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Generating Structural Configurations Having a Number of Specified Eigenfrequencies using a Level Set-Based Topology Optimization Method

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Abstract

This paper presents a topology optimization method, based on the level set method, for vibrating structures that have a number of specified eigenfrequencies. The proposed topology optimization method, which uses level set boundary expressions and is regularized using the Tikhonov regularization method, is described first. An optimization problem for vibrating structures that have a number of specified eigenfrequencies is then formulated. Next, based on this formulation, the topological sensitivities are derived using the adjoint variable method. A new topology optimization algorithm is then constructed that uses the Finite Element Method when solving the equilibrium equations and updating the level set function. Finally, a numerical example is provided to confirm the usefulness of the proposed topology optimization method.

Keywords: topology optimization, level set method, vibration problem, phase field method, structural optimization, optimum design, finite element method, structural analysis, Tikhonov regularization method.

1 Introduction

Vibration characteristics such as eigenfrequencies and eigenmodes are important factors in structural designs. The ability to design mechanical structures that have specified eigenfrequencies facilitates the development of high performance vibratory devices such as specialized sensors and vibromotors. However, trial and error approaches seldom provide optimal configurations for structures whose performance depends on sophisticated control of vibration.

To overcome these issues, topology optimization methods [1] have been proposed for vibration problems, to solve particular design requirements. Topology optimization is the most powerful tool among structural optimization methods because topological changes in the structural configuration are allowed during the optimization process. The topology optimization method was first proposed by Bendsøe and Kikuchi in 1988 [1], employing the homogenization method, and numerous topology optimization methods have been proposed and developed by researchers around the world since then. In these approaches, the SIMP (Solid Isotropic Material with Penalization) method [2] has been the most successful in several industrial fields, since the numerical implementation is straightforward.

Topology optimization methods have also been applied to design problems for vibrating structures. Diaz and Kikuchi [3] and Ma et al. [4] proposed a design method based on the homogenization design method for maximizing the lowest eigenfrequency of a vibrating structure. Nishiwaki et al. [5] and Tcherniak [6] proposed methods that maximize the displacement of a specified port under a periodic load, based on the homogenization design method and the SIMP method, respectively. And Maeda et al. [7] proposed a topology optimization method for structures with specified eigenfrequencies and eigenmode shapes, based on the homogenization design method. However, traditional topology optimization methods such as the homogenization design method and the SIMP method typically provide optimal configurations that include grayscales, areas with intermediate density values that do not belong to either the material or void domains. The issue of grayscales is particularly troublesome in specified eigenfrequency problems, when grayscales are usually widely distributed in the design domain.

To overcome the grayscale issue, several structural optimization methods based on the level set method [8] have been proposed during the last decade, as a new type of structural optimization method. In the level set method, the structural boundaries are represented using the iso-surface of a scalar function called the level set function. Wang et al. [9] and Allaire et al. [10] proposed a shape optimization method based on the level set method that has been applied to deal with a variety of optimization problems, such as minimum mean compliance problems [9, 10], compliant mechanism designs [11] and thermal actuators [12, 13], and maximization of the lowest eigenfrequency [10, 14]. In these methods, the structural configurations are represented using the level set function, which is updated via the Hamilton-Jacobi equation. Consequently, topological changes that generate one or more new holes in the structural domain during the optimization process are not allowed, so the setting of several design parameters pertaining to initial and intermediate configurations crucially affects the utility of the obtained results.

On the other hand, Yamada et al. [15] and Wei et al. [16] proposed a topology optimization method using level set-based structural representations in which the Tikhonov regularization method is applied to regularize the topology optimization problem. In these approaches, the geometrical complexity of the obtained optimal configurations can be controlled by adjusting a regularization parameter. Furthermore, the the Yamada design method shows extremely low dependency with respect to several design parameter settings, such as the initial configuration and finite element mesh size.

This paper presents a topology optimization method, based on the level set method, for vibrating structures that have a number of specified eigenfrequencies. First, the proposed topology optimization method, which uses level set boundary expressions and is regularized using the Tikhonov regularization method, is described. An optimization problem for vibrating structures that have a number of specified eigenfrequencies is then formulated. Next, based on this formulation, the topological sensitivities of the design are derived using the adjoint variable method. A new topology optimization algorithm is then constructed that uses the Finite Element Method (FEM) when solving the equilibrium equations and updating the level set function. Here, the level set function is updated based on a reaction-diffusion equation derived from a phase field concept. Finally, a numerical example of the formulated vibration problem is provided to confirm the usefulness of the proposed topology optimization method.

2 Optimization

2.1 Topology optimization

A fundamental concept of topology optimization is the introduction of a fixed design domain D that consists of a material domain Ω and a void domain $D \setminus \Omega$. The material domain Ω is represented using the following characteristic function χ.

$$\chi(\boldsymbol{x}) = \begin{cases} 1 & \text{if} \quad \boldsymbol{x} \in \Omega \\ 0 & \text{if} \quad \boldsymbol{x} \notin \Omega \end{cases} \tag{1}$$

Using the characteristic function χ, a topology optimization problem is formulated as follows:

$$\inf_{\chi} \qquad F = \int_D f_1(\boldsymbol{x}, \chi)\, \mathrm{d}\Omega + \int_\Gamma f_2(\boldsymbol{x})\, \mathrm{d}\Gamma \tag{2}$$

$$\text{subject to} \quad G = \int_D g(\boldsymbol{x}, \chi)\, \mathrm{d}\Omega - G_{\max} \leq 0, \tag{3}$$

where F is an objective functional, G is a constraint functional, and $G_{\max}$ is the upper limit of the constraint functional.

Now, we address the regularization of the topology optimization problem. It is well known that topology optimization problems are ill-posed problems because the characteristic function χ is allowed to be discontinuous everywhere in the fixed design domain D. Therefore, the design space must be relaxed by using a regularization technique, such as is accomplished in the homogenization design method and by various density approaches. These methods, however, allow intermediate material densities in the obtained optimal configurations, which compromises their utility.

2.2 Level set-baed topology optimization

To regularize the topology optimization and overcome the above issue, we introduce a fictitious interface energy model via the level set method, derived form the phase field method. In the level set method, the structural boundaries are represented using the iso-surface of a scalar function $\phi(\boldsymbol{x})$, which is called the level set function:

$$\begin{cases} \phi(\boldsymbol{x}) > 0 & \text{if} \quad \forall \boldsymbol{x} \in \Omega \setminus \partial\Omega \\ \phi(\boldsymbol{x}) = 0 & \text{if} \quad \forall \boldsymbol{x} \in \partial\Omega \\ \phi(\boldsymbol{x}) < 0 & \text{if} \quad \forall \boldsymbol{x} \in D \setminus \Omega. \end{cases} \tag{4}$$

Using the level set function, the level set based topology optimization problem is formulated as follows:

$$\inf_{\phi(\boldsymbol{x})} \qquad F = \int_D f_1(\boldsymbol{x}, \chi(\phi)) \,\mathrm{d}\Omega + \int_\Gamma f_2(\boldsymbol{x}) \,\mathrm{d}\Gamma \tag{5}$$

$$\text{subject to} \quad G = \int_D g(\boldsymbol{x}, \chi(\phi)) \,\mathrm{d}\Omega - G_{\max} \leq 0 \tag{6}$$

$$-1 \leq \phi(\boldsymbol{x}) \leq 1, \tag{7}$$

where the range of the level set function is subject to a constraint because the regularization term (discussed below) is defined using the gradient of the level set function.

Next, we introduce a fictitious interface energy to the objective functional to regularize the topology optimization problem:

$$\inf_{\phi(\boldsymbol{x})} \qquad F_R = F + R \tag{8}$$

$$\text{subject to} \quad G \leq 0 \tag{9}$$

$$-1 \leq \phi(\boldsymbol{x}) \leq 1, \tag{10}$$

where the regularization term is defined using the regularization parameter $\tau > 0$ as follows:

$$R = \frac{1}{2} \int_D \tau \mid \nabla \phi \mid^2 \,\mathrm{d}\Omega. \tag{11}$$

This regularization technique is known as the Tikhonov regularization method, with details as previously discussed [15].

2.3 Optimization method

The Lagrangian of the above topology optimization problem is derived as follows:

$$\inf_{\phi(\boldsymbol{x})} \qquad \bar{F}_R = F_R + \lambda G, \tag{12}$$

where $\bar{F}_R$ is the Lagrangian with regularization term R and λ is the Lagrange multiplier with respect to the constraint functional G. The Karush-Kuhn-Tucker (KKT) conditions of the topology optimization problem are derived as follows:

$$\bar{F}_R' = 0, \;\; \lambda G = 0, \;\; \lambda \geq 0, \;\; G \leq 0. \tag{13}$$

Although a level set function that satisfies the above conditions is an optimal solution candidate, such solutions are hard to obtain directly. Therefore, a fictitious time t and an appropriate initial level set function $\phi(\boldsymbol{x}, 0)$ are introduced, so that a level set function $\phi(\boldsymbol{x}, t)$ that represents an optimal configuration can be obtained by updating the level set function.

We assume that the updated level set function $\phi(\boldsymbol{x}, t)$ is proportional to the gradient of the Lagrangian:

$$\frac{\partial \phi(\boldsymbol{x}, t)}{\partial t} = -K \bar{F}_R'(\phi, t), \tag{14}$$

where $K > 0$ is a coefficient of proportionality. Therefore, the level set function is updated by using following reaction diffusion equation:

$$\frac{\partial \phi(\boldsymbol{x}, t)}{\partial t} = -K \left(\bar{F}'(\phi, t) - \tau \nabla^2 \phi(\boldsymbol{x}, t) \right). \tag{15}$$

2.4 Optimization problem

We consider a material domain Ω that is filled with a linearly elastic material, where the displacement is fixed at boundary Γ_u. The k-th eigenfrequency, the specified k-th eigenfrequency, and the k-th eigenmode are denoted ω_k, $\bar{\omega}_k$, and $\boldsymbol{u}_k$, respectively. The relationship between the k-th eigenvalue λ_k and the k-th eigenfrequency ω_k is as follows.

$$\lambda_k = \omega_k^2 . \tag{16}$$

The topology optimization problem for vibrating structures that have a number of specified eigenfrequencies is formulated as follows:

$$\inf_{\chi} \qquad F = \sum_{k=1}^{n} \mid \omega_k - \bar{\omega}_k \mid^2 \tag{17}$$

$$\text{subject to} \quad a(\boldsymbol{u}_k, \boldsymbol{v}, \chi) = \lambda_k b(\boldsymbol{u}_k, \boldsymbol{v}, \chi) \tag{18}$$

$$\text{for } \forall \boldsymbol{v} \in U, \;\; \boldsymbol{u}_k \in U, \;\; k = 1, ..., n$$

$$G = \int_D \chi \, \mathrm{d}\Omega - V_{\max} \leq 0 \tag{19}$$

$$-1 \leq \phi(\boldsymbol{x}) \leq 1. \tag{20}$$

The notations in the above formulation are defined as follows:

$$a(\boldsymbol{u}_k, \boldsymbol{v}, \chi) = \int_D \boldsymbol{\epsilon}(\boldsymbol{u}_k) : \boldsymbol{E}\chi : \boldsymbol{\epsilon}(\boldsymbol{v}) \, \mathrm{d}\Omega \tag{21}$$

$$b(\boldsymbol{u}_k, \boldsymbol{v}, \chi) = \int_D \rho \chi \boldsymbol{u}_k \cdot \boldsymbol{v} \, \mathrm{d}\Omega, \tag{22}$$

where $V_{\max}$ is the upper limit of the volume constraint, $\boldsymbol{\epsilon}$ is a linearized strain tensor, $\boldsymbol{E}$ is an elastic tensor, ρ is the material density, and

$$U = \{\boldsymbol{v} = v_i \boldsymbol{e}_i : v_i \in H^1(D) \quad \text{with} \;\; \boldsymbol{v} = 0 \;\; \text{on}\, \Gamma_u\}. \tag{23}$$

2.5 Sensitivity analysis

Using the adjoint variable method, the design sensitivity of the objective functional F' required when updating the level set function is derived.

$$F' = 2 \sum_{k=1}^{n} \mid \omega_k - \bar{\omega}_k \mid \omega_k' \, . \tag{24}$$

Using the relationship between the eigenvalue and eigenfrequency (16), the following equation is derived.

$$\lambda_k' = 2\omega_k \omega_k' \, . \tag{25}$$

In addition, by using the adjoint variable method, the sensitivity of the k-th eigenmode, λ_k, is derived as follows [4, 17]:

$$\lambda_k' = a'(\boldsymbol{u}_k, \boldsymbol{u}_k) - \lambda_k b'(\boldsymbol{u}_k, \boldsymbol{u}_k). \tag{26}$$

Substituting (25) and (26) into (24), we obtain the following:

$$F' = \sum_{k=1}^{n} \frac{\mid \omega_k - \bar{\omega}_k \mid}{\omega_k} \cdot \Big\{ \boldsymbol{\epsilon}(\boldsymbol{u}_k) : \boldsymbol{E}\chi : \boldsymbol{\epsilon}(\boldsymbol{u}_k) - \omega_k^2 \rho \chi \boldsymbol{u}_k \cdot \boldsymbol{u}_k \Big\}. \tag{27}$$

3 Optimization algorithm

Figure 1 shows the flowchart of the topology optimization procedure. As shown in this figure, an initial level set function representing an appropriate initial configuration is set first. In the second step, the governing equation is solved using the FEM. In the third step, the objective functional is calculated and the topology optimization process finishes if it has converged, otherwise the design sensitivities with respect to the objective functional are computed in the fourth step. In the fifth step, the level set function is updated based on the reaction diffusion equation using the FEM and the procedure then returns to the second step.

4 Numerical example

Using the design model shown Figure 2, we examine the utility of the proposed topology optimization method. The fixed design domain is fixed at both side boundaries, and a concentrated mass $M = 1$kg is set at the center of the fixed design domain. The

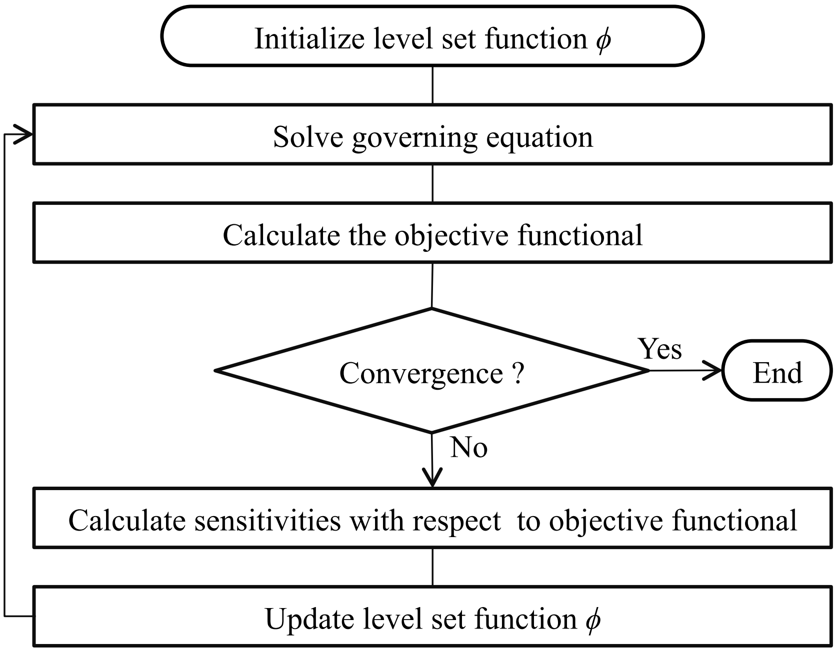


Figure 1: Flowchart

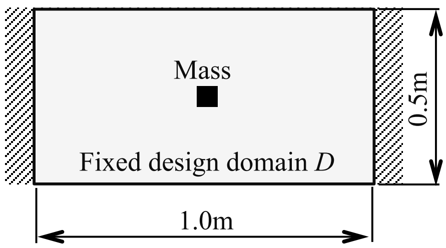


Figure 2: Fixed Design Domain D and Boundary Conditions

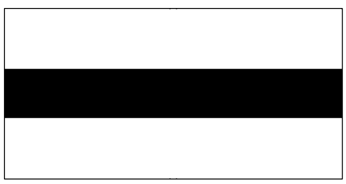
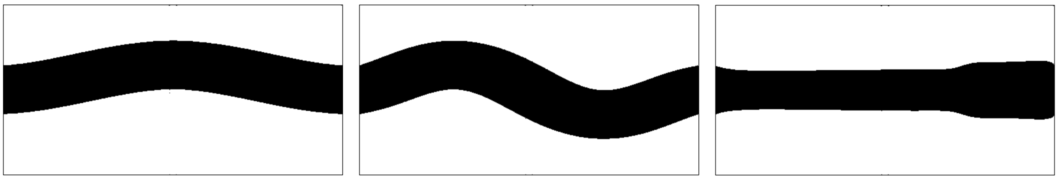


Figure 3: Initial Configuration



(a) 1st Mode (b) 2nd Mode (c) 3rd Mode

Figure 4: Eigenmodes of the Initial Configuration

fixed design domain is discretized using a structural mesh and four-node quadrilateral elements whose length is 5×10^{-5}m. As shown in Figure 3, the initial configuration is a rectangle. The isotropic, linearly elastic material has Young's modulus = 210GPa, Poisson's ratio = 0.3, and a material density = 7,850kg/m^3. The regularization parameter τ is set to 1×10^{-5}, and the upper limit of the volume constraint $V_{\max}$ is set to 25% of the fixed design domain. At this setting, the eigenfrequencies are $\omega_1 = 3.34 \times 10^3$Hz, $\omega_2 = 1.08 \times 10^4$Hz, $\omega_3 = 1.20 \times 10^4$Hz, and each eigenmode is shown in Figure 4. Here, we set the specified eigenfrequencies as noted in the following table. Figure 5 shows an obtained optimal configuration. As shown this figure,

	initial	target: $\bar{\omega}_k$	optimum
1st mode	3.34×10^3	4.00×10^3	4.00×10^3
2nd mode	1.08×10^4	6.00×10^3	6.00×10^3
3rd mode	1.20×10^4	1.00×10^4	9.75×10^3

Table 1: Eigenfrequencies [Hz]

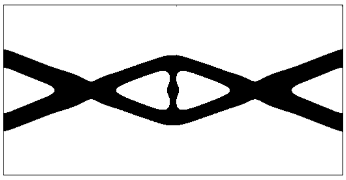
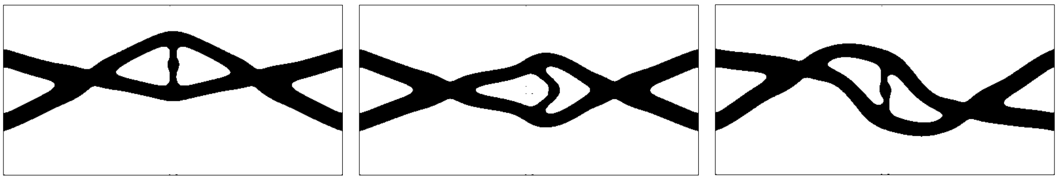


Figure 5: Optimal Configuration

the optimal configuration is clear and smooth, and we can confirm that topological changes in the configuration occurred during the optimization process. The eigenfrequencies of the optimal configuration are listed in Table 4. As this table shows, each target eigenfrequency was appropriately obtained in the optimal configuration. Figure 6 and Figure 7 show the eigenmodes of the optimal configuration and the convergence of the eigenfrequencies, respectively. Comparing Figure 4 with Figure 6, we see that the 2nd mode has switched places with the 3rd mode. We note that an appropriate optimal result was obtained despite this switching of eigenmodes during the optimization procedure.



(a) 1st Mode (b) 2nd Mode (c) 3rd Mode

Figure 6: Eigenmodes of the Optimal Configuration

5 Conclusion

This paper presented a topology optimization method, based on the level set method, for vibrating structures that have a number of specified eigenfrequencies. We achieved following:

(1) A topology optimization problem for a vibrating structure with specified eigenfrequencies was formulated using the level set method.

(2) The topology optimization problem was regularized using the Tikhonov regularization method by introducing a regularization term. The design sensitivities were derived using the adjoint variable method.

(3) Based on our formulation, a topology optimization algorithm was proposed.

(4) A two-dimensional numerical example was provided and we confirmed the validity and utility of the proposed topology optimization method.

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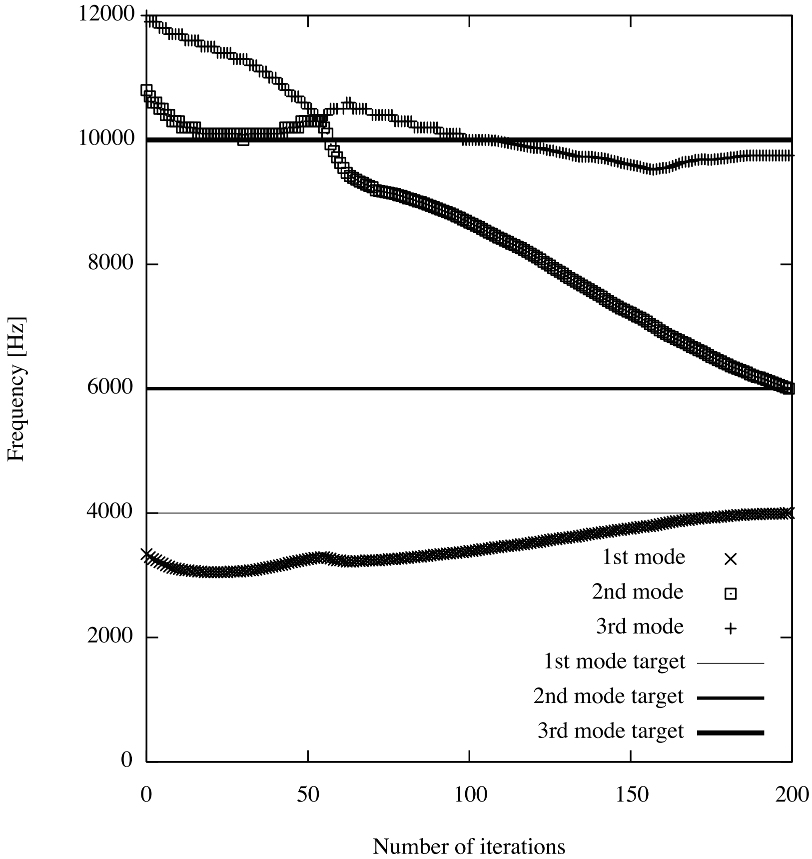

Figure 7: Convergence of the Eigenfrequencies

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