Abstract

Some thirty years ago the computing capability in a design office could not cope with analysing modal dynamics response caused by vibration machines supported on a piled foundation with three-dimensional solid finite elements to represent the soil. As such, the impedance function for the pile-soil-pile interaction was developed over the years for ease of computation. Vibration machines are usually supported by rigid and massive foundations and the appropriate impedance function can be applied for dynamic analysis purposes. This paper describes how an unusual pile foundation, for which the impedance function could not be reliably determined, was modelled with conventional three-dimensional finite elements for the soil and with shell elements for the steel structure. The booster assembly was idealised using beam elements. The mesh was carefully developed and transitioned to minimise the number of solid elements in the model. Eigenvalue extraction and modal harmonic analyses were performed in a reasonable run time using two Intel Xeon X5482 processors. The pile-deck structure was found to behave mainly in a sway mode with some twisting about the vertical. The results enabled engineers to assess the peak response against the performance criteria for the proposed equipment.

Keywords: modal harmonic analysis, pile-soil-pile interaction with three-dimensional finite elements, vibrating machinery foundation.

1 Introduction

Vibration machines are usually supported by foundations that are considered to be rigid and massive. Some typical examples are shown in Figure 1. As a rule-of-thumb, the block type foundation for rotating machine should have a mass, not including the machine, about three times the mass of the machine, and for pile supported foundation the mass, including the pile cap, should be about two and a half times the mass of the machine. Details of foundation design and analysis methodology are given in text book [1] and guideline [2]. The guideline by the
American Concrete Institute (ACI) was comprehensive and engineers can calculate the dynamic response for a range of foundations by the suggested analysis methods – they usually involve an idealised finite element model with the appropriate soil-structure interaction to compute the dynamic response in either the time or frequency domain. Very often the problem is simplified to a single degree of freedom system consisting of a spring-mass-damper model for ease of computation especially when the computing resource is limited to the design engineers. The spring and damping values representing the interaction between the structure and soil can be determined using the elastic half space analogs [3] or the impedance function method [4, 5]. In the case of pile foundation, impedance functions for single pile and pile group can be obtained from the research done by Novak and his co-researchers [6, 7]. They are implemented in DYNA6, a program developed by the Geotechnical Research Centre, University of Western Ontario that computes dynamic response of a range of foundations.

![Figure 1: Examples of rigid foundation types [2]. Top left: Block-type; Top right: Combined block; Bottom left: Table top; Bottom right: Pile-supported](image)

The impedance function for the pile-soil-pile interaction can be readily calculated from established theory or derived from finite element modelling by applying a unit translation/rotation in or about the relevant axes of the rigid block. Gohnert et al [8] derived an equivalent spring system for a raked pile foundation using first principal approach for design pile foundation for vibrating machinery in light of the lack of design methods in the British code, CP2012. However, the method was only applicable to end bearing piles and skin friction was assumed free. Bhatia [9] gave a comprehensive guidance of how finite element method can be applied to different types of machine foundations for design office practices. This includes using continuum elements for soil modelling and suggested how far the boundaries should be in relation to the dimensions of the foundation. However, it does not provide guidance on how to model pile foundation.
Modelling pile-soil-pile interaction and performing modal harmonic analysis using three-dimensional solid finite elements has been perceived to be computationally expensive and time consuming for routine design of this type of foundation. With the advance in computer software and hardware technology, this may no longer be the case. The authors have performed dynamic analysis of pile foundations using block on springs approach when the impedance function can be calculated fairly easily. However, for unusual pile foundations where the impedance cannot be determined or estimated, the three-dimensional finite element approach will be more appropriate. Its feasibility will be demonstrated in a case study in this paper.

2 Analysis Methodology

In general the design of vibrating machine foundation will be such that the dynamic response will be within the elastic stress-strain range, and material non-linearity will not normally be permitted. This paper will not discuss the static analysis and design aspects of pile foundation. For analysing dynamic response, inputs will be required from the equipment supplier, foundation designer and geotechnical engineer. The purpose of the analysis must be defined and confirmed at the beginning of the project. For example, one has to decide whether scenarios such as start-up, machine running at operating speed or emergency shut-down are required for analysis. The design criteria, normally the vibration performance criteria to limit premature failure of machine bearings, will need to be confirmed. If the equipment supplier cannot provide any established limits specific to their equipment, then ISO 10816 [10] will provide some general guidance.

A steady-state linear dynamic analysis should be used to compute the linear response of foundation subjected to continuous harmonic excitation. The modal solution is computationally cheaper than direct-solution in the time domain. Note that it is less accurate than direct-solution if significant material damping is present.

In a modal procedure, the equation of motion of the system projected onto the n-th mode gives:

\[ \ddot{u}_n + c_n \dot{u}_n + \omega_n^2 u_n = \frac{1}{m_n} (f_{1n} + if_{2n})e^{i\Omega t} \]  

where \( u_n \) is the displacement amplitude of mode n in the n-th generalised coordinate
\( \dot{u}_n \) is the velocity amplitude of mode n in the n-th generalised coordinate
\( \ddot{u}_n \) is the acceleration amplitude of mode n in the n-th generalised coordinate
\( c_n \) is the damping associated with mode n
\( \omega_n \) is the undamped frequency of mode n
\( m_n \) is the generalised mass associated with mode n
and \( (f_{1n} + if_{2n})e^{i\Omega t} \) is the forcing in real and imaginary parts associated with mode n.
Once a finite element model of the foundation system is created with the appropriate material properties and boundary conditions, an eigenvalue extraction analysis is performed to extract a number of natural frequencies and associated eigenmodes.

These eigenmodes, normally taken within ±20% of the equipment operating frequency, will be used in the modal harmonic analysis. The unbalance dynamic force, $F_o$, is given by Equation (2) below.

$$F_o = m_r e_m \Omega^2 S_f$$

where $m_r$ is the rotating mass
$e_m$ is the mass eccentricity
$\Omega$ is the circular operating frequency of the equipment
and $S_f$ is the service factor accounting for increased unbalance during the service life of the equipment.

The various balance quality grades (as defined by the product of $e_m$ and $\Omega$) and service factors can be obtained from the ACI report [2].

The modal harmonic analysis will compute for the whole model or report at selected nodal locations the peak response, $u_{peak}$, which can be displacement, velocity and acceleration, from the summation of the modal response between modes $i$ and $j$ as:

$$u_{peak} = \sum_{n=i}^{j} \Phi_n u_n$$

where $\Phi$ is the n-th eigenmode.

A typical analysis procedure will involve running the following steps:

1. Apply gravity – this is optional but it is a good practice for checking static equilibrium and reaction forces for quality assurance purposes.
2. Apply pre-stressing if required – for example, if the structure is being post-tensioned.
3. Eigenvalue extraction analysis or modal analysis – this is to extract the natural frequencies and modes within the specified frequency range of interest.
4. Modal harmonic analysis – applies the appropriate harmonic loads and compute peak responses. When multiple loads are applied and their phase relationship is not known, separate harmonic analyses with the respective loads are performed and the combined peak response can be determined by the root mean square method.

3 Case Study

A proposed rotatory pump/motor assembly (booster unit) was to be mounted on a 2.3m by 2.3m steel square platform, supported by eight H-section (HP310 x 94) steel piles around the perimeter. The purpose of the dynamic analysis was to determine the peak response at the platform bolt mounting locations when the unit
was running at its normal operating speed at 20Hz during the winter season when the top 2.6m (as determined to have a 50 year recurrence interval) of the ground was assumed to be frozen and during the summer season without the frozen soil. Although the platform consisted of a number of I-beams and channel sections welded to the underside of a thick steel plate, it was considered to be relatively flexible compared to a block type concrete foundation. The piles were about 12m in length and were founded on a stiff to hard mudstone. The proposed schematic arrangement is shown in Figure 2.

There was a large diameter (about 1.5m) in-ground central caisson which was founded almost as deep as the pile toes. This caisson was to house the pump barrel vertically so that it was shielded from the harsh weather conditions in this part of the world. This central caisson consisted of double steel casings with the inner closed-end tube connected to the underside of the platform and the outer open-end pipe buried in the ground. The end of the outer pipe was plugged with a slab of reinforced concrete. Sand backfill filled the gap at the base and in the annulus between the outer pipe and the inner tube. The complex arrangement of the central caisson and its close proximity to the piles meant the pile-soil-pile impedance function could not easily be determined from established theory. The connection arrangement of the inner tube, the complex interaction between the outer pipe and the inner tube through the annular sand fill and the interaction between the outer pipe and the embedded soil made it difficult or impossible to idealise the central caisson into an equivalent central pile having the correct stiffness and damping. As such, the whole above ground structure and pile foundation system was analysed in a three-dimensional finite element model. Although the geometric arrangement of the foundation system showed there was a symmetry vertical plane across the diagonal of the square platform that could make some savings in computational efforts, this symmetry was not exploited because there could be anti-symmetric eigenmodes, and rotational harmonic loads about the vertical axis would be applied.

Figure 2: Schematic arrangement of the proposed booster unit foundation. Left: Sectional Elevation. Middle: Typical Pump. Right: Plan View
The proposed booster unit has a dry mass of about 19,000 kg. The rule-of-thumb foundation design suggested that 2.5 times the machine mass was required, that is, about 47,500 kg. The total mass of the proposed steel foundation including the platform deck, piles and caisson was only 15,000 kg, which was below the suggested mass. Some amount of soil would contribute to the foundation mass but it was anticipated to be small. If the frozen ground mass was included, assuming the frozen soil mass within the footprint of the platform would participate in resisting the vibration, the total mass was estimated to be 38,600 kg which was still 19% less than the required mass. It was expected the peak dynamic response for the non-frozen ground would be higher than the frozen ground condition, and furthermore, it would likely to exceed the 2mm/s peak to peak velocity for “Good” machine operation limit, a performance criteria suggested by Baxter and Bernhard [11]. The dynamic analysis using a finite element model would confirm or otherwise the performance expectation.

3.1 Finite Element Model

A three-dimensional finite element model was constructed using the software, ABAQUS. In order to limit the computational efforts, the meshing of the model and the types of element employed were carefully selected. As the dynamic behaviour of the supporting platform was of interest and its welded connection details were well defined, it was finely meshed with 4-node shell elements with the steel material properties assigned to them. Where there was overlapping of plates, the equivalent thickness was assigned to the shell elements. Several isotropic views of the platform are shown in Figure 3. Note that the pile cap plates with the correct mesh transition in the both horizontal directions were also included in the platform. The brackets supporting the inlet and outlet pipes to the booster unit were also included. However, non-structural point mass elements representing the weight of the pipe were applied to the flanges of the pipe support brackets. Figure 3 also shows, for clarity, a portion of the inner tube welded to the underside of the platform plate.

![Platform mesh. Left: Top view. Right: Platform underside view](image-url)
The piles were modelled with 4-node shell elements with the appropriate shell thickness to represent the flange and web. Due to the orientation of the pile caps relative to the H-section piles, the nodes of the regular mesh in the pile cap were not in-line with the nodes at the top of the pile section. That is, there existed a non-matching mesh between the pile and the pile cap. This was overcome by using a tie-contact technique connecting the two node sets together. Since the focus of the dynamic analysis was the peak motion of the platform and not stress response in the individual structural members, any potential for stress discontinuity using this tie-contact technique was not a concern. Figure 4 shows how the piles are connected to the platform via the pile cap plates. The mesh transition helped to reduce the number of three-dimensional finite elements required for the soil below the platform.

Figure 4: Left: An isometric view showing how the piles are connected pile cap plates. Right: A partial cut-out view of the central caisson near the ground surface

The piles were considered to act as both end bearing and floating piles. The pile nodes were fully bonded with the soil nodes. For this type of vibration problem, it was expected the pile deflection would be small and therefore the assumption of no pile-soil separation or slippage was reasonable. Furthermore, some adhesion between the pile and clay could exist allowing some tension to occur in the clay adjacent to the pile. The above behaviour was confirmed from the analysis results.

For the frozen ground condition, the adfreezing stresses on the piles were ignored. It was expected this effect could pre-tension the system thus making the platform structure stiffer which was considered to be less conservative.

The piles and the central caisson were embedded in three layers of soils consisting of granular backfill at the ground surface, a layer of very stiff clay below and a weak mudstone extending to well below the pile toes. The soil layers were modelled by 8-node solid finite elements with full integration scheme. No lateral variation of soil was assumed. The properties used for analysis are summarised in Table 1. Linear elastic isotropic material model was used. For the non-frozen ground condition, it was assumed the piles and the outer pipe would not interact with the backfill materials as well as the first metre of stiff clay below the backfill. It was
reasonable to assume the seasonal freeze-thaw action would cause the near surface stiff clay to break-down thus losing the bonding between the clay and steel structures. In the model, duplicated nodes were used to de-couple the structural nodes and the soil nodes in these surfaces. For the frozen ground condition, fully bonded nodes were used. A separate model was used for this condition. The extent of the soil boundaries is shown in Figure 5. The side boundaries were about four times the width of the platform diagonal plan length and the bottom boundary was about two times the length of the piles. Roller displacement boundary conditions were assigned. Although non-reflecting boundary conditions such as the nodal damping technique [12] could have been used, it was decided it was not necessary because the dynamic solution was not performed in a time domain which did not involve any wave propagation in the model.

![Figure 5: Left: Soil domain geometry model. Top right: Solid element mesh of the soil domain. Bottom right: Close-up view of the foundation mesh](image)

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m³)</th>
<th>Shear Modulus (MPa)</th>
<th>Young’s modulus (MPa)</th>
<th>Poisson’s ratio</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granular backfill (0.8m thick)</td>
<td>1,900</td>
<td>45</td>
<td>122</td>
<td>0.35</td>
<td>0.05</td>
</tr>
<tr>
<td>Stiff clay (between 0.8m and 7m)</td>
<td>2,000</td>
<td>70</td>
<td>196</td>
<td>0.4</td>
<td>0.03</td>
</tr>
<tr>
<td>Mudstone (below 7m)</td>
<td>2,100</td>
<td>130</td>
<td>351</td>
<td>0.35</td>
<td>0.02</td>
</tr>
<tr>
<td>Sand fill in caisson</td>
<td>1,800</td>
<td>-</td>
<td>20</td>
<td>0.3</td>
<td>0.05</td>
</tr>
<tr>
<td>Concrete caisson plug</td>
<td>2,400</td>
<td>-</td>
<td>14,000</td>
<td>0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>Steel</td>
<td>7,850</td>
<td>5,000</td>
<td>13,500</td>
<td>0.35</td>
<td>0.015</td>
</tr>
<tr>
<td>Frozen backfill</td>
<td>1,900</td>
<td>5,000</td>
<td>14,000</td>
<td>0.4</td>
<td>0.015</td>
</tr>
<tr>
<td>Frozen clay</td>
<td>2,000</td>
<td>-</td>
<td>200,000</td>
<td>0.3</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 1: Material properties used in analysis
In regard to the central caisson, the steel inner tube and outer pipe were meshed with 4-node shell elements with the appropriate thickness. The sand fill in the annulus and below the inner tube was modelled with 8-node solid elements. As there could be construction uncertainty and subsequent vibration effect regarding the consistency of the sand fill, a stiffness corresponding to loose sand was assumed. The concrete plug at the base of the outer pipe was also meshed with 8-node solid elements.

The main body of the booster unit was approximated by 2-node beam elements because there was insufficient structural detail to model it using shell elements. Due to the size of the pump barrel and motor housing and the way the unit was mounted to the platform, it was necessary to include the mass and bending stiffness of the booster unit in the model because they could contribute to the inertia and bending behaviour of the system. A series of rigid link elements were used to connect the booster unit to the bolted locations on the platform.

Composite modal damping was applied for the harmonic analysis. This form of damping uses a weighted average of the material damping factors to determine an overall damping factor for each mode.

The entire model was discretised using 44,522 8-node solid elements, 8,549 4-node shell elements and 23 2-node beam elements giving a total of 174,812 degrees of freedom. The global coordinate system (see Figure 5) was defined as follows. The x-y plane is the horizontal plane with the x-axis along the platform diagonal in line with the pipe brackets and the y-axis is along the platform diagonal without the pipe brackets. The z-axis is in the vertical direction.

### 3.2 Eigenvalue Extraction Analysis

The Lanczos method was used to extract 364 eigenvalues and modes for the non-frozen ground condition and 277 eigenvalues and modes for the frozen ground condition. The frequency range was from zero to 27Hz which was adequate in view of the operating frequency of 20Hz. By inspecting the eigenmodes, it can be observed that the soil foundation participated in the excitation although the modes of interest were those involving the platform/piles/central caisson structure. These soil vibration modes would contribute very little in the platform vibration. A plot of the normalised mass is shown in Figure 6 indicating a large number of frequencies of which the majority were due to the vibration of the soil. The lowest fundamental frequency of 3.58Hz was well below the operating frequency of the booster unit. However, there were a number of natural frequencies occurring near the operating frequency.

Figure 7 shows a few eigenmodes mainly due to soil excitation. It should be noted that these modes had insignificant influence on the dynamic response of the platform. These modes could have been eliminated by assuming a massless soil domain – a technique routinely employed for seismic analysis of concrete dams founded on rock. In this case study, the soil mass was included in order to observe the soil dynamic behaviour.
Figure 6: Normalised mass indicates the significant eigenvalues in the system.

Figure 7: Soil excitation modes. Top left: 19.879Hz. Top right: 19.938Hz. Bottom left: 20.016Hz. Bottom right: 20.270Hz.
The fundamental modes of the platform are shown in Figure 8. The dominant excitations were swaying of the platform, twisting about the central caisson, and translation in the vertical direction. The frozen ground condition for the similar eigenmode occurred at a higher frequency because of the higher soil stiffness and the shorter free pile length. It was expected more mode shapes could exist but they would be beyond the upper frequency limit specified in the analysis.

![Figure 8: Significant eigenmodes. Left: Sway mode at 17.622Hz. Middle: Twisting mode at 17.999Hz. Right: Vertical translation mode at 21.332Hz](image)

### 3.3 Modal Harmonic Analysis

The eigenmodes between 16 Hz and 24 Hz, which was within 20% of the operating frequency, were included in the modal harmonic analysis. This frequency range allowed for any potential variation between the natural frequencies of the model and the constructed foundation. The machine unbalance dynamic force under normal operating condition was estimated from the manufacturer specifications. There were two main sources of dynamic force: one due to the rotating pump impellers and the other due to the rotor. It was found the unbalance dynamic force was more significant for the rotor than the impellers. However, both sets of forces were applied in the model as in-phase excitation. This was considered to be a conservative assumption. These cyclic loads, rotating about the vertical axis, were defined at the relevant beam element nodes at the correct elevations.
The peak velocity at the platform as function of forcing frequency for both ground conditions is shown in Figure 9. Also plotted on the figure are several of the vibration severity limits suggested by Baxter and Bernhard [11]. The frozen ground condition gave a much lower peak velocity compared to the non-frozen ground as expected. For the non-frozen ground condition, there were two peak responses on either side of the operating frequency. At the peak response the platform was undergoing sway motion with some twisting about the vertical axis. The peak velocity was just below the “good” limit. Note that the results were for a service factor of one. However, they can be scaled linearly according to the required service factor for detailed performance assessment. A steel structure can tolerate very high peak velocity - of the order of 90mm/s for this case. However, it was the equipment tolerance that had a much tighter velocity limit in order to safeguard the long-term performance within the specified design life of the machine.

The mobilised soil strains for the non-frozen ground condition are shown in Figure 10. It can be seen that only a small amount of soil adjacent to the outer pipe and around the piles was mobilised at this peak frequency. It confined to just below the soil/structure separation zone. The computed shear strains were very small, in the order of 10^{-6}, and not enough to cause any dynamic degradation of the soil shear modulus. Thus for this particular problem, the soil stiffness, and probably the soil damping, was independent of frequency.

![Figure 9: Peak response as function of forcing frequency](image-url)
3.4 Computation Performance

The analysis was performed in two Intel Xeon X5482 processors (that is, eight cores at 3.2GHz clock speed with 32 GB RAM memory). The eigenvalue extraction and modal harmonic analyses took 1,768 CPU seconds or about 29.5 minutes to complete. This fast elapsed time meant a large number of cases could be analysed within a reasonable time frame. Furthermore, it is feasible to carry out mesh convergence and boundary effect studies. However, they were not performed due to project budget constraint.

4 Conclusions and Future Research

It was demonstrated by analysing this unusual pile foundation arrangement that it was feasible to use conventional three-dimensional finite elements to model the dynamic behaviour of the pile-soil-pile system without simplifying the problem using the impedance function approach. Attention to the meshing details and using the appropriate tie-contact technique meant the number of solid elements representing the soil foundation could be controlled so that the analysis could be performed within a reasonable time frame. This dispelled the notion about excessive analysis run time held almost thirty years ago [13] when computing resources for dynamic finite element analysis of soil-structure interaction was at its infancy. Nevertheless, in the authors’ opinion, dynamic analysis of pile foundation based on the impedance function should still be the preferred method if the foundation in question warrants the application of this method.

In the case study it was found that this pile foundation exhibited a number of significant mode shapes with their frequencies close to the operating frequency of
the booster unit. The peak dynamic response at the platform was significantly lower for the frozen ground condition than for the non-frozen ground condition. The computed result could be used by the design engineers for performance assessment purposes.

Further research using this model may be carried out on a mesh convergence study, and how the extent of the boundaries and the element types affect the dynamic behaviour. A sensitivity study using different soil shear moduli and damping values may also be performed.

References