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Paper 65

A Hybrid Metaheuristic for Project Scheduling Problems with Fuzzy Activity Durations to support the Ős-Dráva Water Management Programme

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Abstract

Spatial developmental programs in Hungary are important to support economically lagging regions. As a result of their nature they can be considered as big projects, with long timescales, high levels of capital, causing significant impact on the targeted field. For an effective performance, an accurate planning phase is essentially important. In this paper we concentrate on the project scheduling element of planning, to reduce risk and gain knowledge on the possible, net present value and makespans.

The Ős-Dráva program is a water-management based spatial development program. There have been no reference projects like this in Hungary that can support the scheduling of the problem with applicable information. This means that a highly uncertain case is considered with resource constraints implied. Our goal is to support the programme with an appropriate project scheduling method, resulting in a reliable makespan. For this we present a stochastic programming model and a hybrid metaheuristic for resource constrained project scheduling problems.

The stochastic programming model was presented by Goel and Grossman[1], considering the net present value of subprojects. The hybrid metaheuristic was presented by Danka [2] which is a modification of the model developed by Csébfalvi [3]. In the presented approach, it is assumed that each activity duration is considered as a fuzzy variable which, can be described with an appropriate membership function. The heuristic algorithm produces "robust" resource-feasible schedules which are totally immune against uncertainties in the activity durations. The hybrid algorithm presented is based on the "Sounds of Silence" harmony search metaheuristic developed by Csébfalvi et al. [4, 5].

Keywords: resource-constrained project scheduling, fuzzy project scheduling, robust scheduling, metaheuristics, hybrid methods, spatial development, water management.

1 Introduction

The paper presents a spatial development program that is planned to be carried out in some of the most lagging small regions in Hungary and Europe. The conditions have altered such that support and intervention is indisposable. The work has started in 2005 with a feasibility study that showed the necessity of deeper analysis and planning. In 2006 the Interreg III/A SL-HU-CR/05/4012-106/2004/01/HU-103 tender according to which Aquaprofit corp. and KÖVIZIG has won the rights to perform a complex spatial development plan, based on a watergovernance plan on the river Dráva. A speciality of the situation is that the river is situated as a border between Hungary and Croatia, so the effect of the program will be perceptible in both countries. The main goal of the program is to ensure an economical standard, a higher level of living for the citizens in the affected area, in consideration with the local environments and natures aspects on a sustainable way.

The plan consists of various projects that would be individually large enough to be dealt with. There has never been such a project that could be mentioned as a reference project, so the level of uncertainty of the time scale and the outcomes are high. The goal for our work is to support the management of the program with an appropriate schedule that considers uncertainties in duration times, and the values of the separated projects within. This is highly important because the budget of the program is limited and thus the consideration of the returns of subprojects can increase the efficiency and the lifetime of it. This is crucial because if no further capital is granted for the program the complex realization of can be threatened. For this the subprojects have to be distinguished and performed in a manner so that their returns can be built in the total budget, and reinvested for further subprojects. An efficient way to rank subprojects can be done based on their expected net present value (ENPV). Pervious researches have been done by Láng [6], on scheduling with the consideration of cash-flows of projects. A stochastic programming model was presented by Goel and Grossman [1], considering the net present value of subprojects, that we apply to create logic predecessor-successor relations and extend the relations.

For the resource-constrained scheduling process using fuzzy activity durations we present a hybrid metaheuristic Presented by Danka [2]. The heuristic algorithm produces “robust” resource-feasible schedules which are totally immune against uncertainties in the activity durations. Theoretically the robust schedule searching process is formulated as a mixed integer linear programming problem (MILP) based on the “forbidden set” concept. The output of the model is an optimal (robust) conflict repairing relation set. The initial model was developed by Csébfalvi [3] is insensitive to the “real meaning” of the input parameters, so the originally probabilistic (density function oriented) approach can be replaced by a possibilistic (membership function oriented) approach without essential modifications. The presented hybrid algorithm is based on the “Sounds of Silence” harmony search metaheuristic developed by Csébfalvi at al. [4, 5].

2 The Ős-Dráva program

2.1 The affected area

2.1.1 The geographic definition of the area

The Ős-Dráva program has a planning area of 45250 ha, three small regions and 36 towns. All the three small regions are of the most underprivileged small regions in Hungary. According to the its geographic and historic denomination the area is a part of Ormánság, which is a Transdanubian part of the great Hungarian plane. Traditionally it has always been dominated by agricultural farming, while other sectors were never able to evolve. The region neglects good infrastructural networks and connections to main transportation roots, thus it is an isolated spot in the Hungarian spatial structure.

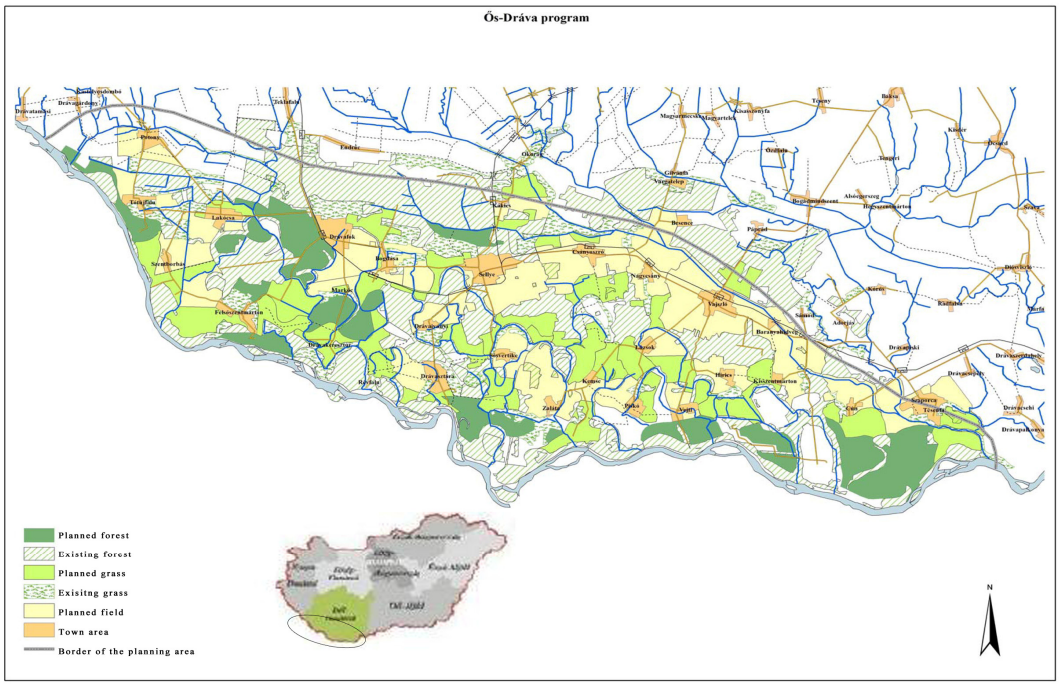

Figure 1: Planning area of the, Ős-Dráva program

2.1.2 The social and economic state of the region

The demographic state of the region reflects a tragic state and the necessity of action. The affected towns demographic relations are worse than the national averages. The intense decrease in the population can be explained by natural losses, and further gone aging has to be considered also, besides the basically low population and number of births.

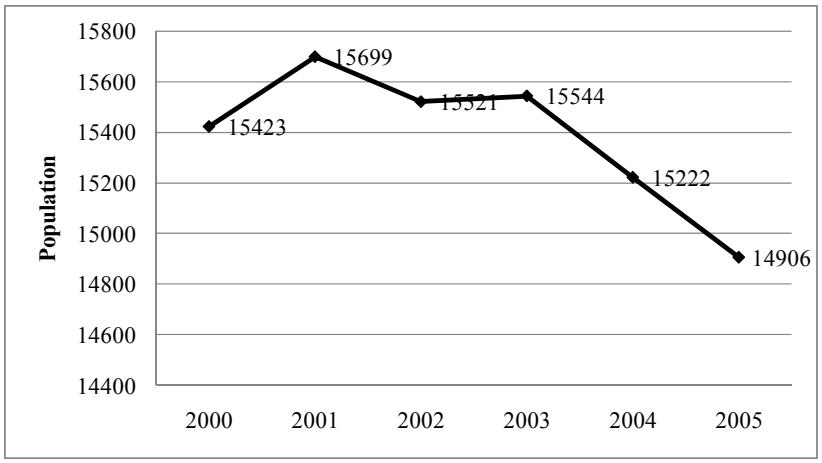

Figure 2: Population in the affected region 2000-2005, Ős-Dráva program [8]

In the past 20 years the shrunk of the labour market and negative visions of the local economy caused a dramatic situation. A mass number of active workers lost their jobs, and was not able to find any employers. This caused that recently there is no active worker in over 50% of households. The unemployment rate is nuanced because of the high percentage of the inactive population. The Central Statistical Office' data shows that the unemployment rate is not much above 10%, but for a clear picture we have to note that the percentage of inactives are above 50% and the employed rate of the whole population is only around 30%. This means that 70% of population in the analysed area lives either from the black market, or from governmental aids and support.

Furthermore it has to be considered that the region is also lagging is most aspects of the economy. Without efficient support it has no visions to improve its declining position in the industry, tourism, culture, and even in agriculture. There is a strong need for highlighting and improving values of the environment and nature. The analysis conducted for the program considers these aspects as well and defines possible solutions for development using them.

2.2 The structure of the program

The Ős-Dráva program consists of three complex studies: a spatial development program which analyses the regions capabilities and current state, defines a vision, and defines priorities and actions through which the realization can be possible. These priorities and actions synthesize the notions in the other two studies.
The second study is a landscape management program, which gives proposals to the appropriate landscape usage through ground applications.
The third study is a theoretical permit plan documentation of the water governance plan for the given section of the Dráva and the affected areas inner water infrastructure. Until now these studies create the most detailed program for the region with the consensus of the locals.

The program gives proposals for the natural environments conscious transformation at once with an economic development program ensuring the local residents with work and a fair livelihood. Various approaches have been synchronized to find the fit between the political, economic environment, the needs of the locals and the ecologic sustainability. The program focuses on important aqueous habitats that have been decaying due to the lack of water supply. With this the natural or close to natural environment can be restorable with a rich and a diverse habitats. This can ensure the development of many traditional farming methods and stable workplaces. The plane forecasts a 50% rate of forests and 10% of aqueous habitats that can at provide least 1500 new workplaces.

There have been numerous researches and development plans for the region without any measurable outcome. This program contains specified instructions for projects, aligns with the requirements of the tender, the possibilities of the residents and natural conditions. The whole programs complexity might even require decades, but deliberate short programs can result in success not harming long term goals. Besides the long makespan of the whole program the capital requirements are also large.

The plan defines five priorities to be accomplished which are the following: economic development, landscape management, tourism development, infrastructure development, organization and institutional development. The five priorities are made up of further 55 subprojects.

The following example illustrates the bounds of the total budget. In the permit plan scheme a planned budget for only one activity of a subproject consists of 280 million forints, while the planned total budget of the program is around 20 billion Forints for more than 55 subprojects. The contrast highlights the crucial importance of a logical budgeting plan, investment decisions, and a well designed schedule considering the returns of the subprojects. Returns of subprojects can be utilized in further stages of the accomplishing of the program. The problem is that the returns of the subprojects can be considered to be highly uncertain.

3 A logic built of some predecessor-successor relations

3.1 Decision tree

The appropriate choice of subprojects leads us to the case of uncertainty. Jonsbraten [7] defines two cases of uncertainty in planning problems. The first category is the project exogenous uncertainty. This case applies for those projects where the scenario tree is fixed and the decision does not affect it. In other words we can say that the returns of a given project are mainly a consequence of factors that are independent from the decision itself. These can be market prices, environmental changes, policy shifts and so on. Figure 3 shows a scenario fixed tree with three time periods and two random variables ξ_1, ξ_2.Both variable can either result in L (Losses) or P (Profits). The uncertainty in the case of variable ξ_1 resolves after the first time

period, while in the case of variable ξ_2 it is only resolved after time period 2. The result of the case is four determined scenarios by the realization of the random variables.

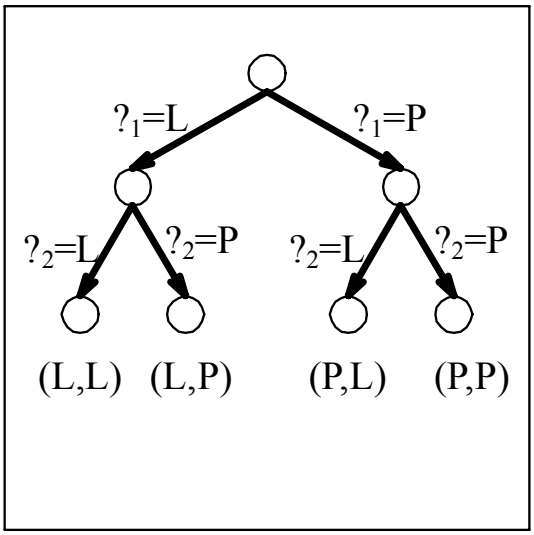

Figure 3: A scenario tree for a problem with exogenous uncertainty

The other category is the so called project endogenous uncertainty. In this case the uncertainty is dependent on the decision itself. In this situation the vagueness is resolved after the decision has been made. The outcome becomes clear as soon as the project has started because there are no outer factors that could exercise any kind of impact on it. An example for this can be a mine planning problem with uncertainty in mineral reserves. As soon as mining has started the amount of reserves contained by the chosen site will be not be a fact of uncertainty anymore.

We consider a complex spatial development program in the Ormánság region of Hungary. The program consists of 5 priorities and 55 subprojects under uncertainty. It has been identified that each of the projects are necessary for the successful development of the region, but their returns are not always evidential. There for we have to categorize the subprojects into ones with certain and uncertain returns.

Each subproject i has a time horizon of Ti which is discretized into Ti time periods. To start the subproject the investment decision has to be made. For simplicity it is assumed that investments are done only once at the starting point of the subproject, while work has to be performed throughout the whole required time. Due to this we cannot assume similar simplicity in the case of returns as in the case of investments, the returns of the subprojects are continuous and not discretized and collected at one determined time period. . Furthermore subprojects are characterized by “value” and “gain”. The “value” of the project can be translated as the total amount of returns we are expecting to have from the total subproject. The “gain” on the contrary is the maximum value that we are expecting only at a given time. The “gain” of the subproject is the highest when no returns have been made before-at the point of investment-and decreases during the duration time of its lifetime. When

cumulative return reaches the “value” of the subproject the “gain” reduces to zero. We assume that the “gain” reduces linearly, as it is visible in Fig.4.

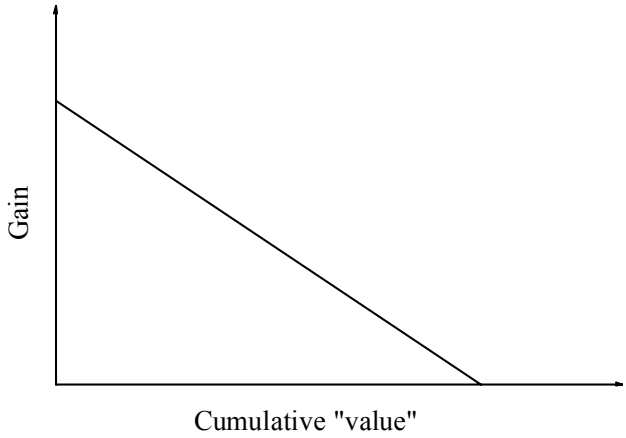

Figure 4: Linear Reservoir Model

We use discrete probability distributions to represent the uncertainty in the “value” and “gain” of each subproject, so all possibilities are represented by “scenarios”, where each scenario is a combination of “values” and “gains” and have a given probability. The objective is to find the decisions that maximize the expected net present value (ENPV) of the subprojects. Figure 5 shows possible combinations for the ENVP.

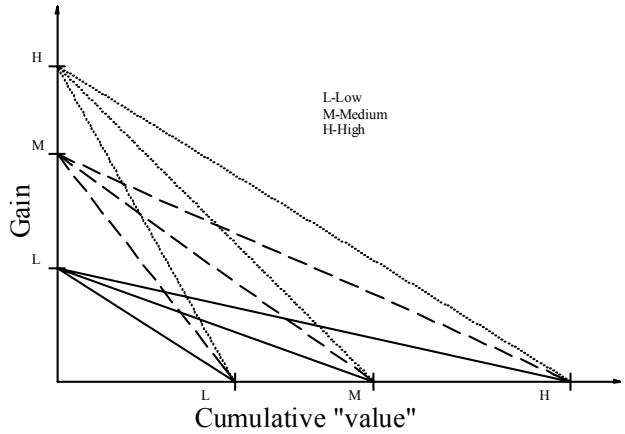

Figure 5: Representation of uncertainty in size and initial deliverability of a subproject.

In most cases of the problem we consider to have endogenous uncertainty, so we can state that uncertainty resolves after the investment have started, so our knowledge about the “value” and “gain” of the subproject will be known deterministically once the work has been started. The uncertainty in returns reduces by investments. This means that the scenario tree is not unique and depends on the decisions. The approach yields that uncertainty reduces earlier for the entire project horizon in the investment decision is done sooner. As investment decisions are spread over the program horizon, we should consider recourse in these.

3.2 Maximising the expected value of subprojects.

In this section we describe a stochastic multistage problem model (SPM) similar to the SPM developed by Goel and Grossmann [5]. The model is a hybrid mixed-integer optimization problem formulated using conditional non-anticipativity constraints

$$Max \sum_{e} p^{e} \left[\sum_{t} \left(c_{1t}^{T} q_{t}^{e} + c_{1t}^{T} q_{t}^{e} + c_{1t}^{T} q_{t}^{e} + \sum_{up} c_{4t,up}^{T} b_{up,t}^{e} \right) \right] \qquad (1)$$

$$A_{t}^{e} q_{t}^{e} \le a_{t}^{e} \qquad \forall (t,e) \qquad (2)$$

$$q_{t}(q_{1}^{e}, q_{2}^{e}, ..., q_{t}^{e}) \le 0 \qquad \forall (t,e) \qquad (3)$$

$$d_{t}(d_{1}^{e}, d_{2}^{e}, ..., d_{t}^{e}) \le 0 \qquad \forall (t,e) \qquad (4)$$

$$\sum_{t} b_{up,t}^{e} \le 1 \qquad \forall (up,e) \qquad (5)$$

$$r_{up,t}(q_{t}^{e}, d_{t}^{e}, b_{up,1}^{e}, b_{up,2}^{e}, ..., b_{up,t}^{e}, y_{1}^{e}, y_{2}^{e}, ..., y_{t}^{e}) \le 0 \qquad \forall (up,t,e) \qquad (6)$$

$$\begin{bmatrix} Z_{t}^{e,e^{1}} \\ q_{t}^{e} = q_{t}^{e^{1}} \\ d_{t+1}^{e} = d_{t+1}^{e^{1}} \\ y_{t+1}^{e} = y_{t+1}^{e^{1}} \\ b_{up,t+1}^{e} = b_{up,t+1}^{e^{1}} \forall up \end{bmatrix} \vee \begin{bmatrix} \neg Z_{t}^{e,e^{1}} \\ q_{t}^{e}, q_{t}^{e^{1}} \ge 0 \\ d_{t+1}^{e}, d_{t+1}^{e^{1}} \ge 0 \\ y_{t+1}^{e}, y_{t+1}^{e^{1}} \in \{0,1\}^{\dim(y)} \\ b_{up,t+1}^{e}, b_{up,t+1}^{e^{1}} \in \{0,1\} \forall up \end{bmatrix}$$

$$\forall (t,e,e^{'}), e < e^{'} \qquad (7)$$

$$Z_{t}^{e,e^{'}} \Leftrightarrow \bigwedge_{up \in G(e,e^{'})} \left[\wedge_{T=1}^{t} (\neg b_{up,T}^{e}) \right] \qquad \forall (t,e,e^{'}), e < e^{'} \qquad (8)$$

$$b_{up,1}^{e} = b_{up,1}^{e^{'}} \qquad \forall (up,e,e^{'}), e < e^{'} \qquad (9)$$

$$y_{1}^{e} = y_{1}^{e^{'}} \qquad \forall (e,e^{'}), e < e^{'} \qquad (10)$$

$$d_{1}^{e} = d_{1}^{e^{'}} \qquad \forall (e,e^{'}), e < e^{'} \qquad (11)$$

$$q_{t}^{e}, d_{t}^{e} \ge 0, \qquad b_{up,t}^{e} \in \{0,1\}, \qquad y_{t}^{e} \in \{0,1\}^{\dim(y)}, \qquad Z_{t}^{e,e^{'}} \in \{True, False\}$$

To understand the model the followings require description. Variable vectors $q_{t}^{e}, d_{t}^{e}, b_{uf,t}^{e}$ and y_{t}^{s} are defined for each time period t in scenario e. q_{t}^{e} is a continuous operational vector such speed of work, or quality. the vector d_{t}^{e} is a continuous investment vector defining the amount of capital to be set available for the project, and $b_{up,t}^{e}$ are binary investment variables whether or not to invest in a subproject with uncertain returns in time period t in scenario e. For all other binary investment variables for time period t in scenario e y_{t}^{s} vector stands. The decisions about subprojects with certain returns are represented by this. Finally the Boolean

variables $Z_t^{e,e'}$ are used to show whether scenarios e and e' are distinguishable or not. Discounted prices are represented by $c_{1t}, c_{2t}, c_{3t}, c_{4t,up}$ corresponding to the earlier mentioned variables: q_t^e, d_t^e, y_t^s and. $b_{uf,t}^e$.

The objective (1) maximizes the ENVP, which is the weighted average of the NVPs of the various scenarios. Constraints (2)-(6) represent investment and operational decisions for the scenarios. For instance constraint (2) stands for an operational decision for given time periods including the linear reservoir model and mass balance constraints. The uncertainty of coefficient matrix of A_t and the vector a_t of all constraints reflect the uncertainty of the problem. Constraint (3) links operational decisions for period t with operational decisions from previous periods. Constraint (4) is a similar on but represents investment decisions in the same manner. Constraint (5) ensures that the investment of a subproject can only be made once throughout the program horizon. Constraint (6) represents big-M and capacity constraints.

(7)-(11) are "non-anticipativity" constraints that link decisions for various scenarios. It is important to note that the total investment is made instantly at the beginning o the time period. On the contrary the operations are performed during the whole time period. For the investment decision we only have information in time period t form investments made in $t-1$. The operations are done with more information that was obtained by the investment in the same time period. $Z_t^{e,e'} \in \{True, False\}$ is a special variable. If the variable is $True$ than scenarios e and e' are indistinguishable after the investment in the given period of t. Based on the similarity and according to non-acitipativity rule the operational decisions must be the same for both scenarios.

Constraint (8) is a related to $Z_t^{e,e'}$ with the investment decisions for uncertain fields. The set $G(e,e') \subseteq UP$ is established on a manner that $up \in G(e,e')$ of and only if the uncertain properties ("value" and "gain") of the uncertain projects are differentiable in the case of e and e'. It means that of $G(e,e')$ is a set of uncertain subprojects whose properties is e are not the same as those in e' The constraint also ensures that if no investments have been done in any projects with uncertain returns which causes e and e' to be different scenarios before time period t ($\bigwedge_{up \in G(e,e')} \left[\wedge_{T=1}^{t} (\neg b_{up,T}^e)\right] = True$) ,then there is no information that could help to differentiate between s and s' scenarios. Because of the lack of information prior to investment decisions in the first period, non-anticipaty requires that decisions at $t=1$ should be same for all scenarios(constraints (9)-(11)).

In the following a small example will be presented to illustrate the models capabilities. Table 1 shows the subprojects 1-6 indicating their "Value" and initial "Gain" measures. Without loss of generality, it is assumed that the makespan is known, which is in this example $T=15$ periods Among the subprojects we can find certain ones (1-5) and one uncertain subproject(6). For simplicity it is assumed that

only the “Vaule” of subproject 6 is uncertain, and has three possible scenarios: Low, Medium and High with probability values associated.

	1	2	3	4	5	6					
						Low	p^{low}	Med	p^{med}	High	p^{high}
Value	400	400	350	200	290	130	0,3	300	0,4	470	0,3
Gain	130	200	100	100	130	130					

Table 1: Subproject properties

The problem has been solved by two methods to illustrate the effectiveness of the proposed method. A deterministic approach has been used (Table 2) to calculate the expected net present value (ENPV) of the example. Scenarios A,B,C correspond to to low, medium and high “values” for subproject 6. Different scenarios define different investment decisions. Considering the three scenarios solutions the ENVP of the deterministic approach is Ft 94,56 million

Using the SPM approach the ENVP of the problem increases to FT 99,55 million. The schedule of the different scenarios become different (Table 3.) and allows the profitability to increase. Investment for subprojects 1,2,3,5 should be dated in year 1, the uncertain subproject should be started in year 2 while subproject 4 should be started later in year 6 or 7 depending on the “Value” of subproject 6. The value of the model in this example is 99,55-94,56=Ft 4,99 million. The explanation for the higher ENVP roots in the dating of the investment decision. Investments cause high costs to the total program so its impact on the NPV is large. If two solutions result in the same revenues, than the one will result in higher NPV that is able to delay the investments. This can be seen comparing Table 2 and 3. It is important to note that the proposed method leads to a positive NPV in all scenarios o contrary to the deterministic approach.

	Scenario		
	A	B	C
Year1	1,2,3,6		
Year3	5	-	-
Year5	-	5	5
Year6	4	-	-
Year7	-	4	-
Year8	-	-	4
NVP (Ft Million)	-24,18	117,26	183,03
ENVP (Ft Million)	94,56		

Table 2: Deterministic solutions

	Scenario		
	A	B	C
Year1	1,2,3,5		
Year5	6		
Year6	4	-	-
Year7	-	4	4
NVP (Ft Million)	15,73	115,97	161,48
ENVP (Ft Million)	99,55		

Table 3: Solutions of the proposed method

The example used for illustration has all possibilities to be extended with uncertainties in "Gain" measures as well as with multiple uncertain subprojects at the same time. The proposed method in this sense is able determine the accurate logic of a scheduling method with supporting it with appropriate predecessor relationships considering the ENPV of the given subprojects.

4 Problem formulation

In this section we describe a robust scheduling model for the resource constrained project scheduling problem with random activity durations that was earlier presented by Danka [8]. The Ős-Dráva project as it was mentioned before can be stated as a highly uncertain program. The uncertainty roots not only in the NPV of the projects but also in the durations. The lack of reference problems and the knowledge about certain operations makes it difficult to indicate the duration times of activities. The reactive way of finding out the total makespan of the program can cause a line of managerial problems. We attempt to use proactive scheduling techniques to find a good baseline schedule using fuzzy duration times. This requires good professional knowledge and experience on the field of the certain subprojects. Using this knowledge we are able to dress up for instance fuzzy triangle with enough information about time estimates. These fuzzy triangles will be able to contain optimistic, most likely and pessimistic time estimates with associated with function of degree to a membership to a set. The higher the value of the function is the higher is the possibility that an element will belong to the set. The result of a fuzzy scheduling method will be a robust fuzzy schedule that is immune to uncertainties in activity durations. Robust scheduling methods have also been considered by Szendrői [9] and Láng [6].

The theoretical model developed by Csébfalvi [3] for resource constrained projects with uncertain activity durations. The optimality criterion is defined as a linear combination (weighted sum) of resource-feasible makespans connected to the key terms of the applied uncertainty formulation. Theoretically the optimal robust schedule searching process is formulated as a multi-objective mixed integer linear programming problem (MOMILP) where the number of objectives corresponds to the number of key terms (parameters) of uncertainty formulation.

In this case the MOMILP was replaced with a MILP by scalarization. The resulting MILP can be solved directly in the case of small-scale projects within reasonable time. The proposed model is based on the so-called "forbidden set" concept. The output of the model is the set of the optimal conflict repairing relations. Obviously, the solution of the problem depends on the choice of the weights for the objective functions.

In order to model uncertain subproject duration of complex programs, we consider the following resource-constrained project-scheduling problem: In our case the Ős-Dráva program consists of N real subprojects $i \in \{1, 2, ..., N\}$ In the following subprojects are also referred as activities.

Each subproject duration D_i, $i \in \{1, 2, ..., N\}$ is a fuzzy variable with a triangular membership function $D_i = \boldsymbol{TRIANGULAR}(D_{1i}, D_{2i}, D_{3i})$.where, using the usual naming convention, D_{pi}, $p \in \{1,2,3\}$ D_{2i}, and D_{3i} are the optimistic, most likely, and pessimistic estimations of the uncertain subproject duration D_i, respectively.

The subprojects are interrelated by precedence constraints: Precedence constraints - as known from traditional CPM-analysis - force an activity not to be started before all its predecessors are finished. A logic build behind of these relations is supported by the model presented in the previous section. The motif behind the structure is the ENPV maximization of program though a correct order of the subprojects. The precedence constraints are given by network relations $i \to j$, where $i \to j$ means that activity j cannot start before activity i is completed. Furthermore, activity $i = 0\,(i = N+1)$ is defined to be the unique dummy source (sink). Let IP_i, $i \in \{1, ..., N+1\}$ denote the set of immediate predecessors for activity i and let NR be the set of the network relations.

Let R denote the number of renewable resources required for carrying out the project. Each resource $r \in \{1, ..., R\}$ has a constant per period availability R_r. In order to be processed, each real subproject $i \in \{1, 2, ..., N\}$ requires $R_{ir} \geq 0$ units of resource $r \in \{1, ..., \mathrm{R}\}$ over its duration.

Let $PS = \{i \to j \mid \mathrm{i} \neq \mathrm{j},\ \mathrm{i} \in \{1, ..., N\}, \mathrm{j} \in \{1, ..., N\}\}$ denote the set of predecessor-successor relations. A schedule is network-feasible if satisfies the predecessor-successor relations:

$$S_i + D_i \leq S_j, \text{ if } i \to j \in PS. \tag{12}$$

Let $\Re$ denote the set of network-feasible schedules. For a network feasible schedule $S \subset \Re$, and let $\mathrm{A_t} = \{i \mid S_i \leq t < S_i + D_i\}, \mathrm{t} \in \{1, ..., T\}$ denote the set of active (working) activities in period t and let

$$U_{t\mathrm{r}} = \sum_{i \in A_t} r_{i\mathrm{r}},\ t \in \{1, ..., T\},\ r \in \{1, ..., R\} \tag{13}$$

be the amount of resource r used in period t.

A network-feasible schedule $S \subset \Re$ is resource-feasible if satisfies the resource constraints:

$$U_{tr} \leq R_r,\ t \in \{1, ..., T\},\ r \in \{1, ..., R\}. \tag{14}$$

Let $\overline{\Re} \subseteq \Re$ denote the set of resource-feasible schedules. As we mentioned, the MILP formulation is based on the forbidden (resource constraint violating) set concept.

A forbidden activity set is identified such that: (1) all activities in the set may be executed concurrently, (2) the usage of some resource by these activities exceeds the resource availability, and (3) the set does not contain another forbidden set as a proper subset. A resource conflict can be repaired explicitly by inserting a network feasible precedence relation between two forbidden set members, which will guarantee that not all members of the forbidden set can be executed concurrently. We note, that an inserted explicit conflict repairing relation (as its side effect) may be able to repair one or more other conflicts implicitly, at the same time. Let F denote the number of forbidden sets. Let RR_f denote the set of explicit repairing relations for forbidden set F_f, $f \in \{1,2,...,F\}$.

Let $RR = \left\{ \bigcup_f RR_f \middle| f \in \{1,2,...,F\} \right\}$ denote the set of all the possible repairing relations. In the forbidden set oriented model (see for example Alvarez-Valdés and Tamarit [10]), a resource-feasible schedule is represented by the set of the inserted conflict repairing relations IR. According to the implicit resource constraint handling, in this model the resource-feasibility is not affected by the feasible activity shifts (movements). In the time oriented model, a resource-feasible schedule is represented by the activity starting times. In this model, according to the explicit resource constraint handling, an activity movement may be able to destroy the resource-feasibility.

Let M_p, $p \in \{1,2,3\}$ denote the optimistic, most likely, and pessimistic makespan of a resource-feasible schedule set, respectively. Let triplet $\{D_p^* | \, p \in \{1,2,3\}\}$ denote the set of project's makespans in the optimal resource-feasible schedule set. Let $\overline{T}$ denote an upper bound of the optimal project's makespan in the pessimistic case ($M_3^* \le \overline{T}$). Let $\overline{T} = \sum_{i=1}^{N} D_{3i}$, which is an "extremely weak" upper bound on the project's pessimistic makespan D_3^*, and fix the position of the unique dummy sink in period $\overline{T}+1$. Naturally, this "weak" upper bound can be replaced by any "stronger" one. In our notation, the time periods are labelled by consecutive $t \in \{0,1,...,\overline{T}+1\}$ integers. Note the convention of starting a subproject at the beginning of a time period and finishing it at the end of it. (According to the applied convention, time period one is the first working period.) Let S_i, $\underline{S}_i \le S_i \le \overline{S}_i$ denote the start time of activity, where $\underline{S}_i$ $\left(\overline{S}_i\right)$ denotes the earliest (latest) starting time of subprojects i by setting $D_i = D_{1i}$ for $i \in \{1,...,N\}$. Because preemption is not allowed, the ordered set $S = \{S_1,..., S_N\}$ defines a schedule for the project. Naturally, the latest starting times are varying in the function of $\overline{T}$.

5 Model description

In the presented mixed integer linear programming (MILP) model the total number of zero-one variables is $|RR|$, and the formulation is based on well-known "big-M" constraints. The presented MILP model is a modified and simplified version of the original forbidden set oriented model developed by Alvarez-Valdés and Tamarit [10].

The model results in a robust "makespan minimal" resource constrained schedule which is not affected by the uncertain subproject durations. Here "makespan minimal" means a schedule for which a given linear combination of the optimistic, most likely, and pessimistic resource constrained makespans is minimal. Naturally, the optimal solution will be a function of the weighting coefficients. We will demonstrate by simulation that, according to the "robust nature" of the central limit theorem (CLT), the distribution of the makespan will be nearly normal in the optimal schedule.

Defining the decision variables:

$$Y_{ij} = \begin{cases} 1 & \text{if } i \to j \text{ inserted} \\ 0 & \text{otherwise} \end{cases}, \; i \to j \in RR \tag{15}$$

the following MILP model arises:

$$\sum_{p=1}^{3} W_i * M_i \to \boldsymbol{min} \tag{16}$$

$$\sum_{i \to j \in RR_f} Y_{ij} \geq 1, \quad f \in \{1,...,F\} \tag{17}$$

$$S_i + D_{pi} \leq S_j + \left(\overline{S}_i - \underline{S}_j + D_{pi}\right) * \left(1 - Y_{ij}\right), \; i \to j \in RR, \; p \in \{1,2,3\} \tag{18}$$

$$S_i + D_{pi} \leq S_j, \; i \to j \in NR, \; p \in \{1,2,3\} \tag{19}$$

$$M_3 \leq \overline{T} + 1 \tag{20}$$

$$Y_{ij} \in \{0,1\}, \; i \to j \in RR \tag{21}$$

The objective function (16) minimizes the linear combination of the optimistic, most likely, and pessimistic resource constrained makespans. Constraint set (17) assures the resource feasibility (explicit or implicit correction is required for all resource conflicts, at least one element must be chosen from each conflict repairing set). Constraint sets (18) take into consideration the precedence relations between activities in the function of the inserted repairing relations. Constraint sets (19) take into consideration the original precedence (network) relations between activities also defined by the SPM model. Finally, constraint (20) specifies an upper bound for the pessimistic project makespan. Note, that the optimal solution is a function of W_p, $p \in \{1,2,3\}$ weights. According to the model construction in the optimal schedule every possible activity movement is resource feasible, and schedule is "robust" because it is invariant to the variability of activity durations. In other words, a non-

critical activity movement (a non-critical delay) or longer (but possible) activity duration is unable to destroy the resource feasibility of the schedule.

For the example problem an optimal solution with relative frequency histogram for the makespan is presented in Figure 6, where row values labelled by P are the theoretical probabilities, and row values labelled by F are the relative frequencies estimated by simulation with setting $SampleSize = 10{,}000$. At the running of example $\{W_1, W_2, W_3\} = \{1, 1, 1\}$ weight triplet was used with default CPLEX options.

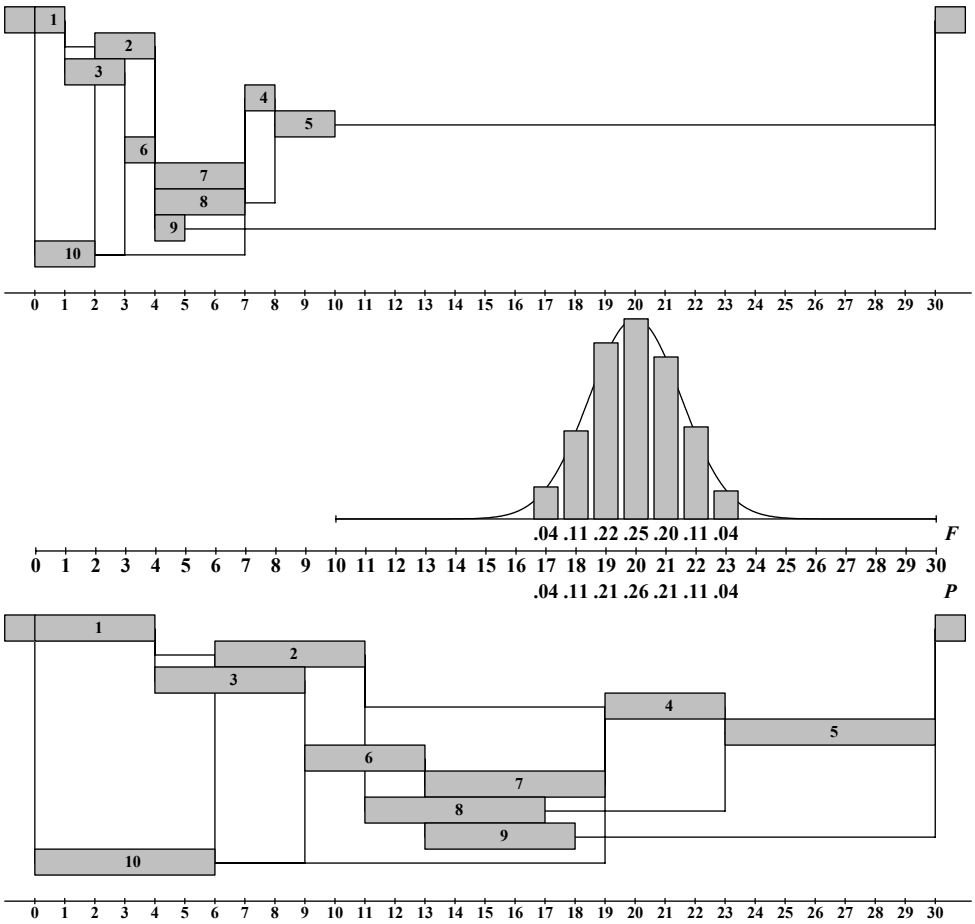

Figure 6: Optimal solution with the relative frequency histogram and the normal density function for the makespan

6 The Algorithm

Harmony search (HS) algorithm was recently developed by Lee and Geem [11] in an analogy with music improvisation process where music players improvise to obtain better harmony. In HS, the optimization problem is specified as follows:

$$\boldsymbol{max}\left\{ f(\boldsymbol{X}) \mid \boldsymbol{X} = \left\{ X_i \mid \underline{X}_i \le X_i \le \overline{X}_i, i \in \{1,2,\ldots,N\} \right\} \right\} \qquad (22)$$

In the language of music, $\boldsymbol{X}$ is a melody, which aesthetic value is represented by $f(\boldsymbol{X})$. Namely, the higher the value $f(\boldsymbol{X})$, the higher the quality of the melody is. In the band, the number of musicians is N, and musician i, $i = \{1,2,\ldots,N\}$ is responsible for sound X_i. The improvisation process is driven by two parameters: (1) According to the repertoire consideration rate (RCR), each musician is choosing a sound from his/here repertoire with probability RCR, or a totally random value with probability $(1-RCR)$; (2) According to the sound adjusting rate (SAR), the sound, selected from his/here repertoire, will be modified with probability SAR. The algorithm starts with a totally random "repertoire upload" phase, after that, the band begins to improvise. During the improvisations, when a new melody is better than the worst in the repertoire, the worst will be replaced by the better one. Naturally, the two most important parameters of HS algorithm are the repertoire size and the number of improvisations. The HS algorithm is an "explicit" one, because it operates directly on the sounds. In the case of RCPSP, we can only define an "implicit" algorithm, and without introducing a "conductor" we can not manage the problem efficiently.

First, we show how the original problem can be transformed into the world of music. Here, the resource profiles form a "polyphonic melody". So, assuming that in every phrase only the "high sounds" are audible, the transformed problem will be the following: find the shortest "Sounds of Silence" melody by improvisation! Naturally, the "high sound" in music is analogous to the overload in scheduling. The "Sounds of Silence" algorithm family was recently developed by Csébfalvi at al. [4, 5]. The presented algorithm, as a new member of the family, a modified version of the original conflict repairing "Sounds of Silence" algorithm, according to the uncertain activity durations. In the adapted harmony search process, the conductor applies a "from shortest to longest" melody generating strategy, which means that the conductor firstly generates an "optimistic" melody, resolves the visible (hidden) resource conflicts using the optimistic durations, after that replaces the shortest durations with the longest ones according to the number of "key melodies". The algorithm exploits the fact, that we can not destroy the resource-feasibility replacing the optimistic durations in a conflict free optimistic solution with longer (for example: most likely or pessimistic) durations.

In the language of music, the RCPSP can be summarized as follows: (1) the band consists of N musicians; (2) the polyphonic melody consists of R phrases and N polyphonic sounds; (3) each $i \in \{1,2,\ldots,N\}$ musician is responsible for exactly one polyphonic sound; (4) each $i \in \{1,2,\ldots,N\}$ polyphonic sound is characterized by the set of the following elements: $\{X_i, D_i, \{R_{ir} \mid r \in \{1,2,\ldots,R\}\}\}$; the polyphonic sounds (musicians) form a partially ordered set according to the precedence (predecessor-successor) relations; (5) each $r \in \{1,2,\ldots,R\}$ phrase is additive for the simultaneous sounds; (6) in each phrase only the high sounds are audible: $\{U_{tr} \mid U_{tr} > R_r, t = 1,2,\ldots,T\}$; (7) in each repertoire uploading (improvisation) step, each $i \in \{1,2,\ldots,N\}$ musician has the right to present (modify) an idea $IP_i \in [-1,+1]$ about X_i where a large positive (negative) value means that the musician want to

enter into the melody as early (late) as possible; (8) in the repertoire uploading phase the "musicians" improvise freely, $IP_i \leftarrow \boldsymbol{RandomGauss}(0,1)$, where function $\eta \leftarrow \boldsymbol{RandomGauss}(\mu,\sigma)$ generates random numbers from a truncated ($-1 \le \eta \le 1$) normal distribution with mean μ and standard deviation σ (9) in the improvisation phase the "freedom of imaginations" is monotonically decreasing step by step, $IP_i \leftarrow \boldsymbol{RandomGauss}(IP_i,\sigma)$, where standard deviation σ is a decreasing function of the progress (see Figure 7-8); (10) each of the possible decisions of the harmony searching process (melody selection and idea-driven melody construction) is the conductor's responsibility; and (11) the band try to find the shortest "Sounds of Silence" melody by improvisation.

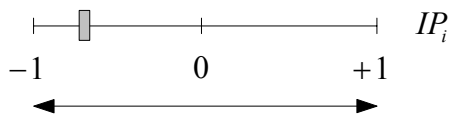

Figure 7: An idea IP_i about the "best" position

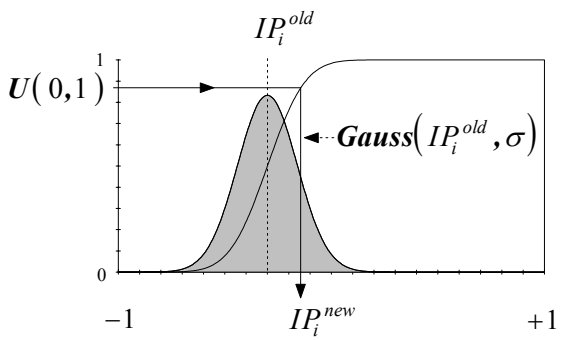

Figure 8: Perturbation of IP_i

The conductor solves a linear programming (LP) problem to balance the effect of the more or less opposite ideas about a shorter "Sounds of Silence" melody. The LP problem, which maximizes the satisfaction of the musicians with the sound positions, is the following:

$$\boldsymbol{min}\left[\sum_{i=1}^{N} IP_i * S_i\right] \tag{23}$$

$$S_i + D_i \le S_j,\ i \to j \in \boldsymbol{PS}, \tag{24}$$

$$\underline{S}_i \le S_i \le \overline{S}_i,\ i \in \{1,2,\ldots,N\} \tag{25}$$

$$D_i = D_{1i},\ i \in \{1,2,\ldots,N\} \tag{26}$$

The result of the optimization is a schedule (melody) which is used by the conductor to define the final starting (entering) order of the sounds (musicians). The conductor generate a soundless melody by taking the selected sounds one by one in the given order and scheduling them at the earliest (latest) feasible start time. After that, using the well-known forward-backward improvement (FBI) methods the conductor tries to improve the quality of the generated melody.

The conflict repairing version of the “Sounds of Silence” algorithm is based on the forbidden set concept. In the conflict repairing version, the primary variables are conflict repairing relations, and a solution will be a makespan minimal resource-feasible solution set, in which every movable activity can be shifted without affecting the resource feasibility.

In the traditional “time oriented” model the primary variables are starting times, therefore an activity shift may be able to destroy the resource feasibility. It is well-known, that the crucial point of a conflict repairing model is the forbidden set computation. In the “Sounds of Silence” algorithm the conductor using a simple (but fast and effective) “thumb rule” to decrease the time requirement of the forbidden set computation. In the forward-backward list scheduling process the conductor (without explicit forbidden set computation) inserts a precedence relation $i \to j$ between an already scheduled activity i and the currently scheduled activity j whenever they are connected without lag.

The result will be schedule without “visible” conflicts. After that, the conductor (in exactly one step) repairs all of the hidden (invisible) conflicts, inserting always the “best” conflict repairing relation for each forbidden set. In this context “best” means a relation $i \to j$ between two forbidden set members for which the lag is maximal. Naturally, the conductor memorizes the most aesthetic soundless melody found so far by computing the duration of the melody according to the key point durations. In the harmony search process, according to the "from optimistic to pessimistic" strategy, the conductor resolves the visible (hidden) resource conflicts using the optimistic durations, after that replaces the optimistic durations with the most likely, and pessimistic ones. The algorithm exploits the fact, that the conductor can not destroy the resource-feasibility, replacing the optimistic durations in a conflict free optimistic solution with longer durations.

After the "best conflict repairing combination" searching phase, the makespan distribution function is generated by simulation. In the simulation phase we replace the membership functions with appropriate density functions (for example: we replace a triangular membership function with a triangular density function and use a triangular random number generator to get duration instances).

In the language of music, the result of the conflict repairing process will be a robust (flexible) “Sounds of Silence” melody, in which the musicians have some freedom to enter to the performance with a shorter or longer sound without affecting the soundless nature of the composition. Note, that the aesthetic value of a melody will be the function of the weights. Easy to imagine, that for a risk-avoider project manager, the shortest pessimistic schedule set may be the most aesthetic melody.

7 Conclusion

In this paper, we have presented a spatial development program for a lagging region in Hungary. The program due to its nature is a unique problem and faces high level of uncertainty and a limited budget comparing to its size. For this we present a metaheuristic for resource-constrained projects with fuzzy activity durations.

To support the model, we have extended it with an effective SPM approach considering expected net present values of subprojects to build effective predecessor relations. The metaheuristic produces “robust” resource-feasible schedules which are totally immune against uncertainties in the activity durations. Theoretically the robust schedule searching process is formulated as a mixed integer linear programming problem which can be solved directly in the case of small-scale projects. The proposed forbidden set oriented metaheuristic, as a new member of the "Sounds of Silence" metaheuristic family, is able to solve larger problem instances within reasonable time. After the best conflict repairing strategy searching phase, the makespan distribution function was generated by simulation. In the simulation phase the membership functions were replaced by the corresponding probability distribution functions. The unified approach presented is able to manage mixed probabilistic-possibilistic-deterministic problems, so it is able to decrease the gap between the theory and the practice.

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