

## **Vulnerability Assessment of Structures in a Low-To-Moderate Seismic Region based on Ambient Vibration Test Modal Data**

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### **Abstract**

Classical methods for structural seismic vulnerability assessment often link directly the probability of a given level of damage to a specific construction type arising from ground motion. Recent methods feature more detailed and physical components that not only allow detailed sensitivity studies to be undertaken, but also provide straightforward calibration for various characteristics of the building stock. However, these models often require an increased amount of input parameters, and thus time and resources, to construct an earthquake damage model based on such analyses. Hence, a balance is needed between the computational intensity and the required data.

In this paper a simplified structural model is proposed to represent the dynamic behaviour for a particular class of structure. The model parameters are estimated directly from an ambient vibration test. The covariance-driven stochastic system identification algorithm (SSI-cov) is employed for modal identification. By using SSI-cov, the uncertainty of the modal parameters can also be delivered in a single measurement.

This process is applied to build a fragility curve of a typical frame building structure according to the first damage grade in a low-to-moderate seismic region. The results show that the source of modelling uncertainty can be explicitly accounted for. The procedure is very simple to implement and can be applied to different structural types.

**Keywords:** vulnerability assessment, fragility curve, ambient vibration, SSI-cov.

## **1 Introduction**

In seismic regions, a vulnerability assessment aims at categorizing key assets – from residential dwelling to strategic infrastructures – and potential threats to such assets in the event of a future earthquake. Seismic vulnerability from the perspective of disaster management means assessing the threats from potential earthquake hazards

to the population and to infrastructures. It may be conducted in the social, economic and environmental fields. One of the major task in seismic vulnerability assessment is to predict the expected damage of a given earthquake in existing structures.

Empirical methods were developed for vulnerability assessment in strong seismic regions with destructive earthquakes e.g. [1]. Empirical method can be mainly classified into two types based on post-seismic inventories to build damage-motion relationships. The first one is damage probability matrix, which expresses in a discrete form the conditional probability of obtaining a damage threshold due to a ground motion intensity. The second one is referred to as continuous vulnerability functions describing the probability of exceeding a given damage given the earthquake intensity. These methods were popular in the past because the cost of wide area studies is relatively low due to the small number of parameters required.

Recent more advanced analytical methods have been developed in complement to the classical ones, e.g. [2, 3, 4]. These methods aim at obtaining capacity and fragility curves of various kinds of structures through structural modelling. Furthermore, the ground motion can be diversified into response spectrum instead of peak ground acceleration and different sources of uncertainties can be accounted for. However for existing constructions, the adaption of structural models must assume a large set of unknown parameters due to lack of understanding about structural plans, ageing and details. A complete literature review for both classical and analytical methods up to 2006 can be found in [5].

Lately, research interest has devoted to incorporating structural dynamic properties into fragility functions [6, 7]. Those parameters can be obtained directly from an ambient vibration test and site experimental data are rather reliable. This method is perfectly suitable to apply in the low-to-moderate seismic regions, where there is often lack of both information on recent earthquake and ground motion record.

This paper further develops the analytical method using experimental modal parameters with a low cost experiment approach. Furthermore, the uncertainty when extracting the modal parameters from the experimental data can be explicitly accounted for. Consequently, facility functions can be obtained with higher confidence.

## **2 A short review of ambient vibration test**

The dynamic characteristics of a structure can be described by modes, which include natural frequency, damping ratio and mode shape. These modes can be identified from physical, measurable quantities such as acceleration, velocity, strain, etc. with acceptable accuracy and under low cost condition using natural (free) excitation sources. The general term for such a test is ambient vibration test (AVT) or operational modal analysis (OMA). Supported by robust mathematical identification models, the identified dynamic properties often reveal the dynamic signature of the construction and can also be correlated to a structural model. AVT has been widely used in civil engineering practiced for various structural health monitoring purposes as well as structural control.

## 2.1 Instrumentation

Vibration transducers can be very sensitive to small movements. For example, a low cost accelerometer can resolve an acceleration of less than  $10^{-5} \text{ m/s}^2$  ( $1 \mu\text{g}$ ) and can sense an amplitude of  $10^{-7} \text{ m}$  ( $0.1 \mu\text{m}$ ) at a frequency of 1 Hz.

Most transducers produce an electrical output signal proportional to the physical quantities to be measured. There are two common technologies utilizing either piezoelectric or capacitive sensing. The piezoelectric accelerometers are capable of measuring very fast acceleration changes but they cannot measure uniform acceleration such as gravity. The piezoelectric effect of either quartz crystal or ceramic material is low-cost and very reliable with high sensitivity and high resolution (a very low internal noise level). Piezoelectric sensors are the most popular in AVT practice. On the other hand, capacitive accelerometers are capable of responding to both uniform acceleration and varying acceleration but with limitation to low frequencies. However, its working range up to several hundred hertz is more than enough for most ambient excitation sources and civil engineering structures. Capacitive accelerometer is often referred to as micro electro mechanical system (MEMS) sensor. Mechanical parts and electronic circuits combined to form a miniature device, typically on a semiconductor chip, with dimension scale can be less than  $100 \mu\text{m}$  in size. MEMS sensors are easy to develop in three directions (triaxial sensors). Furthermore, MEMS sensors are believed to be less sensitive to ambient temperature variances. For civil engineering applications, MEMS sensors are commonly used in seismographs.

Optical fiber as the sensing element that makes use the principle of low-coherence optical interferometry has been employed successfully for output only modal analysis [8, 9]. The measured dynamic strains can be very useful in many vibration-based tasks. One of the drawback of optical sensing is its high cost. The optical sensors are attached to a very expensive interrogation unit with a limited number of channels. But may it soon be affordable with fast technological innovations.

Remote sensing techniques such as GPS, photogrammetry, laser or radar are not yet widely used. However, remote sensing are promising because of its non-contact nature which can be a huge advantage to use for large scale measurement. Some techniques can measure up to a distance of a few hundred meters and other techniques can be used to measure multi-DOFs.

Signal conditioning is not a major problem in the case of a classical experimental modal analysis, such as methods widely used in mechanical engineering, because the intensity of the signals to be measured remains more or less constant. Whereas, an insufficient dynamic range or noise frequently reduces the effectiveness of AVT. Advancement in data acquisition with the use of high resolution analogue-to-digital conversion (ADC) rate, e.g. 24-bit, to copy with various signal level without risking a bad signal-to-noise ratio.

Nowadays, the use of wireless sensor is more and more popular. A triaxial sensor is often embedded in a stand-alone acquisition unit with onboard signal conditioner, digitizer, autonomy power supply, memory card, filter and a possibility even to connect

(by cable) external sensors. The units are communicated by configuring a wireless network. Time synchronization is a rather delicate issue as the synchronization error influences the identified mode shape significantly and has to be controlled under certain limit. Wireless sensors can make tremendous difference in comparison the the wired system because of its fast deployment. Therefore, it is suitable for large scale measurement.

## 2.2 Signal processing and modal analysis

Basically, an AVT consists of three distinctive step, i.e. (i) field measurement, (ii) system identification, (iii) modal parameter extraction. For practical purposes, the distinction between step (ii) and step (iii) is rarely important as they are both computational in nature. Therefore, most of the modal analysis softwares include both these two steps.

AVT works with the fact that no information on the excitation side is measured. This lack of input is translated into a stochastic dynamic load description with mean value equals to zero. And the frequency content of the loading is supposed to be equally distributed over the frequency range of interest. The later is known as white noise assumption. Amplitude is apparently unknown. The white noise is valid for most kind of ambient excitation except for harmonic loading presence. Under the white noise description, the forces at different time instances are uncorrelated.

The output is the measured structural response. The positive power spectra density (PSD) of the measured signals is defined as the Fourier transform of the positive correlation functions. The positive PSD is by definition real and constant under white noise input. Then, it has the form of a frequency response function (FRF). Therefore, we can use the classical input-output modal analysis approaches such as the peak picking (PP) and the complex mode indication function (CMIF).

The peak picking method, as the name suggests, identifies the modal parameters by figuratively selecting the peaks of the averaged normalized PSD function plot. Mathematical speaking, it depends on the modal decomposition properties of the FRF. The CMIF method similarly selects the peaks of the function-of-frequency plot. This function is the singular values of the identified FRF matrix. Since for the case of AVT with unknown excitation, the FRF is replaced by the sum of the identified positive PSD. Therefore, sometimes it is referred to as frequency domain decomposition or FDD.

These two approaches usually yield rather rough estimates and are often referred as nonparametric methods. PP is physically intuitive but often fails when structure has closely spaced modes and when modes are not weakly damped. The accuracy of natural frequency is determined by frequency resolution. The damping calculated by half power bandwidth is unreliable. FDD is in fact an extension of PP with improvement in mode multiplicity separation.

Later development into parametric methods often go through all the about processes. The so-called parametric method is based on a model. The most convenient way to model a vibrating structure is to rearrange the discretized equation of motions

into a state-space model.

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k + w_k \\y_k &= Cx_k + Du_k + v_k\end{aligned}\tag{1}$$

where  $u$  is the input vector,  $y$  is the output vector and  $x$  is the state vector;  $w$  and  $v$  are noises due to modeling and measurement inaccuracies, respectively;  $A$ ,  $B$ ,  $C$  and  $D$  are state matrices. This state-space equation has an exact solution in the free vibration case. In AVT, both excitation input and noise are unknown. From a mathematical point of view, it is impossible to differentiate between those terms in Equation (1). Therefore, the input  $u$  can be included in the noise term and we have the stochastic state-space model:

$$\begin{aligned}x_{k+1} &= Ax_k + w_k \\y_k &= Cx_k + v_k\end{aligned}\tag{2}$$

The covariance-driven stochastic subspace identification (SSI-cov) is developed base on Equation (2) by identifying the system matrix  $(A, C)$  from output-only data with the assumption of white noise. Once the output correlation function is calculated, it can be mathematically decomposed (using single-value decomposition or SVD). Knowledge of  $(A, C)$  suffices to estimate modal parameters of frequency  $f_i$ , damping ratio  $\xi_i$  and mode shape  $\phi_i$  [10].

In system identification parlance, SSI-cov method is often called stochastic realization. The subspace identification method can be considered as an extension of system realization where direct computation of correlation is replaced by geometrical projection. In fact, the notions of covariance and projection are closely related, because they both aim at cancelling out the uncorrelated noise. Subspace identification is more flexible than realization as it makes the combination of measured and unmeasured loads possible. In the practice of AVT, the subspace identification is purely based on treatment of measured time instants and specifically named data-driven stochastic subspace identification or SSI-data [11].

Another popular parametric method in AVT is the (stochastic) poly-reference least squares complex frequency domain (pLSCF), which is also known by its commercial name PolyMAX [12]. In fact, it is the most simple yet efficient method for prediction error minimization. The main advantage of this method is to recognize spurious modes due to over-modelling.

In summary, parametric methods are less intuitive than PP and CMIF but they are more consistent. Moreover they can deal with closely spaced modes as well as highly damped modes. pLSCF is less accurate than SSI methods. SSI-cov and SSI-data are of almost the same accuracy. They are both non-iterative method and can be implemented into a robust reference-based version. Interestingly, the statistical accuracy (covariances) of  $(A, C)$  can also be evaluated [10]. The covariance estimation of extracted modes will be exploited in this paper.

### 3 Dynamic response by mode superposition

Consider a lumped multi DOF elastic system with translational excitation on rigid foundation, thus neglecting the soil-structure interaction. For the purpose of discussion, the structure is assumed to be a shear frame building with equal lumped mass  $m$  in each rigid floor girder and the same story-to-story stiffness  $k$  in the columns of each story.

Mode superposition is based upon the fact that the deflected shape of the structure may be expressed as a linear combination of all the modes:

$$\mathbf{u} = \sum_{n=1}^N \phi_n y_n \quad (3)$$

or in a compact matrix form:

$$\mathbf{u} = \Phi \mathbf{y} \quad (4)$$

with  $N$  is the number of modes and  $\mathbf{y} = \{y_1, y_2, \dots, y_N\}^T$ .

The coefficients  $y_n$  are the modal amplitudes, which vary in time. In the case of a system with lumped masses, the motion of the  $i$ th mass is given by:

$$u_i = \phi_{i1}y_1 + \phi_{i2}y_2 + \dots + \phi_{iN}y_N \quad (5)$$

The modal amplitudes,  $y_n$ , are often referred to generalized coordinates, which may be contrasted with the natural coordinates  $\mathbf{u}$ . Modal analysis is a process of decomposing the equation of motions, using generalized coordinates, so as to obtain a set of differential equations that are decoupled, each of which may be analyzed as a single DOF. The total response of each DOF now can be obtained by solving the  $N$  uncoupled modal equations in the time-domain expressed by the Duhamel integral:

$$y_i = \frac{1}{m\omega_i} \int_0^t u_s''(\tau) \exp[-\xi_i\omega_i(t-\tau)] \sin \omega(t-\tau) d\tau \quad (6)$$

with  $\omega_i$  is the damped angular frequency which is the function of the identified frequency and damping ratio. Mode superposition is an efficient method of analysis for many problems for two main reasons. The first is because the modal summation, given by (3), is usually dominated by the lower modes of vibration (also typical for earthquake response), allowing higher modes to be excluded from the analysis without significant error. The second reason, more importantly, for the effectiveness of mode superposition is that modal analysis can be done experimentally such as the AVT described earlier. Because information on the mass and stiffness of structures is often scarce at large scale assessment.

## 4 Fragility curve estimation based on modal data

According to HAZUS manual [2], the direct physical damage to general building stock after an earthquake can be described by one of the five damage states: none, slight, moderate, extensive and complete. General building stock constitutes typical structures of a given model building type designed to either high-code, moderate-code, low-code or pre-code seismic standards. The term pre-code is referred to constructions that were not seismically designed.

A fragility curve expresses the conditional probability that a building reaches or exceeds a given damage state for a given level of shaking. The ground motion input data can be expressed as response spectrum, peak ground acceleration and/or peak ground displacement.

Building damage functions are often in the form of lognormal distribution [2, 3]. A fragility curve is characterized by a median value of the demand parameter, e.g. spectral displacement, that corresponds to the threshold of the damage state and by the standard deviation associated with that damage state. For structural damage, let  $S_d$  be the spectral displacement. The probability of being in or exceeding a damage state is calculated as:

$$P[d_s | S_d] = \Phi \left[ \frac{\ln(S_d / \bar{S}_{d,ds})}{\beta_{ds}} \right] \quad (7)$$

where  $\bar{S}_{d,ds}$  is the median value of spectral displacement at which the structure reaches the threshold of the damage state  $d_s$ ;  $\beta_{ds}$  is the standard deviation of the natural logarithm of spectral displacement of damage state  $d_s$ ; and  $\Phi$  is the standard normal cumulative distribution function.

### 4.1 Development of damage parameter

In popular vulnerability analysis methods [2, 3], the damage parameters are tight to the interstory drift (interstory displacement divided by story height). This may be considered as nominal shear. Given ground motion one can calculate the floor displacement by mode superposition as described in Section 3. The drift has to be less than a given value at each damage state. In fact the limit value is specific to each categorized structural system. According to [2], the threshold at pre-code seismic design level of the slight damage state can range from 0.0012 to 0.0048 obtained from pushover analyses.

### 4.2 Development of damage state variability

The total lognormal standard deviation is computed by the combination of different contributors. These include the variability in ground motion  $\beta_{GM}$ , the variability in structural model and the variability in damage threshold. The first source of uncertainty, depending on ground motion itself, is aleatory in nature. The second source

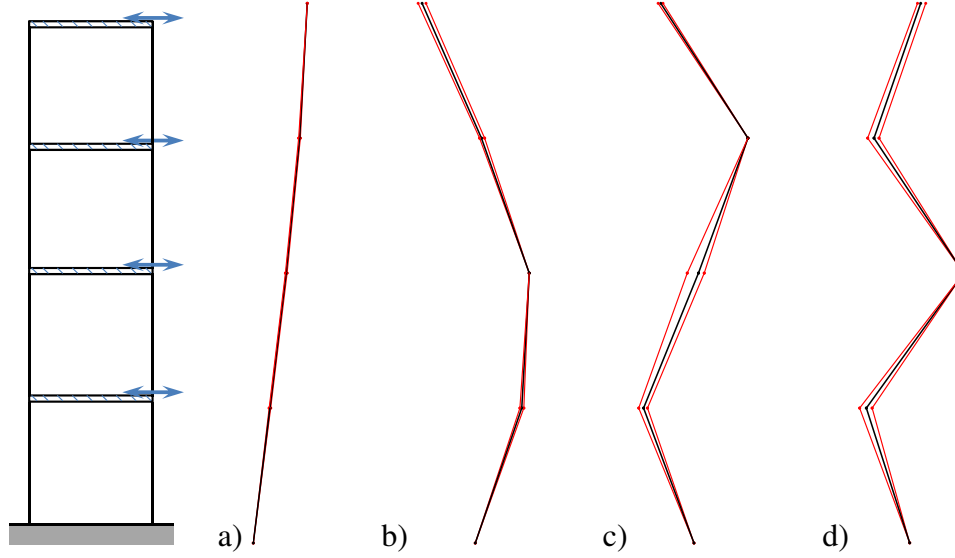


Figure 1: The measurement locations and the identified mode shapes 1 to 4 (black) with estimated  $2\sigma$  uncertainty bound (red).

of uncertainty can be directly estimated from the measurement results as discussed in Section 2. The third source of uncertainty can be further improved with recent effort to have more specific analysis to be carried out. Suppose that these sources of uncertainty are independent then the total standard deviation is delivered by the square root of the sum of component variability squares as mentioned.

## 5 An implementation example

The application of the methodology is presented next to a concrete frame building (Figure 1). The structure is assumed located in a low-to-moderate seismic region. About 100 time histories of the ground shaking between magnitude of 5 and 6 is collected randomly. The building is supposed to belong to pre-code design with 5 floors. The floor beam is assumed rigid. The building frame is a four-story structure of 10 m height in total.

### 5.1 Field testing

The ambient vibration test is first performed to identify the modal parameters. It is assumed that the floor be relatively rigid to the column, one sensor in each floor is enough to characterize the dynamic behavior of the building. The time history of each channel is simulated with about 300 s of measurement time at a sampling frequency of 100 Hz. The measurement locations are shown in Figure 1.

As the structure is assumed to be concentrated in four lump masses, there are four



mode shapes. Half the number of block row of the Hankel matrix is 35 and the expected model order is 30 in step of 2. The covariance-based stochastic subspace system identification algorithm (SSI-cov) [10] is used.

Mode	Frequency [Hz]	Damping ratio [%]
1	1.305±0.007	0.179±0.302
2	3.615±0.012	1.318±0.293
3	5.527±0.014	1.708±0.278
4	6.831±0.015	2.086±0.277

Table 1: Identified eigenfrequencies  $f_i$  and damping ratio  $\xi_i$  with their estimated  $2\sigma$  uncertainty bound.

Using SSI-cov, the deviation of identified frequencies and mode shapes can also be calculated. Table 1 shows the summary of the identified frequencies, damping ratios and their statistical variances. It can be seen that the  $2\sigma$  uncertainty bound of the frequency is quite small for mode 1. The variance of damping ratios differs significantly for all modes. In fact the theoretical background of the sensitivity based covariance estimation of the damping ratio cannot guarantee to yield accurate uncertainty bound [10].

Figure 1 shows the four identified mode shapes with their estimated  $2\sigma$  uncertainty bounds. It can be seen that mode 1 is very precise, which is also confirm with frequency variance. The accuracy of identified fundamental frequency is also important in calculating the spectral displacement value, which was often assumed without field testing. Other modes are fairly accurate. In fact it is a well chosen structure with separated natural frequencies, low damping ratios. Therefore, these modes are almost real with very small imaginary components.

The extracted modal parameters from measurement is quite small for this structure. In reality the uncertainty can be higher depending on the present level of noise and the number of setups needed for structure with many DOFs.

## 5.2 Fragility curve

The building is supposed to be designed not according to seismic code (pre-code) for which the interstory drift limit for the slight damage is supposed to be 0.003 (3000  $\mu\epsilon$ ) often depending on the building type and adjusting by height. About 100 earthquake strong motion records (time histories) of magnitude between 5 and 6 are extracted from different sources (e.g. U.S. Geological Survey, PEER Strong Motion Database, European Strong Motion Database, Japan Earthquake and Volcanic Disaster Prevention Laboratory, New Zealand Geological and Nuclear Sciences, Swiss Seismological Service, etc.). Then the spectral value at fundamental frequency is extracted. Afterward, the interstory drift is calculated and compared to the limit threshold. This can be done repeatedly at different spectral value interval to give the probability of being

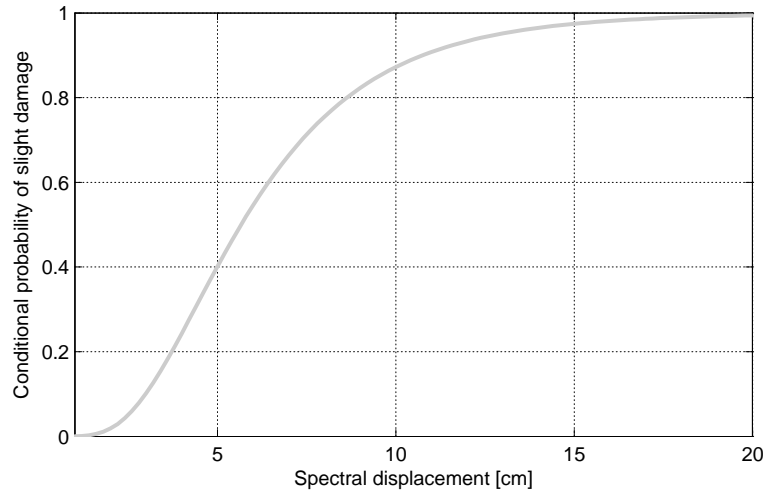


Figure 2: The fragility curve for the slight damage state.

equal or exceeding the limit. The variance determined from this process can be considered as the ground motion uncertainty. The methodology to estimate the fragility curve is similar to the one in [7]. However, it is noted that the uncertainty of the modelling is also contributed since this value can be extracted from the ambient vibration test. The final fragility curve is shown in Figure 2.

## 6 Conclusions

Seismic vulnerability assessment methodologies need to specify all sources of uncertainty in order to give a certain level of confidence. The method based on a modal field testing experiment is able to quantify systematic uncertainty in both the spectrum displacement side with identified fundamental frequency and in structural model side with the uncertainty bound of the modal parameters.

The method is limited to be applicable within the slight damage state until the proportional limit. For low-to-moderate seismic regions, the first damage grade is of important concern.

The ambient vibration test could be further used for related assessment purposes such as updating the fragility curve before and after an earthquake event.

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