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Paper 166

Assessing the Effect of Two Entering Triangular Initial Geometric Imperfections on the Buckling Strength of an Axisymmetric Shell subjected to Uniform Axial Compression

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Abstract

Axisymmetric localised geometric imperfections represent, for axially compressed circular cylindrical shells made from isotropic and homogeneous elastic material, the most adverse buckling configuration. Various axisymmetric forms of single localised defects have been investigated and the entering triangular shape was found to be the worst case. Since multiple localised defects can arise, studying their global effect on the shell buckling strength is required. In this paper two localised entering triangular imperfections are considered. Modelling was performed by means of specialised finite element software. It was shown that further reduction of the buckling stress would occur if two initial defects were present on the shell.

Keywords: buckling, shells, welding, initial geometric imperfections, finite element method, analysis of variance.

1 Introduction

Thin shells are such common structural elements: silos, boilers, containers, tanks,... There exists no manufacturing process that can be used to produce these structures without suffering from various kinds of initial geometric imperfections. Control of manufacturing processes of shells and their optimization makes it possible to decrease the degree of these imperfections, but they could never be completely removed.

Defects affecting the shell initial geometry disturb the shell mechanical behavior. During service life, shell structures may be subjected to various kind of loading, such as axial compression, external/internal pressure, flexure or torsion. For thin cylindrical shells under uniform axial compression, the buckling strength constitutes the most adverse design issue. Calculation of the buckling stress as it could be affected by the presence of initial geometric imperfections represents then an essential task. The pursued objective is to be able to achieve shell structural design with pertinent and relevant margins of safety by integrating some knowledge of the worst imperfections that these structures could experience.

Several studies were reported in the literature that has dealt with the effect of geometric imperfections on strength buckling of thin shell structures. Arbocz and Babcock [1] have studied experimentally buckling of thin cylindrical shells subjected to general forms of imperfections. Koiter [2] made a review study about the effect of geometric imperfections on shell buckling strength. Initial geometric imperfections were recognized to reduce drastically the buckling stress of elastic cylindrical shells when subjected to axial compression. The extent of this reduction depends however on the nature of the actual type of geometric imperfections that affect the shell structure. Therefore, investigation has been motivated by the analysis of buckling in the presence of typical imperfections obtained from modal analysis of measured data or by considering realistic imperfection shapes such as those resulting from welding operations that are performed to assembly shell structures [3]. Steel silos and tanks are constructed from strakes that are welded together to assemble the complete shell structure. At circumferential welds localised geometric imperfections form. Measurements have revealed that mostly axisymmetric imperfections occur in these structures [4]. Importance of the local axisymmetric imperfections occurring at these circumferential welds was first assessed by Bornscheuer et al. [5]. Local axisymmetric depressions were previously recognized to constitute more realistic representations of initial geometric imperfections than full eigenmodes as was stated by many authors [6,7,8], however the first comprehensive study regarding the effect of these geometric imperfections was achieved by Rotter and Teng [8] who have introduced the typical weld depression and studied its effect on the buckling strength. Other valuable studies which had addressed various aspects associated to weld-induced axisymmetric geometric imperfections can be found in References [9-14].

Imperfections in welded shells take different forms as the profile of welding can vary from one shell to another, but a common feature of welds shapes is that they can be characterized by only two parameters: the defect amplitude and its width termed also wavelength. The axisymmetric weld depression that was introduced by Rotter and Teng [9] is among the most easily recognizable such forms. Other forms were proposed after that. Mathon and Limam [15] have compared the relative influence of several localised imperfections on reduction of the buckling stress of shells subjected to axial compression or to flexure, and have shown that a triangular imperfection shape has the most severe effect on buckling strength.

Considering welds at different heights, Rotter [16] was the first to study the interaction problem that could happen between them. Using a rigorous nonlinear analysis Rotter [17] has studied this interaction in the case of the axisymmetric weld depression.

Emphasis will be done in the following on interaction effects that could result from two geometric weld imperfections. The localized initial imperfection geometry is assumed to have a triangular shape. This shape is sharper than those of real imperfections encountered in axisymmetric shells, but some weld depressions could be closer to it when fitted to analytical curves [17].

In almost the entire above mentioned literature, no more than a single localized geometric imperfection was considered. The objective of this study is to investigate how two concomitant localized imperfections might influence the shell buckling strength. The localized geometric imperfections that will be considered are assumed to have an entering triangular shape. It was found in Reference [15] that the entering configuration (peak of the geometric imperfection is directed inwards the axisymmetric shell) yields the most adverse case as to buckling strength reduction in comparison with the outwards configuration of this defect (peak of the geometric imperfection is directed towards the exterior of the shell).

Investigation of the relative effect of the intervening factors on shell buckling strength will be performed according to two steps. At first, a single entering triangular geometric imperfection is considered and the most unfavourable combination in terms of shell aspect ratios (radius over thickness and length over thickness) is determined. Then, with these ratios fixed, two concomitant defects having the same entering triangular shape are assumed to occur on the shell structure. In this second situation, the distance separating the two defects is an additional parameter that will interact with the imperfections amplitude and wavelength. It is interesting then to quantify the relative influence of these factors in order to assess more deeply the imperfect shell buckling resistance

Thin axisymmetric cylindrical shells made from homogeneous and isotropic elastic material are considered. They are assumed to deform under a purely axisymmetric strain state when they are subjected to axially uniform compressive loading.

In order to limit the total number of simulations needed for the parametric study, a design of experiment method using Taguchi approach is applied [18]. Three levels for each of the five intervening factors were selected and a $L_{27}(3^{13})$ Taguchi table was used.

2 Modelling shell buckling under two concomitant localized geometric imperfections

This study deals with the strength of imperfect cylindrical shells subject to uniform axial compression, for which the Eurocode terminology descriptions are used [19-20]. According to this standard, three kind of buckling analyses can be considered. These include the basic analysis termed Linear Bifurcation Analysis (LBA) which gives the reference buckling loads for any considered ideal problem conditions and provides mode shapes that could be used for expanding geometric imperfections. There is also, the Geometrically Nonlinear elastic Analysis (GNA) which gives limit point loads for the perfect structure and enables to determine the importance of geometric nonlinearity for each considered loading case. To perform GNA analysis, it is of crucial importance that the eventuality of bifurcation occurrence should be examined at the end of any iteration in order to determine the nonlinear bifurcation load. The associated nonlinear buckling mode constitutes an additional imperfection form that could be considered in the analysis. The results of this work, which deals with the effect of initial geometric imperfections on shell buckling strength, were derived by using Geometrically Nonlinear Imperfection Analysis (GNIA) with explicit representations of geometric imperfections.

In the context of elastic thin cylindrical shell buckling, a relevant finite element modelling based on Sanders-Koiter shell equations [21] was developed by Combescure [22]. This resulted in the software package called Stanlax which is based on an analytical expansion of circumferential variable contributions and finite element modelling of axial dependant quantities.

In Stanlax modelling of buckling for imperfect cylindrical shells, the actual imperfect shell geometry is assumed to be obtained from the ideal cylindrical geometry by applying an initial small displacement defining the imperfection. The initial geometric imperfections are included in model formulation under the assumption that they yield only small perturbations to the ideal geometry. Stanlax offers the possibility to perform either an LBA analysis through computing the linear Euler buckling mode for the perfect shell or a full non linear iterative computation of the buckling load according to Eurocode GNIA analysis. For shells under axial compression, it was shown that Euler calculation of buckling load is sufficient since it provides almost the same results than those obtained by the more complete non linear analysis for which a lot of iterations are needed.

Stanlax software is used in the following in order to model the imperfect cylindrical axisymmetric shell having a given number of localized initial geometric imperfections.

The shell material is assumed to be linear elastic having Young's modulus E and Poisson's ratio υ. The defects are considered to be localized in the median zone of the shell length and sufficiently far from the shell ends in order to avoid any interaction with the boundary conditions. The selected boundary conditions are those corresponding to clamped shell ends.

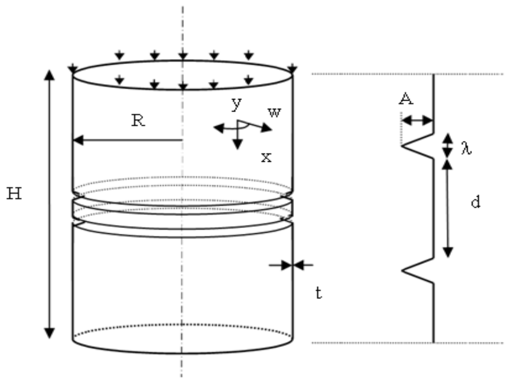

Figure 1: Geometry of the imperfect cylindrical shell showing two localized defects.

During the whole study the shell radius is maintained constant at the value $R = 135\,mm$ while the other parameters are varied. As shown in figure 1, parameters t, H and A designate respectively shell thickness, shell length, defect amplitude and distance separating the two localized geometric imperfections.

To clarify the presentation, the following non dimensionalized parameters associated to the main imperfect shell factors that monitor shell buckling strength are introduced:

- R/t radius to thickness ratio;
- H/R length to radius ratio;
- A/t defect amplitude parameter;
- d/R defect separating interval scale to radius ratio;
- $K = \lambda / \left(1.72\sqrt{Rt}\right)$ parameter fixing the defect wavelength λ.

3 Case study

Let's consider a single triangular geometric imperfection located at the mid height of the shell for which geometric and material properties are given by: $R = 135\,mm$, $H = 405\,mm$, $t = 0.09\,mm$, $E = 70\,GPa$ and $\upsilon = 0.3$. In this case the classical buckling stress is $\sigma_{cl} = 28.233\,MPa$. When, the imperfection amplitude is fixed at $A/t = 1$ and its wavelength at $\lambda = 15\,mm$, Figure 2 presents the evolution of the buckling stress ratio $\alpha = \sigma_{cr} / \sigma_{cl}$ as function of the number of elements with σ_{cr} the actual critical stress and σ_{cl} the classical buckling stress defined as

$$\sigma_{cl} = \frac{E}{\sqrt{3(1-\upsilon^2)}} \times \frac{t}{R} \tag{1}$$

A total number of 100 elements were found sufficient to guarantee FEM model convergence. Another study on the influence of the number of circumferential harmonics on FEM results has enabled to conclude that 25 harmonics are enough to guarantee convergence in the example considered. Other results not shown here have demonstrated that for all single or interacting imperfection cases a total number of 300 elements and a total number of 30 harmonics guarantee well convergence of the finite element model, figure 2.

The effect of a single localized geometric imperfection has been investigated at first in order to determine the most severe defect characteristics as to reduction of buckling strength. These characteristics have then been used to estimate the effect of two interacting defects on the shell buckling stress. Table 1 gives the list of parameters levels that have been considered in the analysis of shell buckling under this coupling situation of two localized geometric imperfections: lower threshold, intermediate value and higher threshold.

Based on table 1, a parametric study regarding the influence of two concomitant initial geometric imperfections has been conducted. This was performed according to a design of experiment method using five factors and three levels for each factor such as they are given by the Taguchi table $L_{27}(3^{13})$. The model so considered is hence linear without any interaction between factors. It has been shown a posteriori that there is no need to consider interaction between factors as the residuals are very small.

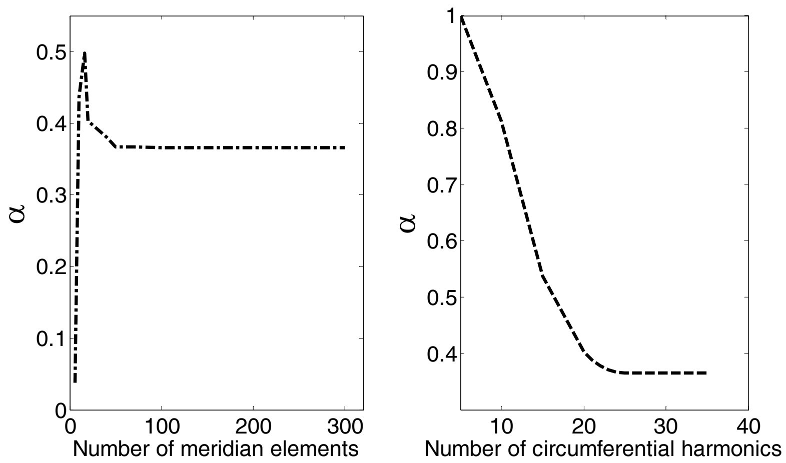

Figure 2: Disretized model convergence as function of the number of elements and the number of circumferential harmonics.

Stanlax software enables to compute the critical stress for each combination associated to a given line of the Taguchi table $L_{27}(3^{13})$. A total set of 27 numerical simulations have been performed. The results are given in Table 2.

Parameter	K	A/t	H/R	d/R
Lower threshold	1	2.0	1	0.000
Intermediate value	2	2.5	2	0.370
Higher threshold	3	3.0	3	0.741

Table 1: Ranges of variation of the considered factors.

By imposing the shell aspect ratio R/t to be greater than 450, for thin shell approximation to be valid, it has been shown in Mathon and Limam [15] that the most adverse case, with regards to the shell buckling strength when considering a single entering triangular geometric imperfection, is obtained for: $R/t = 450$, $H/R = 3$, $K = 2.5$. For these values of shell aspect ratios, imperfection wavelength and $d/R = 0.37$, variations of the buckling stress ratio as function of the amplitude parameter A/t are presented in figure 3. The continuous curve gives the results associated to a single defect. The dashed curve gives in the same conditions of parameters values the results for two interacting triangular geometric imperfections.

As shown in figure 3, the effect of two interacting geometric imperfections is not very important when the parameter A/t is small. But, when A/t increases ($A/t > 1$) the two geometric imperfection configuration yield significant reduction of the shell buckling stress as compared with the single geometric imperfection situation. The most adverse reduction of the shell buckling stress ratio $\alpha = \sigma_{cr} / \sigma_{cl}$ passes from 0.20 in the case of a single defect to only 0.16 in the case of two interacting defects.

Combination number	A/t	d/R	K	R/t	H/R	$\alpha = \sigma_{cr}/\sigma_{cl}$
1	2.0	0.000	1	450	1	0.287
2	2.0	0.000	2	1000	2	0.222
3	2.0	0.000	3	1500	3	0.213
4	2.0	0.370	1	1000	3	0.240
5	2.0	0.370	2	1500	1	0.247
6	2.0	0.370	3	450	2	0.188
7	2.0	0.741	1	1500	2	0.229
8	2.0	0.741	2	450	3	0.261
9	2.0	0.741	3	1000	1	0.809
10	2.5	0.000	1	1000	1	0.254
11	2.5	0.000	2	1500	1	0.192
12	2.5	0.000	3	450	3	0.185
13	2.5	0.370	1	1000	3	0.293
14	2.5	0.370	2	450	1	0.371
15	2.5	0.370	3	1000	2	0.733
16	2.5	0.741	1	450	2	0.550
17	2.5	0.741	2	1000	3	0.184
18	2.5	0.741	3	1500	1	0.686
19	3.0	0.000	1	1500	1	0.223
20	3.0	0.000	2	450	2	0.165
21	3.0	0.000	3	1000	3	0.166
22	3.0	0.370	1	450	3	0.574
23	3.0	0.370	2	1000	1	0.300
24	3.0	0.370	3	1500	2	0.135
25	3.0	0.741	1	1000	2	0.733
26	3.0	0.741	2	1500	3	0.172
27	3.0	0.741	3	450	1	0.619

Table 2: Simulation data layout according to Taguchi $L_{27}(3^{13})$ orthogonal array.

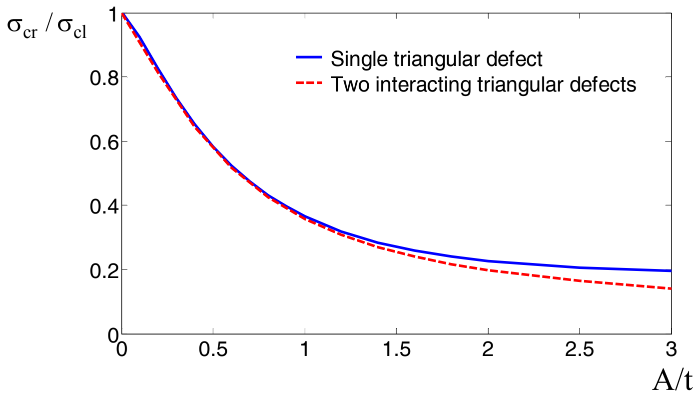

Figure 3: Comparison between a single defect and two interacting defects for the case $R/t = 450$, $H/R = 3$, $K = 2.5$ and $d/R = 0.37$.

Subsequently Analysis of Variance (ANOVA) is performed in order to determine the relative influence of each factor within the ranges of parameters specified by table 1. Figure 4 presents the ANOVA diagram obtained by using simulation results given in table 2.

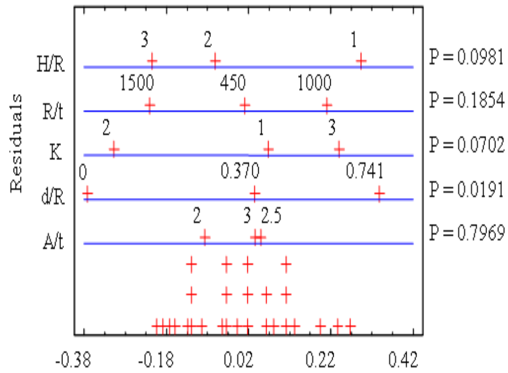

Figure 4: Multifactor ANOVA diagram performed on the five factors.

4 Discussion

Analysis of buckling strength for a single triangular entering geometric imperfection as function of wavelength and amplitude showed that only small effects are associated to parameters H/R and R/t in the range of thin shells. It had been shown also that the wavelength $K = 2.5$ yields the most adverse case. Fixing the imperfection wavelength at this value, the most important parameter is then the imperfection amplitude. Its effect stabilizes however in the most dangerous interval of amplitudes $A/t \in [2,3]$ as the results show in general the existence of a plateau in the buckling curve that gives $\alpha = \sigma_{cr} / \sigma_{cl}$ as function of A/t.

Studying the buckling strength for two concomitant triangular entering geometric imperfections as function of the above parameters plus the distance separating the localized defects has indicated that a large contrast exists between the obtained buckling stress ratios, table 2. The minimum buckling stress ratio is only 0.135 while the maximum reaches 0.733. This gives details why it is important to analyse the influence of distance d separating the defects on the buckling strength within the worst interval of amplitudes $A/t \in [2,3]$. It was observed also from other preliminary simulations that as far as the distance d/R is enough large, its effect on the buckling stress remains very small. This gives explanation for the small values chosen here for the distance d as given in table 1 which are expected to have the greatest effect. This is for instance true when $R = 135\,mm$, $A/t = 2$, $K = 2.5$, $R/t = 450$ and $H/R = 3$ as varying $d = 50\,mm$ to $d = 150\,mm$ increases the buckling stress ratio from 0.17 to 0.21. The gain is significant in this range but when the distance $d > 150\,mm$ the influence of this parameter is found to be too small.

Figure 4 shows that the relative distance between the two localized defects d/R gives the lowest probability which is equal to 1.91%. This percentage is well below the other percentages associated to parameters: A/t (79.69%), K (7.02%), R/t (18.54%) and H/R (9.81%). This signifies that, in the range of parameters values investigated here, when considering the situation of two concomitant initial geometric imperfections, the relative distance between the two geometric imperfections is the most important parameter. It is followed by the geometric imperfection wavelength and by the shell aspect ratios, while the defect amplitude has the lowest influence on shell buckling stress.

Regarding the intrinsic influence of the geometric imperfection amplitude, it is well known that this parameter monitors to a large extent the shell buckling strength as it is shown in figure 3 for $A/t \in [0,2]$. But in the range of parameters values considered here $A/t \in [2,3]$, the amplitude parameter has only a reduced influence as it could be easily seen from the plateau present in the buckling curves of figure 3.

According to the multifactor analysis of variance ANOVA, one can verify that the method of Taguchi applied here without taking into account interaction between factors is acceptable since the residuals are very small (not exceeding 8%).

5 Conclusions

Considering elastic thin cylindrical shells subjected to axial compression and having one or two axisymmetric defects of entering triangular form, numerical simulations based on the finite element method have been performed to quantify shell buckling stress reduction, according to Euler analysis. A set of five factors intervening in the problem have been taken into account. A parametric study according to Taguchi method of design of experiment has been performed in order to determine their relative influence on the shell buckling strength.

It has been shown that two interacting defects could yield further reduction of the critical stress in comparison with the single defect case. In the range of parameters investigated for which the defect amplitude exceeds twice the shell thickness, the distance separating the two triangular geometric defects has been found to have the major influence on critical stress reduction. It is followed by the defect wavelength and by the shell aspect ratios.

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