# The Reinforcement of Multiple-Leaves Masonry Walls using Carbon Fibre Reinforced Polymer: <br> A Theoretical Approach 

L. Anania, C. Giaquinta and G. D'Agata<br>Department of Civil and Enviromental Engineering Faculty of Engineering, University of Catania, Italy


#### Abstract

This paper reports the results obtained by a numerical analysis carried out on multileaf stone masonry very often used in ancient masonry walls, characterized by two or more rigid outer leaves, made of calcareous rock blocks and mortar joints, and a very poor inner leaf of mortar and stone aggregate. First the numerical model was calibrated in order to determine the maximum load that this kind of multi leaf masonry can bear in the presence of eccentric loads by taking into account both the buckling phenomena and the mechanical behaviour of the non-tensile strength material. The enhancement of the global behaviour of the multi-leaf stone masonry reinforced by technology based on the use of C-FRP was assessed with respect to both the unreinforced model and the other technique consisting in the inserting some transversal masonry bricks working as a tie. In the presence of reinforcement, the original scheme of a beam supported at the ends, which has been assumed for the wall, is modified. The proposed reinforcement technology consists in inserting some transversal C-FRP bars through the thickness of the wall. The bar anchorage was performed by stripping the bar extremities of a C-FRP sheet. The obtained C-FRP ribbon was glued on the masonry surface with a CFRP layer applied all along the wall length. The proposed technique provides an enhancement of the global resistance more than three times greater than the original model and one and half times greater than the traditional techniques.


Keywords: structural modelling, composite materials and structures, buckling load, carbon fibre reinforced polymer, seismic retrofit, masonry.

## 1 Introduction

Nowadays, more and more attention is given to the historical or ancient masonry buildings, therefore, seems to be very essential to be able to know the stress in play in the existing structures, as well as to identify the possible and most effectiveness
intervention choice for their strengthening and retrofitting. References in literature are rather scarce on this topic. One of the very common types of masonry used in many European historical center, such as in Italy, is the multi leaf stone masonry walls, in the specific case consisting of three leaves made up of two outer shells and a thick inner core of rubble material. Their behavior is deeply influenced by a wide range of factors, such as the mechanical properties of the leaves, the leaves' dimensions and the way the leaves are connected to each other. Actually, in the last decades we have witnessed the collapse or sometimes the damage of very important monumental buildings due to high compressive loading in multiple-leaf pillars and walls. The causes of the main structural problems of those walls result from the poor or absent connection between the leaves and the weakness of the inner core. Their collapse occurs because of the buckling of the outer leaves and the eccentricity of the axial force due to the inner layer pressure. The present paper illustrates a numerical approach to evaluate the effectiveness of two different repair techniques all based on the inserting of transversal tying through the thickness of the wall; but one of them consisting in inserting CFRP bar through the thickness.

## 2 The limit load evaluation for non-reinforced generic masonry panel

The behavior of structural elements subjected to compression is affected by the slenderness of the element as the collapse may be preceded by a buckling phenomena. In the masonry walls that may occur in the transverse direction where the slenderness of the wall can take such values as not to allow it to coincide the deformed configuration with the un-deformed one. The effects of geometric nonlinearity must be considered each time that the deformations due to the eccentric axial stress are of the same order of magnitude as the eccentricity. In the following study the Sahlin [1] approach was adopted and developed in order to be applied to a masonry lot, considering the instability effects produced by the material of the inner leaf (soil or masonry with stone) on outer elements.
It analyzes the behavior of a transverse strip for masonry having dimensions " 1 "," H ", and " s ", schematized as a cantilever beam loaded on the free extremity by an axial force N having transversal eccentricity " e "; the material is assumed linear elastic with brittle fracture. A local system of curvilinear abscissa along the $\zeta$ deformed axis, $\delta(\zeta)$ represents the eccentricity that the axial force " N " has in the section of abscissa $\zeta$ respect to deformed axis, while "D" represents the maximum value of the eccentricity at the base section (origin of the abscissa).
By assuming the following non-dimensional parameters:

$$
\begin{equation*}
m=\frac{6 \cdot e}{s} ; \bar{D}=\frac{3 \cdot D}{s}-\frac{1}{2} \tag{1}
\end{equation*}
$$

in relation to the value assumed by $\delta(\zeta)$ that is greater or less than $\mathrm{s} / 6$, the section corresponding to the portion $\zeta$ will be zoned or not, in any case cannot be overcome s/2.


Figure 1. Symbols schematization

| $\mathrm{m}=0$ | $e=0$ | N is centered |
| :--- | :--- | :--- |
| $\mathrm{m}=1$ | $e=\frac{s}{6}$ | N is on the border of the inertia core |
| $\mathrm{m}=3$ | $e=\frac{s}{2}$ | N at the top border of the section |
| $\bar{D}=\frac{m-1}{2}$ | $D=e$ | Equal eccentricità on the superior and <br> lower sections, that is the panel is <br> undeformed |
| $\bar{D}=0$ | $D=\frac{s}{6}$ | N is on the border of the central <br> inertia core |
| $\bar{D}=1$ | $D=\frac{s}{2}$ | N on the inferior border of the section |

Table 1.Limit value of $(m)$ and $(\bar{D})$

The panel in relation to the fact that the sections are all entirely reacting, completely choked or only partly reacting, will present different behaviours [2]. But in all cases the solution can be written as (2):

$$
\begin{equation*}
\alpha \cdot H=f(m, \bar{D}) \tag{2}
\end{equation*}
$$

where:
$\alpha \cdot=\sqrt{\frac{N}{E \cdot I}} \quad \mathrm{E}=$ Elastic modulus; $\quad \mathrm{I}=$ Inertia modulus
In the case of all transversal section of masonry panel entirely reacting, $0 \leq m \leq 1$ and $\frac{m-1}{2} \leq \bar{D} \leq 0$, then the equation (2) becomes:

$$
\begin{equation*}
\alpha \cdot H=\arccos \left(\frac{m}{1+2 \cdot \bar{D}}\right) \tag{4}
\end{equation*}
$$

In the case that the axial force applied to the element has eccentricity, on the top section, inside the central core of inertia, and external at the base section, the transversal section of masonry panel partially reacts. Thus, $0 \leq m \leq 1$ and $0<\bar{D}<1$, the equation (2) remains equal only if $H^{*} \leq \zeta \leq H$, where $\zeta=H^{*}$ is the abscissa value representing the crossing point between the section entirely reacting and section partially reacting. Otherwise, if $0 \leq \zeta \leq H^{*}$ the equation (2) becomes:
$\alpha \cdot H=\frac{1}{2} \cdot \sqrt{(1-\bar{D}) \cdot(3-m) \cdot(1+2 \cdot \bar{D}-m)}+\frac{1}{2} \cdot(1-\bar{D})^{3 / 2} \cdot \ln \frac{2-m+\bar{D}+\sqrt{(3-m) \cdot(1+2 \cdot \bar{D}-m)}}{1-\bar{D}}$
By plotting equation (2) the Figure 2 is obtained.


Figure 2. Buckling behavior for a unit strip of masonry panel
Figure 2 shows the maximum value variation of the limit load $\mathrm{N}=\mathrm{N}_{\mathrm{L}}$ of " $\alpha \cdot H$ " function by varying " $m$ ". These diagrams show that with the increasing of the initial eccentricity, the carrying capacity of the element reduces rapidly because of the partialization of the section.
All the previous equations can be written in function of the Eurelian critical load: $N_{E}=\frac{\left(\pi^{2} \cdot E \cdot I\right)}{4 \cdot H^{2}}$, so that the first member of the equation (2) is:

$$
\begin{equation*}
\alpha \cdot H=\frac{\pi}{2} \cdot \sqrt{\frac{N}{N_{E}}} \tag{6}
\end{equation*}
$$

And then:

$$
\begin{equation*}
N=\frac{4}{\pi^{2}} \cdot(\alpha \cdot H)^{2} \cdot N_{E} \tag{7}
\end{equation*}
$$

Therefore we can express the limit load as in (8):

$$
\begin{equation*}
N_{L}=\frac{\left[(\alpha \cdot H)_{\max }^{2} \cdot E \cdot I\right]}{H^{2}} \tag{8}
\end{equation*}
$$

From the previous equation it can be noted that the buckling load depends both on the mechanical features of the material and on the geometry of the wall panel. This equation, is now written taking into account the compressive strength " $\sigma_{\mathrm{k}}$ " of the masonry. The ultimate compression force can be assessed by the expressions (9) and (10) respectively in the case of the entirely reacting section or of the partially reacting section:

$$
\begin{array}{cl}
N_{R}=\frac{\sigma_{k} \cdot s}{1+\frac{6 \cdot D}{s}} \quad \text { when } & e<D \leq s / 6 \\
N_{R}=\frac{3}{4} \cdot \sigma_{k} \cdot s \cdot\left(1-\frac{2 \cdot D}{s}\right) ; \text { when } & s / 6<D<s / 2 \tag{10}
\end{array}
$$

the value of the compression force $N_{R}$ determined in correspondence of the abscissa $D_{L}$ of the limit load gives the value of the ultimate load.


Figure 3. Collapse behavior vs load eccentricity
For " $m$ " values greater than 2 , the crisis occurs for buckling, while $m$ " values lower the crisis occurs for compression strength.

## 3 The limit load assessment for non-strengthened multi leaves masonry

In the case of the multi leaves masonry the load P will be the one applied on top of the vertical wall. Since the wall is formed by two outer walls and an inner layer of filling with generally inconsistent material, in the hypothesis that the stiffness of the inner layer is zero each masonry leaf will be subject to the load $\mathrm{P} / 2$. In the case of stiffness of the inner leaf " $\frac{E A}{h}$ " not negligible, the applied load should be distributed among the three elements proportionally to the respective stiffness (fig.4).


Figure 4. Load transfer mechanism schematization adopted
The load distribution of the $P$ load, respectively on the outer and the inner layer, is given by:

$$
\begin{align*}
& P_{1}=P \cdot \frac{\frac{E A}{h}}{2 \cdot \frac{E A}{h}+\frac{E_{0} A_{0}}{h}}+\frac{\gamma \cdot s \cdot h}{2}  \tag{12}\\
& P_{0}=P \cdot \frac{\frac{E_{0} A_{0}}{h}}{2 \cdot \frac{E A}{h}+\frac{E_{0} A_{0}}{h}} \tag{12}
\end{align*}
$$

In any case in the presence of three leaves we must be also take into account the horizontal load transmitted by the inner layer on the outer ones [3]. This load can be determined:

1. in the case of rubble material in the inner layer, with the equation (11) by applying the Coulomb theory as the sum of the P fraction transmitted on the inner core and the pressure induced by the same filling material, uniformed all along the height because of the thickness (smaller in respect to the height)

$$
\begin{equation*}
p=\gamma_{r} \cdot s_{0} \cdot \operatorname{tg}\left(45+\frac{\varphi}{2}\right) \cdot k_{a}+q \cdot k_{a} \tag{11}
\end{equation*}
$$

Where
$\gamma_{r}=$ specific weight of the material;
$s_{0}=$ thickness of the inner layer;
$k_{a}=$ coefficient of active earth pressure ;
$q=\frac{P_{0}}{s_{0} \cdot 1}$ is the load P fraction transmitted to the inner layer, distributed on
$s_{0} ;$
2. in the case of linear elastic material in the inner layer:, by the equation (12):applying the Elasticity theory,

$$
\begin{equation*}
\sigma_{x}=v \cdot \sigma_{y} \tag{12}
\end{equation*}
$$

Where $\sigma_{y}=\frac{P_{0}}{s_{0} \cdot L}$ is the vertical stress due to the applied load, with $L$ length of the panel;
$v$ is Poisson ratio for masonry chosen, varying according to [4]

### 3.1 Multi leaves masonry with a weak inner core

Now, we consider a given multi leaves panel $\mathrm{H}=300 \mathrm{~cm}$ tall, with a global thickness of 90 cm and 100 cm in width.
This panel geometrical features are very common in old houses in central and southern Italy. The outer leaves $\mathrm{s}=30 \mathrm{~cm}$ thick each, are constituted by limestone block with $\sigma_{k}=0,2 \mathrm{kN} / \mathrm{cm}^{2}$ [5], $E=46,2 \mathrm{kN} / \mathrm{cm}^{2}$ [6]. The inner core, having a thickness of $\mathrm{s}_{0}=30 \mathrm{~cm}$, is constituted by mortar matrix and pebbles, rubble and bricks, all characterized by a friction internal angle variable between $\varphi=45^{\circ}$ and $\varphi=40^{\circ}$, and a specific weight of $\gamma=16 \mathrm{KN} / \mathrm{m}^{3}$. The Young modulus of the of the inner layer of the wall, was assumed variable between zero and $8,0 \mathrm{kN} / \mathrm{cmq}$. The applied load on the top is $P=200 \mathrm{kN}$. The data obtained from the investigation are reported in table 2, where it can be noted, that for high values of shear strength $\left(45^{\circ}\right)$, the collapse occurs for compression for low values of Young modulus, and for buckling when the Young modulus increases.


Figure 5. Static scheme of multi leaves panel

| $\boldsymbol{E}_{\boldsymbol{0}}$ | $\mathbf{0 , 0 0 0}$ | $\mathbf{1 , 3 2 0}$ | $\mathbf{2 , 6 4 0}$ | $\mathbf{3 , 9 6 0}$ | $\mathbf{5 , 2 8 0}$ | $\mathbf{6 , 6 0 0}$ | $\mathbf{7 , 9 2 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{m}$ | 0,417 | 0,765 | 1,113 | 1,460 | 1,806 | 2,152 | 2,496 |
| $\mathbf{P} 1$ | 107,200 | 105,792 | 104,422 | 103,090 | 101,795 | $\mathbf{1 0 0 , 5 3 3}$ | 99,305 |
| $\mathbf{P 0}$ | 0,000 | 2,817 | 5,556 | 8,219 | 10,811 | $\mathbf{1 3 , 3 3 3}$ | 15,789 |
| $\mathbf{q}$ | 0,000 | 9,390 | 18,519 | 27,397 | 36,036 | 44,444 | 52,632 |
| $\mathbf{p}$ | 0,020 | 0,036 | 0,052 | 0,067 | 0,082 | $\mathbf{0 , 0 9 6}$ | 0,110 |
| $\boldsymbol{N}_{\boldsymbol{L}}$ | 696,425 | 451,024 | 271,618 | 147,679 | 68,800 | $\mathbf{2 4 , 6 8 6}$ | 5,161 |
| $\mathbf{N}_{\mathbf{R}}$ | 236,804 | 204,976 | 173,247 | 141,019 | 109,489 | $\mathbf{7 8 , 0 5 9}$ | 46,128 |

Table 2.Data obtained for the given multi leaves wall with rubble material inside and $\phi=45^{\circ}$ by varying the Young modulus

If, we refer at the most representative Young modulus for the inner core chosen equal to $E=6,6 \mathrm{kN}$, by plotting the equations (7),(9) and (10) as function of " $D$ ", the figure 6 is obtained, for the given wall:


Figure 6. Buckling behavior of a given multiple leaf wall with rubble material inner core ( $\mathrm{E}=6,6 \mathrm{kN}$ and $\varphi=45^{\circ}$ )

### 3.2 Multi leaves masonry with linear elastic material inner core

We consider the same previous given multiple leaf. This time the inner leaf is constituted by linear elastic material i.e. limestone. We proceeded to the variation of the Young modulus, so as to evaluate the behavior of the wall for two chosen Poisson ratio, assumed once equal to $v=0,10$ and then $v=0,15$. The data are reported in tables 3 and table 4. So, with the decreasing of the Poisson's ratio, also the contact pressure decreases and then, we will have an increasing in the values of collapse due to the buckling. Also, in this case the increasing of the Young modulus prevails for buckling, due to the higher stiffness of the inner material, so that it can absorb a greater amount of the external loads. This produce an increasing of the pressure transmitted on the outer layers with consequent increasing of the eccentricity.

| $\boldsymbol{E}_{\mathbf{0}}$ | $\mathbf{0 , 0 0}$ | $\mathbf{1 , 3 2}$ | $\mathbf{2 , 6 4}$ | $\mathbf{3 , 9 6}$ | $\mathbf{5 , 2 8}$ | $\mathbf{6 , 6 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{m}$ | 0,50 | 0,71 | 0,92 | 1,12 | 1,33 | 1,53 |
| $\mathbf{P} \mathbf{1}$ | 107,20 | 105,79 | 104,42 | 103,09 | 101,79 | 100,53 |
| $\mathbf{P 0}$ | 7,20 | 10,02 | 12,76 | 15,42 | 18,01 | 20,53 |
| $\mathbf{p}$ | 0,02 | 0,03 | 0,04 | 0,05 | 0,06 | 0,07 |
| $\boldsymbol{N}_{\boldsymbol{L}}$ | 628,82 | 485,38 | 365,81 | 267,85 | 189,30 | 127,93 |
| $\mathbf{N}_{\mathbf{R}}$ | 229,24 | 210,28 | 191,38 | 172,53 | 153,75 | 134,43 |

Table 3.Data obtained for the given multi leaves wall with elastic material inner core and $v=0,10$

| $\boldsymbol{E}_{\boldsymbol{O}}$ | $\mathbf{0 , 0 0 0}$ | $\mathbf{1 , 3 2 0}$ | $\mathbf{2 , 6 4 0}$ | $\mathbf{3 , 9 6 0}$ | $\mathbf{5 , 2 8 0}$ | $\mathbf{6 , 6 0 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{m}$ | 0,76 | 1,07 | 1,37 | 1,68 | 1,99 | 2,30 |
| $\mathbf{P 1}$ | 107,20 | 105,79 | 104,42 | 103,09 | 101,79 | 100,53 |
| $\mathbf{P 0}$ | 7,20 | 10,02 | 12,76 | 15,42 | 18,01 | 20,53 |
| $\mathbf{p}$ | 0,04 | 0,05 | 0,06 | 0,08 | 0,09 | 0,10 |
| $\boldsymbol{N}_{\boldsymbol{L}}$ | 457,04 | 292,79 | 173,71 | 92,42 | 41,59 | 14,00 |
| $\mathbf{N}_{\mathbf{R}}$ | 205,86 | 177,42 | 149,07 | 120,80 | 92,63 | 64,54 |

Table 4.Data obtained for the given multi leaves wall with rubble material inside and $v=0,15$

Now by plotting the equations (7),(9) and (10) as function of "D", for the given wall with $E=6,6 \mathrm{kN}$, figure 7 and figure 8 are obtained:


Figure 7. Buckling behavior of a given multi leaves panel with elastic material inner layer and $v=0,15$


Figure 8. Buckling behavior of a given multi leaves panel with elastic material inner layer and $v=0,10$

## 4 The collapse load evaluation for a multi leaves masonry strengthened by transversal tying

Due to the presence of the filling of the inner layer, the multi leaves masonry walls are subject to collapse because of the detachment of the layers and out-of-plane expulsions due to the buckling caused by the eccentricity of the axial force increased also by the pressure of inner core.
To improve the connection between the leaves and reduce the transversal deformations with a consequent increase of the bearing capacity, and then the buckling load, some transversal ties through the thickness of wall are inserted. Two different kind of ties have been used:

- volcanic tuff stone ties, having rectangular transversal section $20 \times 20 \mathrm{~cm}^{2}$ and $E=100 \mathrm{kN} / \mathrm{cm}^{2}$;
- C-FRP ties with $\mathrm{d}=12 \mathrm{~mm}$ and $E=12.400 \mathrm{kN} / \mathrm{cm}^{2}$

The wall was studied both considering the Coulumb theory and the linear elastic material theory. The previous multiple leaf panel $H=300 \mathrm{~cm}$ tall, $s=30 \mathrm{~cm}$ thickness of the outer layers, $s_{0}=30 \mathrm{~cm}$ tick of the inner core, and 100 cm wide, with specific weight of $\gamma=16 \mathrm{KN} / \mathrm{m}^{3}, \sigma_{\mathrm{k}}=0,2 \mathrm{kN} / \mathrm{cm}^{2}$; for the outer layers was considered an $E$ as $46,2 \mathrm{kN} / \mathrm{cm}^{2}$, The inner core was chosen by a friction internal angle $\varphi=45^{\circ}$ and $E=6,6 \mathrm{kN} / \mathrm{cm}^{2}$. In the case of inner layer constituted by a material similar to linear-elastic materials, the Poisson ratio was assumed equal to $v=0,15$. With the inclusion of these transversal elements, the two outer walls will be connected by bending and tensile force resistant elements. The structure is symmetrical so that the static scheme adopted is the following figure :


Figure 9. Schematization and static scheme of reinforced multi leaves panel

All the types of transversal ties adopted were inserted by a constant distance along the height, each other. The numerical investigation was carried out considering an increasing number of the ties.

### 4.1 Strengthened multi leaves masonry with a rubble material inner core

By solving the scheme of figure 9 and applying Coulomb theory, the data reported in the following tables are obtained, respectively, for the type of the reinforcement applied, in the case of the inner leaf characterized by a E modulus of $6,6 \mathrm{kN}$ and $\phi=$ $45^{\circ}$ :

| Ties no. | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| eccentricity $(\mathbf{c m})$ | 2,33 | 0,85 | 0,55 | 0,41 | 0,33 |
| $\boldsymbol{P 1}$ | 100,53 | 100,53 | 100,53 | 100,53 | 100,53 |
| $\boldsymbol{P 0}$ | 13,33 | 13,33 | 13,33 | 13,33 | 13,33 |
| $\boldsymbol{p}$ | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 |
| $\boldsymbol{N}_{\boldsymbol{L}}$ | 658,27 | 917,20 | 986,11 | 1024,08 | 1045,45 |
| $\boldsymbol{N}_{\boldsymbol{R}}$ | 232,60 | 259,65 | 300,40 | 300,35 | 300,25 |

Table 5.Data obtained for the multi leaves strengthened by tuff volcanic stone ties applying the coulomb theory $\varphi=45^{\circ}$

| Ties no. | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| eccentricity (cm) | 1,90 | 0,99 | 0,69 | 0,54 | 0,46 |
| $\boldsymbol{P 1}$ | 100,53 | 100,53 | 100,53 | 100,53 | 100,53 |
| $\boldsymbol{P 0}$ | 13,33 | 13,33 | 13,33 | 13,33 | 13,33 |
| $\boldsymbol{p}$ | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 |
| $\boldsymbol{N}_{\boldsymbol{L}}$ | 726,22 | 888,74 | 948,62 | 987,62 | 1010,92 |
| $\boldsymbol{N}_{\boldsymbol{R}}$ | 240,45 | 257,01 | 262,64 | 300,59 | 300,09 |

Table 6.Data obtained for the multi leaves strengthened by C-FRP bar ties applying the coulomb theory $\varphi=45^{\circ}$

The collapse behaviour of the multiple leaf wall panel with the chosen Young modulus ( $\mathrm{E}=6,6 \mathrm{kN}$ ), is reported in figures $10-11$ as function of the eccentricity, while figure 12 reports the comparison respect to the un-strengthened model :


Figure 10. Buckling behavior of a panel strengthened by tuff volcanic stone ties


Figure 11. Buckling behavior of a strengthened multi leaves panel by C-FRP ties


Figure 12. Collapse behavior of multiple leaf masonry with incoherent inner core, reinforced by no. 3 tie- comparison among the different typologies of reinforcement

### 4.2 Strengthened multi leaves masonry with a linear elastic plastic material inner core

By solving the scheme of figure 9 , applying linear elastic theory and adopting the $v=0,15, \mathrm{E}=6,6 \mathrm{kN}$ for the inner leaf, the data reported in the following tables are obtained, respectively for the type of the reinforcement applied:

| Ties no. | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| eccentricity (cm) | 2,48 | 0,90 | 0,58 | 0,44 | 0,35 |
| $\boldsymbol{P 1}$ | 100,53 | 100,53 | 100,53 | 100,53 | 100,53 |
| $\boldsymbol{P 0}$ | 20,53 | 20,53 | 20,53 | 20,53 | 20,53 |
| $\boldsymbol{p}$ | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 |
| $\boldsymbol{N}_{\boldsymbol{L}}$ | 633,95 | 906,09 | 975,93 | 1016,61 | 1039,22 |
| $\boldsymbol{N}_{\boldsymbol{R}}$ | 229,65 | 258,52 | 300,49 | 300,12 | 300,17 |

Table 7.Data obtained for the multi leaves strengthened by tuff volcanic stone ties applying the linear elastic theory

| Ties no. | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| eccentricity $(\boldsymbol{c m})$ | 2,03 | 1,06 | 0,73 | 0,58 | 0,49 |
| $\boldsymbol{P 1}$ | 100,53 | 100,53 | 100,53 | 100,53 | 100,53 |
| $\boldsymbol{P 0}$ | 20,53 | 20,53 | 20,53 | 20,53 | 20,53 |
| $\boldsymbol{p}$ | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 |
| $\boldsymbol{N}_{\boldsymbol{L}}$ | 704,91 | 875,96 | 939,40 | 978,15 | 1002,17 |
| $\boldsymbol{N}_{\boldsymbol{R}}$ | 237,77 | 255,59 | 261,84 | 300,09 | 300,36 |

Table 8.Data obtained for the multi leaves strengthened by C-FRP bar ties applying the linear elastic theory

The collapse behaviour is reported in Figures 13-14, while Figure 15 reports the comparison among the buckling load obtained from the investigation:


Figure 13. Behavior of a multi leaves strengthened panel with tuff volcanic stone ties in function of. ties number


Figure 14. Behavior of a strengthened multi leaves panel with C-FRP in function of ties number


Figure 15. Behavior of multiple leaf with limestone inner core, reinforced by no. 3 ties at constant step comparison among the different typologies of reinforcement

## 5 Discussion

In both cases of rubble material or linear elastic material of the inner core, the study highlights that the optimal number of transversal ties to be inserted through the thickness of the wall, in order to avoid unsuitable buckling phenomena, is equal to three. The inclusion of additional ties, in fact, gives a very small increasing of the bearing capacity of the wall.

### 5.1 Ties behavior in tensile

Each ties element mainly works in tensile, so it is necessary to assess the ultimate tensile strength of the transversal ties employed.
So, for the ties used, the ultimate tensile force of the tuff volcanic stone ties, determined according to Italian rules and [8] is $\mathrm{Nu}=10,66 \mathrm{kN}$; while in the case of CFRP, determined according to [9-10] the ultimate tensile force is equal to $\mathrm{Nu}=$ $149,31 \mathrm{kN}$. Table 9 reports the maximum value of tensile force playing in the transversal ties assessed referring to Coulomb Theory and by changing the ties number:

|  |  | Coulomb theory | Elasticity theory |
| :---: | :---: | :---: | :---: |
| Tuff volcanic <br> stone | 1 tie | 18,73 | 17,54 |
|  | 3 tie | 8,32 | 7,79 |
| C-FRP | 1 tie | 16,70 | 17,84 |
|  | 3 tie | 8,17 | 8,73 |

Table 9. Maximum tensile strength $\mathrm{N}_{\text {max }}$ in the ties $(\mathrm{kN})$

It can be seen that the stone tie is not capable, alone of resisting in tensile since the real maximum tensile strength overcomes the ultimate tensile strength, so a minimum number of 3 transversal tying must be provided.

### 5.2 Ties behavior in friction

The tensile force acting in the tie must be less than the friction due to the contact with the wall. The friction load can be determined by $P 1_{d} \cdot f$ where:

$$
\begin{equation*}
P 1_{d}=\sigma_{1} \cdot A_{l} \tag{13}
\end{equation*}
$$

$\sigma_{1}$ is the average compression stress; $A_{l}$ is the contact area of each tie; $f$ the friction coefficient.
In the case of tuff volcanic stone the friction force is equal to $28,14 \mathrm{kN}$ much higher than the tensile force acting in the ties, while in the case of C-FRP the grip is ensured by the presence of the epoxy and improved by a sandblasting of each bar. In any case, in this case to prevent any unthreading of the ties or the de-bonding from the masonry surface the anchorage must be particularly treated. At present we are tested a technique capable of easily facing this matter; it consists in enstrip the bar extremities of the C-FRP bars (fig 16) just after its sandblasting. The obtained CFRP ribbons must be bonded on the masonry surface by other C-FRP sheet glued around.


Figure 16. C-FRP ties prototype to be inserted in the multiple leaf wall

## 6 Conclusions

In this paper the behaviour of a multiple leaf panel was discussed after considering the inner core constituted by incoherent material such as pebbles, rubble in a mortar matrix, and after considering a inner core constituted by limestone material.
In the first case the problem was solved by considering the Coulomb theory in determining the transversal load due to the presence of the inner core, in the latter case the transversal load was determined by applying the Timoshenko theory.
In both cases studied we can see that the collapse behaviour of the non-reinforced panel is deeply influenced by the mechanical proprieties of the inner core, especially by the elastic modulus. In fact when this increases, the transversal load increases too, with unsuitable out plane effects on the outer leaves. Otherwise, the collapse for the buckling depends only on the eccentricity of the vertical load applied. The transversal tying system permits us to change the collapse behaviour of the masonry; in fact in this case the transversal connections highly increase the buckling load so that the collapse occurs only for the compression related to the ultimate compression stress of the masonry.
The proposed technique results are more convenient both for economical reasons and for its realization. The insertion of C-FRP bars will be made by making a small hole through the wall just by using a drill, while the insertion of the tuff volcanic stone bricks need a sort of invasive seaming which is more difficult to realize compared to the former.
From the results obtained it can be said that C-FRP reinforcement shows an excellent resistance, while those in stone (tuff) have low tensile strength. The proposed technique provides an enhancement of global resistance more than three times greater than the original model and 1,5 time greater with respect to traditional techniques. At present we are studying a type of C-FRP ties reinforcement using CFRP plate instead of bars. These plates could be inserted into mortar bed of the walls, thus making this operation even less invasive. This technique is undergoing experimental tests which will be the subject of future publications.

## References

[1] S. Sahlin ; "Diagrams of Critical Stress for Columns of material without tensile Strenght ", National Swedish Institute for Bulding Reserch -, Stoccolma n. 16/1965
[2] N. Augenti "Il Calcolo Sismico Degli Edifici In Muratura", UTET Torino 2004 ISBN 88-7750-942-2
[3] Egermann R, Newald-Burg C. "Assessment of the load bearing capacity of historic multiple-leaf masonry walls". In: Proc 10th Int Brick/Block Masonry Conf. Calgary,Canada; 1994. p. 1603-12.
[4] Elizabeth N. Vintzileou - Iniezioni di malta liquida in muratura a cassetta: Risultati sperimentali e previsione delle caratteristiche meccaniche Università tecnica nazionale di Atene - Dipartimento di Ingegneria strutturale.
[5] Badalà A., Cuomo M., "Determinazione delle proprietà meccaniche della muratura come solido com-posito: risultati sperimentali su muretti di quattro diverse tipologie" Proceedings of "La Meccanica delle Murature Tra Teoria e Progetto", Messina, 1996.A.B. Normal
[6] F. Cluni, V. Gusella "An Approach To The Homogenization Of Multiple Leaf Masonry" Proceedings of 6th European Solid Mechanics Conference, ESMC 2006
[7] J. Pina-Henriques \& P.B. Lourenço et al. "Testing and modelling of multipleleaf masonry walls under shear and compression"
[8] D.M. D.M.LL.PP. del 20/11/1987 Norme tecniche per la progettazione, esecuzione e collaudo degli edifici in muratura e per il loro consolidamento
[9] CNR-DT. 203/2006 rules "Istruzioni per la Progettazione, l'Esecuzione ed il Controllo di Strutture di Calcestruzzo armato con Barre di Materiale Composito Fibrorinforzato"
[10] CNR-DT. 200/2004 rules "Istruzioni per la Progettazione, l'Esecuzione ed il Controllo di Interventi di Consolidamento Statico mediante l'utilizzo di Compositi Fibro-rinforzati".

