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Proceedings of the Eleventh International Conference
on Computational Structures Technology,
B.H.V. Topping, (Editor),
Civil-Comp Press, Stirlingshire, Scotland

Paper 78

Detection of Multiple Cracks in Beams using the Superposition Property

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Abstract

This paper presents the phenomenon that appears in beams with multiple damages in respect to natural frequencies. Damaged beams with different boundary conditions were analysed by the authors, both in analytical way and using the finite element method. The considered damages are open cracks, affecting the whole width of the beam and have various levels of depth. For all types of cracks with a small span the vibrations are non-linear, except for the case of cracks placed symmetrically in the cross-section of the beam, for which we have linear vibrations. For cracks with large span the vibrations are linear. However, we have demonstrated that the linear approach is valid, either periods or pseudo-periods appear, because the vibration is at least pseudo-harmonic and a stabile pseudo-period is obtained.

Keywords: vibration, damages, natural frequency, finite element method, superposition principle.

1 Introduction

Damages influence the dynamic behaviour of structures, changing their geometry, and consequently their mechanical and dynamic characteristics such as natural frequencies, mode shapes, damping ratio and stiffness. These features, used in damage detection, are identified from measured response time-histories (most often accelerations or strains) or spectra of these time-histories. Our research is focused on damage detection using natural frequencies and their spectra because this data can be most easy acquired and processed.

Damage detection using natural frequency shifts is largely presented in literature [1, 2 and 3]. The methods based on frequency change can be classified in two categories: methods limited to damage detection and methods destined to detect, locate and quantify damages. Literature reviews, [2, 4], affirm that all methods based on natural frequency changes belonging to the second category are model-based, typically relying on the use of finite element models. Due to the low sensitivity of frequency shift owed to damage, very precise measurements are required. Often, the changes in frequency caused by structural damage are smaller than those observed between the undamaged structure and the mathematical model. This makes almost impossible to discern between inadequate modelling and changes due to damage, consequently the use of models being difficult to be applied (performed), [5] and [6]. Other problems in using natural frequencies shifts to detect damages reside in the fact that natural frequencies are sensitive to temperature and/or loads changes applied to the structure.

One of the major problems in damage location is the reliance on the finite element model. This model is also an important strength because the very incomplete set of measured data requires extra information from the model to be able to identify damage location [6]. The quality of the damage location assessment is critically dependent upon the updated model being physically meaningful [6]. This requires model validation using a control set of data not used for the updating, or using differences between the damaged and undamaged response data in the damage location algorithm [5].

In this paper we studied the case of symmetric supported beams (simple supported and double clamped) as well as asymmetric supported beams (cantilever beam, respectively clamped at one end and simple supported at the other), all having multiple damages. The undamaged beams are considered with constant cross-section and stiffness. First analytic methods were used for the undamaged beams, afterwards a Finite Element Analysis performed for similar beams. We obtained very close results by both methods for all analysed support types, confirmed also be experimental measurements, for which we used modular equipment with a single accelerometer. This confirmed the correctness of the Finite Element Method approach.

The second step was to define the frequency changes of the first ten weak-axis bending vibration modes for different types of cracks. These types of cracks are characterized by position on the beam, depth and span. We demonstrated that each damage have a unique signature, or pattern, in the natural frequency shift spectrum for asymmetric supported beams. For symmetric supported beams, due to the effect of symmetry, there are always two locations providing one pattern. The relation between crack parameters and frequency changes was also confirmed by measurement.

Afterwards, for beams similar to that previously studied, a series of cracks were realized and the dynamic behaviour analysed. So, the effect of multiple cracks on frequency changes was defined. Finally, the effect of multiple cracks on several beams was compared with the sum of effect of individual similar cracks.

The investigations lead to the conclusion that the superposition principle is valid, so that frequency changes of a beam with multiple cracks have a similar effect the sum of frequency changes produced by each individual crack. Statistical methods where developed to extract the parameters of the individual cracks from data obtained by measurements on multiple damaged beams. This property can be used to detect and assess damages in beams, even for complex cases.

2 Analytical investigation

The aim of the analytical investigations was to gain as much as possible information about the dynamical behaviour of Euler-Bernoulli beam and to correlate them with information obtained by means of the finite element method, in order to understand the phenomenon occurring by appearance of discontinuities in the beam's structure [7]. The study comprises the first ten natural frequencies on the weak-axis bending vibration modes.

The beam's geometrical characteristics are: length L, width B and height H. Consequently, the beam has the cross-section A and the moment of inertia I. The mechanical characteristics are: mass density ρ, Young's modulus E and Poisson's ratio ν. The undamaged beams are considered with constant cross-section and stiffness.

The analysed beam is a steel one, having the following geometrical characteristics: length $L = 1.8\ m$; width $B = 0.05\ m$ and height $H = 0.02\ m$. Consequently, for the undamaged state the beam has the cross-section $A = 10^{-3}\ m^2$ and the moment of inertia $I = 333.333 \cdot 10^{-6}\ m^4$. The material parameters of the specimens are: mass density $\rho = 7850\ kg/m^3$; Young's modulus $E = 2.0 \cdot 10^{11}$ N/m^2 and Poisson's ratio $\nu = 0.3$.

2.1 Undamaged symmetric supported beams

The study of symmetric supported beams comprises two cases: simple supported beam and double clamped beam.

The natural frequencies for the undamaged simple supported beam ($f^{UA}{}_{SSn}$) may be determined according to (1):

$$f^{UA}{}_{SSn} = \frac{1}{2\pi} \cdot \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EI}{\rho A}} \tag{1}$$

where,

n=1, 2…10, represents the first ten vibration mode;
L [m], length of the beam;
E [N/m²], Young's modulus;
I [m⁴], moment of inertia;
ρ [kg/m³], mass density;
A [m²], cross-section

The natural frequencies for the undamaged double clamped beam ($f^{UA}{}_{DCn}$) are given with relation (2);

$$f^{UA}{}_{DCn} = \frac{1}{2\pi} \cdot \frac{\left(n + \frac{1}{2}\right)^2 \pi^2}{L^2} \sqrt{\frac{EI}{\rho A}} \tag{2}$$

The first ten natural frequencies for the considered undamaged simple supported beam and double clamped beam, determined by the relations (1) and (2), are presented in Table 1.

Analysed Cases	Natural Frequencies [Hz]				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
$f^{UA}{}_{SSn}$	14.128	56.514	127.156	226.055	353.211
$f^{UA}{}_{DCn}$	31.543	88.320	173.072	286.101	427.386
Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
$f^{UA}{}_{SSn}$	508.624	692.294	904.221	1144.404	1412.845
$f^{UA}{}_{DCn}$	596.927	787.322	1020.780	1275.092	1557.661

Table 1. First ten natural frequencies for undamaged beam

2.2 Undamaged asymmetric supported beams

For the undamaged asymmetric supported beams have been choose the cases: cantilever beam and clamped at one end and simple supported at the other end.

The natural frequencies for the undamaged cantilever beam ($f^{UA}{}_{Cn}$) and the natural frequencies for the undamaged beam clamped at one end and simple supported at the other end are given with relation (3);

$$f^{UA}{}_{Cn} = f^{UA}{}_{CSn} = \frac{1}{2\pi} \cdot \frac{\lambda_n^2}{L^2} \sqrt{\frac{EI}{\rho A}} \tag{3}$$

where,

λ_n represents the first ten solutions of equation: $1 + \cos\lambda \cdot \cosh\lambda = 0$, for undamaged cantilever beam, and the first ten the solutions of equation: $\tan\lambda - \tanh\lambda = 0$, for undamaged beam clamped at one end and simple supported at the other end.

Analysed Cases	Natural Frequencies [Hz]				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
$f^{UA}{}_{Cn}$	5.033	31.543	88.320	173.072	286.101
$f^{UA}{}_{CSn}$	22.071	71.525	149.232	255.195	389.415
Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
$f^{UA}{}_{Cn}$	427.386	596.927	787.322	1020.780	1275.092
$f^{UA}{}_{CSn}$	551.892	742.626	961.617	1208.865	1484.370

Table 2. First ten natural frequencies for undamaged beam

The results for the first ten natural frequencies of undamaged cantilever beam and undamaged beam clamped at one end and simple supported at the other end determined by the relations (3), are presented in Table 2.

2.3 Considerations regarding the analytical calculation of damaged beams

Analytical investigation for the beams with damages is based on the equation (4) developed by the authors [7, 8 and 9]. The considered damages are open cracks, affecting the whole width of the beam and have various levels of depth.

$$f^{DA}{}_{n}(x,y) = f^{UA}{}_{n} \cdot \left(1 - \frac{c_1 \cdot H}{L} \cdot \left(\frac{\delta(y)}{H-\delta(y)} \right)^{\frac{3}{2}} \cdot c_2 \cdot \frac{\overline{G} \cdot \overline{L}^2}{6} \cdot \left(\overline{\phi_n''}(x)_n \right)^2 \right) \tag{4}$$

where,

$f^{DA}{}_n(x,y)$, represents the natural frequency for the damaged beam, of the weak-axis bending for vibration mode n with damage located at position x and $H\text{-}\delta(y)$ damage depth;

$f^{UA}{}_n$, represents the natural frequency for the undamaged beam, of the weak-axis bending for vibration mode n;

H *[m]*, height of the beam weighted by coefficient c_1;

L *[m]*, length of the beam;

$\delta(y)$ *[m]*, depth of the damage;

c_2, coefficient taking in consideration the support type of the beam. For simply supported beam $c_2=0.5$, for double clamped beam and cantilever beam $c_2=1$, and $c_2=0.875$ for the beam clamped at one end and simple supported at the other end;

$\frac{\overline{G} \cdot \overline{L}^2}{6}$, sum of areas of bending moments acting on the beam, calculated for normalized weight $\overline{G}$ and length $\overline{L}$;

$\overline{\phi_n''}(x)_n$, the mode shape curvature for vibration mode n.

2.4 Damaged beams

In this paper, it was considered a damaged beam with three cases: first case, a damaged beam with the damage located at $x_1=0.056L$ with $\delta(y_1)=0.25H$ damage depth; second case, the damage is located at $x_2=0.5L$ with $\delta(y_2)=0.30H$ damage depth and third case, the same beam with two damages located at $x_1=0.056L$ with $\delta(y_1)=0.25H$ damage depth and $x_2=0.5L$ with $\delta(y_2)=0.30H$ damage depth. These three cases are applied for simply supported beam, double clamped beam, cantilever beam and for the beam clamped at one end and simple supported at the other end. Clamped end is considered the origin of the coordinate system.

Applying the formula (4) with the value of coefficient $c_1=42$ for the height of $H=0.02\ m$, natural frequencies can be calculated for the damaged beam.

Table 3 presents the first ten natural frequencies for damaged beam, with damage located at $x_1=0.056L$ and $\delta(y_1)=0.25H$ damage depth. In line $f^{DA}{}_{SSn}$, natural frequencies values are entered for simply supported beam, line $f^{DA}{}_{DCn}$ contains natural frequencies values for double clamped beam, line $f^{DA}{}_{Cn}$ contains natural frequencies values for cantilever beam and in line $f^{DA}{}_{CSn}$, natural frequencies values are entered for the beam clamped at one end and simple supported at the other end.

Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
$f^{DA}{}_{SSn}$	14.125	56.464	126.918	225.356	351.660
$f^{DA}{}_{DCn}$	31.281	87.899	172.670	285.879	427.355
$f^{DA}{}_{Cn}$	4.969	31.288	87.898	172.670	285.879
$f^{DA}{}_{CSn}$	21.895	71.179	148.859	254.953	389.879
Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
$f^{DA}{}_{SSn}$	505.769	687.719	897.657	1135.839	1402.590
$f^{DA}{}_{DCn}$	596.868	786.783	1019.002	1271.899	1551.053
$f^{DA}{}_{Cn}$	427.355	596.868	786.783	1019.003	1271.289
$f^{DA}{}_{CSn}$	551.878	742.292	960.392	1206.048	1479.258

Table 3. First ten natural frequencies for damaged beam, $x_1=0.056L,\ \delta(y_1)=0.25H$

Table 4 presents the first ten natural frequencies for damaged beam, with damage located at $x_2=0.5L$ and $\delta(y_2)=0.30H$ damage depth and Table 5 contains the first ten natural frequencies for damaged beam, with two damages, first located at $x_1=0.056L$ with $\delta(y_1)=0.25H$ damage depth and second damage located at $x_2=0.5L$ with $\delta(y_2)=0.30H$ damage depth.

Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
$f^{DA}{}_{SSn}$	13.974	56.514	125.769	226.055	349.357
$f^{DA}{}_{DCn}$	31.292	88.320	171.162	286.101	422.725
$f^{DA}{}_{Cn}$	5.021	31.192	88.319	171.184	286.101
$f^{DA}{}_{CSn}$	21.927	71.436	148.004	254.837	386.243
Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
$f^{DA}{}_{SSn}$	508.624	684.740	904.221	1131.918	1412.845
$f^{DA}{}_{DCn}$	596.927	778.757	1020.780	1261.180	1557.661
$f^{DA}{}_{Cn}$	422.722	596.927	778.758	1020.780	1261.180
$f^{DA}{}_{CSn}$	551.121	736.575	960.047	1199.014	1482.182

Table 4. First ten natural frequencies for damaged beam, $x_2=0.5L,\ \delta(y_2)=0.30H$

Analysed Cases	Natural Frequencies [Hz]				
	Vibration Mode (n)				
	1	2	3	4	5
f^{DA}_{SSn}	13.971	56.464	125.531	225.356	347.806
f^{DA}_{DCn}	31.030	87.899	170.76	285.879	422.695
f^{DA}_{Cn}	4.956	30.937	87.897	170.781	285.879
f^{DA}_{CSn}	21.751	71.089	147.632	254.595	386.184
Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
f^{DA}_{SSn}	505.769	680.165	897.657	1123.353	1402.590
f^{DA}_{DCn}	596.868	778.218	1019.002	1257.376	1551.053
f^{DA}_{Cn}	422.692	596.868	778.218	1019.003	1257.376
f^{DA}_{CSn}	551.107	736.240	959.047	1196.197	1477.182

Table 5. Beam with 2 damages: $x_1=0.056L;\ \delta(y_1)=0.25H$ and $x_2=0.5L;\ \delta(y_2)=0.30H$

3 Numerical investigation

The beam presented in the chapter 2 was analysed, both in the undamaged and damaged case, by using the finite element method. The damage was considered of 2 mm wide and it is shown in figure 1.

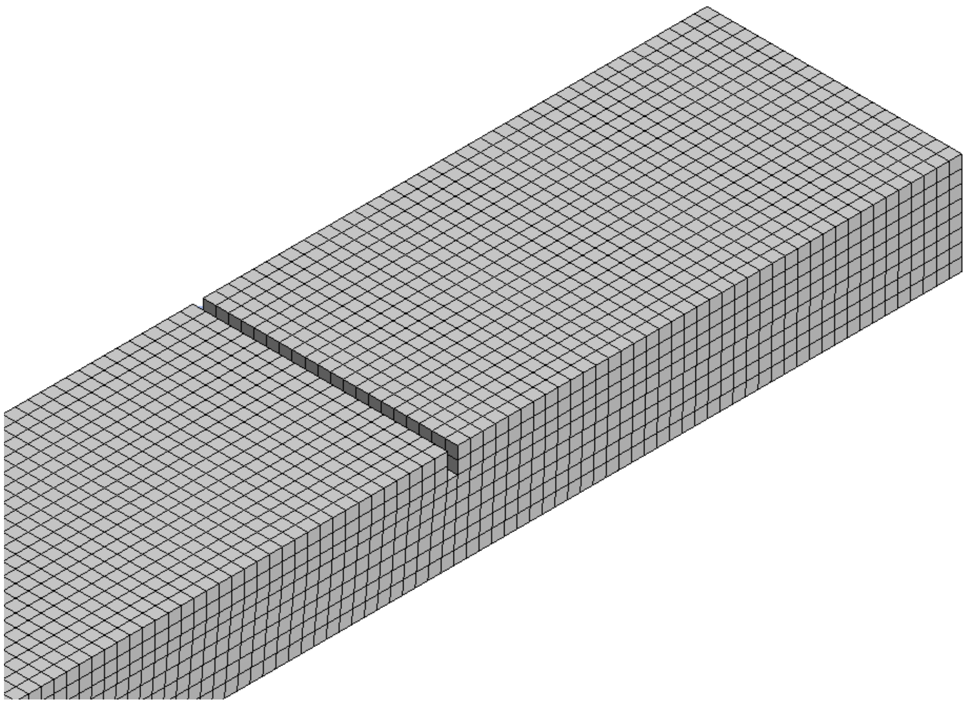

Figure 1: Detail of modelled beam with damage.

For each type, simply supported beam, double clamped beam, cantilever beam and clamped one end and simply supported the other end beam, modal analysis have been applied so for the undamaged beam, as well as for the damaged beam with the same location of the damage and the same damage depth like presented in the chapter 2 and shown in figure 2.

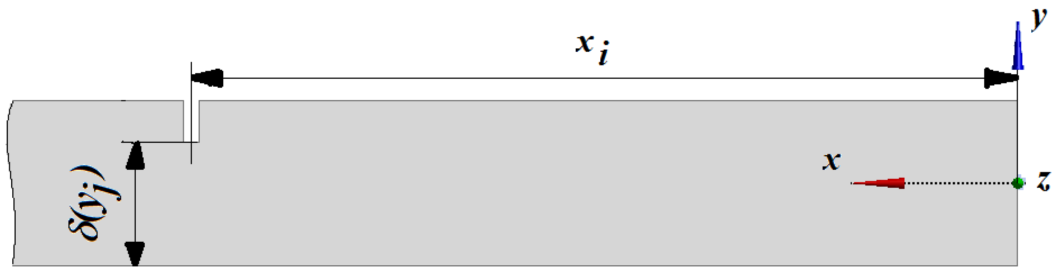

Figure 2. Detail of damaged beam

The first ten natural frequencies of the weak-axis bending modes , for the undamaged beam are presented in Table 6, simply supported beam (f^U_{SSn}), double clamped beam (f^U_{DCn}), cantilever beam (f^U_{Cn}) and for the beam clamped at one end and simple supported at the other end (f^U_{CSn}).

Analysed Cases	Natural Frequencies [Hz]				
	Vibration Mode (n)				
	1	2	3	4	5
f^U_{SSn}	14.126	56.472	126.945	225.390	351.592
f^U_{DCn}	32.086	88.351	172.957	285.396	425.417
f^U_{Cn}	5.039	31.564	88.301	172.816	285.211
f^U_{CSn}	22.090	71.529	149.065	254.518	387.646
Analysed Cases	Natural Frequencies [Hz]				
	Vibration Mode (n)				
	6	7	8	9	10
f^U_{SSn}	505.278	686.117	893.727	1127.674	1387.480
f^U_{DCn}	592.694	786.845	1007.440	1254.003	1526.014
f^U_{Cn}	425.208	592.495	786.703	1007.412	1254.157
f^U_{CSn}	548.148	735.668	949.800	1190.089	1456.034

Table 6. First ten natural frequencies for undamaged beam. Numerical results

Tables 7, 8 and 9 presents the natural frequencies for the damaged beam (figure 3). In line f^D_{SSn}, natural frequencies values are entered for simply supported beam, line f^D_{DCn} contains natural frequencies values for double clamped beam, line f^D_{Cn} contains natural frequencies values for cantilever beam and in line f^D_{CSn}, natural frequencies values are entered for the beam clamped at one end and simple supported at the other end.

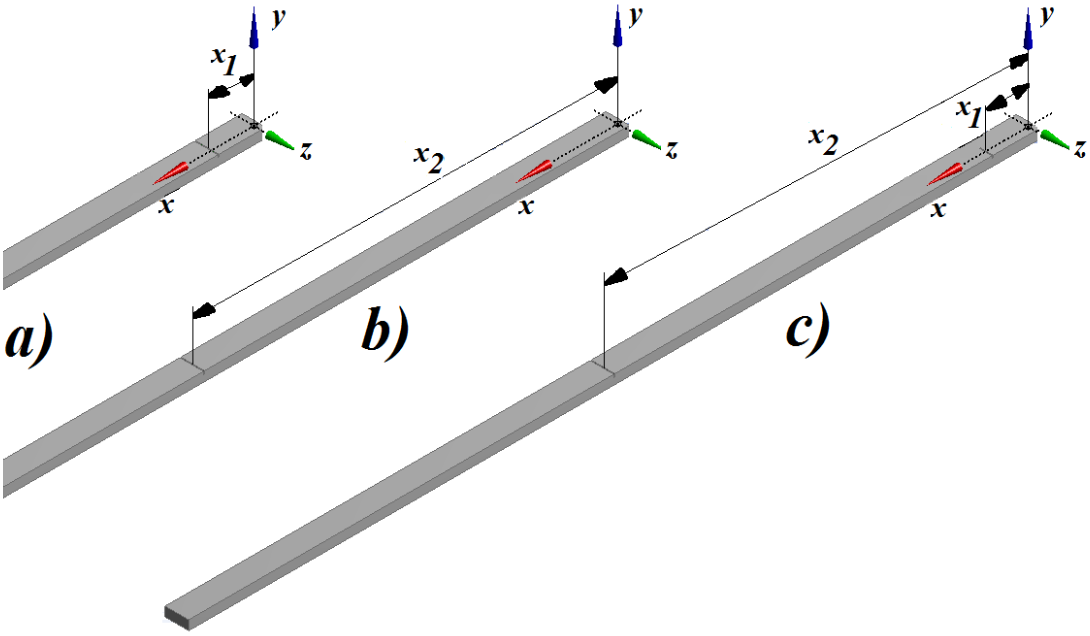

Figure 3. Analysed cases: a) beam with damage at $x_1=0.056L$, $\delta(y_1)=0.25H$; b) beam with damage at $x_2=0.5L$, $\delta(y_2)=0.30H$; c) beam with two damages $x_1=0.056L$, $\delta(y_1)=0.25H$ and $x_2=0.056L$, $\delta(y_2)=0.25H$

Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
f^D_{SSn}	14.123	56.423	126.712	224.707	350.088
f^D_{DCn}	31.822	87.933	172.558	285.174	425.397
f^D_{Cn}	4.975	31.311	87.885	172.419	284.992
f^D_{CSn}	21.884	71.133	148.640	254.240	387.581
Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
f^D_{SSn}	502.542	681.807	887.694	1120.018	1378.587
f^D_{DCn}	592.687	786.419	1005.949	1250.634	1520.608
f^D_{Cn}	425.192	592.492	786.300	1005.954	1250.910
f^D_{CSn}	548.188	735.398	948.641	1187.275	1451.534

Table 7. First ten natural frequencies for damaged beam, $x_1=0.056L$, $\delta(y_1)=0.25H$

In Table 7 are presented the first ten natural frequencies for the damaged beam, (f^D_{SSn}) simply supported beam, (f^D_{DCn}) double clamped beam, (f^D_{Cn}) cantilever beam and (f^D_{CSn}) beam clamped at one end and simple supported at the other end. The damage is located at $x_1=0.056L$ and $\delta(y_1)=0.25H$ damage depth.

In Table 8, the damage is located at $x_2=0.5L$ with $\delta(y_2)=0.30H$ damage depth and in Table 9, the damages are located at $x_1=0.056L$ with $\delta(y_1)=0.25H$ damage depth and $x_2=0.5L$ with $\delta(y_2)=0.30H$ damage depth.

Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
f^{D}_{SSn}	13.973	56.472	125.599	225.389	347.941
f^{D}_{DCn}	31.841	88.355	171.126	285.398	421.072
f^{D}_{Cn}	5.027	31.217	88.301	170.992	285.209
f^{D}_{CSn}	21.937	71.436	147.722	254.128	384.261
Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
f^{D}_{SSn}	505.272	678.466	893.708	1116.648	1378.437
f^{D}_{DCn}	592.690	778.997	1007.422	1241.800	1525.967
f^{D}_{Cn}	420.813	592.487	779.052	1007.389	1241.959
f^{D}_{CSn}	547.323	729.384	948.403	1180.150	1453.948

Table 8. First ten natural frequencies for damaged beam, $x_2=0.5L$, $\delta(y_2)=0.30H$

Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
f^{D}_{SSn}	13.970	56.423	125.372	224.707	346.475
f^{D}_{DCn}	31.576	87.938	170.738	285.177	421.046
f^{D}_{Cn}	4.963	30.966	98.884	170.597	284.990
f^{D}_{CSn}	21.729	71.043	147.300	253.860	384.187
Analysed Cases	**Natural Frequencies [Hz]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
f^{D}_{SSn}	502.537	674.477	887.667	1109.146	1378.517
f^{D}_{DCn}	592.683	778.583	1005.931	1238.663	1520.543
f^{D}_{Cn}	420.799	592.483	778.533	1005.931	1239.019
f^{D}_{CSn}	547.341	729.112	947.288	1177.379	1449.763

Table 9. Beam with 2 damages: $x_1=0.056L;\ \delta(y_1)=0.25H$ and $x_2=0.5L;\ \delta(y_2)=0.30H$

4 Relative frequency shift

To evaluate the results of modal analyses obtained with finite element method and to compare them with the analytical results, given by the formula (4), it was introduced the term: relative frequency shift. Generally, the relative frequency shift is defined with relation (5):

$$\Delta f_n = \frac{f^U{}_n - f^D{}_n}{f^U{}_n} \cdot 100 \quad [\%] \tag{5}$$

and taking in consideration the formula (4), the relative frequency shift will be (6):

$$\Delta f^A{}_n(x,y) = \frac{c_1 \cdot H}{L} \cdot \left(\frac{\delta(y)}{H - \delta(y)} \right)^{\frac{3}{2}} \cdot c_2 \cdot \frac{\overline{G} \cdot \overline{L}^2}{6} \cdot \left(\overline{\phi_n''}(x)_n \right)^2 \cdot 100 \quad [\%] \tag{6}$$

where,

Δf_n [%], represents the relative frequency shift obtained from modal analysis, or obtained from measured natural frequencies, for vibration mode *n*;

$f^U{}_n$ [Hz], represents the natural frequency for the undamaged beam, of the weak-axis bending for vibration mode *n*;

$f^D{}_n$ [Hz], represents the natural frequency for the damaged beam, of the weak-axis bending for vibration mode *n,* with damage located at position *x* with *δ(y)* damage depth;

$\Delta f^A{}_n(x,y)$ [%], relative frequency shift obtained from analytic calculation for vibration mode *n* and damage located at position *x* with *H-δ(y)* damage depth.

For the first ten natural frequencies, at each location of the damage and each depth of the damage, are calculated the relative frequency shift with (5), or (6) and the results are represented in diagrams. The curves formed by segments unifying this pair of values just indicate the tendency of the shift in frequency and are used to make easier comparison with the shift of natural frequencies obtained by measurements. These diagrams, for each location of damage and each level of depth have a unique signature, or pattern, in the natural frequency shift spectrum for asymmetric supported beams. For symmetric supported beams, due to the effect of symmetry, there are always two location providing one pattern.

Figures 4, 5, 6 and 7 present the relative frequency shift diagrams for all the analysed cases.

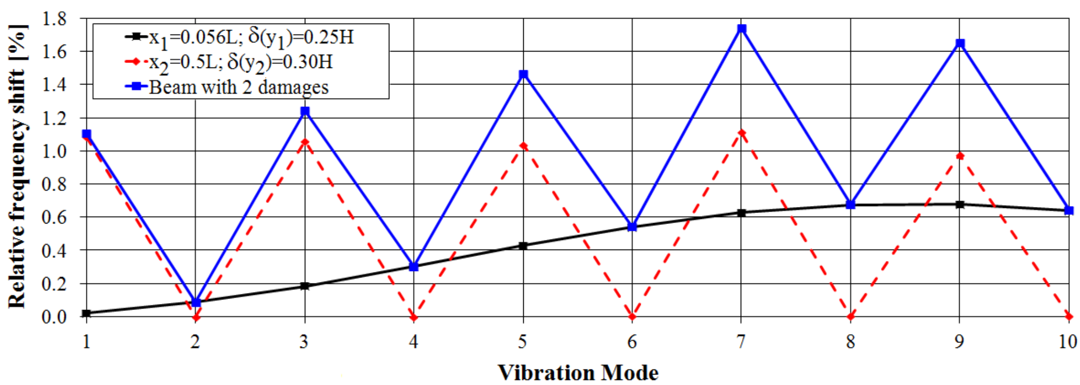

Figure 4. Relative frequency shift for simple supported beam with damages

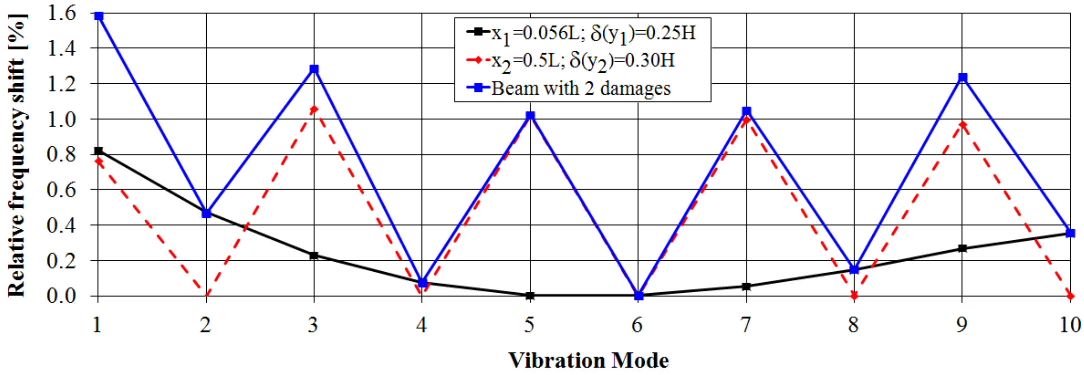

Figure 5. Relative frequency shift for double clamped beam with damages

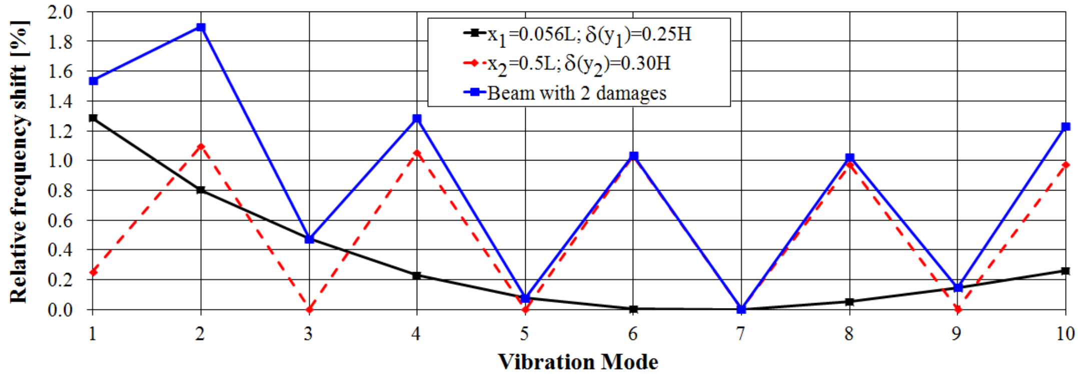

Figure 6. Relative frequency shift for cantilever beam with damages

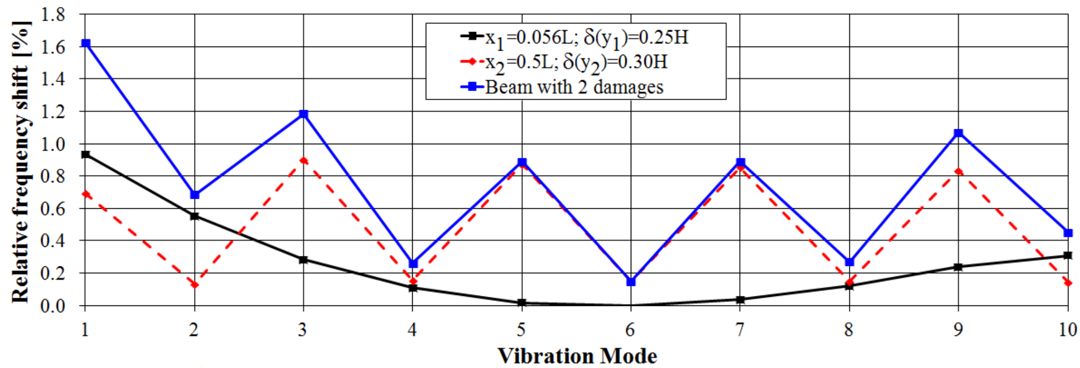

Figure 7. Relative frequency shift for beam with damages, clamped at one end and simple supported at the other end

5 The superposition property

By using the relative frequency shift principle, is very simple to demonstrate the superposition property of natural frequency changes in beams with two or more damages. For cases investigated, do the sum of relative frequency shift obtained for damage located at $x_1=0.056L$ with $\delta(y_1)=0.25H$ damage depth and $x_2=0.5L$ with $\delta(y_2)=0.30H$ and compare it with relative frequency shift obtained for damaged beam with two damages, respectively damage located at $x_1=0.056L$, $\delta(y_1)=0.25H$ and $x_2=0.5L$, $\delta(y_2)=0.30H$.

Tables 10 and 11 present the relative frequency shift for damaged beam with one damage, damage located at $x_1=0.056L$, $\delta(y_1)=0.25H$ (Table 10) and damage located at $x_2=0.5L$, $\delta(y_2)=0.30H$ (Table 11). In Table 12 are presented the relative frequency shifts for damaged beam with two damages ($x_1=0.056L$, $\delta(y_1)=0.25H$ and $x_2=0.5L$, $\delta(y_2)=0.30H$) and the sum of relative frequency shift for damaged beam with one damage, presented in Table 10 and Table 11.

Analysed Cases	**Relative frequency shift [%]**				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
$\Delta f_{SSn}(x_1,y_1)$	0.022	0.086	0.184	0.303	0.428
$\Delta f_{DCn}(x_1,y_1)$	0.824	0.473	0.231	0.078	0.005
$\Delta f_{Cn}(x_1,y_1)$	1.285	0.802	0.472	0.230	0.077
$\Delta f_{CSn}(x_1,y_1)$	0.933	0.554	0.285	0.109	0.017
Analysed Cases	**Relative frequency shift [%]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
$\Delta f_{SSn}(x_1,y_1)$	0.541	0.628	0.675	0.679	0.641
$\Delta f_{DCn}(x_1,y_1)$	0.001	0.054	0.148	0.269	0.354
$\Delta f_{Cn}(x_1,y_1)$	0.004	0.001	0.051	0.145	0.259
$\Delta f_{CSn}(x_1,y_1)$	-0.003	0.037	0.122	0.236	0.309

Table 10. Relative frequency shift for damaged beam, $x_1=0.056L$, $\delta(y_1)=0.25H$

Analysed Cases	**Relative frequency shift [%]**				
	Vibration Mode (n)				
	1	**2**	**3**	**4**	**5**
$\Delta f_{SSn}(x_2,y_2)$	1.084	0.000	1.061	0.001	1.039
$\Delta f_{DCn}(x_2,y_2)$	0.764	-0.004	1.059	-0.001	1.021
$\Delta f_{Cn}(x_2,y_2)$	0.254	1.098	0.001	1.055	0.001
$\Delta f_{CSn}(x_2,y_2)$	0.694	0.131	0.901	0.153	0.873
Analysed Cases	**Relative frequency shift [%]**				
	Vibration Mode (n)				
	6	**7**	**8**	**9**	**10**
$\Delta f_{SSn}(x_2,y_2)$	0.001	1.115	0.002	0.978	0.003
$\Delta f_{DCn}(x_2,y_2)$	0.001	0.997	0.002	0.973	0.003
$\Delta f_{Cn}(x_2,y_2)$	1.034	0.001	0.973	0.002	0.973
$\Delta f_{CSn}(x_2,y_2)$	0.150	0.854	0.147	0.835	0.143

Table 11. Relative frequency shift for damaged beam, $x_2=0.5L$, $\delta(y_2)=0.30H$

Analysed Cases	Relative frequency shift [%] Vibration Mode (n)				
	1	2	3	4	5
$\Delta f_{SSn}(x_1,y_1;\ x_2,y_2)$	1.105	0.086	1.239	0.303	1.455
$\Delta f_{SSn}(x_1,y_1)+\ \Delta f_{SSn}(x_2,y_2)$	1.106	0.086	1.245	0.304	1.466
$\Delta f_{DCn}(x_1,y_1;\ x_2,y_2)$	1.589	0.468	1.283	0.077	1.027
$\Delta f_{DCn}(x_1,y_1)+\ \Delta f_{DCn}(x_2,y_2)$	1.588	0.469	1.289	0.077	1.026
$\Delta f_{Cn}(x_1,y_1;\ x_2,y_2)$	1.527	1.893	0.473	1.284	0.077
$\Delta f_{Cn}(x_1,y_1)+\ \Delta f_{Cn}(x_2,y_2)$	1.539	1.900	0.743	1.285	0.077
$\Delta f_{CSn}(x_1,y_1;\ x_2,y_2)$	1.635	0.680	1.184	0.259	0.892
$\Delta f_{CSn}(x_1,y_1)+\ \Delta f_{CSn}(x_2,y_2)$	1.628	0.685	1.186	0.263	0.890

Analysed Cases	Relative frequency shift [%] Vibration Mode (n)				
	6	7	8	9	10
$\Delta f_{SSn}(x_1,y_1;\ x_2,y_2)$	0.542	1.696	0.678	1.643	0.646
$\Delta f_{SSn}(x_1,y_1)+\ \Delta f_{SSn}(x_2,y_2)$	0.542	1.743	0.677	1.657	0.644
$\Delta f_{DCn}(x_1,y_1;\ x_2,y_2)$	0.002	1.050	0.150	1.223	0.359
$\Delta f_{DCn}(x_1,y_1)+\ \Delta f_{DCn}(x_2,y_2)$	0.002	1.052	0.150	1.242	0.357
$\Delta f_{Cn}(x_1,y_1;\ x_2,y_2)$	1.037	0.002	1.038	0.147	1.207
$\Delta f_{Cn}(x_1,y_1)+\ \Delta f_{Cn}(x_2,y_2)$	1.037	0.002	1.024	0.147	1.231
$\Delta f_{CSn}(x_1,y_1;\ x_2,y_2)$	0.147	0.891	0.264	1.068	0.431
$\Delta f_{CSn}(x_1,y_1)+\ \Delta f_{CSn}(x_2,y_2)$	0.147	0.891	0.269	1.072	0.452

Table 12. Relative frequency shift, $x_1=0.056L,\ \delta(y_1)=0.25H;\ x_2=0.5L,\ \delta(y_2)=0.30H$

Figures 8, 9, 10 and 11 demonstrate the superposition property for damaged beam with two damages regardless of supporting type, location of damage and level of damage depth by using relative frequency shift diagrams. Finally, the effect of multiple cracks on several beams was compared with the sum of effect of individual similar cracks.

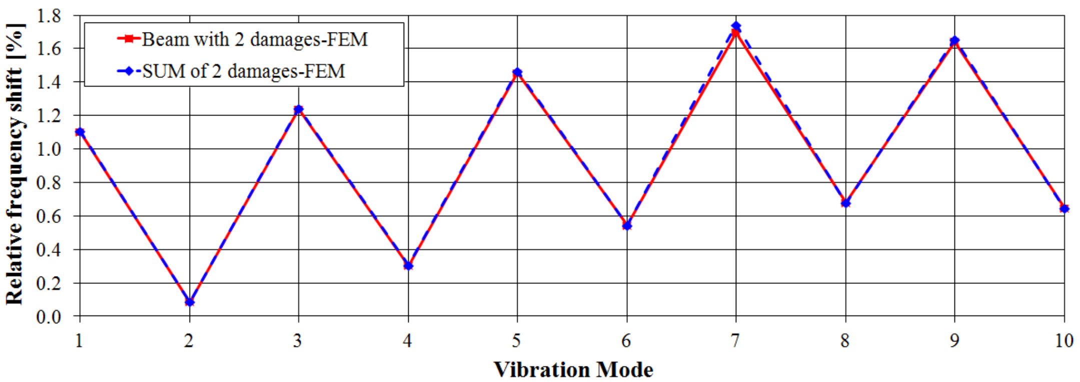

Figure 8. Superposition property for simply supported beam

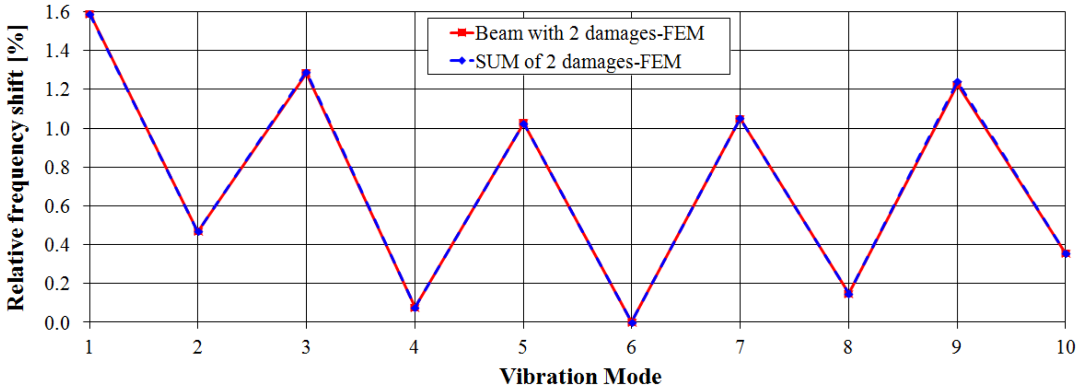

Figure 9. Superposition property for double clamped beam

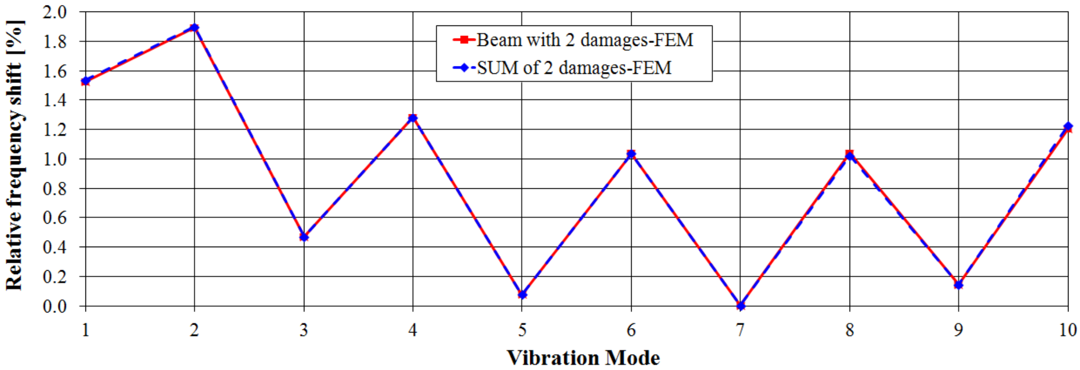

Figure 10. Superposition property for cantilever beam

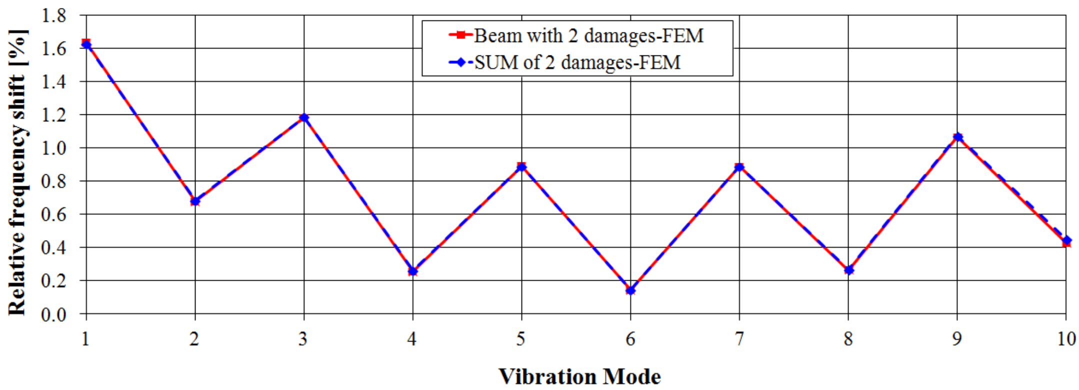

Figure 11. Superposition property for damaged beam with clamped one end and simply supported the other end

Analysing the Figures 8 - 11, can notice that, practically, there is no difference between relative frequency shifts values obtained through the two methods.

6 Method validation

We conducted experiments on the beam to measure the first ten natural frequencies of the undamaged beam, and afterwards, the changes due to different types of damages. The damages were done by saw cutting, the cuts being approximately 2 mm wide. For the simulation of the damage, one initially made a cut $x_1=0.51L$ with the depth between $\delta(y_1)>0.25H$ and $\delta(y_2)<0.33H$, and then, in other location, $x_2=0.1L$, another damages were done with the depth between $\delta(y_1)>0.33H$ and $\delta(y_2)<0.42H$. The beam is a steel one, having the following geometrical characteristics: length $L = 1.0\ m$; width $B = 0.03\ m$ and height $H = 0.0042\ m$. During the tests, the beams where fixed in a milling machine, see Figure 12, which assured sufficient rigidity to the clamping system. The excitation of the structure in view of determining the own frequencies was realised both by hitting with a hammer and by forcing out of the equilibrium position; for the same beam the results were similar. The measurement system composed by a laptop, a NI cDAQ-9172 compact chassis with NI 9234 four-channel dynamic signal acquisition modules and a Kistler 8772 accelerometer mounted on the free end of the beam was used. Virtual instruments created in LabVIEW acquired the time history of acceleration, stored it and realized the spectral analysis.

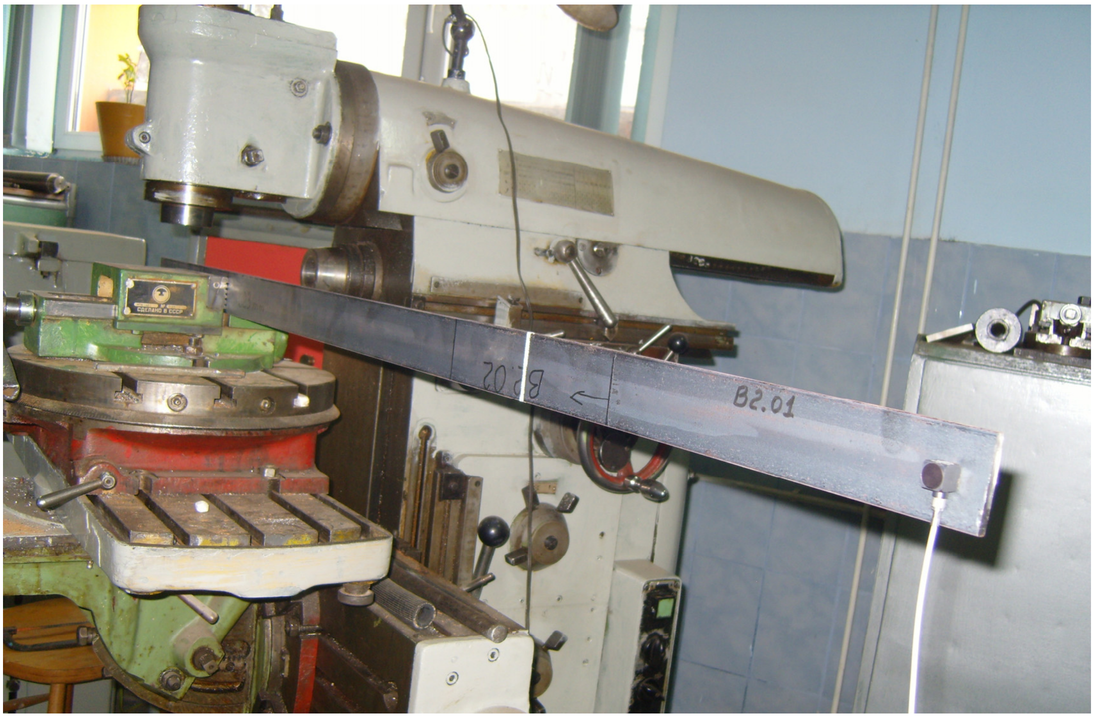

Figure 12. Experimental stand

Damage location and depth evaluation for each case are presented in figure 13 and 14.

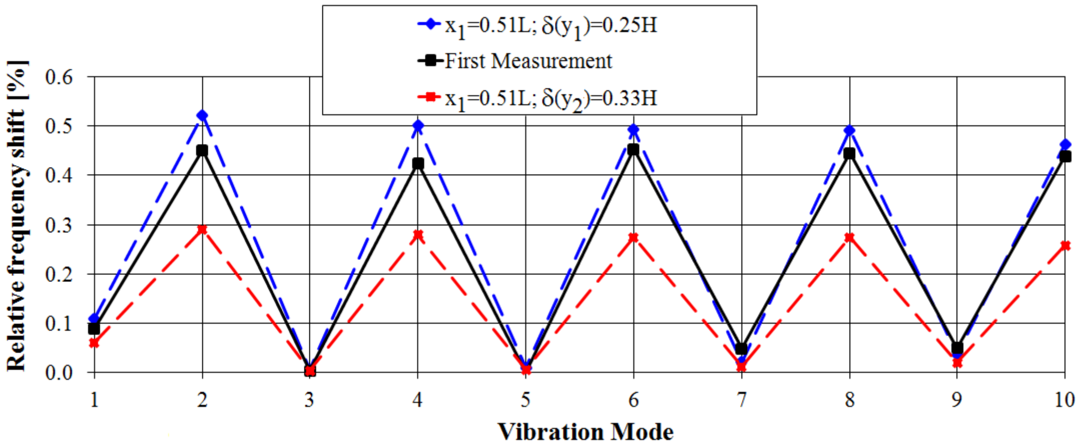

Figure 13. Damage location and depth evaluation for first measurement

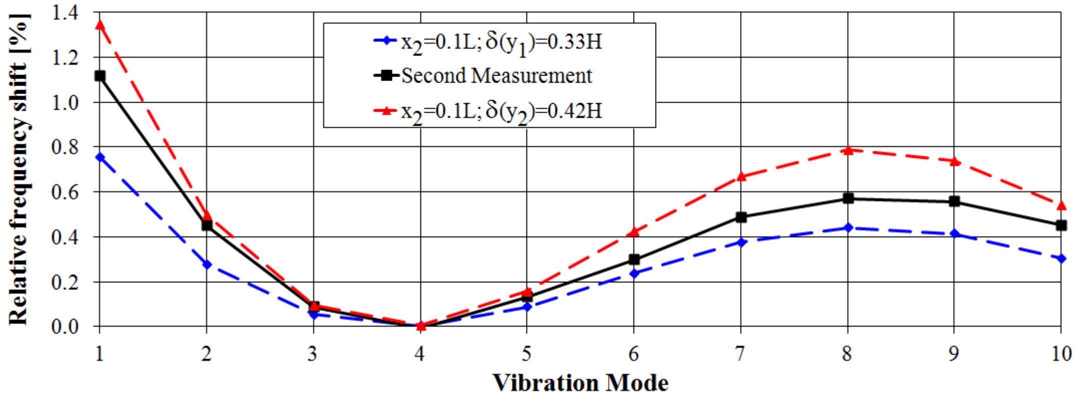

Figure 14. Damage location and depth evaluation for second measurement

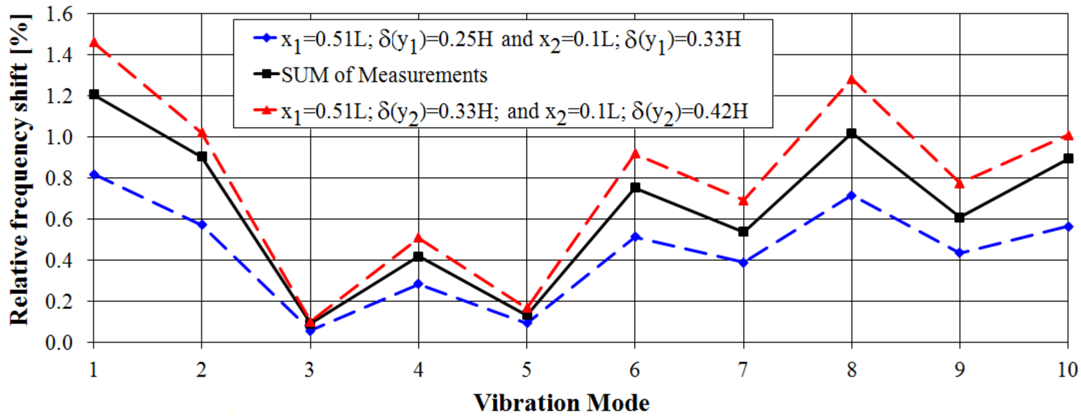

Figure 15. Superposition property applied on measured cantilever beam

For the second measurement, in order to apply the relative frequency shift, the measured natural frequencies in formula (5) for the damaged beam was considered as basis for calculating, in order to obtain the relative frequency shift for the second measurement (Figure 14). Thus, the superposition property can be demonstrated by laboratory measurements. The investigations lead to the conclusion that the superposition principle is valid, so that frequency changes of a beam with multiple cracks have a similar effect the sum of frequency changes produced by each individual crack (see Figure 15).

7 Conclusions

The method, proposed in this paper, applicable to beams with open cracks, is based on certain phenomena characteristic to the dynamic behaviour of beams, highlighted as a result of several analytical, numerical and experimental studies developed by the authors. The method allowed the location of defects with a precision of approximately 1% without requiring laboratory conditions, as it implies the use of one single accelerometer and is appropriate for practical applications.

Based on laborious work by the authors, they managed an analytical expression of the phenomenon that occurs in damaged beams. Statistical methods where developed to extract the parameters of the individual cracks from data obtained by measurements on multiple damaged beams. This property can be used to detect and assess damages in beams, even for complex cases.

The considered damage are open cracks, affecting the whole width of the beam and have various levels of depth. Each damage has a unique signature, or pattern, in the natural frequency shift spectrum for asymmetric supported beams. For symmetric supported beams, arising from the effect of symmetry, there are always two locations providing one pattern. The relation between crack parameters and frequency changes was also confirmed by measurement

Acknowledgements

The authors gratefully acknowledge the support of the Managing Authority for Sectoral Operational Programme for Human Resources Development (MASOPHRD), within the Romanian Ministry of Labour, Family and Equal Opportunities by co-financing the project “Excellence in research through postdoctoral programmes in priority domains of the knowledge-based society (EXCEL)” ID 62557.

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