press, Stirlingshire, Scotland

Finite Fracture Mechanics for the Assessment of Failure Loads of Adhesive Joints

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Abstract

In this paper a finite fracture mechanics approach for calculating failure loads of adhesively bonded joints is presented. The approach based on classical linear elasticity solutions for adhesively bonded single lap joints and uses a combined energy and stress criterion. Parameter studies are performed and it is shown, that the effects of the major geometrical parameters predicted by the present approach agree well with the effects known from experiments. Additionally, a direct comparison of the outlined criterion to experimental results from literature is performed and shows good agreement. Even the effect of the adhesive layer thickness on the failure load is incorporated.

Keywords: adhesive joints, single lap joint, failure load, finite fracture mechanics.

1 Introduction

Adhesive joints have several advantages over other joining techniques, such as riveting, bolting or welding [1]. For example the possibility of large surface joining of thin-walled structures with lower stress concentrations, very low thermal effects or the ability to join dissimilar materials. Moreover, adhesive joints lead to smoother bonding regions and can provide a sealing functions.

For the widespread use of adhesive joints precise and effective methods for design and dimensioning are required. Besides a precise prediction of failure loads, failure models should allow for studying the effects of the main geometrical and material parameters on the failure load of a joint. The present work shall give a contribution to the prediction of failure loads of adhesively bonded joints. Focus is set on a very commonly used joint shape, the simple single lap joint (SLJ).

Most of the current failure models for adhesive joints are either based on strength of materials approaches, fracture mechanics approaches or numerical implementations of cohesive zone models. In strength of materials approaches the problem arises that they cannot be directly applied to stress concentrations with singularities, as they arise at bimaterial notches in adhesive joints. This is possible for fracture mechanics approaches but assumptions on the shape and size of defects in failure prone regions of the adhesive joints are required. Cohesive zone models can be used for analysis of adhesive joints as well. In these models failure is analysed by considering a local non-linear traction-separation law, that describes damage and crack onset. In the simplest form of this law, it is controlled by two parameters: a cohesive strength, that must be exceeded to initiate damage, and a fracture toughness. Cohesive zone models are typically embedded in numerical simulations and thus allow for analysis of various problems. However, it should be noted that these simulations are non-linear and numerically demanding. Further problems arise in the determination of the proper traction-separation law and the involved parameters.

In this work the hybrid criterion by Leguillon [2] in the framework of finite fracture mechanics is used for the determination of failure loads of adhesive bonded single lap joints. Finite fracture mechanics considers the spontaneous formation of cracks of finite size, if a crack formation criterion is fulfilled. The hybrid criterion demands a simultaneous fulfilment of a strength criterion on the full area of the considered crack and an energy criterion. Hence, it can be understood as a combination of strength of materials approaches with fracture mechanics. The hybrid criterion allows for failure load assessment of structures with and without stress singularities. For evaluation two parameters a needed, a strength σ_c and a fracture toughness G_c. In the limit cases of a sharp crack or a homogeneous stress state in a sufficiently large body the hybrid criterion matches Griffith's criterion respectively classical strength of material approaches. The hybrid criterion has been successfully applied to several structural situations, e.g. laminates [3, 4, 5], notched/cracked specimens [2, 6] or joints [7, 8].

For the mechanical model of the single lap joint in this work classical linear elasticity solutions are used, that allow for a closed-form analytical formulation of the hybrid criterion. This enables an efficient assessment of the failure load. The given criterion is used to study the effect of geometrical parameters, as adherend thickness, overlap length or adhesive layer thickness, on the failure load of the joint. These dependencies are shown, compared to effects known from experiments and discussed, whereas special attention is given to the effect of the adhesive layer thickness.

2 Theoretical background

2.1 Theory of the hybrid criterion

In this work the hybrid criterion will be used in the following formulation: A crack of constant width and finite crack length a is initiated, when a strength criterion $f(\sigma_{ij}) \geq \sigma_c$ ($f : \mathbb{R}^3 \rightarrow \mathbb{R}$) is fulfilled on all points x of the crack surface Ω_c and simultaneously

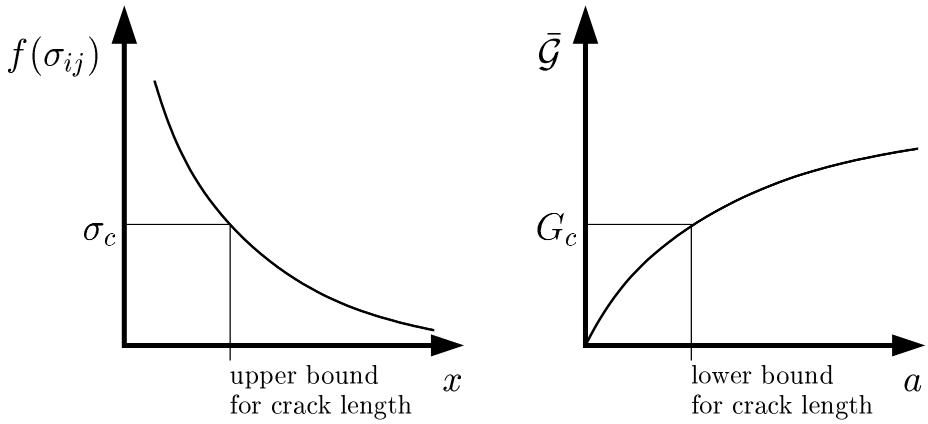


Figure 1: Visualization of the hybrid criterion. The stress criterion yields the upper bound for the crack length and the energy criterion yields the lower bound of the crack length. Both criteria must be fulfilled simultaneously. When both bounds coincide the lowest load fulfilling both criteria is found – the failure load.

the incremental energy release rate $\bar{\mathcal{G}}$ reaches the fracture toughness G_c:

$$f(\sigma_{ij}) \geq \sigma_c \quad \forall\, x \in \Omega_c \quad \wedge \quad \bar{\mathcal{G}}(a) \geq G_c \tag{1}$$

The smallest load for all kinematically admissible crack lengths, that leads to the satisfaction of both criteria given, is the failure load F_f:

$$F_f = \min_{F,a} \left\{ F \,|\, f(\sigma_{ij}) \geq \sigma_c \quad \forall\, x \in \Omega_c \quad \wedge \quad \bar{\mathcal{G}}(a) \geq G_c \right\} \tag{2}$$

In the present case the strength criterion is a monotonically decreasing function of the distance x from the crack origin and the incremental energy release rate is a monotonically increasing function of the crack length a, for expected crack lengths not exceeding quarter overlap length. Hence the strength criterion is is an upper bound for the crack length and the energy criterion is a lower bound for the crack length, as it is shown in Figure 1. In the optimum, when $F = F_f$, both bounds coincide.

2.2 Modeling of the single lap joint

In this work a symmetric single lap joint as shown in Figure 2 is considered. Let the joint have the width b, adherend height h, adhesive layer thickness t and an overlap length L. The joint is loaded by an axial force F in horizontal direction. Both the adherend and the adhesive layer are considered as homogeneous, isotropic continua with linear-elastic material behaviour. Young's modulus of the adherend is denoted as E_x and the one of the adhesive is denoted E_a. Correspondingly, G_a and ν_a are the shear modulus and Poisson's ratio of the adhesive.

Peel stresses occuring due to the eccentricity of the acting forces play an important role in the failure behaviour of single lap joints and must be taken into account failure

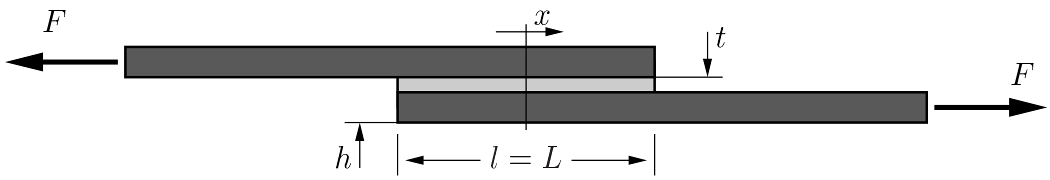


Figure 2: Geometry of the considered symmetric single lap joint under axial loading.

load assessments of single lap joints [9]. The classical work by Goland and Reissner [10] is the most simple model for an SLJ that takes these stresses into account. Explicit solutions for the shear and peel stresses are given in their work. The adherends are considered as beams and the elastic layer is treated by simplified kinematic relationships. Let u_i be the horizontal displacements and w_i the vertical displacements of the adherends 1 (upper adherend) and 2 (lower adherend), then the following relationships are assumed to hold:

$$\gamma = \frac{\partial u}{\partial z} = \frac{u_1 - u_2}{t} \tag{3}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = \frac{w_1 - w_2}{t} \tag{4}$$

In their study Goland and Reissner introduced a non-linear moment factor to take large deformations of the adherend into account. These deformations lead to reduced lever arms of the acting forces. This moment factor has been subject to numerous scientific discussions [11, 12, 13]. Under consideration of the corrections given by Tsai and Morton [12] the expression for the shear and peel stresses in the adhesive layer are:

$$\tau^{GR} = -\frac{F}{8b}\left(\lambda_\tau\left(1+3k\right)\frac{\cosh\lambda_\tau x}{\sinh\lambda_\tau\frac{L}{2}} + \frac{3}{L}\left(1-k\right)\right) \text{ mit } \lambda_\tau = \sqrt{\frac{8G_a}{E_x h t}}, \tag{5}$$

$$\begin{aligned}\sigma_0^{GR} =& \frac{Fh\lambda_\sigma}{\Psi b}\left(\left(\frac{\lambda_\sigma}{2}R_1 k + \frac{2}{L}k'\sinh\left(\lambda_\sigma\frac{L}{2}\right)\sin\left(\lambda_\sigma\frac{L}{2}\right)\right)\sinh\left(\lambda_\sigma x\right)\sin\left(\lambda_\sigma x\right)\right.\\ &\left.\ldots + \left(\frac{\lambda_\sigma}{2}R_2 k + \frac{2}{L}k'\cosh\left(\lambda_\sigma\frac{L}{2}\right)\cos\left(\lambda_\sigma\frac{L}{2}\right)\right)\cosh\left(\lambda_\sigma x\right)\cos\left(\lambda_\sigma x\right)\right)\end{aligned} \tag{6}$$

with

$$\lambda_\sigma = \sqrt[4]{\frac{6E_a\left(1-\nu^2\right)}{E_x h^3 t}}, \tag{7}$$

$$R_1 = \cosh\lambda\frac{L}{2}\sin\lambda\frac{L}{2} + \sinh\lambda\frac{L}{2}\cos\lambda\frac{L}{2};\ R_2 = \sinh\lambda\frac{L}{2}\cos\lambda\frac{L}{2} - \cosh\lambda\frac{L}{2}\sin\lambda\frac{L}{2}, \tag{8}$$

$$\Psi = \frac{1}{2}\left(\sinh\lambda L + \sin\lambda L\right), \tag{9}$$

$$k = \frac{1}{1 + 2\sqrt{2}\tanh\left(\sqrt{\frac{3(1-\nu^2)}{2}}\frac{L}{2h}\sqrt{\frac{F}{E_x h b}}\right)}, \quad k' = \frac{1}{L}\left(\left(1-k\right)h + t\right). \tag{10}$$

The solution by Goland-Reissner bases on a very simplified modeling approach and assumes the strains and hence stresses to be constant over the adhesive layer thickness. The boundary condition of vanishing shear stresses at the unloaded interfaces is not fulfilled as well. Despites these limitations the Goland-Reissner model still represents an important approach for gaining insight into the behaviour of single lap joints under axial loading.

An extended analysis, that considers non-constant stresses over the adhesive layer thickness was given by Ojalvo and Eidinoff [14]. In their modeling the effect of the adhesive layer is included and enhanced kinematic relationships are used. This relationships consider the effect of the slope w_i' of the adherends on the shear strain in the adhesive layer and bases on the definiton of the infinitesimal strain tensor.:

$$\gamma_{1,2} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{u_1 - u_2}{t} + w_{1,2}' \tag{11}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = \frac{w_1 - w_2}{t}. \tag{12}$$

The modelling yields shear- and peel stresses in the adhesive layer. The peel stress changes over the adhesive layer thickness: the shear stress in the interface to the upper adherend is denoted as τ_1^{OE} and τ_2^{OE} is the shear stress in the lower interface.

$$\tau_0^{\text{OE}} = \frac{F}{bL\Omega^2}\left(\Omega\zeta\frac{\cosh\left(\Omega\frac{2x}{L}\right)}{\sinh\left(\Omega\right)} + \Omega^2 - \zeta\right), \tag{13}$$

$$\tau_{1,2}^{\text{OE}} = \tau_0^{\text{OE}} \mp \frac{G_a t}{2E_a}\cdot\frac{\partial\sigma_0^{\text{OE}}}{\partial x}, \tag{14}$$

$$\sigma_0^{\text{OE}} = \frac{F}{bL}\left(A_1\sinh\left(\alpha_1\frac{2x}{L}\right)\sin\left(\alpha_2\frac{2x}{L}\right) + A_2\cosh\left(\alpha_1\frac{2x}{L}\right)\cos\left(\alpha_2\frac{2x}{L}\right)\right), \tag{15}$$

where

$$\Omega = \lambda\sqrt{2\left(1 + 3\left(1 + \frac{t}{h}\right)^2\right)}; \quad \zeta = 2\lambda^2\left(1 + 3\left(1 + \frac{t}{h}\right)^2 k\right), \tag{16}$$

$$\lambda = \sqrt{\frac{G_a L^2}{4E_x th}}; \quad \alpha_{1,2} = \sqrt{\pm\frac{3t\lambda^2}{2h} + \frac{1}{2}\sqrt{\frac{3E_a L^4}{2E_x th^3}}}. \tag{17}$$

The unknowns A_1 and A_2 in the upper equations have to be determined by the boundary conditions:

$$\frac{L^3}{8}\left.\frac{\partial^3\sigma_0^{OE}}{\partial x^3}\right|_{x=\frac{L}{2}} - 6\frac{Lt\lambda^2}{2h}\left.\frac{\partial\sigma_0^{OE}}{\partial x}\right|_{x=\frac{L}{2}} = -k\frac{3E_a L^3}{2E_x th^2}\left(1 + \frac{t}{h}\right) \tag{18}$$

$$\frac{L^2}{4}\left.\frac{\partial^2\sigma_0^{OE}}{\partial x^2}\right|_{x=\frac{L}{2}} = k\frac{3E_a L^3}{2E_x th^2}\left(1 + \frac{t}{h}\right). \tag{19}$$

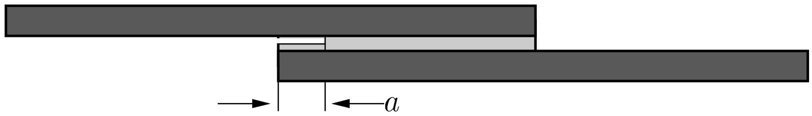


Figure 3: Crack orginating from the reentrant corner of the upper adherend and the adhesive layer.

To use these stress solutions for the computation of the failure load an appropriate stress criterion is needed. In this work a maximum principal stress criterion is used in the hybrid criterion:

$$\begin{aligned}\sigma_c \stackrel{!}{=} f(\sigma_{ij}) &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{\sigma_0}{2} + \sqrt{\left(\frac{\sigma_0}{2}\right)^2 + \tau^2}\end{aligned} \tag{20}$$

Besides a solution for the stresses in the adhesive layer the incremental energy release rate must be given as well to formulate the hybrid criterion given previously. In a single-lap joint cracks typically initiate from the reentrant corners of the adherends and the adhesive. In the following it is assumed that horizontal cracks initiate in the reentrant corner of the upper adherend and the adhesive, as shown in Figure 3. It is assumed that the adhesive layer and the lower adherend below the considered crack are completely relieved. This allows for using the same solutions for the stress distribution as in the uncracked case and is a typical assumption for obtaining energy release rates of lap joints [15]. In case of the Goland-Reissner model the incremental release rate can be given in closed-form analytical form by integrating the differential release rate over the finite crack length a or by using the compliance of the cracked and uncracked structure:

$$\bar{\mathcal{G}}_{(GR)} = \frac{1}{2}\frac{\Delta C}{A}F^2 = \frac{1}{2}\frac{C_{\text{cracked}} - C_{\text{initial}}}{ab}F^2 \tag{21}$$

$$\begin{aligned}&= \frac{F^2}{4Ehb^2}\frac{1}{a}\left[(1+6k)\,a + \frac{2\,(1+3k)}{\lambda_\tau}\left(\coth\left(\lambda_\tau\frac{l-a}{2}\right) - \coth\left(\lambda_\tau\frac{l}{2}\right)\right)\right. \\ &\quad + \frac{6\,(1-k)}{\lambda_\tau^2}\frac{a}{l\,(l-a)} + \\ &\quad \left. + \frac{6k}{\lambda_\sigma}\left(\frac{\cosh\left(\lambda_\sigma(l-a)\right) - \cos\left(\lambda_\sigma(l-a)\right)}{\sinh\left(\lambda_\sigma(l-a)\right) + \sin\left(\lambda_\sigma(l-a)\right)} - \frac{\cosh\left(\lambda_\sigma l\right) - \cos\left(\lambda_\sigma l\right)}{\sinh\left(\lambda_\sigma l\right) + \sin\left(\lambda_\sigma l\right)}\right)\right].\end{aligned} \tag{22}$$

The incremental energy release rate of the Ojalvo-Eidinoff solution unfortunately cannot be obtained in closed-form analytical manner. However, it can be obtained by computing the integral of the differential energy relase rate numerically:

$$\bar{\mathcal{G}}_{(OE)} = \frac{1}{a} \int\limits_{L=l-a}^{l} G(L)\mathrm{d}L = \frac{1}{a} \int\limits_{L=l-a}^{l} \left(\frac{t}{2E_a}\sigma_{\max}^2 + \frac{t}{2G_a}\tau_{\max}^2 \right) \mathrm{d}L, \tag{23}$$

where $\sigma_{\max}$ and $\tau_{\max}$ are the peak stress at the end of the overlap and depends on the overlap length L, which is the variable of integration.

With the expressions for the stresses and the incremental energy release rate the optimization problem (2), needed to determine the failure load, is completly defined. Using a computer algebra system, as e.g. MATHEMATICA, it can be solved quickly. It takes about .3s to solve the optimization problem of the Goland-Reissner model on a standard workstation computer. The Ojalvo-Eidinoff model takes around 10 times longer as the expressions of the stresses are more complex and the incremental energy release rate must be evaluated numerically.

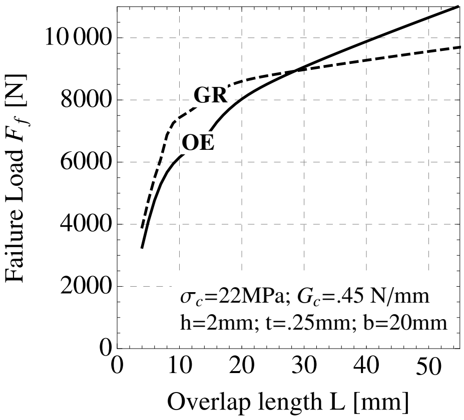
3 Failure load predictions

In the following the failure load predictions by the given hybrid criterion are shown. The predictions of the two underlying models are compared with one another and with parameter effects known from experiments. Moreover, a direct comparison of the failure load predictions of the given criterion to experimental results from literature is shown and discussed.

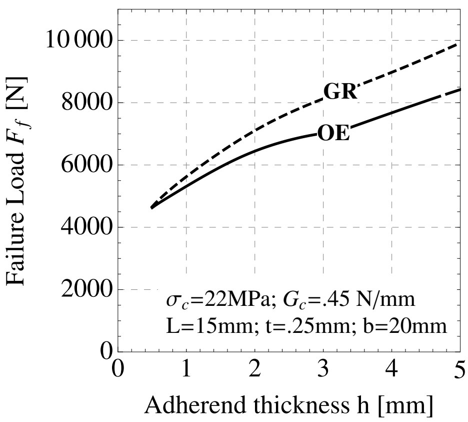
3.1 Effect of geometrical parameters

The main geometrical parameters of a single lap joint are the overlap length L, the thickness of the adherends h and the adhesive layer thickness t. The overlap length of the adherends, essentially the length of the adhesive layer, is known to play an important role on the failure load, such that higher overlap lengths lead to increased failure loads. But for high overlap length this effect declines and the failure load converges and so in practise there is an upper limit for the overlap length. The effect of the adherend thickness is to increase the failure load of single lap joints, as with stiffer adherends the load is distributed more evenly in the adhesive layer. The effect of the adhesive layer thickness has been subject to numerous studies in literature. From experiments it is known that the failure load of adhesive joints decreases with increasing adhesive layer thicknesses. But most analytical and numerical approaches to predict failure given in literature fail to cover this effect [16, 17, 18].

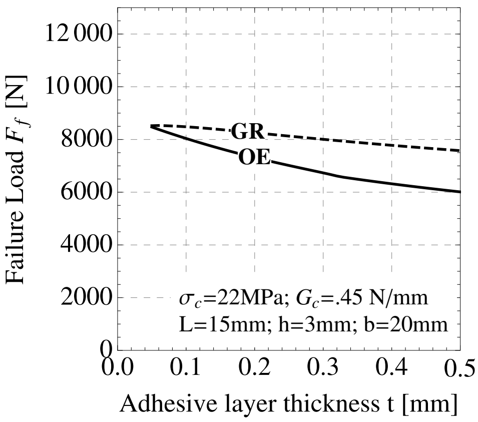
Figures 4 show the failure load predictions by the current approach for an exemplaric joint with steel adherends and an epoxy structural adhesive. The material parameters are chosen according to Table 1. The parameters L, h and t are examined in a typical range to show their effect on the failure loads of single lap joints. All



(a)



(b)



(c)

Figure 4: Predicted effect on the failure load of the three geometrical parameters: overlap length, adherend thickness and adhesive layer thickness. GR stands for the Goland-Reissner model and OE for the Ojalvo-Eidinoff model.

	Adhesives		Adherends
	Redux 326	Hysol 9321	High strength steel
Young's modulus E [GPa]	4.44	3.87	210
Poisson's ratio ν [-]	0.35	0.36	-
Strength σ_c [$\frac{\text{N}}{\text{mm}^2}$]	50.9	22.0	-
Fracture toughness G_c [$\frac{\text{N}}{\text{mm}}$]	0.35	0.45	-

Table 1: Material parameters of the experimentally tested single lap joints in [18, 19, 20]. Data which are not needed here are left out (-).

diagramms show curves according to the Goland-Reissner model as dashed lines and the ones of the Ojalvo-Eidinoff model as solid lines.

Diagram 4(a) shows the effect of the overlap length predicted by the present criterion. For both models an increase of the failure load with increased overlap lengths of the joint is predicted. The Ojalvo-Eidinoff model predicts a stronger effect for higher overlap lengths. In diagram 4(b) the prediction of the effect of the adherend thickness can be seen. Both models yield higher failure loads for increased adherend thicknesses. Where the curve of the Goland-Reissner model has a steeper slope. The effect of the adhesive layer thickness as predicted by both models is shown in diagram 4(c). Both models predict a decrease of the failure load with increasing adhesive layer thicknesses. The effect is more pronounced in the Ojalvo-Eidinoff model.

These predicted dependencies of the failure load on the geometrical parameters agree well with effects observations from literature.

Beside a study of the effect of the geometrical parameters, a direct comparison to experimental results from literature is performed. Two studies are considered: the study by da Silva et al. on the effect of the overlap length from 2004 [19] and the study on the effect of the adhesive layer thickness from 2006 [20]. In the first study experiments with single lap joints composed of steel adherends with a bismaleimide adhesive (Redux 326) were tested. In the second study tests were performed on single lap joints with steel adherends and an epoxy structural adhesive (Hysol 9321). The material parameters of the used materials are summarized in Table 1. The strength and the fracture toughness needed to evaluate the hybrid criterion were taken from the studies [19, 20] resp. from a comparative study which uses the same experimental data [17, 18].

In Figure 5 the comparison of the experimental failure loads and the failure loads predicted by the present approach are shown. Figure 5(a) shows the dependency on the overlap length. The Goland-Reissner and the Ojalvo-Eidinoff model both predict an increase of the failure load with increased overlap lengths. Whereas the latter gives a better prediction of the effect of the overlap length. The loads predicted by both models are always lower than the experimental results.

In Figure 5(b) the study of the effect of the adhesive layer thickness can be seen. Both models predict a decrease of the failure loads for higher adhesive layer thick-

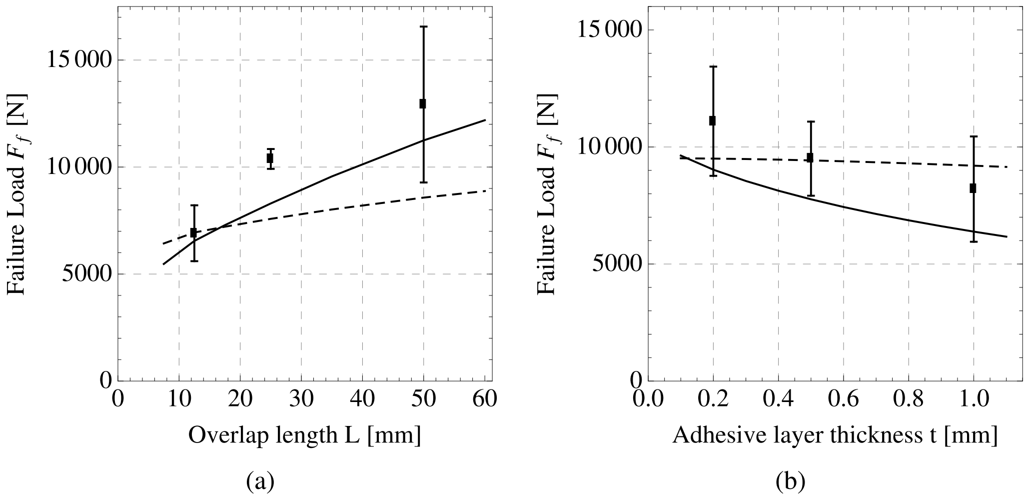


Figure 5: Comparison of the failure load prediction of the current criterion with experimental results. (Dashed line: Goland-Reissner model, solid line: Ojalvo-Eidinoff model)

nesses. But the effect predicted by the Goland-Reissner model is somewhat too weak. The model yields failure loads being lower and higher than the experimental values. The reported values are always within the error bars of the experimental results. However, the Ojalvo-Eidinoff model covers the effect of the adhesive layer thickness very well. The predicted failure loads are 23%, 23% and 28% lower than the experimentally obtained failure loads. The prediction is conservative.

It is particularly interesting that the adhesive layer thickness effect cannot be covered by commonly used analytical solutions for the strength of single lap joints found in literature, as shown by da Silva et al. [17, 18]. The consideration of the energy balance in the hybrid criterion however allows for a correct prediction of the thickness effect, if the underlying model of the joint is sufficiently realistic.

4 Conclusion

In this paper the hybrid criterion in the framework of finite fracture mechanics was used to predict the failure load of adhesively bonded single lap joints. To model the mechanical behaviour of the single lap joints two closed-form analytical linear-elasticity solutions were used. Beside the classical Goland-Reissner model the enhanced Ojalvo-Eidinoff model was used. Both models give solutions for the peel and shear stress distribution in the adhesive layer. On the basis of the expressions for the stresses and the incremental energy release rate the optimization problem of the hybrid criterion was formulated. Parameter studies were performed to study the effect of the geometrical parameters overlap length, adherend thickness and adhesive layer thickness. The obtained effects coincide well with observations known from practice.

Additionally, a comparison of the predicted failure loads with experimental results from the literature was performed. The results show that: (1) especially the Ojalvo-Eidinoff model yields a very good prediction of the failure loads; (2) the effect of the overlap length and even the effect of the adhesive layer thickness are covered; and (3) that the predicted loads are always conservative.

References

[1] R. Adams, J. Comyn, W. Wake, *Structural adhesive joints in engineering*, Springer, 1997.

[2] D. Leguillon, "Strength or toughness? A criterion for crack onset at a notch", *European Journal of Mechanics-A/Solids*, 21(1): 61–72, 2002.

[3] J. Hebel, R. Dieringer, W. Becker, "Modelling brittle crack formation at geometrical and material discontinuities using a finite fracture mechanics approach", *Engineering Fracture Mechanics*, 77(18): 3558–3572, 2010.

[4] J. Andersons, S. Tarasovs, E. Spārniņš, "Finite fracture mechanics analysis of crack onset at a stress concentration in a UD glass/epoxy composite in off-axis tension", *Composites Science and Technology*, 70(9): 1380–1385, 2010.

[5] J. Hebel, W. Becker, "Numerical analysis of brittle crack initiation at stress concentrations in composites", *Mechanics of Advanced Materials and Structures*, 15(6-7): 410–420, 2008.

[6] P. Cornetti, N. Pugno, A. Carpinteri, D. Taylor, "Finite fracture mechanics: a coupled stress and energy failure criterion", *Engineering Fracture Mechanics*, 73(14): 2021–2033, 2006.

[7] P. Cornetti, V. Mantič, A. Carpinteri, "Finite Fracture Mechanics at elastic interfaces", *International Journal of Solids and Structures*, 2012.

[8] D. Leguillon, J. Laurencin, M. Dupeux, "Failure initiation in an epoxy joint between two steel plates", *European Journal of Mechanics-A/Solids*, 22(4): 509–524, 2003.

[9] L. da Silva, A. Öchsner, *Modeling of adhesively bonded joints*, Springer Verlag, 2008.

[10] M. Goland, E. Reissner, "The stresses in cemented joints", *Journal of Applied Mechanics*, 11(1): A17–A27, 1944.

[11] M. Tsai, J. Morton, "An evaluation of analytical and numerical solutions to the single-lap joint", *International Journal of Solids and Structures*, 31(18): 2537–2563, 1994.

[12] M. Tsai, J. Morton, "A note on peel stresses in single-lap adhesive joints", *ASME, Transactions, Journal of Applied Mechanics*, 61(3): 712–715, 1994.

[13] L. Hart-Smith, "Stress analysis- A continuum mechanics approach (in adhesive bonded joints)", *Developments in adhesives- 2.*, 1–44, 1981.

[14] I. Ojalvo, H. Eidinoff, "Bond thickness effects upon stresses in single-lap adhesive joints", *AIAA Journal*, 16(3): 204–211, 1978.

[15] S. Krenk, "Energy release rate of symmetric adhesive joints", *Engineering Fracture Mechanics*, 43(4): 549–559, 1992.

[16] D. Gleich, M. Van Tooren, A. Beukers, "Analysis and evaluation of bondline thickness effects on failure load in adhesively bonded structures", *Journal of Adhesion Science and Technology*, 15(9): 1091–1101, 2001.

[17] L. da Silva, P. das Neves, R. Adams, J. Spelt, "Analytical models of adhesively bonded joints–Part I: Literature survey", *International Journal of Adhesion and Adhesives*, 29(3): 319–330, 2009.

[18] L. da Silva, P. das Neves, R. Adams, A. Wang, J. Spelt, "Analytical models of adhesively bonded joints–Part II: Comparative study", *International Journal of Adhesion and Adhesives*, 29(3): 331–341, 2009.

[19] L. da Silva, R. Adams, M. Gibbs, "Manufacture of adhesive joints and bulk specimens with high-temperature adhesives", *International Journal of Adhesion and Adhesives*, 24(1): 69–83, 2004.

[20] L. da Silva, T. Rodrigues, M. Figueiredo, M.D. Moura, J. Chousal, "Effect of adhesive type and thickness on the lap shear strength", *The Journal of Adhesion*, 82(11): 1091–1115, 2006.