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Numerical Analysis of Railway Track Vibrations

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Abstract

The numerical prediction models are used both for evaluation of the track-soil interaction forces as well as for prediction of the ground-borne vibrations. The dynamic track-soil interaction forces are calculated using a detailed train model and the dynamic behaviour of the layered spring-damper system and the through-soil coupling of the sleepers for the soil model [1,2,3,6,7]. The calculation of the ground-borne vibration level at the distance is based on the viscous-elastics soil model, too. In the frequency domain free-field response numerical results are presented using the response spectra and the frequency response function (FRF) of the viscoelastic soil medium at the distance, [6,8,9,10,11]. In the next step these functions can be applied to the structure (*e.g.* engineering and building) dynamic response calculation arising railway traffic using the relevant computational building structure model.

Keywords: microtremor, railway traffic effects on structures, prediction dynamic half space and structures response models, in situ experimental tests, ground vibration and structure response spectra, spectral analysis.

1 Introduction

The growing traffic volume, the higher population density and the diminishing distance between the track and the structure can be considered to be responsible for increasing vibration nuisance due to railway traffic. Regarding to the increasing amount of complaints related to vibration nuisance due to traffic, this type of problems is becoming more and more important. The increasing interest and awareness for the problem of vibrations in the built environment due to traffic among the population and the local and state authorities has triggered off the need for a better insight in the physical phenomena involved in the problem and for an estimate for the expected vibration levels.

Empirical models show a close relationship to a set of experimental data but the application of the model is limited to similar conditions. Also these models do not always provide insight in the influence of specific parameters. Numerical models allow the influence of various parameters to be investigated but a validation of the model with experimental data is required to verify the underlying theoretical assumptions. Even though the validation focuses on traffic induced vibrations, the numerical prediction model can be generally applicable to other types of vibration sources.
The dynamic train-track interaction is a coupled problem, contrary to vehicle-road interaction problems, that requires the simultaneous solution of the equations of motion of the train and the track. The train-track interaction forces due to the track unevenness are computed using a flexibility formulation. A two-dimensional linear vehicle model with a limited number of DOF is coupled to a linear elastic longitudinal invariant track model, which allows a solution of the equations of motion in the frequency-wavenumber domain. The transfer functions between the track and the soil and the computed interaction forces are used to compute the response at any arbitrary point in the free field [8,12,13].
An analytical expression for the spectral density of ground vibration as functions of distance from both roadways and railways respectively is formulated in terms of rail and wheel roughness, vehicle characteristics, track- soil interaction forces and the frequency response function for the ground. The use of the random process theory to predict the level of ground vibration in the vicinity of railways via calculation of the spectrum of vibration at half - space point is possible by the two principal ways: ***(i)*** - using a computer implementation of the theoretical expression for the rail roughness spectrum, the vehicle mass distribution spectrum and a model of vehicle dynamics and track-soil interaction and the frequency response function (FRF) of the ground by a method involving integral transform, ***(ii)*** - using average response force spectrum derived from experimental data for authorized railway category with corresponding track profile and the FRF of the ground and calculate response spectrum vibration at point by the same way as mentioned in (*i*). The random process theory in the dynamic ground properties investigation can be utilized as well. Via the input signal (due to traffic) measurement into the ground and the output signal measurement passing through the ground, *frequency response function, elastic and attenuation characteristics of the ground* can be obtained, [4,15,16].

2 Description track model - numerical approach

A numerical prediction model for ground-borne vibrations due to railway traffic on ballasted track requires the modelling of several components, as indicated in Figure 2.1. The train consists of the car-body, the bogie(s) and the wheelset(s). The wheelset is in contact with the rail and both are exposed to surface deterioration leading to defects which have an important influence on the wheel-rail interaction forces. The track consists of the rail, discretely supported by railpads and sleepers. The sleepers are founded on a ballast layer which is placed on the substrate. The track-soil interaction forces cause wave propagation to occur and induce ground-borne vibrations.

The literature review [15,17,18] shows that prediction models still use simplifications to predict the ground-borne vibrations due to railway traffic. Most assumptions are introduced for the computation of the track-soil interaction forces. Some models consider these forces to be point loadings proportional to the deflection curve of a rail on an elastic foundation. Other models assume a frequency independent stress distribution beneath the sleeper for which the time history is derived from a train-track model with the sleeper support modelled as rigid or as spring-damper systems. Soil models in literature vary from approximate solutions, including only the surface wave contribution, to horizontally layered viscoelastic half-space models. Few models have developed a systematic procedure to account for the rail roughness.

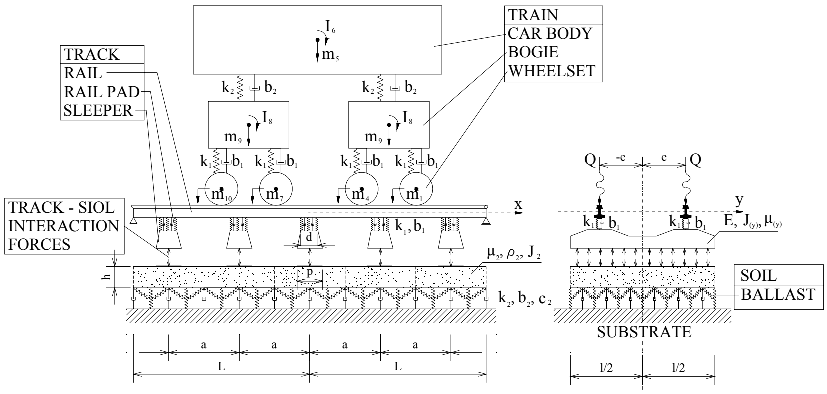

Figure 2.1: Components of the train-track-soil system

This paper presents a numerical prediction model which calculates the ground-borne vibration level due to railway traffic in two steps. The first step determines the dynamic track-soil interaction forces using a detailed train model and the dynamic behaviour of the layered spring-damper system and the through-soil coupling of the sleepers are accounted for in the soil model. The prediction of the ground-borne vibration level in the second step is based on the same soil model.

The car-body, the bogie and the wheelset are modelled as rigid bodies connected by springs and dampers. The wheelset is connected to the rail with a linearized Hertzian spring. The rail is modelled as a hinged Rayleigh beam with rotational inertia. The rail is supported discretely by sleepers modelled as rigid bodies with spring-damper systems representing the railpads. The sleeper is modelled as a short Rayleigh beam resting on flexural mass layer supported by discretely Pasternak spring-damper systems representing the elastic and attenuation characteristics of the railway ballast and substrate soils. As a result, the model evaluates the track-soil interaction forces in terms of the spectral density function which is often used as the statistical description of the rail roughness. This calculations followed by a second step in which the spectral density of the level of ground-borne vibrations is determined by FRF between track and unbounded soil.

There is another approach [5,14,16,19] to solve this problem modelling the ballast and the substrate by the use of the dynamic stiffness matrix of a set of rigid sleepers at the surface of a layered halfspace. This matrix is determined via the boundary element method using the rigorous Green's functions. The calculation of the track-soil interaction forces is performed for a number of artificial irregularity profiles derived from the spectral density function of the rail roughness. The statistical analysis of the resulting forces in the time domain leads to the spectral density function describing the track-soil interaction forces. The next step involves the determination of the response at a certain distance from the track using the same rigorous Green's function of the layered halfspace. The stress distribution between the sleeper and the halfspace is calculated and used to evaluate the free field response.

<u>Proposed prediction theoretical model</u> for vertical track vibration numerical program *Interaction*, [20] consists of three parts:

- model of vehicle
- model of train-track interaction
- model of track (sleepers/ballast and subsoil).

The frequency characteristics method (input-output) was used for calculation modelled feedback linear dynamic system train-track-soil system parameters. Final products of the numerical calculations are vehicle, rail, sleeper, railpads and ballast frequency response functions (also sleepers deflection and bending moment in time domain) using spectral density functions (SDF) of the rail roughness used by railway operators or experimentally measured in situ for case study. An important example of non-linear behaviour is the wheel-rail contact but also the railpads and the suspension of the train can deflect in a non-linear manner. Although the results presented in this study are limited to linear analyses. Also in this model dynamic response of the sleepers is symmetrical to longitudinal axis of the track (rail roughness coherence function for left and right rail is equals to ≈ 1).

2.1 Track model

The track model commonly found in the literature represents the rail as infinite Timoshenko, Euler or Rayleigh beam on a continuous uniform support, Figure 2.2a. The beam is taken as uniform flexural rigidity *EI,* rail mass per unit m_r and distributed sleeper mass m_s. The railpad stiffness and viscous damping constant per unit length are taken to be k_1 and b_1, respectively; the corresponding parameters for ballast are k_2 and b_2. An harmonic point force $p(t) = P \cos \omega t$ is assumed to run at constant velocity v along the rail. The FRF of the track excited to a harmonic force for proposed prediction model are discussed in this section. The track model consists of two parts: (i) model of sleeper with ballast and subballast, and (ii) track model.

2.1.1 Sleeper model

The sleeper is modelled as a short symmetrical Rayleigh beam (E_2I_2,A_2) resting on flexural mass layer (ρ_b) supported by discretely Pasternak spring-damper systems ($k_2(y),c_2(y),b_2$) representing the elastic and attenuation characteristics of the railway substrate soils, Figure 2.2b. Viscous damping (b_2) is constant per unit length of the sleeper. The mass layer has height of *h,* width *a* length *l* and specific mass

ρ_b(kgm^3).The sleeper symmetrical excitation by force Q_1 is taken into account at the contact area sleeper-railpads. The motion equation of sleeper deflection v_2 in time domain is

$$E_2 I_2(y)\frac{\partial^4 v_2}{\partial y^4}+\mu_2(y)\frac{\partial^2 v_2}{\partial y^2}-I_2(y)\rho_2\frac{\partial^4 v_2}{\partial y^4 \partial t^2}+b_2\frac{\partial v_2}{\partial t}-c_2(y)\frac{\partial^2 v_2}{\partial y^2}+k_2(y)v_2=\delta\left(y-\frac{l\pm e}{2}\right),$$

where

(2.1)

$$\mu_2(y)=A_2(y)\rho_2+a.h\,\rho_2\,; \qquad Q_2=\delta\left(y-\frac{l\pm e}{2}\right)e^{j\omega t}\,.$$

The equation (2.1) after Fourier transformation gives equation for frequency response function

$W_2(y,i\omega)=W_{v_2}^{Q_1}(y,i\omega)$ ‹upper index Q_1 means *input* and inferior v_2 *output* in frequency domain

$$E_2 I_2 W_2^{////}-\left(c_2-I_2\rho_2\omega^2\right)W_2''+\left(k_2-\mu_2\omega^2+i\omega b_2\right)W_2=\delta\left(y-\frac{l\pm e}{2}\right). \qquad (2.2)$$

The equation (2.2) can be solved by *Ritz method* as follows

$$W_2(y,j\omega)=\sum_{j=0}^{n}\alpha_i(j\omega)\varphi_i(y),$$

$$\left[(A\varphi_i,\varphi_j)\right]\{\alpha_j\}_{n+1}=\{(f,\varphi_j)\}_{n+1},$$

where base $\varphi_i(y)=\begin{Bmatrix}1 \;\; for \;\; i=0 \\ \sin\dfrac{(2i-1)\pi\, y}{l}\end{Bmatrix}$, *for* $i = 1..n$, $n = 3$

and $$(A\varphi_i,\varphi_j)=\int_0^{\frac{l}{2}}\left\{EI_2\varphi_i''\varphi_j''-\left(-I_2\rho_2\omega^2+c_2\right)\varphi_i'\varphi_j'+\left(k-\mu\omega^2+i\omega b_2\right)\varphi_i.\varphi_j\right\}dy\,,$$

$$(f,\varphi_j)=\int_0^{\frac{l}{2}}\delta\left(y-\frac{l-e}{2}\right)dy=\begin{Bmatrix}1 \;\; for \;\; i=0 \\ \sin\dfrac{(2i-1)\pi(l-e)/2}{l}\end{Bmatrix} \; for\; i=1..n, \;\; n=3.$$

2.1.2 Track model

The rail is modelled as a symmetrical *Rayleigh hinged beam* ($E_1 I_1,\mu_1,\rho_1$) supported by sleepers and connected with rail by discretely spring-damper systems (k_1,b_1). Two different track supports model is possible take in to account: (a) *as continuous* *elastic layer*, Figure 2.2c or (b) *as track on a discrete support*, Figure 2.2d. The stiffness and damping for both supports model have uniform dimension per rail unit length. The rail length 2*L* of 25 m for common rail track is sufficient for numerical calculations results accuracy.

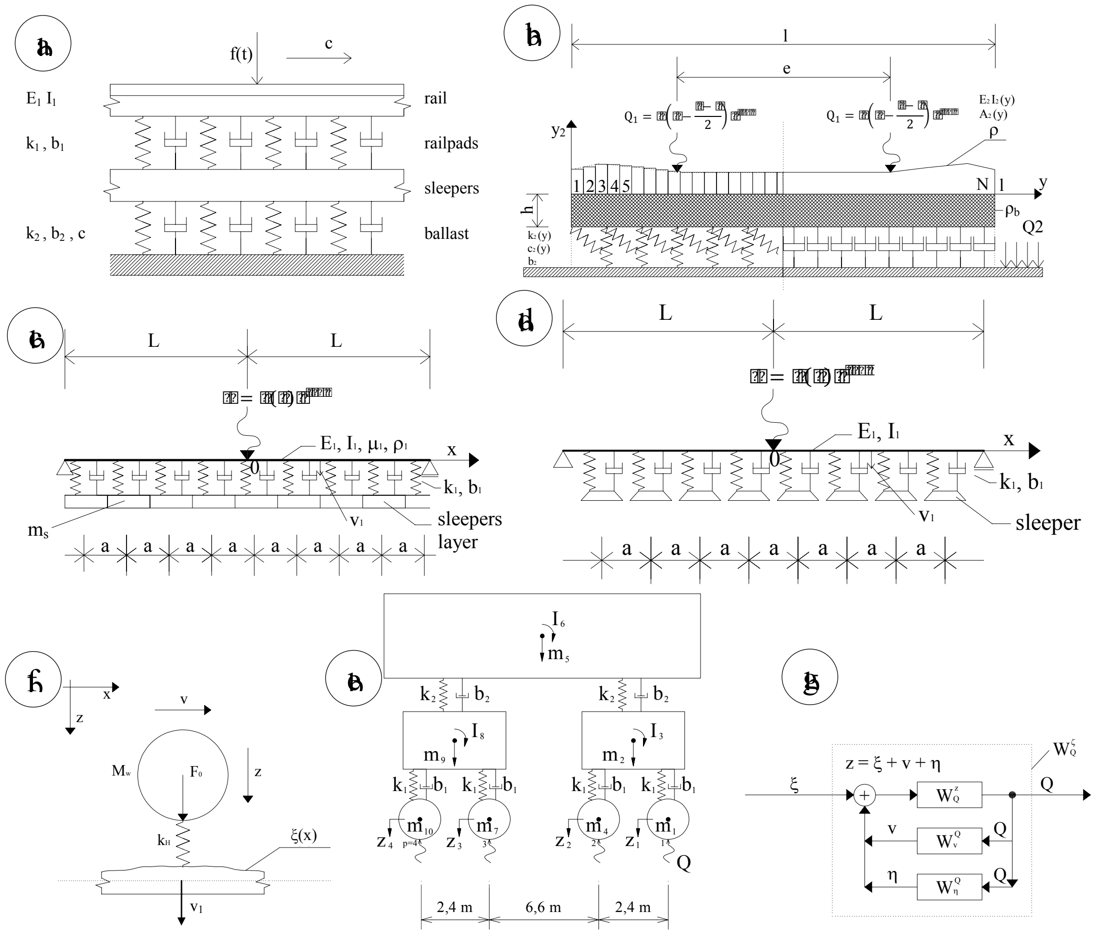

Figure 2.2: The railway components model for numerical analysis

The motion equation of rail *FRF* $W_{v_1}^{Q}(x, j\omega) = W_1(x, j\omega)$ in frequency domain is

$$E_1 I_1 W_1^{////} - I_1 \rho_1 \omega^2 W_2^{''} + \left\{ \frac{k_1}{1 + k_1 W_2\left(\frac{l-e}{2}, j\omega\right)^{-1}} - \mu_1 \omega^2 + j\omega b_1 \right\} W_1 = \delta(x), \tag{2.3}$$

$W_1(x, j\omega) = W_1^{''}(x, j\omega) = 0, \quad for \;\; x = \pm L.$

The equation (2.3) is solved in the same way as (2.2) with respect to the different base

$$\varphi_i(x) = \left\{ \sin \frac{(2i-1)\pi(x+L)}{2L} \right\}, \qquad for \quad i = 1..n, \quad n = 16.$$

2.1.3 Vehicle model

The three types vehicle models are proposed for numerical calculation FRF displacements of the wheel centres (a) *four axles - two bogies vehicle* with 10 degree of freedom with 10 degree of freedom, Figure 2.2e, (b) *two axles vehicle without bogies* with 4 degree of freedom, and (c) *one axle vehicle model* with 3 degree of freedom. The motion equations of vehicle wheel centres displacements z in time domain are

$$[M]\{\ddot{z}\} + [B]\{\dot{z}\} + [K]\{z\} = \{E\}^i e^{j\omega t}, \tag{2.4}$$

where: $[M]$ is stiffness matrix, $[K]$ - mass matrix, $[B]$ - damping matrix, $\{z\}$ - vector of unknowns dynamic displacements and angular rotations, $\{E^i\}$ - *i-th* column of unit matrix. The motion equations in frequency domain after the Fourier transform of the equation (2.4) is

$$\left([K] + i\omega[B] - \omega^2[M]\right)_{N,N} = \{E\}_N^j \tag{2.5}$$

from which *FRF* of wheel centres displacements

$$\left[W_z^Q(j\omega)\right]_{p,p} \leftarrow \left([K] + i\omega[B] - \omega^2[M]\right)_{N,N},$$

where: N is degree of freedom, p - number of axles and $\leftarrow$ is math operator for choose matrix elements with indexes belonging to the wheel centres displacements and laying on the intersections matrix rows and columns.

2.1.4 Wheel-rail interaction model

The spring between wheel and rail takes account of *Hertzian deformation* in the contact region. Its stiffness k_H is the incremental stiffness under the dead load Q carried by the wheel. According to the Hertz definition the contact force is

$$F = k_H y^{3/2},$$

where: y is contact area deformation and k_H is the Hertz contact constant, Figure 2.2g. For numerical computations it needs to do linearization of this equation for F at the close area surrounding a force acting point, than

$$k_{lin} = \frac{dF}{dy} = \frac{3}{2} k_H^{2/3} F^{1/3}.$$

It is shown in [21] that for F = 75 kN and wheel of 1m diameter is

k_{lin} = 1,4. 109 [N/m] for new wheels,
k_{lin} = 1,6. 109 [N/m] for worn-out wheels.

Finally, the solution for FRF of the linear dynamic system model in which on input are rail roughness ξ and on output are wheel forces Q enable calculations of the matrix FRF according to scheme as shown Figure 2.3. The dynamic displacement of the wheel z is defined by $z = \xi + v + \eta$, where ξ represents rail roughness, v - rail vertical deflection and η - wheel and rail contact deformation in contact location. The matrix of the interaction wheel-rail system *FRF* $\left[W_Q^{\xi}(j\omega)\right]_{p,p}$ is therefore

$$\left[W_Q^{\xi}\right] = \left\{\left[W_z^{Q}\right] - \left[W_{v_1}^{Q}\right] - \frac{1}{k_{lin}}[E]\right\}^{-1} . \tag{2.6}$$

2.1.5 Track irregularities

The track irregularities are great source of the track and vehicles dynamic excitations. Such excitation arises from discrete irregularities such as wheel flats and rail joints as well as periodic irregularities such as corrugation of the railhead. It is assumed that excitation of the track arises from a wheel passing over a sinusoidal irregularity on the rail head (Figure 2.2g). The response to both discrete and periodic non-sinusoidal profiles can be found by Fourier analysis from the response to a sinusoidal profile. The stochastic theory analysis enables to define irregularities by PSD function by

$$S_{\xi}(\Omega) = f(\Omega), \tag{2.7}$$

where: $\Omega = 2\pi / L$, and L is the wavelength of the corrugation. The function (2.7) can be also written by

$$S_{\xi}(\Omega) = A.\Omega^{-a}, \tag{2.8}$$

where *A, a* are empirical (or experimental) constants. The distance x is used as the independent variable to define ξ (x). Corresponding to the frequency f [Hz], with time as the independent variable, the cyclic frequency of path ΔF is defined with the units [cycles/m]. The spectral density function $S_{\xi}(\Omega)$ of the rail roughness is expressed in [mm^2/(cycles/m)] = [mm^3]. As an example of the experimental measured track irregularities PSD results [20] is defined by

$$S_{\xi}(f) = 9{,}527\left(\frac{F}{0{,}04}\right)^{-2{,}101},$$

presenting PSD track irregularities on the *CSD* railways line in section between stations *Přelouč - Řečany.*

2.1.6 Calculation of arbitrary FRF and standard deviation

The frequency response function of arbitrary dynamic system part is calculated by rule of *FRF* summing as follows

$$W_Q^I = W_{Q1}^I . W_{Q2}^{Q1} . W_{Q3}^{Q2} .... W_{Qi+1}^{Qi} ..... W_Q^{Qn} \quad . \tag{2.9}$$

The same way is used for summing of the FRF matrix. E.g. consider a dynamic system with a defined input rail roughness ξ producing a defined output sleeper deflection v_2, than the FRF is given by

$$W_{v_2}^{\xi} = W_Q^{\xi} . W_{Q_1}^{Q} . W_{v_2}^{Q_1} .$$

Consider a dynamic response of the track due to moving vehicle with speed c [m/s] when the wheel rolls over a rail having a roughness ξ described by power spectral densities S_ξ (PSD). The standard deviation σ (or *rms* value) of arbitrary quantity Q_i beneath i-th wheel is calculate by integration of a diagonal element i,i of the PSD matrix of quantity Q_i by

$$\sigma_{Q_{i,i}}^2 = \int_0^\infty S_{Q_{i,i}} d\omega \rightarrow \sigma = \sqrt{\sigma_{Q_{i,i}}^2}$$

and required PSD matrix unknowns quantity Q_i is defined by

$$\left[S_{Q_{i,i}} \right]_{p,p} = \left[W_Q^{\xi} \right] . \left[S_\xi \right] . \left[W_Q^{\xi} \right]^T , \tag{2.10}$$

where the roughness PSD matrix is given by

$$\left\lfloor S_\xi \right\rfloor_{p,p} = S_\xi \left[R \right]_{p,p} \tag{2.11}$$

in which $[R]$ is the geometrical matrix with elements

$$R_{i,j} = \exp\left\{ j \frac{\omega}{c} \left(a_i - a_j \right) \right\} , \tag{2.12}$$

where: c is a vehicle velocity, ω - angular frequency, a_i - the *first* and *i-th* wheels distance.

2.1.7 The response of track resting on a continuous rail supports

An advantage of the continuous track model (Figure 2.2c,d) is that enables for arbitrary track variables to calculate [20]: FRF, input spectra and standard deviation of the track dynamic response parameters - dynamic forces Q, Q_1, Q_2, dynamic deflections v_1, v_2, wheel centre dynamic displacement z and dynamic bending moment in rail and sleeper. As an example on Figure 2.3 is plot of numerical calculations results for vertical track receptance of rail deflection v_1 due to wheel contact forces Q.

Table 2.1 <u>Parameters used in calculation</u>

Sleeper SB8: $k_2 = 49{,}40$ Mpa; $b_2 = 0{,}023$ Mpa.s; $c_2 = 0$; $h = 0{,}45$ m; $a = 0{,}55$; $\rho_b = 0{,}0017$ Mkg.m^{-3}.

Rail R65: $L = 12{,}12$ m; $k_1 = 217\,000$ Mpa; $b_1 = 0{,}037$ Mpa.s; $E = 210000$Mpa; $I_1 = 3{,}6.10^{-5}$ m^4; $m_r = 65$ kg.m^{-1}.

Vehicle: SKODA E 699; 10 degree of freedom; 4 axles; $k_H = 1{,}5.10^9$ [N/m]; mass of axle:1250 kg; mass of bogie: 4750 kg; mass of body casing: 23.500 kg; bogie inertia moment: $5{,}5.10^3$ kgm^2; bogie casing inertia moment: 5.10^5 kgm^2; $k_1 = 7{,}266.10^5$ Nm^{-1} spring stiffness: $k_2 = 9{,}5.10^6$ Nm^{-1}; $k_1 = 7{,}266.10^5$ Nm^{-1}; spring damping: $b_1 = 7{,}37.10^4$ Nm^{-1};$b_2 = 3{,}68.10^4$ Nm^{-1}; axle base: 2,8 m; boggie base:10,3m.

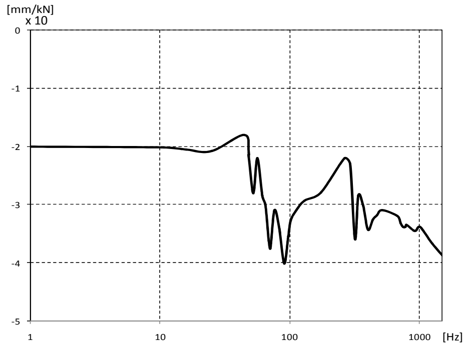

Figure 2.3: The vertical track receptance of rail deflection v_1

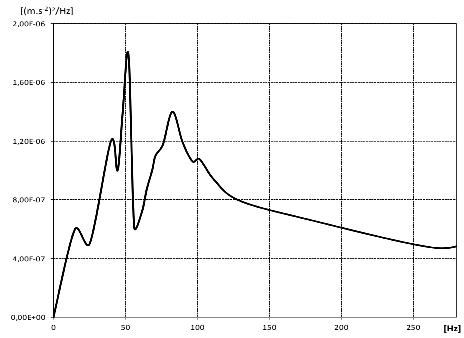

Figure 2.4: The input PSD $G_{\ddot{v}_2\ddot{v}_2}(\omega)$

From proposed model it is possible to calculate also sleeper dynamic deflection or acceleration PSD as the input spectra $S_{v_2v_2}(\omega)$ or PSD $G_{\ddot{v}_2\ddot{v}_2}(\omega)$ into the ballast and roadbed, see Figure 2.4. To predict the level of ground vibration in the vicinity of railways it needs to calculate the response spectrum at distance point on the surface of a linear viscous - elastic half space $S_{ww}(\omega)$ via the frequency response function (FRF) - $H_{ik}(\omega)$ of the ground by a method involving integral transform,($w \equiv z$).

3 Prediction models for ground vibration propagation from railways

Prediction models for ground vibration from railway train to nearby region involve consideration of two processes: (1) the vibration generation process, and (2) the vibration propagation process. These processes should be treated separately. The frequency response function (transfer function) of the ground can be defined by both analytical and experimental method involving application of the spectral analysis to calculate of the ground response spectrum vibration at halfspace point. The random process theory for the dynamic ground properties investigation can be utilized as well. Via the input signal (e.g. due to traffic) measurement into the ground and the function and elastic and attenuation characteristics of the ground can be obtained.

3.1 The analytic - experimental approach

The literature review [15,17,18] shows that the models still use simplifications to predict the ground-borne vibrations due to railway traffic. The analytical prediction model [22] results have given good agreement with measured ones and are applicable for real engineering needs and it is suitable to be mentioned it here. In this model impacts from the wheels passing over the rail joints are assumed to generate the damped free vibrations in the rail. These generated vibrations are then transmitted to ballast, roadbed and ground. If the frequencies of the generated vibrations are much higher than the natural frequency of the ballast, the input to the ballast is given from the envelope function of the vibration generated at the rail. The ground vibration recorded at a distance from a railway is analysed assuming it to be a random and statistically stationary function of time. Perhaps the most descriptive representation of the traffic influence on a half space is provided by a response spectrum. We consider response at a point due to random line excitation it needs to apply random process theory to predict the level of ground vibration at the distance by vibration spectrum calculation via FRF of the ground using a method involving integral transform.

On the surface of a linear viscous-elastic half space, the displacement response spectrum $S_{ww}(\omega)$ can be expressed in terms to the input spectrum of sleeper displacement $S_{v_2v_2}(\omega)$, [4] by

$$S_{ww}(\omega)=\left|H(y_s,\omega)\right|^2 S_{v_2v_2}(\omega), \tag{3.1}$$

where $|H(y_s,\omega)|$ is the frequency response function magnitude for the half space medium and w is the displacement of the surface measured at a distance y_s from an applied force F. In this approach it can be used as the input spectra to the ballast and roadbed $S_{v_2v_2}(\omega)$ or PSD $G_{\ddot{v}_2\ddot{v}_2}(\omega)$ calculated by numerical models, e.g. mentioned in Section 2.

<u>The analytic-experimental approach</u> suggests the test and the theory data combination to calculate the prediction level of ground vibration. In this process as an input signal can be used accelerations spectra (or spectral densities) derived from experimental data bank for authorized railway category with corresponding rail profile or accelerations spectrum $\overline{S}_{\ddot{w}\ddot{w}}(f)$ measured at nearest ground point to the track for *individual case study,* ($\ddot{w}=\partial^2 w/\partial t^2$). The frequency response function (transfer function) of the ground can be derived via experimental impulse seismic method (ISM) or cross-hole test data, from which *elastic and attenuation parameters of the ground* can be obtained, too. The measuring output response acceleration spectrum at the distance $S_{\ddot{w}\ddot{w}}(f)$ due to input accelerations spectrum $\overline{S}_{\ddot{w}\ddot{w}}(f)$ the FRF - $H(f)$ is possible derived, using eq. $S_{\ddot{w}\ddot{w}}(f)=\left|H(f)\right|^2\overline{S}_{\ddot{w}\ddot{w}}(f)$ in accordance with (3.1).

An experimental case study of ground vibration transmission from a railway [23,24] was carried out adjacent to the ŽSR railway Bratislava - Vienna, track No.1 (No.2) in Bratislava (BA). The track is straight and well situated on level ground (sandy loam - 3,5 m and gravel sand - 12,0 m). This permits the ground to be modelled as a damped, viscoelastic half space. As an example on Figure 3.1 are depictured the input and output PSD at the ground points BK1 and BK3 and halfspace soil medium - FRF. The truck and adjacent ground region vibrations were excited by the bulk granular materials train, 76,0 km/h.

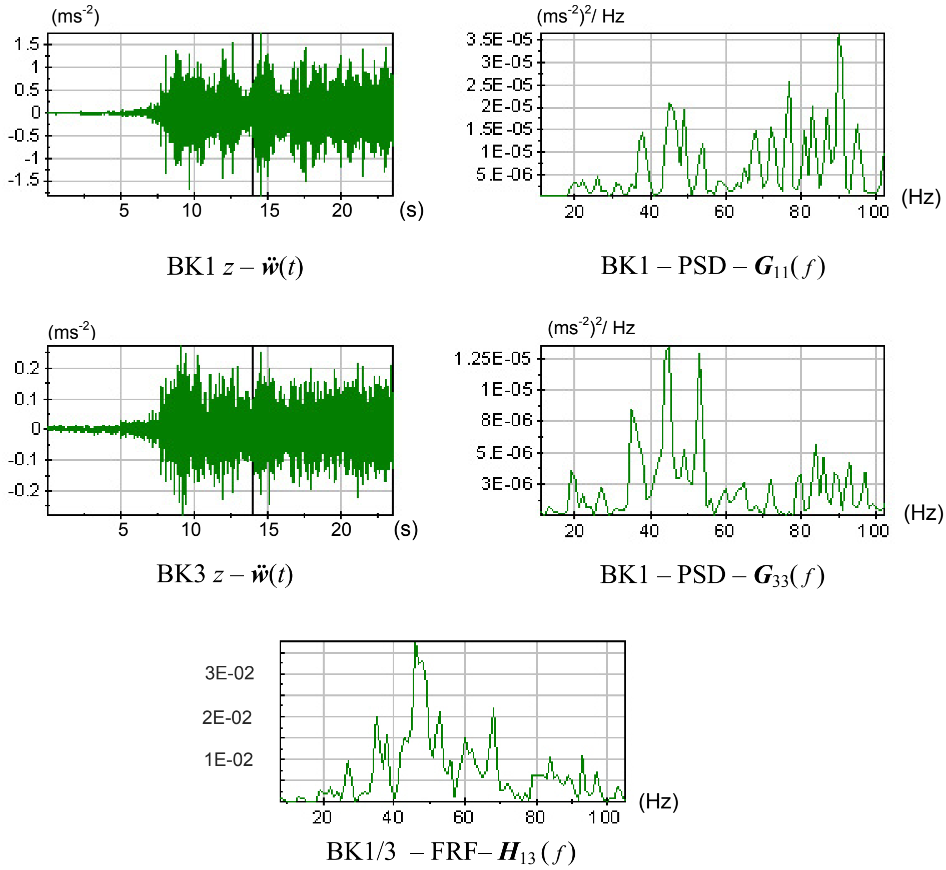

Figure 3.1 The accelerations time histories, PSD and FRF of ground vibration at points BK1, BK3 due to train of Bulk Carriers of Granular Materials, (76.0 km/h).

4 Conclusions

This paper presents an overview of numerical prediction model for the ground-borne vibrations at the distance point of the visco elastic halfspace due to railway traffic. The numerical and numeric-experimental approach for ground response at the distance calculation procedure were introduced. Based on the results presented in this paper the following conclusions can be drawn:

- The track numerical model can account for many parameters of the train-track-soil interaction problem. The frequency domain analysis enables the model to perform calculations with linear behaviour. Final products of the numerical calculations are vehicle, rail, sleeper, railpads and ballast frequency response functions (also sleepers deflection and bending moment in time domain) using spectral density functions of the rail roughness.

- To predict the level of ground vibration in the vicinity of railways it needs to calculate the response spectrum at distance point on the ground surface $S_{ww}(f)$ via the frequency response function FRF - $H_{ik}(f)$of the ground by a method involving integral transform.
- The modelling of the soil as a viscoelastic halfspace represents the key feature of the prediction model. This soil model is used both for the evaluation of the track-soil interaction forces as well as for the prediction of the ground-borne vibrations.

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