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Development and Implementation of Numerical Strategies for Nonlinear Dynamic Analysis of Risers using the Finite Element Method

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Abstract

In this paper, the finite element method is applied to obtain the nonlinear dynamic behaviour of risers using a bi-dimensional Reissner frame element. The proposed methodology is based on the minimum potential energy theorem written regarding nodal positions. The risers are slender pipes used in the offshore industry to link the seabed with the vessel. In this paper the buoyancy load is given by the Archimedes principle. The self-weight is applied as a distributed element force. The current velocities and accelerations are calculated using the linear Airy theory for waves. The hydrodynamic forces are modelled according to the modified Morison equation. The vessel movements are applied as boundary conditions of displacement at the top of the riser. The temporal integration is performed using the classical Newmark's method as the formulation is total Lagrangian. Extensive tests of the basic formulation are performed in order to identify its limitations for riser applications. Some modifications are proposed in order to overcome the detected limitations. Examples are used to validate the final proposed strategy.

Keywords: finite element method, riser, surge motions, heave motions, nonlinear dynamic analysis.

1 Introduction

In the last years the offshore industry has increased its interest regarding safety in the drilling and transportation operations. The design of the risers requires that with all load cases, the structure should present small bending. Moreover, the normal force should be positive (tension) along the riser's extension.

The design of riser structures implies to combine a variety of loads and factors. Generally, the design includes hydrodynamic forces, the self-weight, the floater motions and the soil-structure interactions. In this work, special attention is given to

the imposed displacements applied at the top of risers, which represents the motions of the floated vessel.

The API standard establishes that axial motions stimulate extreme dynamic phenomena in catenary moorings. Chatjigeorgiou [1] noted that imposed displacements amplify significantly the bending moment especially at the touch-down region and that the heave excitation can produce compression in some parts of the structure. These effects can be mitigated by using control devices, such as heave compensators or dynamic positioning systems. However, these alternatives may not be feasible or simply unnecessary because the analysed structure does not present compression.

The detection of compression is essential for design. However, the application of the imposed displacements can be a difficult task, due to numerical instabilities that may arise. To be able to apply these boundary conditions, in this work, it is proposed to employ a technique of softening (exponential decay), which is commonly used for mesh adaptation of numerical models of fluids [2].

Moreover, the initial static position is achieved from an undeformed straight riser with a large elasticity modulus that is gradually reduced until the real one at the desired position and natural static pretension.

Generally, the structural behaviour of Risers has been obtained using analytical formulations, finite difference methods or finite element methods. One may find analytical solutions in Seyed and Patel [3], Atadan et al [4] and recently Lenci and Callegari [5]. Researches using finite difference methods are the works reported by Nordgren [6], Howell [7] and Chatjigeorgiou [1]. Many authors use Finite Element Method (FEM) to solve Risers. Highlight the works of Yazdchi and Crisfield [8], McNamara [9], Patel and Sarohia [10]. Alternative and efficient solutions are presented by Vaz et al [11] and Pesce et al [12].

The present work use the FEM formulated in the Lagrangian form to obtain the non-linear static and dynamic behaviour of risers. The proposed methodology is based on the minimum potential energy theorem written regarding nodal positions as described by Coda and Greco [13]. Therewith, the velocity, acceleration and strain are extracted directly from positions and not from the displacements. A non-dimensional space is created and the reference and current positions are mapped from this space by two mappings.

The structure is discretized using bi-dimensional bar elements with Reissner kinematic. It is adopted the Green strain measure with the Second Piola Kirchhoff stress and quadratic potential strain energy. The classical Newmark's algorithm is used to obtain the response of the structure on the time.

The considered external actions in the simulation are self-weight, buoyancy load, top tension, add mass, drag force and vessel motions. The add mass and drag forces are modelled with the modified Morison equation. The properties of the current in the surface region follow the linear Airy theory for waves. The displacement boundary conditions are applied with the aid of a softening technique.

2 Positional Mapping

2.1 Reissner Kinematic

The riser is discretized with bi-dimensional bar elements. Figure 1 shows the mapping of the middle line of a quadratic bar element. The initial and current positions for the points of the middle line of the element can be mapped from a non-dimensional space using the following expressions

$$\begin{aligned} f_{0i}^{m} &= x_i^m(\xi, X_{li}) = \phi_l(\xi) X_{li} \\ f_{1i}^{m} &= y_i^m(\xi, Y_{li}) = \phi_l(\xi) Y_{li}, \end{aligned} \tag{1}$$

where x_i^m and y_i^m are the continuum coordinates of the any point of the middle line in the reference and current configurations, X_{li} and Y_{li} are the discrete coordinates of the nodal points in the middle line in the reference and current configurations, respectively. $\phi_l(\xi)$ is the shape function.

Figure 2 shows that any point out of the middle line is restricted by the expressions

$$\begin{aligned} x_i(\xi, \eta) &= x_i^m(\xi) + g_i^0(\xi, \eta), \\ y_i(\xi, \eta) &= y_i^m(\xi) + g_i^1(\xi, \eta), \end{aligned} \tag{2}$$

where the vector g^0 is defined to be orthogonal to the middle line, but the vector g^1, in the current configuration, is not necessary orthogonal to the middle line. These vectors take into account the influence of shear deformation.

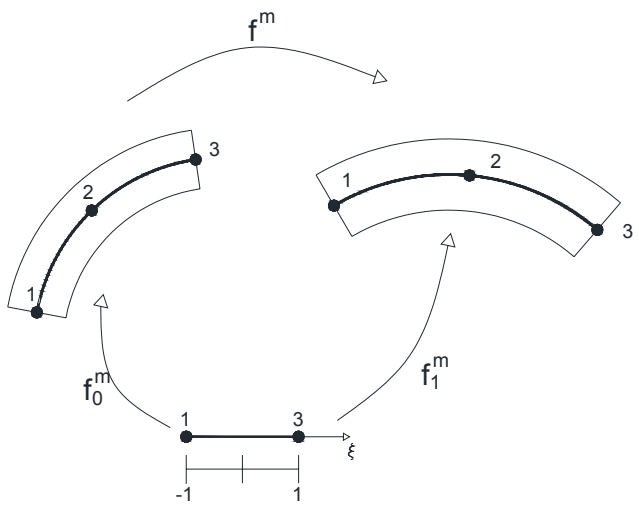

Figure 1: Mapping of the middle line of a quadratic element.

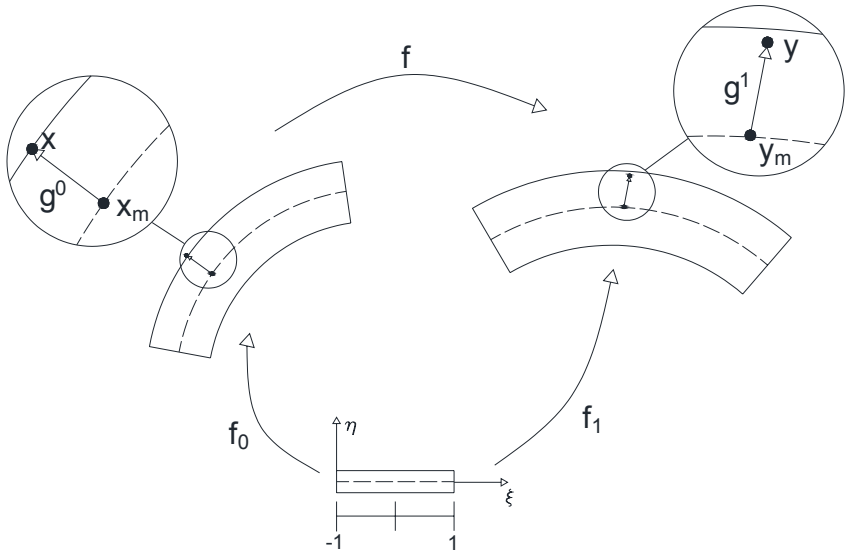

Figure 2: Mapping of the points out of the middle line.

Generally, it is made the assumption that there is a constant reference height, h_0, for bar elements, with this the vectors g^0 and g^1 are written as

$$g_1^0(\xi,\eta) = \frac{h_0}{2}\eta\, Cos(\phi_a(\xi)\Theta_a^0),$$
$$g_2^0(\xi,\eta) = \frac{h_0}{2}\eta\, Sin(\phi_a(\xi)\Theta_a^0), \tag{3}$$

and

$$g_1^1(\xi,\eta) = \frac{h_0}{2}\eta\, Cos\big(\phi_a(\xi)\Theta_a\big),$$
$$g_2^1(\xi,\eta) = \frac{h_0}{2}\eta\, Sin\big(\phi_a(\xi)\Theta_a\big), \tag{4}$$

where Θ_a^0 and Θ_a are the angles in the reference and current configurations of the cross section, respectively, ξ and η represent the non-dimensional coordinates, such as shows the Figure 2.

The reference mapping is described by the first of the Equations (2) plus Equations (3). The current mapping is given by the second of the Equations (2) plus Equations (4).

To complete the definition of the configuration mapping of a structure one needs to describe the deformation gradient, as follow:

$$A_{ij}^0 = \begin{bmatrix} \frac{\partial x_1}{\partial \xi} & \frac{\partial x_1}{\partial \eta} \\ \frac{\partial x_2}{\partial \xi} & \frac{\partial x_2}{\partial \eta} \end{bmatrix},$$

$$A_{ij}^{1} = \begin{bmatrix} \frac{\partial y_1}{\partial \xi} & \frac{\partial y_1}{\partial \eta} \\ \frac{\partial y_2}{\partial \xi} & \frac{\partial y_2}{\partial \eta} \end{bmatrix}, \tag{5}$$

where x and y are given by the Equations (2).

2.2 Saint-Venant-Kirchhoff Constitutive Law

The Green strain measure can be writing in function of the Equations (5) as:

$$\mathbb{E} = \frac{1}{2}(\boldsymbol{A}^T\boldsymbol{A} - \boldsymbol{I}) = \frac{1}{2}(\mathbb{C} - \boldsymbol{I}), \tag{6}$$

where $\boldsymbol{A} = \boldsymbol{A}_1\boldsymbol{A}_0^{-1}$ and $\boldsymbol{A}^T = \boldsymbol{A}_0^{-T}\boldsymbol{A}_1^T$, $\mathbb{C}$ represents the right Cauchy stretch of forth order and $\boldsymbol{I}$ is the identity matrix. It is adopted the Saint-Venant-Kirchhoff constitutive law, then the quadratic strain energy per unit of initial volume is given as:

$$u_e = \frac{1}{2}(\mathbb{E}\ \mathrm{C}\ \mathbb{E}). \tag{7}$$

With this, the energy conjugate of the Green strain, called Second Piolla-Kirchhoff stress, is writing as:

$$S = \frac{\partial u_e}{\partial \mathbb{E}} = \mathrm{C}\ \mathbb{E}. \tag{8}$$

The u_e can be written as:

$$u_e = \frac{G}{(1-2\upsilon)}\left[(1-\upsilon)\left({E_{11}}^2 + {E_{22}}^2\right) + 2\upsilon(E_{11}E_{22}) + (1-2\upsilon)({E_{12}}^2 + {E_{21}}^2)\right], \tag{9}$$

where υ is the Poisson ratio, G is the shear modulus given by $G = E/(2\ (1+\upsilon))$, E Young modulus and E_{ij} represents a component of Green tensor $\mathbb{E}$. The Equation (9) is simplified to plane strain.

2.3 Minimum Energy Theorem

Bower [14] places that to find the equilibrium configuration of a discrete system, it is possible to identify a suitable set of generalize coordinates q_i, and then to express the potential energy in terms of these $\Pi_0(q_i)$. The equilibrium values of the generalized coordinates could then be determined by using the fact that the potential energy is stationary at equilibrium, which result in a set of equations $\partial\Pi_0/\partial q_i = 0$ that could be solved for q_i.

Energy dissipation is possible in some mechanical due to, for example, non-conservative forces or damping. To preserve the principle of the conservation of energy. It is necessary to consider the input and output of energy. Hence, the total energy of a system can be written as:

$$\Pi = \Pi_0 - Q, \tag{10}$$

where Q can be stated as the quantity of energy withdrawn from the simple conservative idealized energy Π_0.

For a structural problem associated with a fixed reference system the ideal potential energy is written as:

$$\Pi_0 = U_e + K + Q - P. \tag{11}$$

In Equation (11) U_e is the strain energy which is written as function of the specific strain energy u_e, Equation (9), as:

$$U_e = \int_{V_0} u_e \, dV_0. \tag{12}$$

K is the kinetic energy given by:

$$K = \frac{1}{2} \int_{V_0} \rho_0 \dot{y}_i \dot{y}_i \, dV_0. \tag{13}$$

P represents the potential energy of the conservative forces, i.e.,

$$P = F_i Y_i + \int_{S_0} p_i \, y_i dS_0, \tag{14}$$

where F_i is the force or moment applied in "i" direction, Y_i is the associated current position, p_i is the distributed force applied in "i" direction, y_i is the current position of the middle line points and dS_0 is the initial differential length of the element. Finally, the dissipative term is written in its differential form as:

$$\frac{\partial Q}{\partial Y_i} = \int_{V_0} \frac{\partial \hat{q}(x,t)}{\partial Y_i} dV_0 = \int_{V_0} \lambda_m \, \dot{y}_i dV_0 - \int_{S_0} q_i \, dS_0, \tag{15}$$

where $\hat{q}$ is the specific dissipative functional, λ_m is a proportionality constant, $\dot{y}_i$ is the velocity of any point and q_i are the non-conservative distributed forces

According to the above, the minimum potential energy theorem can be applied to the potential energy Π_0, given by (11), such that

$$\frac{\partial \Pi}{\partial Y} = \frac{\partial U_e}{\partial Y} - \boldsymbol{F} + \boldsymbol{M}\ddot{Y} + \boldsymbol{C}\,\dot{Y} = 0. \tag{16}$$

The result of Equation (16) is the equilibrium position, Y, solving the problem.

3 Hydrodynamic Forces

The hydrodynamic forces can be classified as Buoyancy add mass and drag forces. In this work, the buoyancy force is adopted to be equal to the weight of the displaced fluid. It acts on floating riser in the upward vertical direction. According with Yazdchi and Crisfield [8], Seyed and Patel [3] and Bergan and Martisen [15] the buoyancy force in finite elements may result of the integration of the pressure in the surface in contact with the water. However, Hosseini et al. [16] showed that, to the normal boundary conditions of the riser, the Archimedes' principle provides accurate results and is employed here. The buoyancy force is a non-conservative load distributed on the length of the element and is considered in the last term of Equation (15) as q_2 in the vertical coordinate direction.

The Add Mass and Drag Force are modelled using the modified Morison equation [17], which is written as:

$$d\boldsymbol{F} = \rho\pi\frac{D^2}{4}dz\left[C_m a_f - (C_m - 1)a_b\right] + \rho\, C_d\,\frac{D}{2}\;dz\left|u_f - u_c\right|\left(u_f - u_b\right), \qquad (17)$$

where ρ is the external fluid density, D is the external diameter of the cylinder, C_m and C_d are the add mass and drag coefficients, a_f and a_b are the external fluid and body horizontal accelerations, u_f and u_c are the external fluid and body horizontal velocities and dz is the differential length in the vertical direction of the riser. In Equation (17) $d\boldsymbol{F}$ is a non-conservative force distributed in the length of the riser and is considered in the last term of Equation (15) as q_1, in the horizontal direction.

4 Strategy to apply the displacement boundary conditions

When an offset is applied at the top of a riser, in the numerical model it is simulated as an imposed displacement at the end node. After this, an iterative process is used to search the equilibrium configuration, such that the final result meet the boundary conditions. However, the imposed displacement introduces an energetic disturbance in a single element. In other words, the initial configuration used in the search of the minimum energy is not smooth lending to an unstable procedure.

To avoid this problem, we proposed a strategy that allows smoothing the configuration of the structure after the application of the boundary condition. The method uses a proportional distribution of the imposed displacement in a k-node to all i-nodes of the structure, according to the following equation:

$$\boldsymbol{u}^i = \frac{\sum_{k=1}^{nc} a_{ik}\boldsymbol{u}^k}{\sum_{k=1}^{nc} a_{ik}}, \qquad (18)$$

where the index k represents the node of the boundary with displacement imposed, the i index indicates all nodes different of k-nodes , $\boldsymbol{u}^k$ is the imposed displacement in the k-node, $\boldsymbol{u}^i$ is the proportional displacement imposed in the i-node, nc is the number of k-nodes and a_{ik} is the inverse of the square of the current distance between each node i and node k.

5 Results

5.1 Example 1

Initially, it is obtained the static behaviour of a polyethylene cantilever beam with tubular cross section that is under effect of a vertical load at the free end and of the corresponding hydrostatic forces at 100.0 m water depth. The beam has 20.0 m of long and capped ends.

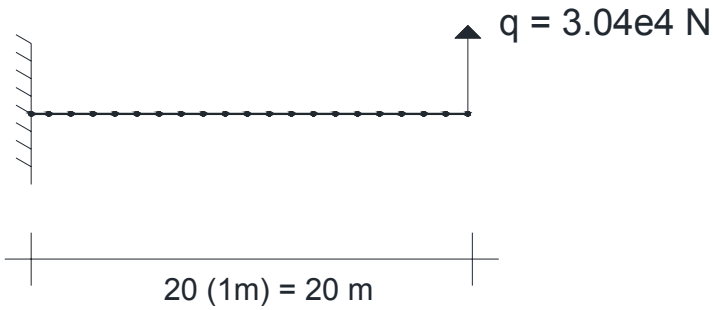

Figure 3. Cantilever beam with capped ends and vertical load.

The discrete model uses 10 quadratic finite elements such that there is a node at every meter, see Figure 3. Other parameters of the model are shown in Table 1.

This academic example permits to performance the numerical verification of the formulation and the hydrostatic loads. Yazdchi and Crisfield [8] solved this example using a co-rotational formulation with two dimensional finite elements.

Modulus of elasticity, E	$2.0e9\ N/m^2$
Poisson's ratio, ν	0.25
Unit weight of pipe, γ_r	$1.2e4\ N/m^3$
Unit weight of sea water, γ_w	$1.025e4\ N/m^3$
Internal riser diameter, D_i	$0.80\ m$
External riser diameter, D_e	$0.85\ m$

Table 1. Parameters of the example 1.

The vertical load q is applied perpendicular to the beam's centreline. Figure 4 shows the results with two load cases, q and $-q$. It is observed that exist a good agreement between the results encountered by Yazdchi and Crisfield [8] and the results obtained in this work.

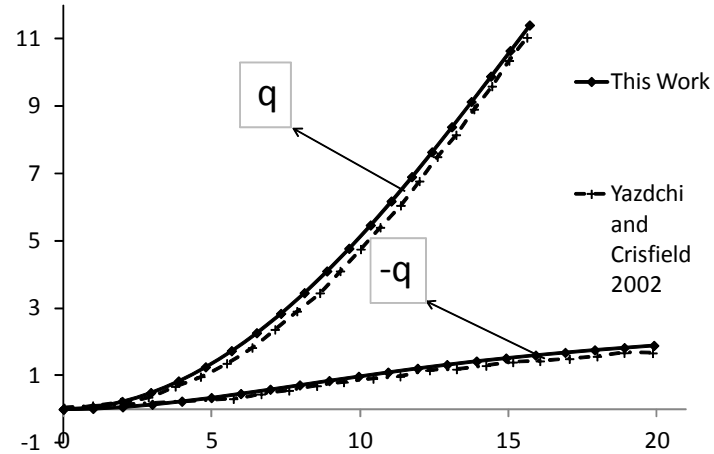

Figure 4. Deformed configuration of a cantilever pipe beam with load q and –q.

5.2 Example 2

The American Petroleum Institute Committee (API) seeking the standardization of the analysis for offshore structures defined a set of example problems to serve as benchmark of the dynamic and static analysis of riser structures. The results were published in the API Bulletin [18]. The example 500-20-1D was selected among the cases shown in the API Bulletin, which is also modelled by Patel and Sarohia [10]. The parameters of the example are shown in the Table 2.

The riser used in this example is connected to a frictionless ball-joint at the upper and lower end containing mud. Waves are assumed to act in the direction of the positive offset, and the upper end is forced by vessel motion in the surge direction. The initial position assumed in this work is shown in Figure 5. The Vessel motion is sinusoidal with the same period of the wave. Top tension is constant. In the second part of this example the vessel motion is applied in the heave direction.

Figure 6 shows the deformed configurations. The circular marker represents the minimum values of the dynamic analysis. The triangular marker symbolizes the maximum values of the dynamic analysis. The lines without marker correspond to the deformed of the static analysis. The dashed lines are the results of the API bulletin [18]. The dotted lines describe the results obtained by Patel and Sarohia [10]. Finally, the continuous lines show the behaviours obtained in this work.

Figure 7 follows the same graphic notation that Figure 6. This shows the bending stress of the riser along its length.

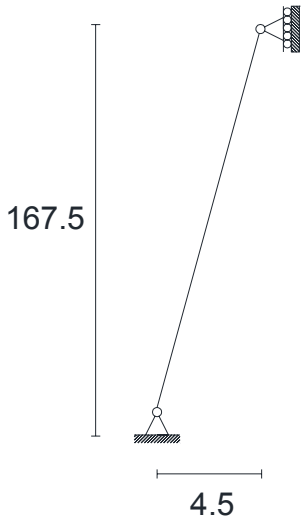

Figure 5. Initial configuration, adopted, in the example 2.

The results of the three works shown in Figure 6 and Figure 7 are similar to the literature dates in the maximum dynamic values and in the static values, but the behaviour in the minimum dynamic values is different. This variation may be due to the different formulations used by the authors.

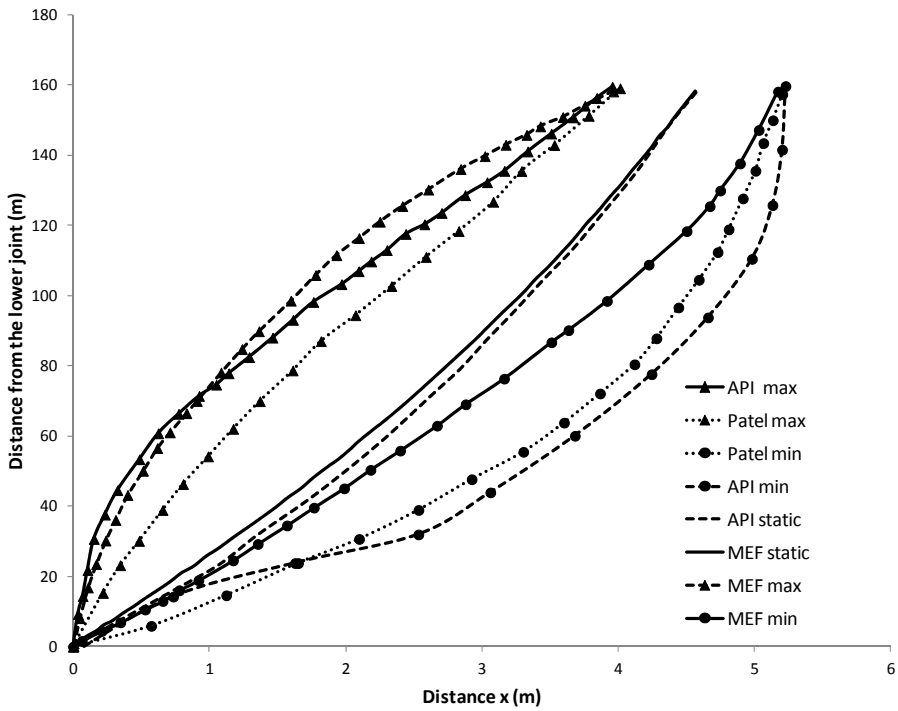

Figure 6. Deformed configurations of example 2. (API 500-20-1D).

Distance from mean sea level to riser support ring	$15.24\ m.$
Distance from sea floor to bottom ball joint	$9.144\ m.$
Water depth	$152.4\ m.$
Riser pipe outer diameter	$0.4064\ m.$
Riser pipe wall thickness	$0.01587\ m.$
Choke line outer diameter	$0.1016\ m.$
Choke line wall thickness	$0.01651\ m.$
Buoyancy material outer diameter	$0.6096\ m.$
Modulus of elasticity of riser pipe	$2.1 * 10^{11} N/m^2$
Density of sea water	$1025\ Kg/m^3$
Density of mud	$1438\ Kg/m^3$
Drag coefficient	0.7
Added mass coefficient	1.5
Effective diameter for wave/current load	$0.6604\ m.$
Density of buoyancy material	$160.2\ Kg/m^3$
Current at surface	$0.2574\ m/s$
Surface vessel static offset	$4.572\ m.$
Weight per unit length of riser joint in air	$2282\ N/m$
Wave height	$6.096\ m.$
Wave period	$9\ s.$
Vessel surge amplitude	$0.6096\ m.$
Vessel surge phase angle	$15°$

Table 2. Input parameters of example 2 (case 500-20-1D of the API bulletin).

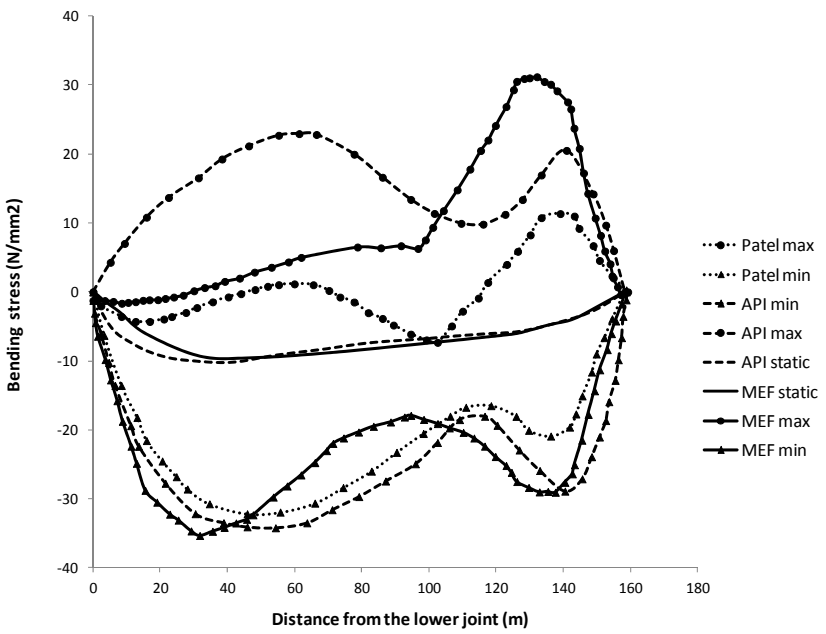

Figure 7. Bending stress for example 2 (API 500-20-1D).

As an additional contribution of this work, it is computed the response for the structure of the example 500-20-1D of the API bulletin a displacement boundary condition in heave direction is applied. The vertical movement follows sinusoidal form with a amplitude of $0.3\ m$ and a period of $6\ s$. The numerical modelling was carried out for a predefined period of time with a step of $0.005\ s$.

In Figure 8 is shown the sequence of the deformed configurations of the risers, as time progresses. In this figure is noted the appearance of the phenomenon of buckling. This can be confirmed in the following two figures (Figure 9 and Figure 10), which describe the bending stress and normal force along of the length of the riser with forced heave movement. Figure 9 shows a significant increase of the bending stress in relation to the obtained with the surge movement. Note in Figure 10 that compression forces appear. As already mentioned, large bending stress or compressive forces are not desired in design of riser[1], nevertheless, it is important that the numerical tool used in the analysis of the structure allows to determine its behaviour under these conditions.

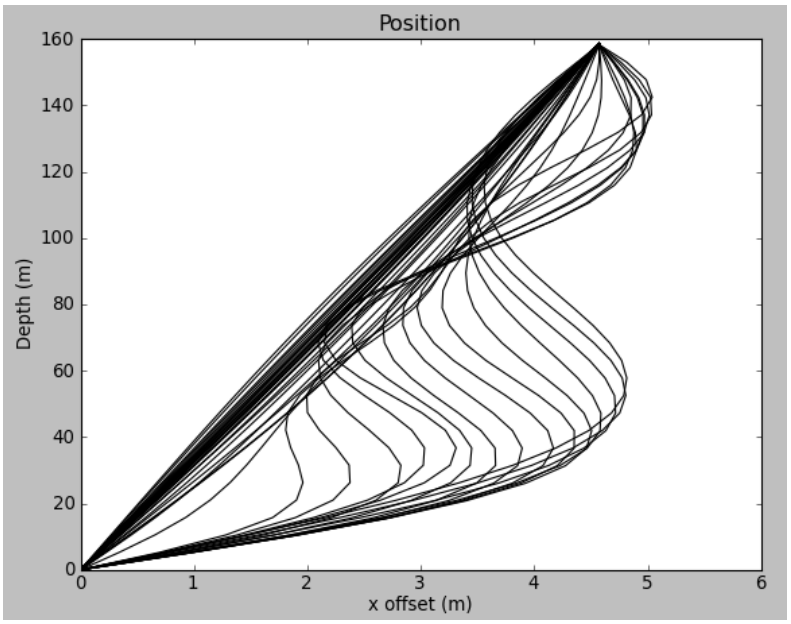

Figure 8. Sequence of the deformed configurations of the riser, with the time advance, under forced excitation in the heave direction.

[1] When the bending stress is very large or there are compressive forces in the riser, it is necessary to apply design strategies such as using the heave compensators, change the vessel type to reduce heave motions, attach buoyancy modules...etc.

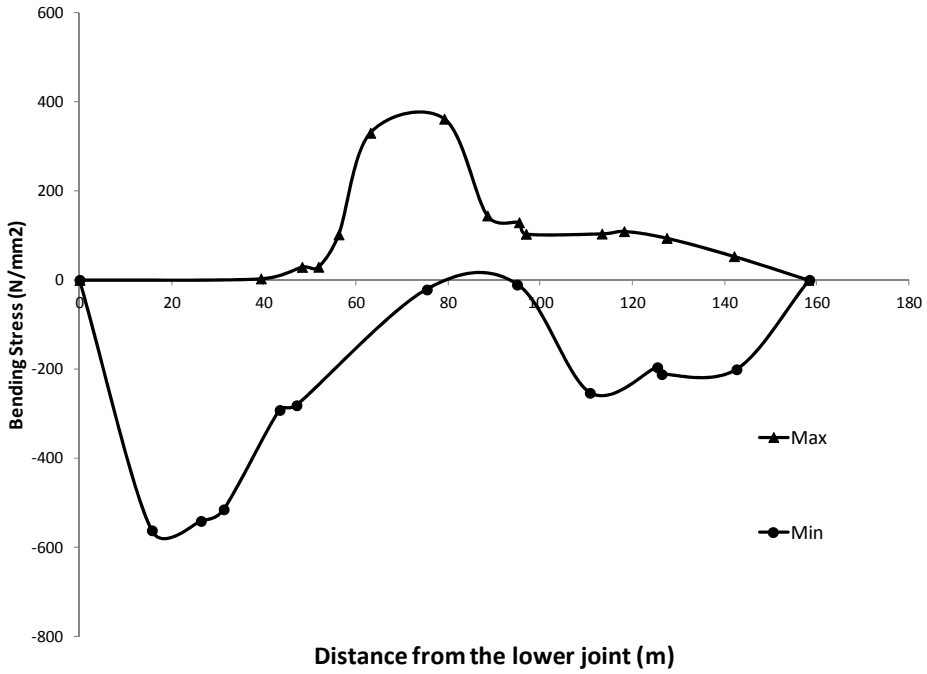

Figure 9. Bending stress in the riser under forced excitation in heave direction.

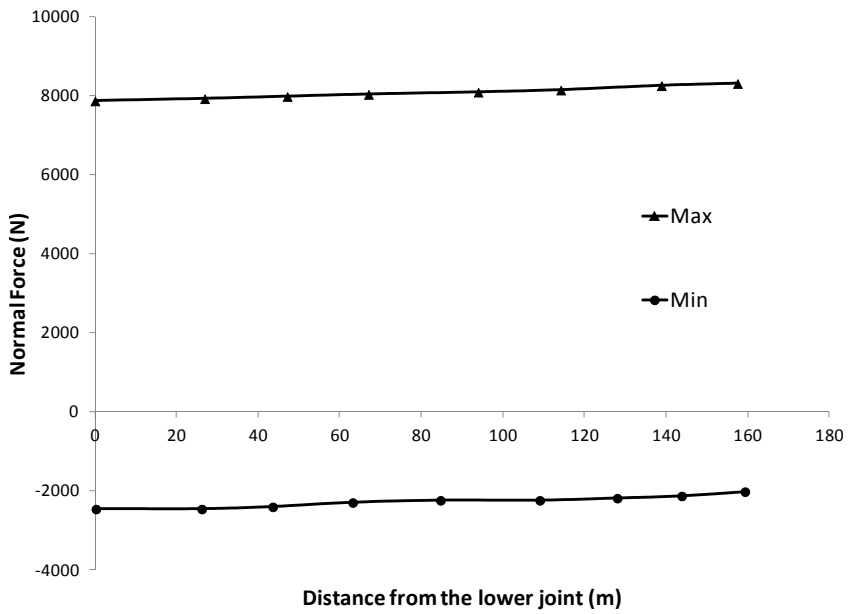

Figure 10. Normal force in the riser under forced excitation in heave direction.

6 Conclusions

The finite element formulation based in the positions together with the proposed strategies enables the dynamic and static behaviour of the riser structures to be obtained, consistent with the literature.

The displacement boundary conditions in the heave direction may cause an increase in the bending stress and the appearance of the compression, especially in the almost vertical risers. Our formulation is able to capture this behaviour and therefore allows designers to choose other engineering solutions to avoid compression.

References

[1] I. K. Chathigeorgiou, "A finite differences formulation for the linear and nonlinear dynamics of 2D catenary risers", Ocean Engineering, v. 35, p. 616-636, 2008.

[2] R. A. K. Sanches, "Sobre o acoplamento fluido-casca utilizando o método dos elementos finitos" (Doutor). Engenharia de Estruturas, Universidade de São Paulo, 2011.

[3] F. B. Seyed, M. H. Patel, "Mathematics of Flexible Risers Including Pressure and Internal Flow Effects", Marine Structures, v. 5, p. 121-150, 1992.

[4] A. S. Atadan, et al., "Analitical and numerical analysis of the dynamics of a marine riser connected to a floating platform", Ocean Engineering, v. 24, n. 25, p. 111-131, 1997.

[5] S. Lenci, M. Callegari, "Simple analytical model for the J-lay problem", Acta Mechanica, v. 178, p. 23-29, 2005.

[6] R. P. Norgren, "On computation of the motion of elastic rods", Journal of Applied Mechanics, p. 777-780, 1974.

[7] C. Howell, "Numerical analysis of 2D non-linear cable equations with application to low tension problems", International Journal of Offshore Polar Engineering, v. 2, p. 110-113, 1992.

[8] M. Yazdchi, M.A. Crisfield, "Buoyancy forces and the 2D finite element analysis of flexible offshore pipes and risers", International Journal for Numerical Methods in Engineering, v. 54, p. 61-68, 2002.

[9] J.F. Mcnamara, P.G. O'Brien, S.G. Gilroy, "Non-linear analysis of flexible risers using hybrid finite elements", International Conference on Ocean, Offshore and Artic Engineering (OMAE). Tokyo, Japan, 1986. p.

[10] M.N. Patel, S. Sarohia, " Finite-element analysis of the marine riser", Engineering Structures, v. 6, p. 175-184, 1984.

[11] M.A. Vaz, J.A. Witz, M.H. Patel, "Three dimensional transient analysis of the installation of marine cables", Acta Mechanica, v. 124, p. 1-26, 1997.

[12] C.P. Pesce, C.D.A. Martins, L.M.Y. Silveira, "Riser-soil interaction: local dynamics at TDP and a discussion on the eigenvalue and the VIV problems", Journal of Offshore Mechanics and Artic Engineering, v. 128, p. 39-55, 2006.

[13] H.B. Coda, M.A. Greco, “A simple FEM formulation for large deflection 2D frame analysis based on position deescription”, Computer Methods in Applied Mechanics and Engineering, v. 193, p. 3541-3557, 2004.
[14] A.F. Bower, “Applied Mechanics of Solids”, United States of America: 2010.
[15] P.G. Bergan, K.M. Mathisen, “Hydrostatic loading and stability analysis of structures with large displacements”, International Journal for Numerical Methods in Engineering, v. 22, p. 575-595, 1986.
[16] S.A. Kordkheili Hosseini, H. Bahai, M. Mirtaheri, “An updated analysis of three-dimensional flexible riser structures”, Ocean Engineering, v. 38, p. 793-803, 2011.
[17] J.O. Morison. et al., “The force exerted by surface waves on piles”, Petroleum Transactions, v. 189, p. 149-54, 1950.
[18] American Petroleum Institute A. P. I., “Comparison of marine drilling riser analysis”, API Bulletin 2J, 1977