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Modeling Shakedown Plastic Strains of Subballast and Subgrade Materials in the Concrete Slab Track System of High-Speed Trains

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Abstract

Recently, the Korean government is driving a research and development project for a high-speed railway system which is capable of operating in the maximum speed over 400 km/h. Besides the vehicle system, the track and railroad system should also be built properly to cope with the sustainable operation of the high-speed trains. The concrete slab track system is being introduced to reduce the cost of frequent maintenance in the conventional ballast track. This introduction also demands a new geotechnical assessment on the performance of the subsoil materials. It is of interest that we should estimate and control cumulative plastic strains in total operation period. The stress distribution in the subsoil materials beneath the concrete slab track becomes more sophisticated than the ballistic track system. The shakedown phenomena, in which the permanent deformation rate at a number of load cycles has become negligibly small and the cyclic response has become elastic, should be understood for better design of railway track.

This study, described in this paper, investigated the pattern of the development of the shakedown plastic strains of the subsoil materials subjected to dynamic loads induced by high-speed trains on the stiff concrete slab track system. A constitutive model that captures only the envelope of the maximum plastic deformations generated during the cyclic loading process was employed. The accumulated plastic displacement measured in a series of full-scale model tests was compared with the predicted results using the proposed model.

Keywords: shakedown, subgrade, concrete slab track.

1 Introduction

Recently, Korean government is driving a R&D project to develop a high-speed railway system which is capable of operating in the maximum speed over 400 km/h. Besides the vehicle system, the track and railroad system should also be built properly to cope with the sustainable operation of the high-speed trains. For the newly built lines in Korea, the concrete slab track system is commonly adopted to reduce the cost of frequent maintenance in the conventional ballast track. This introduction also demands a new geotechnical assessment on the performance of the subsoil materials. It is of interest that we should estimate and control cumulative plastic strains in total operation period. The stress distribution in the subsoil materials under the concrete slab track becomes more sophisticated than the ballistic track system. The shakedown phenomena, in which the permanent deformation rate at a number of load cycles has become negligibly small and the cyclic response has become elastic, should be understood for better design of railway track.

Because of the present rapid expanding of high-speed trains in the world, the slab track railway is becoming popular. The relatively new slab track is being applied worldwide on an increasing scale for high-speed railway lines, especially in Europe and Asia. Japan and France were the initiators of high-speed technology. The French network mainly comprises of traditional ballasted track, whereas Japan primarily focused on slab track, with their well-known J-Slab. Studies on life cycle cost and availability have shown that non-ballasted tracks have great advantages. In Europe and other parts of the world, the German Rheda2000 is the leading solution. However these systems could be further optimized, whereby the design cannot be seen separate from the bearing capacity and properties of the subgrade [1]. In the past new projects were mainly assessed on the basis of investment costs, whereas today the principle of life cycle costing is strongly emerging. Although ballasted concepts are still widely used in high-speed operation, they will lose attractiveness in favour of slab track systems due to this new attitude.

With the growth of traffic intensity it becomes more and more difficult to carry out maintenance and renewal work. In The Netherlands night time possessions often last no longer than 5 hours. On the future high speed line in Korea (435 km from Seoul to Pusan) the maximum effective possession is estimated at no more than 1 ½ hours per night. In this respect the current increase in popularity of low-maintenance track designs is not surprising. From the above reasons, Korean government is driving a R&D project, to develop a high-speed railway system which is capable of operating in the maximum speed over 400 km/h.

Due to the large complexity of slab track design, a rational design and comprehensive knowledge for concrete slab track requires the elaboration of predictive methods aimed at assessing the long term performance of this particular kind of Geotechnical structure. One of the major specific concern is to determine the permanent deformations (plastic strains) of the subballast and subgrade layer induced by the application of the repeated traffic loading. However, the stress distribution and calculation in the latter layers become more sophisticated than ballistic track system.

These permanent deformations of subbalast and subgrade materials will cease to occur after a number of load cycles. In other words, after a number of cycles with plastic deformation, the whole track bed will respond purely elastically to the remaining load cycle. If this happens, then we consider the structure to have shakedown. The shakedown theory has been widely documented, but its implementation into railway engineering requires certain modification and adaptations with respect to the special and mostly sharper requirements of railway track especially for high speed networks [2]. The application of shakedown concept to study the behaviour of elastic-plastic track bed subjected to repeated or cyclic loading conditions is suitable way for providing genuinely the better understanding in the design of railway track.

With the development of computing technology, a large number of papers and reports document the use of finite element analyses for modelling rutting and permanent deformations based on shakedown principles in road pavements. However, only few references have paid attention to the dynamic behaviour of railway substructure under moving vehicles. To date, no overall framework has been established to explain satisfactorily with shakedown theory the behaviour of substructure granular materials under the complex repeated loading which they experience [3, 4].

In this paper, finite element analyses were carried out for a three dimensional (3D) slab track model to investigate plastic strain based on shakedown concept with load repetition. All layers were modelled using a constitutive model that captures only the envelope of the maximum plastic deformations generated during the cyclic loading process. The finite element code, ABAQUS, was used for modelling and simulation. The track system consists of rails with rail pads, sleeper, concrete slab, subsoils. The experimental data obtained from the full scale model test were compared with the computed results of finite element analyses.

2 Mechanism of Accumulation of Plastic Strains

2.1 Shakedown Plastic Deformation

A railway embankment is exposed to a large number of load applications during its service life. Although the permanent deformation is normally a fraction of the total deformation produced by each load repetition, the gradual accumulation of a large number of these small plastic deformation increments could lead to eventual failure within a viewpoint of maintenance. Though, for an individual train axle passage, the plastic deformation caused by these mechanisms is relatively small, the cumulative effect over a very large number of train axle passages may be considerable. In this respect, a model that simulates the plastic deformation behaviour during each individual load cycle is unattractive because such a model would require a vast amount of computer processing time to calculate the deformations corresponding to common track deterioration periods. This problem can be overcome by employing a model that captures only the envelope of the maximum plastic deformations generated during the cyclic loading process, which allows evaluating the deformation accumulation by means of relatively large load cycle increments.

When an elastic-plastic structure is subjected to a cyclic load, three distinctive situations may occur [5]. First, if the load magnitude applied is so low that nowhere in the structure is deforming plastically, then the behaviour will be entirely elastic. Second, if the load is larger than the elastic limit so that some part of the structure is deforming plastically but is less than a critical limit, the plastic deformation will cease to occur after a number of load cycles. In other words, after a number of cycles with plastic deformation, the entire structure will respond purely elastically to the remaining load cycles. If this happens, then we consider the structure to have 'shakedown', and the critical limit below which shakedown can occur is termed as a 'shakedown limit'. Third, if the applied load is greater than the shakedown limit, then the structure will continue to exhibit plastic strains for however long the load cycles are applied. If this occurs, then the structure would eventually fail owing to fatigue or excessive plastic deformation. Therefore for structures under variable loads, the shakedown limit provides a rational criterion for design.

2.2 Formulations

General stress-strain relationship in the incremental formulation can be given as

$$\dot{\sigma}_{ij} = D_{ijkl}(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^p_{kl}) = D_{ijkl}(\dot{\varepsilon}_{kl} - \dot{\bar{\varepsilon}}^p m_{kl}) \tag{1}$$

where $\dot{\sigma}_{ij}$ is the stress rate, $\dot{\varepsilon}_{kl}$ is the strain rate, $\dot{\varepsilon}^p_{kl}$ is the plastic strain rate, $\bar{\varepsilon}^p$ is the effective plastic strain measure, m_{kl} is the flow direction, and D_{ijkl} is the elastic stiffness tensor. The history parameter, $\bar{\varepsilon}^p$, and the yield function, $f(\bar{\sigma}, \bar{\varepsilon}^p)$, where $\bar{\sigma}$ is an effective stress measure, have to satisfy the Kuhn-Tucker conditions:

$$f(\bar{\sigma}, \bar{\varepsilon}^p) \le 0, \quad \dot{\bar{\varepsilon}}^p \ge 0, \quad \dot{\bar{\varepsilon}}^p f(\bar{\sigma}, \bar{\varepsilon}^p) = 0 \tag{2}$$

If both the initial boundary of the elastic domain and the history formulation are isotropic, an isotropic yield function can be used as

$$f(\bar{\sigma}, \bar{\varepsilon}^p) = \bar{\sigma} - H(\bar{\varepsilon}^p) = 0 \tag{3}$$

where $H(\bar{\varepsilon}^p)$ is the strength as a function of the history parameter and the effective stress measure. Eq. (3) and its inverse form yield

$$\bar{\sigma} = H(\bar{\varepsilon}^p) \text{ or } \bar{\varepsilon}^p = K(\bar{\sigma}) \tag{4}$$

where $K(\bar{\sigma}) = H^{-1}(\bar{\sigma})$. From Eq. (4), the rate of the plastic strain invariant, $\dot{\bar{\varepsilon}}^p$, is given by

$$\dot{\bar{\varepsilon}}^p = K'(\bar{\sigma})\dot{\bar{\sigma}} \tag{5}$$

where $K'(\bar{\sigma}) = dK(\bar{\sigma})/d\bar{\sigma}$. In a given time period, $[t, t+\Delta t]$, for a single cycle of loading, the incremental plastic strain invariant, $\Delta\bar{\varepsilon}^p$, is computed as

$$\Delta\bar{\varepsilon}^p = \int_t^{t+\Delta t} \dot{\bar{\varepsilon}}^p dt = \int_t^{t+\Delta t} K'(\bar{\sigma})\dot{\bar{\sigma}} dt \tag{6}$$

Under the repeated loading condition such that the amplitude of the cyclic loading, $\bar{\sigma}_{cyc}$, and the shakedown level, $\bar{\sigma}_{sh}$, remain constant, Eq. (6) can be transformed into

$$\Delta\bar{\varepsilon}^p = \int_{\bar{\sigma}_{sh}}^{\bar{\sigma}_{cyc}} K'(\bar{\sigma}) d\bar{\sigma} \tag{7}$$

During a number of load cycle, ΔN, the accumulated plastic strain, $\Delta\bar{\varepsilon}^p_{\Delta N}$, is simply equal to $\Delta N \Delta\bar{\varepsilon}^p$ such that

$$\frac{\Delta\bar{\varepsilon}^p_{\Delta N}}{\Delta N} = K(\bar{\sigma}_{cyc}) - K(\bar{\sigma}_{sh}) \tag{8}$$

The evolution of plastic deformation is considered over a large number of load cycles, N, such that it can be regarded as a continuous process. When the amplitude of the cyclic loading, $\bar{\sigma}_{cyc}$, is above the shakedown level, $\bar{\sigma}_{sh}$, the internal material structure will alter during the loading process, and thus the shakedown level can evolve by means of history parameter, $\bar{\varepsilon}^p$. For a long-term cyclic loading process with N, Eq. (8) becomes

$$\frac{d\bar{\varepsilon}^p}{dN} = K(\bar{\sigma}_{cyc}) - K(\bar{\sigma}_{sh}(\bar{\varepsilon}^p)) \tag{9}$$

For example, if there is no evolution of $\bar{\sigma}_{sh}$ during cyclic loading, the same amount of the plastic strain increment is accumulated such that the total plastic strain increases linearly with the number of load cycle, N, as illustrated in Fig. 1.

Over a number of cycles, gradual decreasing and eventually vanishing plastic strains can be modelled via the evolution of the shakedown level, $\bar{\sigma}_{sh}$. As the shakedown level increases, the value of $\bar{\sigma}_{sh}$ approaches the value of $\bar{\sigma}_{cyc}$, which corresponds to diminishing plastic strain increment. Within the context that the plastic deformation is generation when the cyclic amplitude exceeds the shakedown level, the plastic strain per cycle can be represented as

$$\frac{d\bar{\varepsilon}^p}{dN} = K_*(\bar{\sigma}_{cyc} - \bar{\sigma}_{sh}(\bar{\varepsilon}^p)) \tag{10}$$

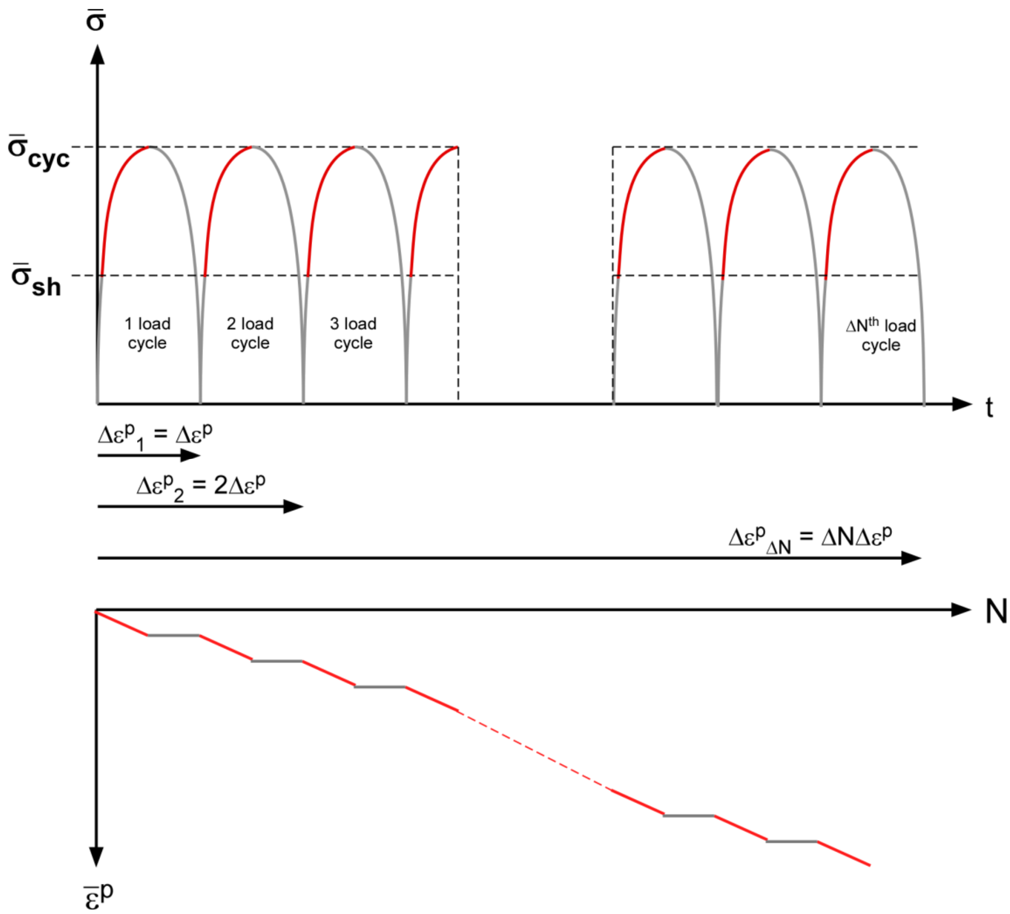

Figure 1: Generation of incremental plastic strain caused by repeated loadings

This alternative expression has a great advantage in formulation because Eq. (10) is essentially identical to the viscoplastic model of Perzyna [6-8]. Consequently, this analogy attains the adoption of the conventional viscoplastic model to represent the generation of the plastic deformation caused by cyclic loadings. In the finite element modelling, the adoption of Eq. (9) presumably regards the constant level of $\bar{\sigma}_{cyc}$ to avoid a coupled behaviour of plasticity and creep. This limitation appears reasonable because the weight of the trains is unlikely varying significantly in an operation period.

The soil subjected to cyclic loadings responds differently with regard to the relative amplitude of the cyclic loadings. Figure 2 illustrates three distinctive phases of soil deformation. When the cyclic amplitude, $\bar{\sigma}_{cyc}$, is lower than the shakedown level, $\bar{\sigma}_{sh}$, the soil deforms elastically. If the cyclic amplitude is higher than the shakedown level, the soil exhibits plastic deformation that after a specific number of cycles turns into the elastic deformation.

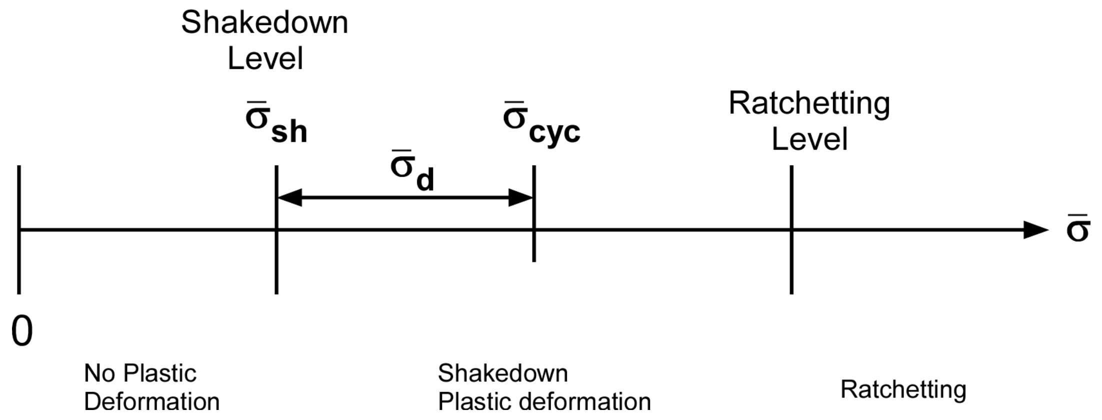

Figure 2: Various deformation responses in cyclic loading

The cyclic amplitude is even higher than ratchet level, the plastic deformation is continuously increasing with the increasing number of load cycles, and eventually the soil reaches an unstable failure. Herein, it is assumed that the cyclic amplitude remains lower than ratchet level so that the plastic deformation ceases to increase after a large number of load cycles. Under a condition that the cyclic amplitude is higher than the shakedown level, it is useful to designate a parameter, $\bar{\sigma}_d$, which is a effective stress measure between the cyclic amplitude and shakedown level as illustrated in Fig. 2. Using this parameter, a power function representing K_* in Eq. (10) can be employed as

$$\frac{d\bar{\varepsilon}^p}{dN} = \left[A(\bar{\sigma}_d)^n[(m+1)\bar{\varepsilon}^p]^m\right]^{\frac{1}{m+1}} \tag{11}$$

where A, m, and n are material constants, and $\bar{\varepsilon}^p$ is the accumulated plastic strains. Eq. (11) essentially shares the strain hardening form of the power law model describing the creep law in ABAQUS. Integration of Eq. (11) yields

$$\bar{\varepsilon}^p = B(\bar{\sigma}_d)^n(N)^{m+1} \tag{12}$$

where $B = A(m+1)^{2m+1}$ and N is the number of load cycles. The determination of plastic strain increments requires the direction of plastic flow. This can be fulfilled by employing a plastic potential, g, as

$$g = q - p\tan\psi \tag{13}$$

where ψ is the dilation angle, p is the mean normal stress, q is the deviator stress. Initial shakedown level and its evolution have been described by a Drucker-Prager yield surface, f, given as

$$f = q - p\tan\beta - d \tag{14}$$

where β is the slope of the linear yield surface in the p-q stress plane, and d is the cohesion of the material, which relates to the effective stress measure, $\overline{\sigma}_d$, as

$$d = (1 - \tfrac{1}{3}\tan\beta)\overline{\sigma}_d \tag{15}$$

The effective stress measure, $\overline{\sigma}_d$, is assumed as an equivalent uniaxial compression stress so that

$$\overline{\sigma}_d = (q_{cyc} - p_{cyc}\tan\beta)/(1 - \tfrac{1}{3}\tan\beta) \tag{16}$$

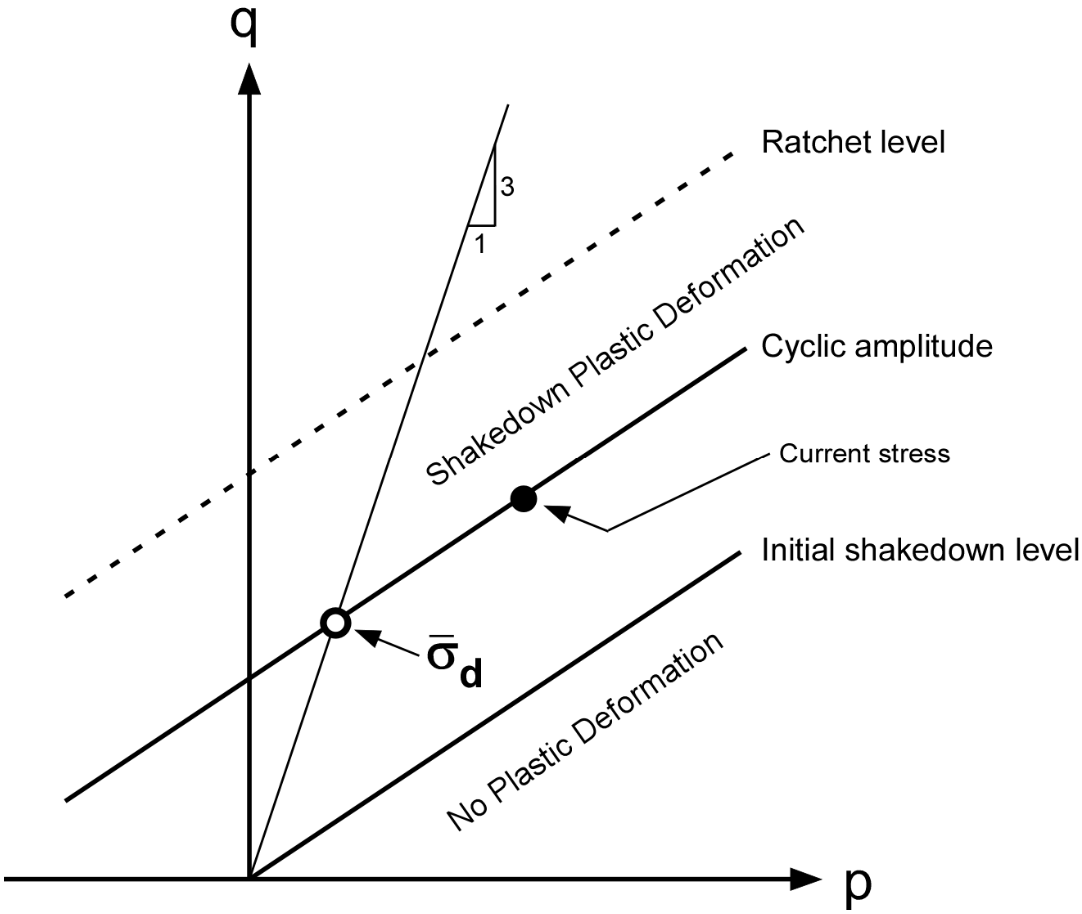

Figure 3: Effective stress measure using Drucker-Prager yield criterion

Incremental plastic strain in cyclic loading is derived from the plastic potential, g.

$$\frac{d\varepsilon_{ij}^{p}}{dN} = \frac{1}{\Lambda}\frac{d\bar{\varepsilon}^{p}}{dN}\frac{\partial g}{\partial \sigma_{ij}} \tag{17}$$

where Λ is a proportional factor. Since $d\varepsilon_{ij}^{p}$ is obviously work conjugate to σ, the proportional factor becomes

$$\Lambda = \frac{1}{\bar{\sigma}_d}\sigma_{ij}\frac{\partial g}{\partial \sigma_{ij}} \tag{18}$$

3 Simulation of Full-scale Loading Tests

3.1 Full-scale Loading Tests

A full-scale model loading tests were conducted in Korea railway research institute (KRRI). This test aimed at analyzing the effect of the thickness of reinforced roadbed on the displacement in the concrete slab track. Figure 4 shows the concrete slab track used for tests (left) and the actuators (right). The concrete slab track consists of track concrete layer (TCL) and hydraulically stabilized base (HSB). The reinforced roadbed including sub-ballast and crushed stone layers is located beneath the concrete slab. The granular soils were carefully selected to meet the regulation designated in Korean railway specification. Both upper and lower subsoil layers were compacted using well-graded sand or sand with gravel. In three different testing sections, A, B, and C, thickness of the reinforce roadbeds were 20, 30, and 50 cm as shown in Fig. 4. Five sleepers were equally spaced by 65 cm and embedded in the concrete slab.

Figure 4: Full-scale loading test of concrete slab track

The concentrated forces were applied to the rails at the location of the middle sleeper. Magnitude of the concentrated force is determined by considering the impact factor which increases as the train velocity increases. Frequencies of the load cycle, f, were determined by f = V/d where V is the train velocity and d is the spacing between two bogies. Frequency of 4 Hz was chosen for the train velocity of 200 km/h, 6 Hz for 300 km/h, and 8 Hz for 400 km/h. The number of load cycles is chosen as the annual average of the number of train passages in Korea. Prior to full-scale loadings, the soil conditions of each section were comprehensively investigated. Field tests include LFWD test, field density test, cyclic plate load test, direct arrival test, and cross-hole seismic test. During loading, the vertical displacement, velocity, and acceleration were measured at some specific depths within the subsoil layer.

3.2 Finite element simulations

The constitutive model described previously has been applied in the simulation of the full-scale loading test. Figure 5 shows the finite element mesh used in this study. The model has 3.6-m thick subsoil and 50-cm thick concrete layers, TCL and HSB, which have the thickness of 24 and 30cm, respectively. The measurement data was obtained from two different sections: section A with 20-cm thick reinforced roadbed and section B with 40-cm thick reinforced roadbed. The rail was modelled as an Euler-Bernoulli-type beam with the UIC60 sectional properties. The magnitudes of the concentrated load in section A and B were equivalent to the forces induced by the train moving at the velocity of 300 and 400 km/h, respectively. Elastic pad between the rail and sleeper were modelled using a linear spring.

Track Component	ρ (t/m3)	E	ν	β_0 (°)	ψ (°)	A	n	m
Rail	7.8	210 GPa	0.3	-	-	-	-	-
Pad	-	40 kN/mm	-	-	-	-	-	-
Sleeper	2.3	29.1 GPa	0.2	-	-	-	-	-
TCL	2.3	34.0 GPa	0.2	-	-	-	-	-
HSB	2.3	12.9 GPa	0.2	-	-	-	-	-
Roadbed	2.0	180 MPa	0.2	20	5	0.00029 (6.5E-7)	0.62	-0.825
Subsoil	2.0	125 MPa	0.3	15	5	0.00012 (1.2E-6)	0.36	-0.972

* ρ = density; E = Young's modulus (or spring constant for pad); ν = Poisson's ratio; β_0 = The slope of frictional yield line in Drucker-Prager model; ψ = Dilatancy angle; A, n, and m = material constants for the shakedown plastic model. The values of A in parenthesis are used to simulate the full-scale loading test.

Table 1. Model parameters for finite element simulations

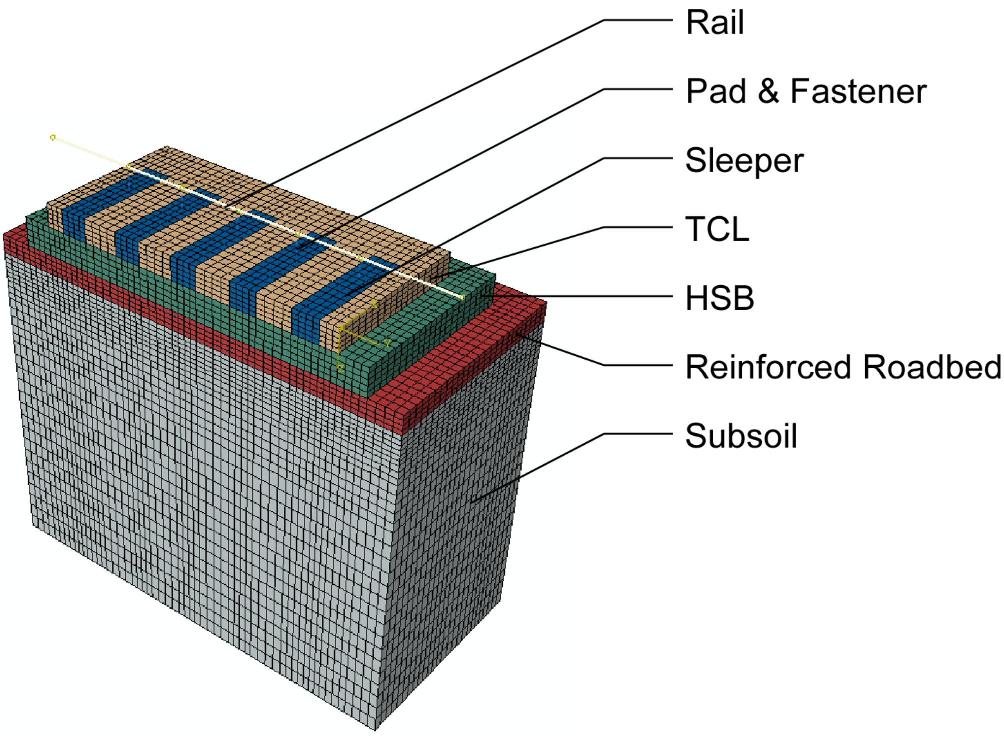

Figure 5: Finite element mesh used for simulation of the full-scale loading test

Model parameters used for simulations were summarized in Table 1. Linear elastic material was assumed to represent the concrete slab, roadbed and subsoil. The shakedown plasticity model was only applied to the roadbed and subsoil. Due to lack of basic laboratory testing data, the parameters, A, m, and n, of Eq. (11) were estimated by using the experimental data from literature.

A recent research [9] shows that the plastic deformation caused by cyclic loading can be expressed by an empirical relationship as

$$\bar{\varepsilon}^{p}(N) = a(N) \cdot b(p_{cyc}, q_{cyc}) \tag{19}$$

where a(N) is a shape function depending on the number of cycles N, and b is the function describing the influence of the level of stress. According to Hornych et al. (2004), the measured permanent deformation of unbound soils can be successfully described when they chose the equation proposed by Sweere [10] for a(N) and the equation by Gidel et al. [11] for b, given as

$$a(N) = \alpha(N)^{\gamma} \tag{20}$$

$$b = \left(\frac{L_{\max}}{p_a}\right)^{\mu} \frac{1}{\left(\kappa + \frac{s}{p_{cyc}} - \frac{q_{cyc}}{p_{cyc}}\right)} \tag{21}$$

where $L_{\max} = \sqrt{p_{cyc}^2 + q_{cyc}^2}$, p_{cyc} and q_{cyc} are the stresses corresponding the maximum cyclic stress, and α, β, μ, m, and s are material constants. Hornych et al. [9] suggested the optimal values of material constants for unbound granular materials (UGM), which can be regarded as the material used in the reinforced roadbed, and sand, as summarized in Table 2. At a given cyclic stress, p_{cyc} = 50 kPa and q_{cyc} = 25 kPa, the model parameters of the shakedown plasticity corresponding to the data in Table 2 are summarized in Table 1. Figure 6 compares the empirical relationships for UGM and sand and the predicted values using Eq. (11) used in this study. It should be noted that a good agreement between empirical relationship and shakedown plastic model may not be valid under the different combination of cyclic stresses.

Material	Function a		Function b		
	α	γ	μ	κ	s
UGM	1.98×10^{-3}	0.175	0.62	2.41	46.82 kPa
Sand	7.73×10^{-3}	0.028	0.36	1.60	26.21 kPa

Table 2. Material constants of empirical permanent deformation relationships

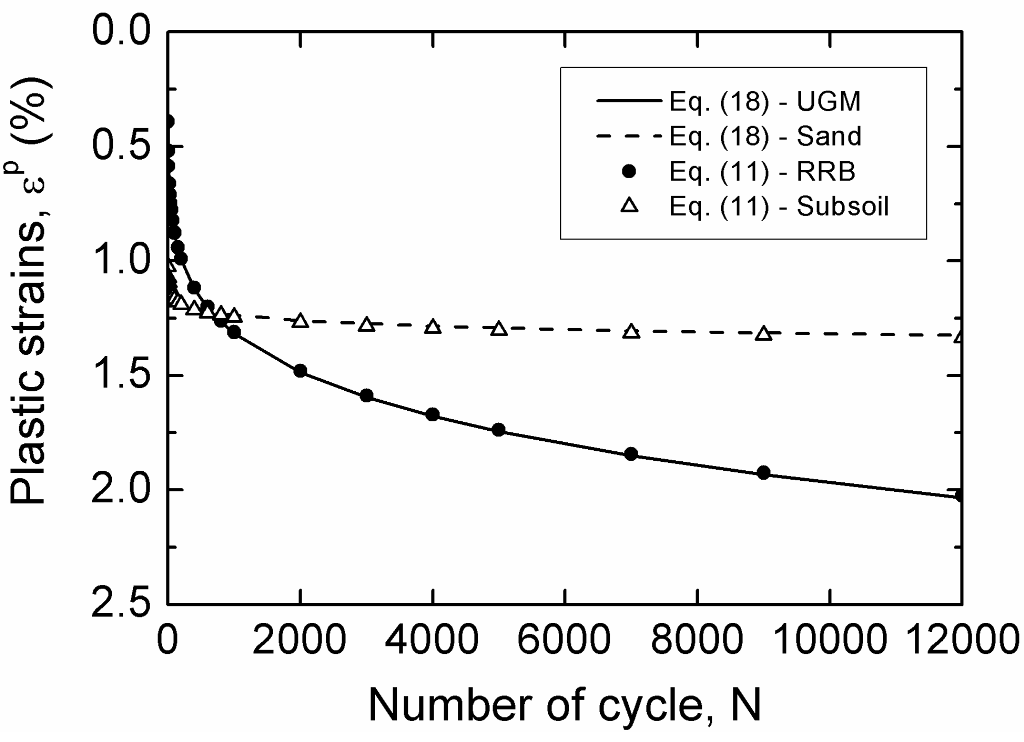

Figure 6: Shakedown plastic strains in reinforced roadbed and subsoil

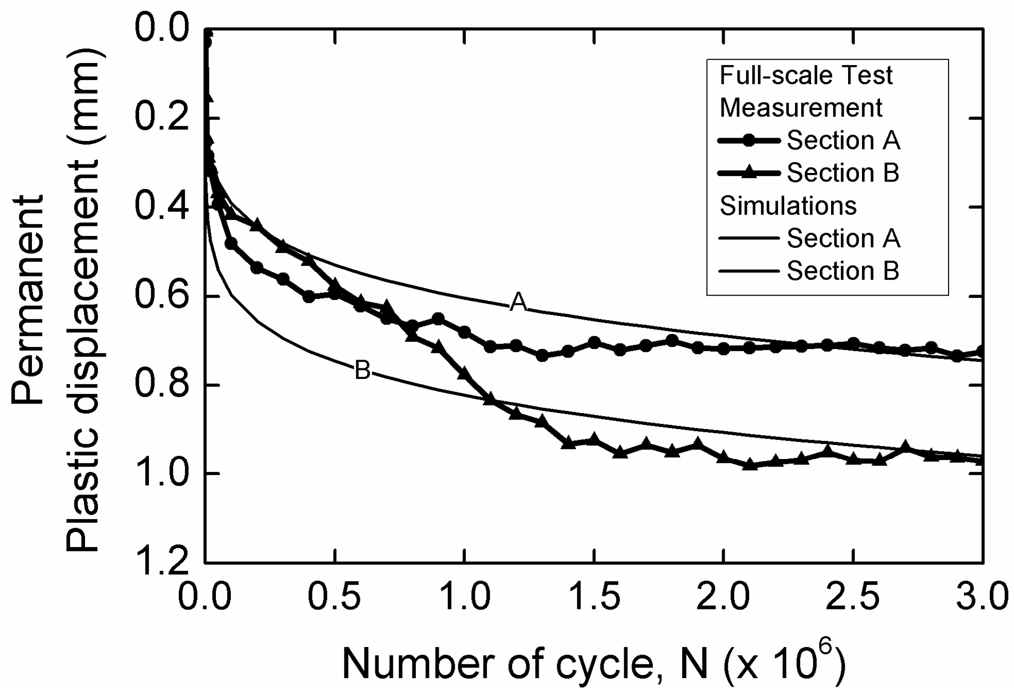

Figure 7: Measured and predicted shakedown plastic strains in full-scale test

Figure 7 compares the computed plastic displacement using the shakedown plastic model with the measurement data in the full-scale loading test at the section A and B. The predicted results show a good agreement with measurement, whereas the values of A in Eq. (11) needs to be corrected to accommodate the unexpectedly small displacement observed in the full-scale test. Corrected values of A are indicated in Table 2. However, general decreasing patterns of the plastic deformation during cyclic loadings can be successfully simulated using other model parameters estimated from the literature. Obviously, it is necessary to intensively investigate basic plastic behaviour of geomaterial used in the railway for better simulation of permanent displacement during long-term operation.

4 Conclusions

This paper investigated the pattern of the development of the shakedown plastic strains of the subsoil materials subjected to dynamic loads induced by high-speed trains on the stiff concrete slab track system. A shakedown plasticity model that captures only the envelope of the maximum plastic deformations generated during the cyclic loading process was developed. The model parameters were estimated using the empirical relationships based on the experimental data. The accumulated plastic displacement measured in a series of full-scale model tests was compared with predicted results using the proposed shakedown model. The predicted plastic displacement shows a reasonably good agreement with the measured values.

Experimental investigation, is still required, into the shakedown plastic behaviour of geomaterials used in a high-speed railway track to enhance the proposed model.

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