Paper 138

Proceedings of the Eighth International Conference
on Engineering Computational Technology,
B.H.V. Topping, (Editor),
Civil-Comp Press, Stirlingshire, Scotland

Vehicle Platoon Control Based on a Multi-Leader Vehicle Following Model

E. Kita, H. Shimizu, S. Nishiyama and Y. Wakita
Graduate School of Information Sciences
Nagoya University, Japan

Abstract

Grouping vehicles into platoons is a method of increasing the capacity of roads. Platoons decrease the distances between vehicles using electronic, and possibly mechanical, coupling. The automated highway system is a proposal for one such system, where cars organize themselves into platoons.

In this study, the Chandler-type multi-leader vehicle following model is used for the velocity control of five vehicles platoon. In this case, each follower vehicle has one, two, three or four leader vehicles. The Chandler-type multi-leader vehicle following model controls the velocity according to the velocity differences from each vehicle to all leader vehicles. Stability analysis of the models indicates that maximum total sensitivity to leader vehicles depends only on the platoon lead vehicle and the nearest leader vehicle. The traffic simulation reveals the model effectiveness.

Keywords: platoon, Chandler Model, multi-leader vehicle following model, stability analysis.

1 Introduction

Grouping vehicles into platoons is a method of increasing the capacity of roads. In this study, we will discuss the velocity control of the vehicles in the platoon.

Many researchers have presented the control algorithm of vehicle behavior in the platoon group[1, 2, 3]. In their studies, the vehicle velocity is controlled according to the information from the nearest lead vehicle alone. On the other hand, the present algorithm controls the vehicle velocity according to the information from the multiple leader vehicles[4, 5, 6, 7]. In this study, the velocity is controlled according to the multi-leader extension of the Chandler-type vehicle following model[4]. In the Chandler model, the vehicle acceleration rate depends on the velocity difference between

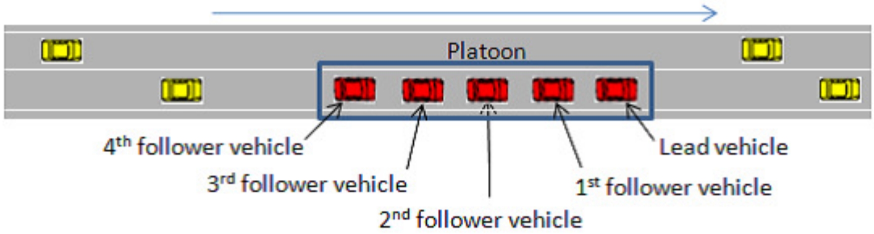

Figure 1: Vehicle platoon

the vehicle and its nearest leader vehicle.

In this study, Chandler-type multi-leader vehicle following model is applied for the velocity control of the five-vehicle platoon. When follower vehicles have multiple leader vehicles, the processing of information from all leader vehicles is very complicated. Therefore, the stability analysis of the vehicle following model and the maximization of the sensitivities for the leader vehicles determine adequate and smaller input data set. The effectiveness of the parameter sets is discussed in the traffic flow simulation.

The remaining of this paper is organized as follows. The vehicle platoon group and general formulation of the stability analysis are explained in the section 2. Stability analysis of the follower vehicles is explained in section 3. Numerical results are shown in section 4. Finally, the conclusions are summarized in section 5.

2 Vehicle Platoon and Stability Analysis

2.1 Vehicle Following Model

In this study, the platoon of five vehicles is considered as an example. The first vehicle is named as the lead vehicle of the platoon and the other vehicles are as first, second, third and fourth follower vehicles (Fig.1).

Chandler-type multi-leader vehicle following model is defined as follows:

$$\ddot{x}_n(t+\Delta t) = \sum_{j=1}^{m} a_j(\dot{x}_{n-j}(t) - \dot{x}_n(t)) \tag{1}$$

where the notation $x_n(t)$ denotes the nth vehicle position at the time t and the notation $a_j > 0$ denotes the nth vehicle driver's sensitivity to the $(n-j)$th vehicle. The parameter m is the number of the leader vehicles for the nth vehicle. The upper dot $(\dot{})$ and $(\ddot{})$ denote the first- and second-derivatives with respect to the time, respectively.

In this model, the vehicle acceleration rate $\ddot{x}_n$ is controlled according to the velocity difference between the vehicle and the leader vehicle; $\dot{x}_{n-j}(t) - \dot{x}_n(t)$. Chandler model[4] takes $m = 1$ and Bexelius model[8] and Wakita model[9] take $m > 1$.

2.2 Stability Analysis

In equation (1), we will consider as the stable state that all vehicles move at the same velocity v_0.

Let y_n be a small deviation from the steady state velocity;

$$\dot{x}_n = v_0 + y_n \tag{2}$$

Substituting equation (2) to equation (1), we have

$$\dot{y}_n(t + \Delta t) = \sum_{j=1}^{m} a_j (y_{n-j}(t) - y_n(t)). \tag{3}$$

Fourier series of y_n is given as

$$\begin{aligned} y_k(n,t) &= \exp(i\alpha_k n + zt) \\ \alpha_k &= \frac{2\pi}{N}k, \quad k = 0, 1, 2, \ldots, N-1, \end{aligned} \tag{4}$$

where N and i denote total number of vehicles and the imaginary unit, respectively.

From equations (3) and (4), we have

$$e^{\Delta t z} z - \sum_{j=1}^{m} a_j (e^{j i \alpha_k} - 1) = 0. \tag{5}$$

Applying Taylor series expansion $(e^{\Delta t z} \cong 1 + \Delta t z)$ to equation (5), we have

$$\Delta t z^2 + z - \sum_{j=1}^{m} a_j (e^{j i \alpha_k} - 1) = 0. \tag{6}$$

On a critical curve, the real part of the imaginary number z is zero and therefore, $z = iv$. Substituting it to equation (6), we have

$$\Delta t = \frac{\sigma_c}{\sigma_s^2} \tag{7}$$

where

$$\sigma_c = \sum_{j=1}^{m} a_j \{1 - \cos(j\alpha_k)\} \tag{8}$$

$$\sigma_s = \sum_{j=1}^{m} a_j \sin(j\alpha_k) \tag{9}$$

Figure 2: First follower vehicle in platoon

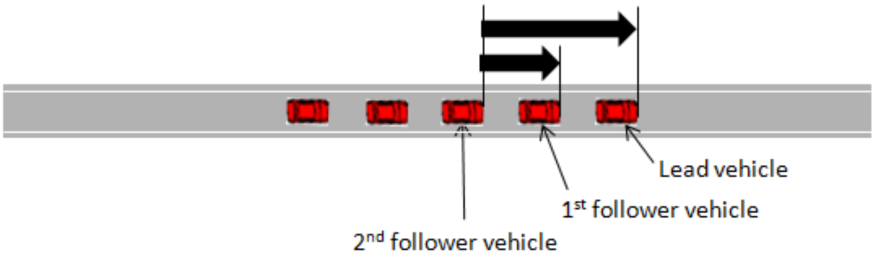

Figure 3: Second follower vehicle in platoon

3 Stability Analysis for Follower Vehicle

3.1 First Follower Vehicle

The first follower vehicle is the vehicle just behind the lead vehicle (Fig.2). It has the lead vehicle as only one leader vehicle.

Substituting $m = 1$ to equation (7), we have

$$\Delta t = \frac{1}{2a_1 \cos^2(\alpha_k/2)}. \tag{10}$$

Substituting $\alpha_k = 0$ to equation (10), we have the stability condition

$$a_1 \leq \frac{1}{2\Delta t}. \tag{11}$$

Taking $\Delta t = 1$ at the above equation, we have

$$0 \leq a_1 \leq \frac{1}{2}. \tag{12}$$

This result shows that, in this case, the maximum sensitivity with respect to one leader vehicle is $b_1 = a_1 = 0.5$.

3.2 Second Follower Vehicle

The second follower vehicle has two leader vehicles (Fig.3). Substituting $m = 2$ to equation (7), we have

$$\Delta t = \frac{a_1 + 2a_2(1 + \cos\alpha_k)}{2(a_1 + 2a_2 \cos\alpha_k)^2 \cos^2(\alpha_k/2)}. \tag{13}$$

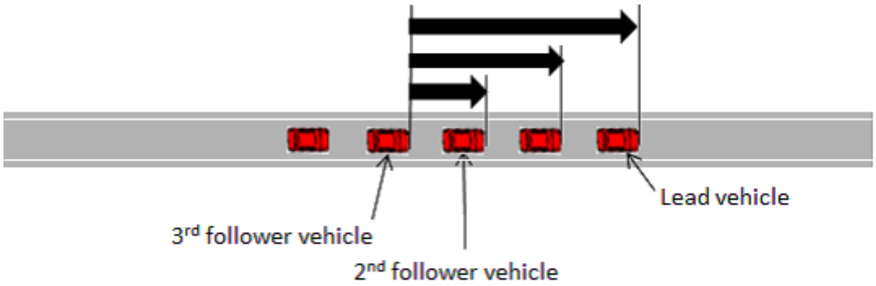

Figure 4: Third follower vehicle in platoon

Substituting $\alpha_k = 0$ to the equation (13), we have the stability condition

$$\frac{(a_1 + 2a_2)^2}{a_1 + 4a_2} \leq \frac{1}{2\Delta t}. \tag{14}$$

Taking $\Delta t = 1$ at the above equation, we have

$$8a_2^2 + 4(2a_1 - 1)a_2 + (2a_1^2 - a_1) \leq 0. \tag{15}$$

Since the sensitivities a_1 and a_2 are real numbers and $a_1, a_2 \geq 0$, the following conditions should be satisfied.

$$0 \leq a_1 \leq \frac{1}{2} \tag{16}$$

The sensitivity a_2 is calculated from equation (15) as follows.

$$0 \leq a_2 \leq \frac{(1 - 2a_1) + \sqrt{1 - 2a_1}}{4} \tag{17}$$

The driver's total sensitivity is defined as follows.

$$b_2 \equiv a_1 + a_2 \leq \frac{(1 + 2a_1) + \sqrt{1 - 2a_1}}{4}. \tag{18}$$

The total sensitivity b_2 takes the maximum value $(b_2)_{\max} = 9/16$ at $a_1 = 3/8$ and $a_2 = 3/16$.

3.3 Third Follower Vehicle

The third follower vehicle has three leader vehicles (Fig.4). Substituting $m = 3$ to equation (7), we have

$$\Delta t = \frac{a_1 + 2a_2(1 + \cos\alpha_k) + a_3(4\cos^2\alpha_k + 4\cos\alpha_k + 1)}{2(a_1 + 2a_2\cos\alpha_k + a_3(4\cos^2\alpha_k - 1))^2\cos^2(\alpha_k/2)}. \tag{19}$$

Substituting $\alpha_k = 0$ to the above equation and taking $\Delta t = 1$, we have

$$18a_3^2 + (12a_1 + 24a_2 - 9)a_3 + (2a_1^2 + 8a_2^2 + 8a_1a_2 - a_1 - 4a_2) \leq 0. \tag{20}$$

Since $a_1, a_2, a_3 \geq 0$, the following conditions should be satisfied.

$$0 \leq a_1 + a_2 \equiv b_2 \leq \frac{9}{16} \tag{21}$$

$$0 \leq a_3 \leq \frac{(3 - 4a_1 - 8a_2) + \sqrt{9 - 16(a_1 + a_2)}}{12} \tag{22}$$

The driver's total sensitivity is defined as follows.

$$b_3 \equiv a_1 + a_2 + a_3 \leq \frac{(3 + 8a_1 + 4a_2) + \sqrt{9 - 16(a_1 + a_2)}}{12} \tag{23}$$

The total sensitivity b_3 takes the maximum value $(b_3)_{\max} = 2/3$ at $a_1 = 1/2$, $a_2 = 0$ and $a_3 = 1/6$.

3.4 Fourth Follower Vehicle

The fourth follower vehicle has four leader vehicles (Fig.5). Substituting $m = 4$ to equation (7), we have

$$\Delta t = \frac{\begin{array}{r} a_1 + 2a_2(1 + \cos\alpha_k) + a_3(4\cos^2\alpha_k + 4\cos\alpha_k + 1) \\ +4a_4(1 + \cos\alpha_k)(1 + \cos 2\alpha_k) \end{array}}{\begin{array}{l} 2\{a_1 + 2a_2\cos\alpha_k + a_3(4\cos^2\alpha_k - 1) \\ \quad +4a_4\cos\alpha_k\cos 2\alpha_k\}^2\cos^2(\alpha_k/2) \end{array}}. \tag{24}$$

Substituting $\alpha_k = 0$ to the above equation and taking $\Delta t = 1$, we have

$$\begin{aligned} 32a_4^2 + (16a_1 + 32a_2 + 48a_3 - 16)a_4 + 2a_1^2 + 8a_2^2 + 18a_3^2 & \\ +8a_1a_2 + 12a_1a_3 + 24a_2a_3 - a_1 - 4a_2 - 9a_3 &\leq 0. \end{aligned} \tag{25}$$

Since $a_1, a_2, a_3, a_4 \geq 0$, the following conditions should be satisfied.

$$0 \leq 3a_1 + 4a_2 + 3a_3 \leq 2 \tag{26}$$

$$0 \leq a_4 \leq \frac{2(1 - a_1 - 2a_2 - 3a_3) + \sqrt{2(2 - 3a_1 - 4a_2 - 3a_3)}}{8} \tag{27}$$

The driver's total sensitivity should be defined as follows.

$$\begin{aligned} b_4 &\equiv a_1 + a_2 + a_3 + a_4 \\ &\leq \frac{2(1 + 3a_1 + 2a_2 + a_3) + \sqrt{2(2 - 3a_1 - 4a_2 - 3a_3)}}{8} \end{aligned} \tag{28}$$

The sensitivity a_4 takes the maximum $(a_4)_{\max} = 1/2$ at $a_1 = a_2 = a_3 = 0$. The total sensitivity b_4 takes the maximum value $(b_4)_{\max} = 3/4$ at $a_1 = 1/2$, $a_2 = 0$, $a_3 = 0$, and $a_4 = 1/4$.

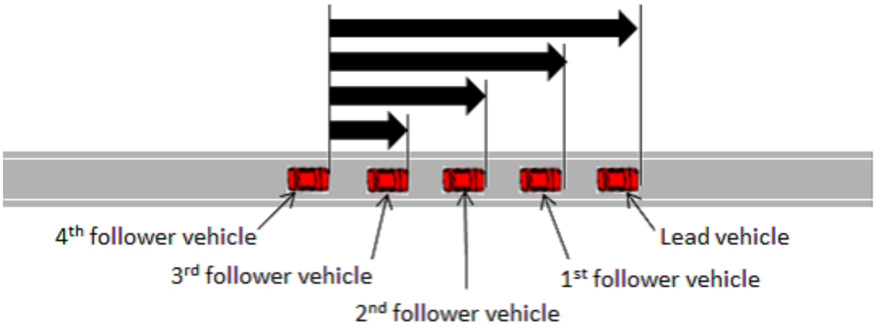

Figure 5: Fourth follower vehicle in platoon

4 Numerical Examples

Traffic flow simulations are performed according to the single- and multi-leader vehicle following models. The results are compared in order to discuss the effectiveness of the multi-leader vehicle following model.

4.1 Models

Model 1

The model 1 is the single-leader vehicle following model. Each vehicle changes its acceleration rate according to the velocity difference between the vehicle and the nearest leader vehicle alone. The acceleration rates for the first, second, third and fourth follower vehicles are given as follows.

$$\ddot{x}_{1st}(t+\Delta t) = a_{1st\to lead}(\dot{x}_{lead}(t) - \dot{x}_{1st}(t)) \tag{29}$$
$$\ddot{x}_{2nd}(t+\Delta t) = a_{2nd\to 1st}(\dot{x}_{1st}(t) - \dot{x}_{2nd}(t)) \tag{30}$$
$$\ddot{x}_{3rd}(t+\Delta t) = a_{3rd\to 2nd}(\dot{x}_{2nd}(t) - \dot{x}_{3rd}(t)) \tag{31}$$
$$\ddot{x}_{4th}(t+\Delta t) = a_{4th\to 3rd}(\dot{x}_{3rd}(t) - \dot{x}_{4th}(t)) \tag{32}$$

where the parameter $x_{lead}, x_{1st}, x_{2nd}, x_{3rd}$ and x_{4th} denote the positions of the lead, first, second, third and fourth follower vehicles, respectively. The parameter $a_{A\to B}$ denotes the sensitivity from the vehicle A to the vehicle B. In this case, the vehicles follow the single vehicle following model. According to the section 3.1, the sensitivity is specified as $a_{A\to B} = 1/2$.

Model 2

The model 2 is the multi-leader vehicle following model. The sensitivities are given according to the results in section 3. The acceleration rates for the first, second, third

Vehicle	Sensitivities
1st	$a_{1st\to lead} = \frac{1}{2}$
2nd	$a_{2nd\to 1st} = \frac{3}{8}$, $a_{2nd\to lead} = \frac{3}{16}$
3rd	$a_{3rd\to 2nd} = \frac{1}{2}$, $a_{3rd\to lead} = \frac{1}{6}$
4th	$a_{4th\to 3rd} = \frac{1}{2}$, $a_{4th\to lead} = \frac{1}{4}$

Table 1: Sensitivities of model 2

and fourth follower vehicles are given as follows.

$$\ddot{x}_{1st}(t+\Delta t) = a_{1st\to lead}(\dot{x}_{lead}(t) - \dot{x}_{1st}(t)) \tag{33}$$

$$\ddot{x}_{2nd}(t+\Delta t) = a_{2nd\to lead}(\dot{x}_{lead}(t) - \dot{x}_{2nd}(t)) + a_{2nd\to 1st}(\dot{x}_{1st}(t) - \dot{x}_{2nd}(t)) \tag{34}$$

$$\ddot{x}_{3rd}(t+\Delta t) = a_{3rd\to lead}(\dot{x}_{lead}(t) - \dot{x}_{3rd}(t)) + a_{3rd\to 2nd}(\dot{x}_{2nd}(t) - \dot{x}_{3rd}(t)) \tag{35}$$

$$\ddot{x}_{4th}(t+\Delta t) = a_{4th\to lead}(\dot{x}_{lead}(t) - \dot{x}_{4th}(t)) + a_{4th\to 3rd}(\dot{x}_{3rd}(t) - \dot{x}_{4th}(t)) \tag{36}$$

The sensitivities are listed in Table 1.

4.2 Simulation Results

A traffic flow on a one-way road is simulated with Chandler-type multi-leader vehicle following model. The delay time is taken as $\Delta t = 1$. Initial velocities for all vehicles are 50(m/s). The lead vehicle velocity decreases suddenly to 45 (m/s) at time 1(s) and then, increases again to 50 (m/s) by the acceleration rate 2.5(m/s^2). The lead vehicle velocity is given as follows.

$$\dot{x}_{lead} = \begin{cases} 50 & (0 \le t < 1) \\ 45 + 2.5(t-1) & (1 \le t < 3) \\ 50 & (3 \le t) \end{cases} \tag{37}$$

Simulation results are shown in Figs.6 and 7, respectively. The figures are plotted with the time as the horizontal axis and the vehicle velocity as the vertical axis, respectively.

In all cases, the maximum velocity reduction is observed at the first follower vehicle and decreases gradually from the first to the fourth follower vehicles. The velocity histories in Figs.6 and 7, however, are very different.

In the model 1, as shown in Fig.6, The follower vehicle velocity reduction occurs at some time interval because the velocity is changed according to the velocity of the nearest leader vehicle alone.

In the model 2, the velocity is changed according to the velocities of both the lead and the nearest leader vehicles. The following vehicles notice the velocity reduction of the lead vehicle sooner than them in the model 1. As shown in Fig.7, therefore, the follower vehicle velocity reduction occurs much sooner than the model 1.

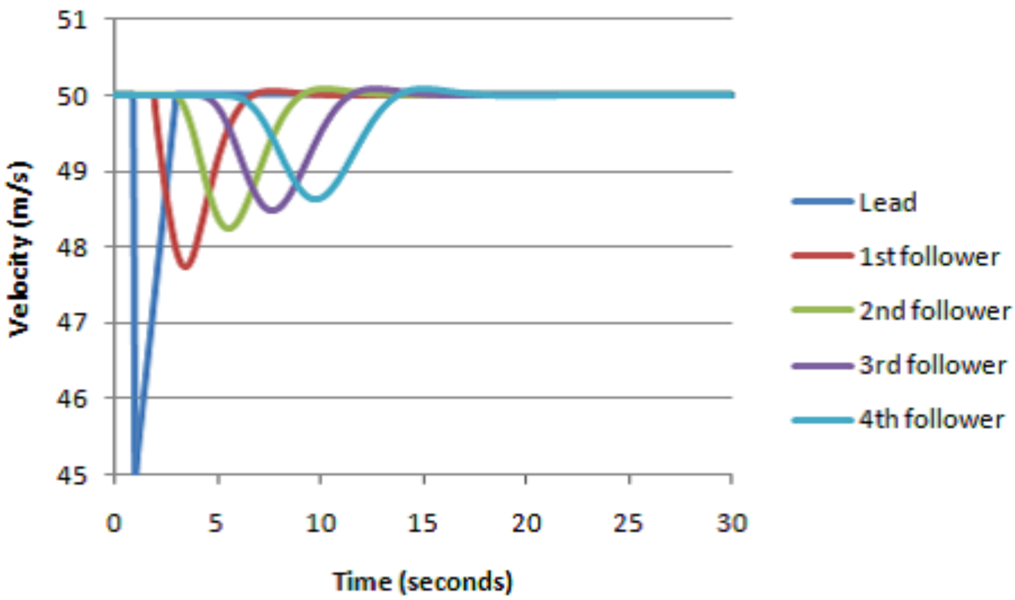

Figure 6: Velocity history in simulation 1 by model 1

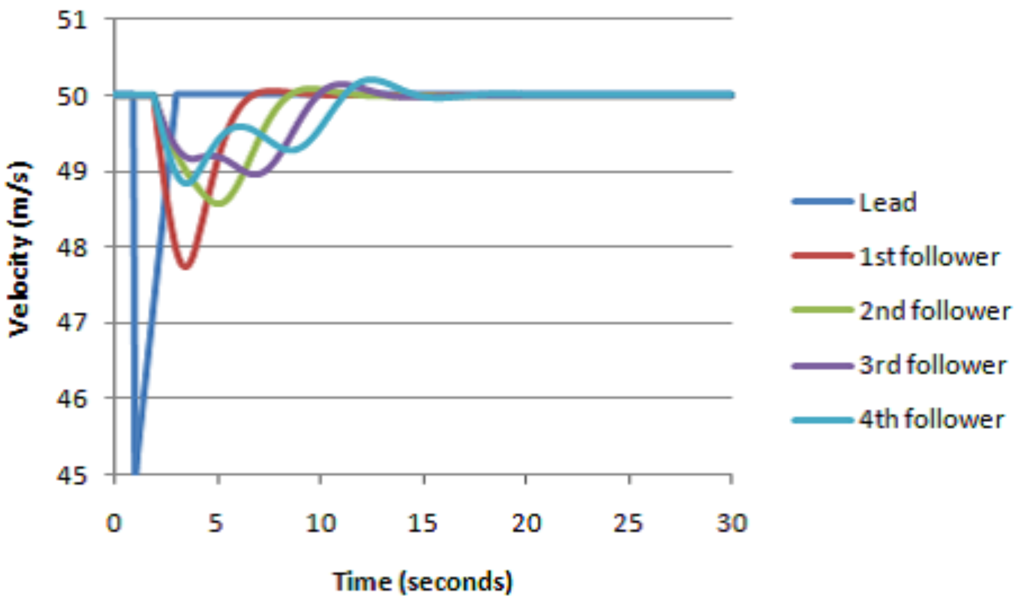

Figure 7: Velocity history in simulation 1 by model 2

5 Conclusion

In the case of the five-vehicle platoon, the mathematical model of the vehicle velocity control was presented in this study. The velocity control model was defined as the extension of the Chandler-type vehicle following model, in which the vehicle acceleration rate is defined as the product of the driver's sensitivity and the velocity difference between the vehicle and its nearest leader vehicle.

The stability analysis of the control model showed that the maximum total sensitivities depend on the velocity information of only two vehicles; i.e., the platoon lead vehicle and the nearest leader vehicle. The simulation results according to the single- and multi-leader vehicle following model were compared. The results showed that the velocity fluctuation of the follower vehicles can be saved if they move according to the multi-leader vehicle following model.

References

[1] A. Galstyan and K. Lerman. A stochastic model of platoon formation in traffic flow. In *Proceedings of the TASK workshop, Santa Fe, NM*, 2001.

[2] Yi Jiang, Shuo Li, and Daniel E. Shamo. A platoon-based traffic signal timing algorithm for major-minor intersection types. *Transportation Research Part B: Methodological*, 40(7):543 – 562, 2006.

[3] Manabu Omae, Takeki Ogitsu, Noritsugu Honma, and Keisuke Usami. Automatic driving control for passing through intersection without stopping. *International Journal of Intelligent Transportation Systems Research*, 8:201–210, 2010.

[4] R. E. Chandler, R. Herman, and E. W. Montroll. Traffic dynamicsGstudies in car-following. *Operations Research*, 6(2):165–184, 1958.

[5] G. F. Newell. Nonlinear effects in the dynamics of car following. *Operations Research*, 9(2):209–229, 1961.

[6] R. L. Bierley. Investigation of an intervehicle spacing display. *Highway Research Record*, (25):58–75, 1963.

[7] M. Bando, K. Hasebe, K. Nakanishi, A. Nakayama, A. Shibata, and Y. Sugiyama. Phenomenological study of dynamical model of traffic flow. *Journal of Physics I France*, 5:1389–1399, 1995.

[8] S. Bexelius. An extended model for car-following. *Transportation Research*, 2(1):13–21, 1968.

[9] Y. Wakita, T. Iguchi, H. Shimizu, T. Tamaki, and E. Kita. Comparison of zipper and non-zipper merging patterns near merging point of roads. *International Journal of Natural Computing Research*, 1(3):19–29, 2010.