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Development of a Computational Tool for the Dynamic Analysis of the Pantograph-Catenary Interaction for High-Speed Trains

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Abstract

High-speed railway overhead systems are subjected to tight functional requirements to deliver electrical energy to train engines, while their reliability and maintenance periods have to be increased. The quest for interoperability of different pantographs in existing and projected catenary systems puts an extra level of demand on the ability to control their dynamic behaviour. Also the quality of the current collection, the loss of contact and consequent arching, not also limit the top velocity of highspeed trains but have implications in the deterioration of the functional conditions of these mechanical equipment's. To address such important aspects for the design and analysis of the pantograph-catenary system, it is necessary to develop reliable, efficient and accurate computational procedures that allow capturing all the relevant features of their dynamic behaviour. This work presents a computational tool able to handle the dynamics of pantograph-catenary interaction using a fully threedimensional methodology. In order to exploit the advantages of using a multibody formulation to model the pantograph, a high-speed co-simulation procedure is setup in order to communicate with the finite element method, which is used to model the catenary. A contact model, based on a penalty formulation, is selected to represent the interaction between the two codes. The methods and methodologies developed here are used in realistic operation conditions for high speed trains. The purpose is to study the dynamic behaviour of the pantograph-catenary system in single and multiple pantograph operation scenarios and to assess the conditions that limit the increase of the trainset speed.

Keywords: railway dynamics; multibody systems; finite elements, contact mechanics, co-simulation.

1 Introduction

A limitation on the velocity of high-speed trains concerns the ability to supply the proper amount of energy required to run the engines, through the catenarypantograph interface [1]. Due to the loss of contact not only the energy supply is interrupted but also arcing between the collector bow of the pantograph and the contact wire of the catenary is observed, leading to a deterioration of the functional conditions of the two systems. Higher contact forces would lead to less incidents of loss of contact but would also lead to higher wear of the catenary contact wire and pantograph registration strip [2]. A balance between contact force characteristics and wear of the energy collection system is the objective of improving contact quality.

The quest for higher speeds for train operations requires changes in the catenaries and pantographs in which the increase of the wave travelling speed on the contact wire plays a central role, being this achieved by decreasing the weight per unit of length of the contact wire and increasing its tension [1]. For instance, to achieve the world record of 574.8 km/h by a French high-speed train, the contact wire used was made of copper with a cross-section of 150 mm² and subjected to an axial tension of 40000N, being the voltage in the line increased from 25 kV to 31 kV [3]. To allow for the Sinkansen to operate at 360 km/h, in lines that were designed for an operation up to 240 km/h not only the tension of the contact wire is increased from 14600 N to 19600 N but the tension on the messenger and auxiliary wires also changed in order to maintain the total tension at 53900 N [4].

Another important aspect of the catenary design is to maintain as constant as possible the stiffness of the contact wire to transversal loading by the pantograph registration strip [5, 6]. Different types of catenaries exist with alternative topological arrangements. The compound catenary, commonly used in the Japanese Sinkansen, guarantees an almost uniform stiffness while maintaining the contact wire at constant height, without requiring pre-sag. The stitch wire catenaries, such as the French LN1 and the German Re330, use the stitch wire to improve the uniformity of stiffness around the steady-arms while in the simple catenaries, such as the French LN2 or Italian C270, the stiffness is controlled via the dropper distance around the steady-arms. For both stitch and simple catenary types there is a pre-sag of 1/1000 to further improve the uniformity of the stiffness [5].

The overhead catenary system is a very lightly damped structure in which the damping characterization is important, in particular when the trains are equipped with multiple pantographs [5]. Different studies show that the evaluation of the pantograph-catenary contact quality is highly dependent on the amount of structural damping considered for the catenary structural elements [7]. However, it is also recognized that the estimation of the structural damping of the catenary is still a challenge. The catenary system dynamics exhibits small displacements about the static equilibrium position. The only source of nonlinearities results from the slacking of the droppers. Therefore, the linear finite element method has all features necessary to the modeling of this type of systems, provided that the nonlinear effects are suitably modeled as nonlinear forces, in this case, the dropper slacking can be handled by adding corrective terms to the system force vector.

The aerodynamic forces due to the direct effect of the wind on the overhead contact line, direct effect on the pantograph components and indirect effect due to the additional motion of the carbody imparted to the base of the pantograph influence the dynamics of the pantograph and catenary [5, 8-10]. Any dynamic

analysis methodology that aims at allowing a realistic modeling of the pantograph must allow handling these aerodynamic loads. In any case, the problem of characterizing and controlling the pantograph aerodynamics, although addressed in several technological projects, is still an open issue for research [8, 11].

Different pantographs are currently used in train vehicles in the World, with the exception of the Sinkansen 500 series telescopic pantograph, are of the scissors type. Ongoing research for the development of new pantograph's with active electronic control [12] is in place. The topology of a pantograph must address three stages of its operation: lift the pan head to contact wire height and compensation for spans with lower catenary heights; handle the displacements and frequencies associated to steady-arms passage; deal with the displacements and frequency excitations due to the dropper passage and to higher frequency excitations [5]. Typically the pantograph head, with its suspension, is responsible for handling the high-frequency excitations, up to 20 Hz, while the lower stage, including the pneumatic bellow, deal with the low frequency excitations, below 5 Hz. Some other effects such as the pantograph bow flexibility and aerodynamics may result in the contact force to exhibit frequency contents over 20 Hz [13]. Due to the range of motion of the pantograph mechanical components and to the nonlinear elements present on its construction, multibody methods are well suited to handle the pantograph dynamics [8]. Special models based on the use of lumped masses can still be used in the framework of linear finite element methods [1], however, the use of multibody methods ensure that both lumped mass and detailed nonlinear pantograph models can still be used in the analysis of the pantograph-catenary interaction problem. Hybrid methodologies in which the catenary dynamics is evaluated using a linear finite element model and the pantograph dynamics is obtained using a real prototype in a test bench are hardware-in-the-loop alternatives to fully computational oriented approaches [14, 15].

The interaction of the pantograph and catenary is achieved through the contact of the pantograph registration strip on the catenary contact wire. The modeling of contact between the registration strip and the contact wire can be done using unilateral kinematic constraints [16, 17], which does not require the estimation of any contact law parameter but prevents any loss of contact to be detected. Alternatively, penalty formulations can be used [5, 18] with no limitations on how contact may develop but requiring that the penalty terms of the contact law are estimated. In any case, the use of different methods to handle the dynamics of the catenary and pantograph requires that either a single code in which both methods are implemented is developed or that a co-simulation strategy between the two codes is implemented [19, 20]. The contact modeling plays a central role in the establishment of the co-simulation strategies [19].

The quality of the pantograph-catenary contact required for high-speed train operations is quantified in current regulations [21, 22]. The norm EN50367 specifies the following thresholds for pantograph acceptance.

•	Mean contact force (F_m)	$F_m = 0.00097 v^2 + 70 N$
•	Standard deviation (σ_{max})	$\sigma_{max} < 0.3 F_m$
•	Maximum contact force(F_{max})	F_{max} <350 N

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- Maximum CW uplift at steady-arm (d_{up}) $d_{up} \le 120 \text{ mm}$
- Maximum pantograph vertical amplitude(Δ_z) $\Delta_z \leq 80 \text{ mm}$
- Percentage of real arcing (*NQ*)

A limitation of the operational speed of the trains is the wave propagation velocity on the contact wire, *C*, which is given by

$$C = \sqrt{F/\rho} \tag{1}$$

NO≤0.2%

where F is the tension of the contact wire and ρ is the contact wire mass per length unit. When the train speeds approach the wave propagation velocity of the contact wire the contact between the pantograph and the catenary is harder to maintain due to increase in the amplitude of the catenary oscillations and bending effects. In order to avoid the deterioration of the contact quality the train speed, V, current regulation impose a limit of V=0.7C.

In this work a computational methodology is proposed enabling the dynamic analysis of pantograph-catenary interaction. The finite element method is used for the dynamic analysis of the catenary and a multibody dynamics approach is used for the dynamic analysis of the pantographs, regardless of being lumped or multibody models. A co-simulation environment is setup to run interference between the independent catenary and pantograph dynamic analyses. The methods proposed in this work are demonstrated in the framework of the application of the regulation EN50367 to a study case of multiple pantograph operation in high-speed trains between a generic catenary and a high speed pantograph. This case addresses one of the limiting factors in high-speed railway operation that is the need to use more than a single pantograph for current collection. The disturbance that the pantographs cause on each other dynamics worsens the quality of the pantograph-catenary contact.

2 Catenary dynamics

High-speed railway catenaries are periodic structures that ensure the availability of electrical energy for the train vehicles running under them. Typical constructions, such as the one presented in Figure 1, include the masts (support, stay and console) which serve as support for the registration arms and messenger wire, the steady arms, which not only support the contact wire but also ensure the correct stagger, the messenger wire, the droppers, the contact wire and, eventually, the stitch wire.

Depending on the catenary system installed in a particular high-speed railway all the elements or only some of them may be implemented, as shown in Figure 2. However, in all cases both messenger and contact wires are tensioned with high axial forces not only to limit the sag and guarantee the appropriate smoothness of the pantograph contact by controlling the wave travelling speed but also to ensure the stagger of the contact and messenger wires.



Figure 1: General structural and functional elements in a high speed catenary.



Figure 2: Different type of high-speed catenary implementations: a) Stitch wire catenary; (b) Simple catenary; (c) Sinkansen catenary.

The motion of the catenary is characterized by small rotations and small deformations, in which the only nonlinear effect is the slacking of the droppers. The axial tension on the contact, stitch and messenger wire is constant and cannot be neglected in the analysis. Therefore, the catenary system is modelled with linear finite elements in which the dropper nonlinear slacking is modelled with compensating forces added to the force vector along with the pantograph contact forces and the constant line tensioning forces.

All catenary elements, contact and messenger wires are modelled by using Euler-Bernoulli beam elements [5]. Due to the need to represent the high axial tension forces the beam finite element used for the messenger, stitch and contact wire, designated as element i, is written as

$$\mathbf{K}_{i}^{e} = \mathbf{K}_{L}^{e} + F \, \mathbf{K}_{G}^{e} \tag{2}$$

in which \mathbf{K}^{e}_{L} is the linear Euler-Bernoulli beam element, *F* is the axial tension and \mathbf{K}^{e}_{G} is the element geometric matrix. The droppers and the registration and steady arms are also modelled with the same beam element but disregarding the geometric stiffening. The mass of the gramps that attach the droppers to the wires are modelled here as lumped masses. In order to ensure the correct representation of the wave propagation 4 to 6 elements are used in between droppers to appropriately model the contact and messenger wires. There is no special requirement on the number of elements required to model each dropper, registration or steady-arm.

Using the finite element method, the equilibrium equations for the catenary structural system are assembled as

$$\mathbf{M}\mathbf{a} + \mathbf{C}\mathbf{v} + \mathbf{K}\mathbf{d} = \mathbf{f} \tag{3}$$

where **M**, **C** and **K** are the finite element global mass, damping and stiffness matrices of the finite element model of the catenary. Usually proportional damping is used to evaluate the global damping matrix as $\mathbf{C} = \alpha \mathbf{K} + \beta \mathbf{M}$ with α and β being suitable proportionality factors [23], or the damping matrix of each finite element, i.e, $\mathbf{C}^e = \alpha^e \mathbf{K}^e + \beta^e \mathbf{M}^e$ with α^e and β^e being proportionality factors associated with each type of catenary element, such as dropper, messenger wire, stitch wire, etc. The nodal displacements vector is **d** while **v** is the vector of nodal velocities, **a** is the vector of nodal accelerations and **f** is the vector with the force vector, written as

$$\mathbf{f} = \mathbf{f}_{(g)} + \mathbf{f}_{(t)} + \mathbf{f}_{(c)} + \mathbf{f}_{(d)}$$
(4)

which contains the gravity forces, $\mathbf{f}_{(g)}$, and line tensioning forces, $\mathbf{f}_{(t)}$, which are always constant plus the pantograph contact forces, $\mathbf{f}_{(c)}$, and the dropper slacking compensating terms, $\mathbf{f}_{(d)}$. Equation (3) is solved for \mathbf{x} or for \mathbf{a} depending on the integration method used.

In this work Equation (3) is solved with the integration of the nodal accelerations using a Newmark family integration algorithm [24, 25]. The contact forces are evaluated for $t+\Delta t$ based on the position and velocity predictions for the FE mesh and on the pantograph predicted position and velocity. The finite element mesh accelerations are calculated by

$$\left(\mathbf{M} + \gamma \,\Delta t \,\mathbf{C} + \beta \,\Delta t^2 \mathbf{K}\right) \mathbf{a}_{t+\Delta t} = \mathbf{f}_{t+\Delta t} - \mathbf{C} \tilde{\mathbf{v}}_{t+\Delta t} - \mathbf{K} \tilde{\mathbf{d}}_{t+\Delta t}$$
(5)

Predictions for new positions and velocities of the nodal coordinates of the linear finite element model of the catenary are found as

$$\tilde{\mathbf{d}}_{t+\Delta t} = \mathbf{d}_t + \Delta t \, \mathbf{v}_t + \frac{\Delta t^2}{2} (1 - 2\beta) \mathbf{a}_t \tag{6}$$

$$\tilde{\mathbf{v}}_{t+\Delta t} = \mathbf{v}_t + \Delta t \left(1 - \gamma\right) \mathbf{a}_t. \tag{7}$$

Then, with the acceleration $\mathbf{a}_{t+\Delta t}$ the positions and velocities of the finite elements at time $t+\Delta t$ are corrected by

$$\mathbf{d}_{t+\Delta t} = \tilde{\mathbf{d}}_{t+\Delta t} + \beta \,\Delta t^2 \mathbf{a}_{t+\Delta t} \tag{8}$$

$$\mathbf{v}_{t+\Delta t} = \tilde{\mathbf{v}}_{t+\Delta t} + \gamma \,\Delta t \,\mathbf{a}_{t+\Delta t} \tag{9}$$

The dropper slacking is also corrected in each time step, if necessary. Although the droppers perform as a bar during extension their stiffness during compression is either null or about 1/100th of the extension stiffness, to represent a residual resistance to buckling at high speed. As the droppers stiffness is included in the stiffness matrix K as a bar element, anytime one of them is compressed such contribution for the catenary stiffness has to be removed, or modified. In order to keep the dynamic analysis linear the strategy pursued here is to compensate the contribution to the stiffness matrix by adding a force to vector f(d) equal to the bar compression force

$$\mathbf{f}_{(d)t+\Delta t} = \mathbf{K}_{dropper}^{e} \mathbf{B}\tilde{\mathbf{d}}_{t+\Delta t}$$
(10)

where the Boolean matrix \mathbf{B} simply maps the global nodal coordinates into the coordinates of the dropper element.

The correction procedure expressed by using Equations (6) through (10) and solving Equation (5) is repeated until convergence is reached for a given time step, i.e., until $|\mathbf{d}_{t+\Delta t} - \tilde{\mathbf{d}}_{t+\Delta t}| < \varepsilon_d$ and $|\mathbf{v}_{t+\Delta t} - \tilde{\mathbf{v}}_{t+\Delta t}| < \varepsilon_d$ being ε_d and ε_v user defined tolerances. Note that the criteria of convergence of the nodal displacements must imply convergence of the force vector also, i.e, the balance of the equilibrium equation right-hand side contribution of the dropper slacking compensation force with the left-hand-side product of the dropper stiffness by the nodal displacements in Equation (3).

The computational implementation of the finite element procedure outlined by Equations (3) through (10), in the context of the catenary dynamic analysis, requires that the maximum number of iterations allowed for the correction process to be 6 or higher. If a maximum number of iterations is set to be below 6 there is the danger that the droppers exhibit residual compression forces during the dynamic analysis, with all implications that such error has over the evaluation of the pantograph-catenary contact force.

3 Pantograph dynamics

The roof pantographs used in high-speed railway applications are characterized as mechanisms with three loops ensuring that the trajectory of head is in a straight line, perpendicular to the plane of the base, while the pantograph head is maintained levelled. The pantographs are always mounted in the train in a perfect vertical alignment with the centre of the boggies of the vehicle in order to ensure that during curving the centre of the bow does not deviate from the centre of the railroad. The mechanical system that guarantees the required characteristics of the trajectory of the pantograph head during rising is generally made up by a four-bar linkage for the lower stage and another four-bar linkage for the upper stage. Another linkage between the head and the upper stage of the pantograph ensures that the bow is always levelled. In order to control the raise of the pantograph one bar of the lower four-bar linkage is actuated upon by a pneumatic actuator.

The numerical methods used to perform the dynamic analysis of the pantograph must be able to represent the important details of the system, including mechanisms and compliances and to evaluate their correct dynamics. Two different types of models are generally used to represent pantographs: lumped mass and multibody. Each of them has advantages and shortcomings in their use that are discussed hereafter.

3.1 Multibody dynamic analysis of pantographs

A typical multibody model, as also a multibody pantograph model, is defined by a collection of rigid or flexible bodies that have their relative motion constrained by kinematic joints and is acted upon by external forces. The forces applied over the system components may be the result of springs, dampers, actuators or external applied forces describing gravitational, contact/impact or other forces. A wide variety of mechanical systems can be modelled as the schematic system represented in Figure 3.



Figure 3: Generic multibody system.

Let the configuration of the multibody system be described by *n* Cartesian coordinates \mathbf{q} , and a set of *m* algebraic kinematic independent holonomic constraints $\boldsymbol{\Phi}$ be written in a compact form as [26].

$$\Phi(\mathbf{q},t) = \mathbf{0} \tag{11}$$

Differentiating Equation (11) with respect to time yields the velocity constraint equation. After a second differentiation with respect to time the acceleration constraint equation is obtained

$$\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} = \mathbf{v} \tag{12}$$

$$\Phi_{\mathbf{q}}\ddot{\mathbf{q}} = \gamma \tag{13}$$

where Φ_q is the Jacobian matrix of the constraint equations, υ is the right side of velocity equations, and γ is the right side of acceleration equations, which contains the terms that are exclusively function of velocity, position and time.

The equations of motion for a constrained multibody system (MBS) of rigid bodies are written as

$$\mathbf{M\ddot{q}} = \mathbf{g} + \mathbf{g}^{(\mathbf{c})} \tag{14}$$

where **M** is the system mass matrix, $\ddot{\mathbf{q}}$ is the vector that contains the state accelerations, **g** is the generalized force vector, which contains all external forces and moments, and $\mathbf{g}^{(c)}$ is the vector of constraint reaction equations. The joint reaction forces can be expressed in terms of the Jacobian matrix of the constraint equations and the vector of Lagrange multipliers

$$\mathbf{g}^{(c)} = -\boldsymbol{\Phi}_{\mathbf{q}}^{T}\boldsymbol{\lambda} \tag{15}$$

where λ is the vector that contains *m* unknown Lagrange multipliers associated with *m* holonomic constraints. Substitution of Equation (15) in Equation (14) yields

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{T}\boldsymbol{\lambda} = \mathbf{g}$$
(16)

In dynamic analysis, a unique solution is obtained when the constraint equations are considered simultaneously with the differential equations of motion with proper set of initial conditions. Therefore, equation (13) is appended to equation (16), yielding a system of differential algebraic equations that are solved for $\ddot{\mathbf{q}}$ and λ . This system is given by

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_{q}^{T} \\ \mathbf{\Phi}_{q} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_{r} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} \end{bmatrix}$$
(17)

In each integration time step, the accelerations vector, $\ddot{\mathbf{q}}$, together with velocities vector, $\dot{\mathbf{q}}$, are integrated in order to obtain the system velocities and positions at the next time step. This procedure is repeated up to final time will be reached.

The solution of the multibody equations of motion and their integration in time is depicted in Figure 4. The set of differential algebraic equations of motion, Equation

(17) does not use explicitly the position and velocity equations associated to the kinematic constraints, Equations (11) and (12), respectively. Consequently, for moderate or long time simulations, the original constraint equations are rapidly violated due to the integration process. Thus, in order to stabilize or keep under control the constraints violation, Equation (17) is solved by using the Baumgarte Stabilization Method or the augmented Lagrangean formulation and the integration process is performed using a predictor–corrector algorithm with variable step and order. Due to the long simulations time typically required for pantograph-catenary interaction analysis, it is also necessary to implement constraint violations correction methods, or even the use of the coordinate partition method for such purpose [26].



Figure 4: Flowchart with the forward dynamic analysis of a multibody system

One of the criticisms that the multibody model of the pantograph made of rigid bodies connected by perfect kinematic joints, as the one presented here, is that it only has 2 degrees-of-freedom, i.e., the raising of the pantograph and the head suspension. In fact, laboratory dynamic analysis of real pantographs leads to FRF measurements in all pantographs that exhibit the presence of three resonances, implying that a basic requirement for the pantograph models is to have three degrees of freedom, at least. The third degree-of-freedom of the multibody pantograph may be associated with the clearance or with bushings existing in the joints [27, 28] or even with the flexibility of one or more of the bodies considered in this model [29, 30]. In any case, the minimal requirements for modelling multibody pantographs that can have a realistic behaviour in the complete frequency range of their operation, i.e. in the 0-20 Hz range, are still to be identified.

3.2 Lumped Mass Pantograph Model

Alternatively to a full multi-body pantograph model another modelling method is commonly used consisting on a lumped mass approach. The lumped mass pantograph model, depicted on Figure 5(b), is composed of a simple series of lumped masses linked consequently to a ground by a spring/damper element. Although in the literature pantograph models are presented with two, three of more mass stages, for high-speed train applications there is a minimal requirement of three stages to well represent the system.



Figure 5: Lumped mass pantograph model: a) Laboratory parameter identification procedure; b) Three stage lumped mass model; c) Parameter values

While the multi-body pantograph models can be built with design data alone, for example with data obtained from technical drawings complemented with measured physical characteristics from selected components, the lumped mass pantograph model parameters, in example of those presented on Figure 5(c), must be identified experimentally. In this sense, the lumped mass pantograph model can be thought as a transfer function in a winch an experimental procedure, presented on Figure 5(a) is used. The idea is to excite the contact strips of the desired pantograph to model with prescribed motions of known frequency and amplitude while measuring the response of the pantograph namely the contact forces on the registration strip and positions, velocities and accelerations at prescribed points of the mechanical pantograph. This acquired data is then held to build the frequency response functions (FRF) of the pantograph. The lumped mass model parameters are identified in such way that the FRF of the model is matched to the experimentally acquired [31].

It is important to note that in spite of the simplicity of their construction and fidelity of their dynamic response, the lumped mass models are commonly used by operators, manufacturers and homologation bodies instead of more complex models. The only part of the lumped mass model that as a physical interpretation is the upper stage, winch limits the use of this type of models for any application that requires modifications on the pantographs structure or mechanics. From this point of view, this is where the multibody pantograph models come in. Provided they are able to

adequately represent the dynamics of the implied system, including a match with the FRF experimental data, they constitute an irreplaceable tool.

When using lumped mass pantograph models it is possible to use the same finite element code to solve the equations of motion of both pantograph and catenary. For this purpose, the pantograph is considered a linear system and it's equations of motion must be assembled in the same way the catenary's equations are, as expressed in Equation (3). In this assembled form the equations of motion of the pantograph can be solved along the finite elements code either by adding them to the catenary's equations or by solving them separately in parallel to the finite element code.

It should be noted that the lumped mass pantograph model can also be easily integrated on a multibody application.

4 Contact Model

The contact involved in the pantograph-catenary interaction is related to the contact between the pantograph's registration strip and the catenary contact wire. The efficiency of the electrical current transmission and the wear prediction of either the registration strip or the contact wire are deeply influenced by the quality of the contact. This implies that the correct modelling of the contact mechanics involved between these two systems is crucial for its correct and efficient evaluation.

The contact between the registration strip of the pantograph and the contact wire of the catenary, from the contact mechanics point of view, consists in the contact of a cylinder made of copper with a flat surface made of carbon having their axis perpendicular as shown in Figure 6. The contact problem can be treated either by a kinematic constraint between the registration strip and the contact wire or by a penalty formulation. In the first procedure the contact force is simply the joint reaction force of the kinematic constraint [16, 17]. With the second procedure the contact force defined in function of the relative penetration between the two cylinders [27, 32]. The use of the kinematic constraint between contact, being this approach valid only if no contact losses exist. The use of the penalty formulation allows for the loss of contact and it is the method of choice for what follows.



Figure 6: Pantograph-catenary contact: (a) Pantograph bow and catenary contact wire; (b) Cross-section of the contact wire; (c) Cross-section of the registration strip

The contact model used here is based on a contact force model with hysteresis damping for impact in multibody systems. In this work, the Hertzian type contact force including internal damping can be written as [33]:

$$F_N = K\delta^n \left[1 + \frac{3(1-e^2)}{4} \frac{\dot{\delta}}{\dot{\delta}^{(-)}} \right]$$
(18)

where *K* is the generalized stiffness contact, *e* is the restitution coefficient, $\dot{\delta}$ is the relative penetration velocity and $\dot{\delta}^{(-)}$ is the relative impact velocity. The proportionality factor *K* is obtained from the Hertz contact theory as the external contact between two cylinders with perpendicular axis. Note that the contact force model depicted by Equation (18) is one of the different models that can be applied. Other continuous contact force models are presented in references [34-36].

In the application that follows the contact is considered purely elastic, i.e., the restitution coefficient e=1, the generalized stiffness defined to be K=50000 N/m and power of the penetration is n=1. The relative penetration, δ , is evaluated taking into account the nodal displacements of the beam finite element in which contact occurs and its shape functions. For the efficiency of the computer code it is important that the numbers of the finite elements that are in contact with each of the registration strips are kept track of so that unnecessary searches for contact are avoided.

5 Co-Simulation of Multibody and Finite Elements

The interaction between the multibody pantograph and the finite elements catenary systems represents a coupled problem. Generally the dynamic analysis of pantograph-catenary systems is done with both models using the same formulation. However, since the pantograph exhibits large displacements and rotations during its operation it is not advisable to model this system with finite elements method, plus when compared the use of finite elements to preform dynamic analysis of nonlinear systems to the use of multibody methodology the finite elements method leads to larger computational time costs. Also, in spite of being possible to build a catenary model in multibody formulation there is no reliable model that can handle all its complex details. Furthermore the modelling of catenary systems using finite element formulation is nowadays used by the railways industry, with validated results. The analysis of pantograph-catenary interaction is then consequently done by two standalone and independent codes, the multibody pantograph and the finite elements catenary, both running in a co-simulation environment. There is still the option of using a lumped mass pantograph model which can be handled either by the multibody or finite element formulation giving the possibility do avoid cosimulation.

The structure of the communication between the pantograph and the catenary codes is shown in Figure 7. The multibody pantograph code (PAT) provides the catenary finite element code (CAT) with the positions and velocities of the

pantograph's registration nodes. These nodes correspond to the registration strip extremities. The CAT code then calculates the position of the point of contact on the registration strip using geometric interference and the contact force using an appropriate contact model. These calculations are then communicated from CAT to PAT code. Each code handles independently their equations of motion of each subsystem, and apply the contact force on the contact point both shared between them.



Figure 7: Co-simulation between a finite element and a multibody code

The catenary is modelled as a dynamic linear system integrated with a Newmark family numerical integrator [24] using a fixed time step where a prediction of the positions and velocities of not only the catenary but also the pantograph are needed. On the other side the multibody pantograph is a nonlinear dynamic system handled with variable time step and multi-order integrators winch in order to proceed with its dynamic analysis need not only the positions and velocities of the pantograph but also the contact force and its application point at different time instants during the integration period. Therefor there is the need of one of the codes to make a prediction to a forthcoming time before advancing to a new time step. Either the PAT predicts a contact point and force of contact or the CAT predicts the registration strip's position and velocity. It this work it was chosen the PAT code to make the predictions using a simple linear extrapolation based on stored data of the last time steps. The reasons for being the PAT code to make a prediction are that PAT code uses a variable time step and an extrapolation/interpolation algorithm would already have to be used to estimate the contact position and force on a forthcoming or past time instant that is not multiple of the fixed CAT code time step.

The compatibility between the two integration algorithms imposes that the state variables of the two sub-systems are readily available during the integration time and also that a reliable prediction of the contact forces is available at any given time step. The accuracy of this prediction relies upon the choosing of small enough time steps either of the fixed CAT code time step or the maximum allowed PAT code time step winch is variable. The key of this co-simulation procedure between PAT and CAT codes is their time integration step which must be such that a correct dynamic analysis is ensured.

6 Pantograph-catenary case study

The numerical procedure for the dynamic analysis of the pantograph-catenary interaction is demonstrated in this work using generic, but realistic, models for the

catenary and pantographs. The typical data required to build a finite element model of a simple catenary, such as that presented in Figure 2(b), is presented in Table 1.

General								
Catenary height [m]		1.4 Contact wire		ire height [m]	5.08			
Number of spans		25	Number of	f droppers/span	8			
Nº spans at C.W. heigh	nt	20 Inter-dropper distance [m]		6.75				
Span length [m]		50-54 Stagger [m]		0.20				
Damping α	0.0027-0.027 Damping β			0				
	Contact Wire	Messe	nger Wire	Droppers	Steady Arms			
Material	Cu Bz II ((braided)	Bz II (braided)				
Section [mm ²]	150	66		12	120			
Mass [kg/m]	1.33	0	.605	0.11	1.07			
Tension [N]	20000	1	4000	-	-			
Claw with	dropper	dropper		-	C.W.			
Claw mass [kg]	0.195	0.165		-	0.200			
Length [m]	-		-	1.25-1.075	1.24			
Angle w/horiz.	-		-	90°	-10°			

Table 1: Geometric and material properties of a generic simple catenary

Using the data contained in Table 1 a finite element model of the generic simple catenary is obtained, being different views shown in Figure 8. Several models of the same catenary are developed with different proportional damping factors to allow studying the variation of the pantograph-catenary contact quality in face of the structural energy dissipation of the catenary.



Figure 8: Finite element model of a generic catenary with the sag highlighted.

In conjunction to the built catenary model a lumped mass pantograph model, presented on Figure 5 (c) and (b) was used using a multibody approach.

A limiting factor in the current collection is the need to operate with multiple pantographs to allow for the collection of enough energy to run all the engines required the train. However, the contact quality of the pantograph-catenary interaction is perturbed due to the mutual influence of the leading and trailing pantographs in each other. In this work, the operation of multiple pantographs in catenaries with low and moderate damping is considered. Different vehicles of high-speed trains have different lengths and, consequently, the separation distance may change accordingly. The typical separations between pantographs shown in Figure 9 reflect how multiple train units operate and constitute the scenarios to which the methodologies proposed in this work are applied.



Figure 9: Multiple pantograph operations of high-speed trains with typical distances between pantographs.

In all scenarios the pantographs move along the catenary in a tangent track at 300 km/h. In the initial part of the analysis the pantographs are raised until their bows touch the contact wire. In order to disregard this transient part of the dynamic response, only the contact forces that develop in the pantograph between 400 and 800 m, and the droppers and steady arms that exist in this range are used in the analysis of results. The contact forces are filtered with a cut-off frequency of 20 Hz before being post-processed, as specified by the appropriate regulation [21, 22].

Figure 10 depicts the characteristics of the contact forces that develop between the pantographs and catenaries, with different proportional damping.

The results show that the amplitude of the contact forces in the trailing pantographs is always larger than what the leading pantographs exhibit, being that difference higher for lightly damped catenaries. The contact force results, as shown in Figure 10, hide some of the important results of the analysis that are used in design and in homologation of pantographs. In Figure 12 the statistic values of the contact force are overviewed for the different pantograph separations, running in the two catenaries considered before.



Figure 10: Pantograph-catenary contact force for several pantograph separations in catenaries with different proportional damping: (a) α =0.00275; (b) α =0.0275.



Figure 11: Statistical quantities of the pantograph-catenary contact force in catenaries with different proportional damping: (a) α =0.00275; (b) α =0.0275.



Figure 12: (Continued) Statistical quantities of the pantograph-catenary contact force in catenaries with different proportional damping: (a) α =0.00275; (b) α =0.0275.

The first important observation of statistical quantities depicted in Figure 12 is that the standard deviation of the contact force for all pantographs, running on the lightly damped catenary, is always larger than 30% of the mean contact force. These



Figure 13: Histograms of the pantograph-catenary contact force in catenaries with different proportional damping: (a) α =0.00275; (b) α =0.0275.

values imply that the trains using these pantographs would not be allowed to run at a speed of 300 km/h in the catenary system. However, the pantographs can be used, with the current operational setup, in the catenary with normal damping. In both cases, the multiple pantograph operation with a separation of 200 m shows the worst contact force characteristics for the trailing pantograph, i.e., the trailing pantograph exhibits larger maximum forces, lower minimum forces and larger standard deviations. None of the pantographs exhibits any contact loss.

Another characteristic of the contact force that is worth being analysed is its histogram. Figure 13 presents the histograms of all pantographs for all separations considered in this work.

The histograms show that for a lightly damped catenary the contact forces not only have large variations, as observed also in Figure 10, but also that the number of occurrences of contact forces in each range considered is high, i.e., even away from the mean contact force the existence of higher or lower contact forces is not sporadic. For a catenary with average damping the contact force magnitude is closer to the mean contact force. In all cases considered, the mean contact force is always 150 N, which satisfies the regulations.

One of the reasons why the contact force characteristics has to stay inside a limited range concerns the potential interference between the pantograph head and the catenary mechanical components. The steady-arm uplift, shown in Figure 14, and the dropper axial force, depicted in Figure 15, are measures of the catenary performance and of its compatibility with the running pantographs.



Figure 14: Typical steady-arm uplift in catenaries with different proportional damping for two different separations of pantographs: (a) α =0.00275; (b) α =0.0275.



Figure 15: Typical mid-span dropper forces in catenaries with different proportional damping for two different separations of pantographs: (a) α =0.00275; (b) α =0.0275.

The steady-arm uplift is lower than 7 cm in all cases depicted in Figure 14. Although not represented, the maximum uplift of all steady arms of the catenary is also lower than the 12 cm limit allowed for the type of catenary used. The droppers exhibit slacking for the lightly damped catenary, as seen for the axial forces shown in Figure 15. For the catenary with average damping, only the passage of the trailing pantograph, when the separation is 200m, has some slacking. It is interesting to notice that the position of the contact wire of the catenary is disturbed even before the pantograph bow passes. This is because the traveling wave speed is higher than the train speed. For lightly damped catenaries this disturbance is higher. For longer catenary sections, it is expected that the trailing pantograph may affect the contact of the leading pantograph due to the wave traveling speed of the contact wire.

The mutual influence of the pantographs in each other's contact quality is better understood when displaying the contact force characteristics as shown in Figure 16, for the lightly damped catenary and Figure 17, for the average damped one. In both cases the statistical values of the contact force of a single pantograph operation are also presented to better understand the problem.

For a lightly damped catenary the perturbation of the trailing pantograph over the leading pantograph exist but are low. However, the trailing pantograph contact forces are clearly affected by the leading pantograph, being the influence enhanced by the decrease of the catenary damping. The tendency exhibited for the lightly damped catenary is the for a pantograph separation of 31 m the trailing pantograph being this influence due to the wave traveling speed in the contact wire. For a pantograph separation of 200 m the trailing pantograph contact quality is clearly affected in both

lightly and average damped catenaries. The results suggest that the critical distance between pantographs, at least for the catenary design considered in this work, is 200m. These results are in agreement with the findings of Ikeda who studied the multiple pantograph operation for the Japanese Sinkensen pantograph-catenary interaction and found the same critical separation [4]. Thus it is suggested that regardless of the type of construction of the catenary a critical separation distance between the pantographs exists and that the distance is close to 200 m.







Figure 17: Statistical quantities associated to the contact force for a catenary with average damping (α =0.0275): (a) Leading pantographs; (b) Trailing pantographs.

7 Conclusions

A computational approach, based on the co-simulation of linear finite element and general multibody codes, is presented and demonstrated in the framework of the pantograph-catenary interaction of multiple pantograph operations in high-speed trains. It was shown that the use of linear finite elements are enough to allow for the correct representation of the catenary provided that the wire tension forces are accounted for in the stiffness formulation and that the droppers slacking is properly represented via the force vector. Minimal requirements for the catenary finite element modeling include the use of Euler-Bernoulli beam elements with axial tensioning for the catenary messenger and contact wire with a discretization enough to capture the deformation wave traveling of the contact wire. It was also shown that the use of multibody dynamics methods allow capturing all the important dynamic features of the pantographs.

Spatial pantograph models require the use of multibody dynamics procedures to capture all their dynamic features. In the particular case of using unidirectional lumped mass pantograph models the equations of motion of the system can be solved together with the finite element equilibrium equations. The lumped mass models result from a laboratory identification of the system that represents the pantograph prototype and it is, by definition, a validated model. A multibody pantograph model made of rigid bodies and perfect kinematic joints does not have the minimum features to represent correctly the real system. It is a topic of research to identify the minimum features required for a multibody pantograph model, being the existence of bushings and clearances in the joints and, eventually, the flexibility of the bow and arms, some of the modeling features that need to be accounted for.

When multibody pantograph models are used, the co-simulation between the finite element and multibody codes must be ensured. The use of a contact penalty formulation demonstrates to be enough to obtain all main contact features.

The application of the procedures to multiple pantograph operations, in highspeed railway vehicles, allowed the identification of the important quantities of the dynamic response that are required for the pantograph homologation and for the operational decisions. The catenary damping plays a fundamental role in the pantograph-catenary contact quality. This parameter leads to higher maximum contact forces, lower minimum contact forces, eventually to contact losses, and to higher standard deviations of the contact forces. All these characteristics of the contact force lead to the rejection of the operation of multiple pantograph units at the required speed of 300 km/h in lightly damped catenaries. It is also concluded, that for operations in average damped catenaries all standard separations between the pantographs lead to acceptable contact forces. As a general tendency, it was observed that for smaller pantograph separations the trailing pantograph affects the quality of the leading pantograph contact due to the wave travelling speed of the contact wire. For larger pantograph distances it is the leading pantograph that affects adversely the contact quality of the trailing pantograph. In any case, all results show that the critical separation between pantographs is 200 m, i.e., it is at this separation that the leading pantograph has a greater influence in the contact quality of the trailing one.

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References

- [1] A. Collina, and S. Bruni, "Numerical Simulation of Pantograph-Overhead Equipment Interaction," Vehicle System Dynamics, 38 (4), 261-291, 2002
- [2] G. Bucca, and A. Collina, "A Procedure for the Wear Prediction of Collector Strip and Contact Wire in Pantograph-Catenary System," Wear, 266 ((1-2)), pp. 46-59, 2009
- [3] F. Pupke, "Installation of Contact Wire (CW) for High Speed Lines Recommendations", Secondary Installation of Contact Wire (CW) for High Speed Lines – Recommendations, January 25, Houston, Texas, 2010
- [4] K. Ikeda, "Optimization of Overhead Contact Lines for Sinkansen Speed Increases," JR East Technical Review, 12, 6469, 2008
- [5] G. Poetsch, J. Evans, R. Maisinger, W. Kortum, W. Baldauf, A. Veitl, and J. Wallaschek, "Pantograph/Catenary Interaction and Control," Vehicle System Dynamics, 28, 159-195, 1997
- [6] J. P. Mentel, "Technical developments in superstructure," Proc. Communication at Highspeed 2008: 6th World Congress on High Speed Rail.
- J. Ambrósio, J. Pombo, A. Facchinetti, S. Bruni, J.-P. Massat, and H. Dupuis, "Key Parameters for Pantograph/Catenary Numerical Models," PantoTRAIN Technical Report D1.1, UNIFE, Brussels, Belgium, 2010
- [8] J. Pombo, J. Ambrósio, M. Pereira, F. Rauter, A. Collina, and A. Facchinetti, "Influence of the Aerodynamic Forces on the Pantograph-Catenary System for High Speed Trains," Vehicle System Dynamics, 47 (11), 1327-1347, 2009
- [9] F. Kiebling, R. Puschmann, and A. Schmieder, Contact Lines for Electric Railways, Siemens AG, Berlin, Germany.
- [10] J. Pombo, and J. Ambrósio, "An Alternative Method to Include Track Irregularities in Railway Vehicle Dynamic Analyses," Nonlinear Dynamics, 68, 161-176, 2012
- [11] EUROPAC, "EUROPAC Publishable Final Activity Report," EUROPAC Technical Report, SNCF, Paris, France, 2008
- [12] R. M. Goodall, S. Bruni, and A. Facchinetti, "Active Control in Railway Vehicles," International Journal of Railway Technology, 1 (1), 57-85, 2012
- [13] A. Collina, A. L. Conte, and M. Carnevale, "Effect of Collector Deformable Modes in Pantograph-Catenary Dynamic Interaction," Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 223 (1), 1-14, 2009
- [14] A. Facchinetti, and B. Bruni, "Hardware-in-the-Loop Hybrid Simulation of Pantograph-Catenary Interaction," Journal of Sound and Vibration, DOI: 10.1016/j.jsv.2012.1001.1033, 2012
- [15] J. Ambrósio, J. Pombo, M. Pereira, P. Antunes, A. Mósca, "Recent Developments in Pantograph-Catenary Interaction Modelling and Analysis",

International Journal of Railway Technology, 1(1), 249-278, 2012. doi:10.4203/ijrt.1.1.12

- [16] J.-H. Seo, H. Sugiyama, and A. A. Shabana, "Three-Dimensional Deformation Analysis of the Multibody Pantograph/Catenary Systems," Nonlinear Dynamics, 42, 199-215, 2005
- [17] J.-H. Seo, S.-W. Kim, I.-H. Jung, T.-W. Park, J.-Y. Mok, Y.-G. Kim, and J.-B. Chai, "Dynamic Analysis of a Pantograph-Catenary System Using Absolute Nodal Coordinates," Vehicle System Dynamics, 44 (8), 615-630, 2006
- [18] F. Rauter, J. Pombo, J. Ambrósio, M. Pereira, A. Bobillot, and J. Chalansonnet, "Contact Model for the Pantograph-Catenary Interaction," JSME International Journal of System Design and Dynamics, 1 (3), 447-457, 2007
- [19] J. Ambrósio, J. Pombo, F. Rauter, and M. Pereira, 2008, "A Memory Based Communication in the Co-Simulation of Multibody and Finite Element Codes for Pantograph-Catenary Interaction Simulation," Multibody Dynamics, Bottasso C.L., Ed., ed., Springer, Dordrecht, The Netherlands, 211-231.
- [20] M. Schaub, and B. Simeon, "Pantograph-Catenary Dynamics: An Analysis of Models and Techniques," Mathematical and Computer Modelling of Dynamic Systems: Methods, Tools and Applications in Engineering and Related Sciences, 7 (2), 225-238, 2001
- [21] E. standard, "Railway applications Current collection systems Technical criteria for the interaction between pantograph and overhead line," CENELEC European Committee for Electrotechnical Standardization, Brussels, Belgium, 2012
- [22] EN 50367, "Railway applications Current collection systems Technical criteria for the interaction between pantograph and overhead line," ed.^{eds.}, 2006
- [23] T. Hughes, The Finite Element Method: Linear Static and Dynamic Finite Element Analysis, Prentice-Hall, Englewood Cliffs, New Jersey. 1987
- [24] N. Newmark, "A Method of Computation for Structural Dynamics," ASCE Journal of the Engineering Mechanics Division, 85 (EM 3), 67-94, 1959
- [25] K.-J. Bathe, Finite element procedures, Prentice Hall, Englewood Cliffs, N.J., 1996
- [26] P. E. Nikravesh, Computer-Aided Analysis of Mechanical Systems, Prentice-Hall, Englewood Cliffs, New Jersey. 1988
- [27] P. Flores, Kinematics and dynamics of multibody systems with imperfect joints : models and case studies, Springer, Berlin. 2008
- [28] J. Ambrósio , and P. Verissimo, "Improved Bushing Models for Vehicle Dynamics", Multibody System Dynamics," Multibody Systems Dynamics, 22 (4), 341-365, 2009
- [29] J. Ambrósio, F. Rauter, J. Pombo, and M. Pereira, 2010, "A Flexible Multibody Pantograph Model for the Analysis of the Catenary-Pantograph Contact," Multibody Dynamics: Computational Methods and Applications, Arczewski et al., (Ed.), ed., Springer, Dordrecht, The Netherlands, 1-27.
- [30] N. Zhou, and W. Zhang, "Investigation of the Influence of the Pan-Head Elasticity on Pantograph-Catenary Dynamic Performance," Proc. In

Proceedings for the 21st Symposium on Dynamics of Vehicles on Roads and Tracks.

- [31] C. P. Vyasarayani, T. Uchida, A. Carvalho, and J. McPhee, "Parameter identification in dynamic systems using the homotopy optimization approach," Multibody Syst Dyn, 26 (4), 411-424, 2011
- [32] C. Pereira, A. Ramalho, and J. Ambrósio, "A Critical Overview of Internal and External Cylinder Contact Force Models," Nonlinear Dynamics, 63 (No. 4), 681-697, 2011
- [33] H. M. Lankarani, and P. E. Nikravesh, "Continuous Contact Force Models for Impact Analysis in Multibody Systems," Nonlinear Dynamics, 5, 193-207, 1994
- [34] K. Lee, "A Short Note for Numerical Analysis of Dynamic Contact Considering Impact and a Very Stiff Spring-Damper Constraint on the Contact Point," Multibody Systems Dynamics, 26 (4), 425-439, 2011
- [35] P. Flores, M. Machado, M. T. Silva, and J. M. Martins, "On the Continuous Contact Force Models for Soft Materials in Multibody Dynamics," Multibody Systems Dynamics, 25 (3), 357-375, 2011
- [36] S. Djerassi, "Three-Dimensional, One Point Collision with Friction," Multibody Systems Dynamics, 22 (2), 173-195, 2012