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Stability, Breathing and Design of Steel Girders subjected to Repeated Loading

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Abstract

Two ways to making steel construction more competitive: (i) to save steel, and (ii) to reduce fabrication expenses. An analysis of the impact of initial imperfections and the conclusion regarding the need to apply heat straightening to plate elements is undertaken. The post-buckled behaviour of slender webs and an experimental investigation into its partial "erosion" under the effect of many times repeated loading. A simple and user-friendly design method based on the authors´ experiments, for webs subjected to repeated (i) predominantly shear and (ii) partial edge loading.

Keywords: saving steel, saving fabrication expenses, thin-walled construction, effect of initial imperfections, stability, repeated loading, cumulative damage, fatigue cracks, "erosion" of post-buckled behaviour, user-friendly design.

1 Two ways to making steel construction more competitive

There are two ways to increasing the competitiveness of steel structures, viz.
(i) To save steel – via thin-walled structural systems,
(ii) To reduce fabrication expenses – by way of economic-fabrication structures;
and the best result is achieved by combining both of them.

As thin-walled systems are composed of slender plate elements, which are always liable to buckle, it is obvious that a successful way to their application is always paved by a successful solution to the corresponding stability problems.

The economic-fabrication strategy can be implemented by choosing a simple and easily fabricated structural system and a reduction, or complete elimination, of some costly fabrication procedures. And in this respect an important question ought to be asked, viz. should we straighten the plate elements (for example the webs of steel plate and box girders) of which the structural system is composed, which, as a result of the fabrication of the system (in particular as a result of the welding procedures used), always exhibit an initial curvature?

And a correct answer to this question can be given just in the light of the results of an extensive investigation into the effect of various configurations and magnitudes of initial curvatures on the limit state of thin-walled steel girders, which again bring us back to solution of stability problem.

2 Stability

2.1 Thin-walled construction and stability limitations involved

One of the most promising trends in our striving to save steel is to use thin-walled structures, i.e. structural systems made of slender (usually plate) elements. Of course, it can be argued at this juncture that by decreasing the thickness of the plate elements, we do reduce the consumption of steel, but on the other side by lessening the plate thickness we automatically enlarge the slenderness of the plate and thereby make it more liable to buckle, so that the limit state of the system is then substantially reduced by stability phenomena.

This is very true, but the quantitative impact involved to a large extent depends on the way we look at the problem.

For more than half a century the stability problems of plated structures were studied via the so-called linear buckling theory based on the following two basic assumptions:

(i) It was assumed for all plate elements to be "ideal", i.e. without any initial imperfections, i.e. to be fabricated as perfectly as they were presented on the designer's drawings.

(ii) As far as the stability limit state of the plate elements is concerned, it was assumed that the elements remained perfectly plane up to the so-called critical load, under which they suddenly buckled and collapsed.

The critical loading was determined as the first eigen value of a relatively simple eigen-value problem related to linear partial differential equations, the solution being more or less simple, mathematically elegant, and therefore is even nowadays liked and preferred by a good many investigators.

For example, for a web panel in shear (and shear in the stability behaviour of the webs of steel plated girders plays a very important role), the critical load τ_{cr} can be written as follows:

$$\tau_{cr} = \frac{\pi^2 E}{12(1-\upsilon^2)} \frac{1}{\lambda^2} k_\tau , \qquad (1)$$

where E is Young's modulus of steel, v its Poisson's ratio, $\lambda = b/t$ the slenderness of the web panel, b being its depth and t thickness, and k_τ a buckling coefficient, which is a function of the geometry of the panel and its boundary conditions.

For example, for a square web panel simply supported on all boundaries, k_τ = 9.34, so that for λ = 200, which is nothing out the ordinary in up-to-date steel construction, τ_{cr} = 44.3 MPa, i.e. less than one third of the shear yield stress of mild steel, this highlighting the importance of the stability-induced limit state reduction, at least if this is looked at "through the glasses" of linear buckling theory.

The situation is however remedied by, and the promising idea of thin-walled construction can be materialized thanks to, the miracle of post-buckled behaviour, in the light of which a thin-walled plated system behaves like a (so called) super-smart structure, i.e. like one which is able not only to diagnose its own situation, but also to generate a means of powerful defence. In the case of plates and plated systems, this self-defence is brought about by the stabilising effect of membrane stresses, which come into play when plate deflections are of the order of plate thickness, and thereafter very considerably slow down any further deflection growth and therefore also substantially increase the load-bearing capacity of the system. The critical load, provided by linear buckling theory, has no practical meaning for the limit state of the structure because, thanks to the post-buckled behaviour, the ultimate strength of the system is usually very significantly (and in the case of slender webs subjected to combined shear and bending even several times) higher than the linear-buckling-theory critical load.

That is why a great attention has been internationally paid to research on the post-buckled behaviour and ultimate strength of slender webs, flanges and other plate elements, the Czech research always striving to play a useful role in these activities.

For example, the authors of this paper and their co-workers spent about three decades in investigating the post-critical reserve of strength and ultimate load behaviour of steel plate girders, box girders, thin-walled columns etc.

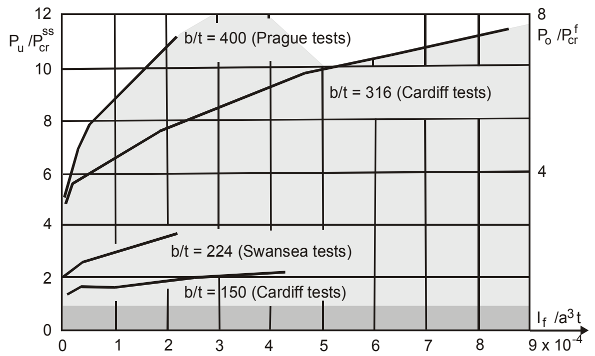


Figure 1: Swansea, Cardiff and Prague shear girder results.

Among other things plenty of attention was paid, during the last decades, to the derivation of an ultimate load approach to the design of the webs of steel plate and box girders. It was found out, using both theoretical and experimental investigations, that the (so-called) critical load, determined via linear buckling analysis, was in no relation to the actual ultimate strength of the web, since in a great majority of cases there always existed a very substantial post-critical reserve of strength. This is demonstrated in Figure 1, where the results of tests on shear girders carried out by (i) K.C. Rockey and one of the authors in Swansea and Cardiff and (ii) by the authors of this paper in Prague are plotted. They are plotted in terms of the flange rigidity parameter $I_f /a^3 t$, (I_f being the second moment of area of the flange, b the width of the web panel and t the thickness of the web sheet) because it was found out that flange stiffness very substantially affected the ultimate load of the test girders. On the left-hand vertical axis, the ultimate load is related to the critical load of a web simply supported on all boundaries, while on the right-hand vertical axis the critical load of a clamped web is used in the denominator of the ratio ultimate strength/critical load. But it can be seen in the figure that, whether we consider the former or the latter value of the critical load, the post-buckled reserve of strength is always very great.

It should be mentioned at this juncture that the Czechoslovak Design Code ČSN 73 1401 was probably the first in which the design of slender plate elements, and of members using them, i.e. of plate, box and similar girders, was completely based on a full exploitation of the post-critical reserve of strength.

2.2 Initial imperfections in thin-walled girders and stability limitations involved

It is practically impossible to fabricate welded steel plated structures without their plate elements exhibiting initial curvatures. Therefore it is understandable that various standards require that this initial "dishing" be kept under control via prescribed tolerances, and that in the case of need the magnitude of the initial curvature be reduced by straightening, usually heat straightening.

But is this really indispensable and desirable?

Of course, we are not the first to pose this question;

The problem was already dealt with some time ago by the Task Group "Tolerances in Steel Plated Structures", sponsored by the IABSE, chaired by Prof. Ch. Massonnet [1], with the first of the authors of this paper being a member of the Task Group. And the observations made during, and the conclusions drawn from, the activities of the Task Group are very much of interest even now.

It is true that discussions within the Task Group exhibited some differences of opinion. However, these differences notwithstanding, the Task Group tended to the opinion that heat straightening in steel plated structures is not desirable.

And the authors of this paper endorse the above stand point, the reasons being twofold:

(i) The procedure of straightening is rather costly (not only directly, but also due to blocking some space in the steel fabricator which can be used for other operations) and therefore would not be compatible with the aforesaid economic-fabrication strategy.

(ii) While it is understandable that the straightening of webs should frequently be employed for aesthetical and psychological reasons, it is not certain that the actual stability behaviour of a girder with a straightened web is better than that of the original girder. To straighten the web of a girder by heat treatment only means that one initial imperfection (initial web curvature) is replaced by another initial imperfection (additional residual stresses induced by the heat treatment applied). The aforementioned Task Group also turned attention to another important fact; namely, while in the case of compressed plates (e.g. the compression flanges of steel box girder bridges) the influence of an initial curvature on load-carrying capacity can be significant (in ordinary cases even 20%), with webs subjected to combined shear and bending the same effect is (of course, when the initial "dishing" is not too large – and it should not be when the plated structure is fabricated in a good enough steel fabricator) much less important.

But is it really so?

If we desire to go as far as entirely to disregard the effect of unavoidable initial web curvatures, we must be certain that the effect of initial web "dishing" larger than usually adopted tolerances will not imperil the safety of steel plated girders.

And for this reason the first author, J. Melcher, J. Kala and Z. Kala, have started an extensive investigation into the impact of various (but practically important) configurations and magnitudes of web initial curvature on the limit state of steel girders. About its results and conclusions it is reported in detail in [2].

Various types of girders were used in the analysis, among them being also the girders tested by the authors in their investigation into the "breathing" problem (see section 3 herebelow), and various configurations and magnitudes of the web initial curvature were considered.

The study was based on an application of elasto-plastic large-deflection theory, the ANSYS program being used in the analysis.

The Euler method based on proportional loading in combination with the Newton-Raphson method was used. The girder was modelled, in a very minute manner, by means of a mesh of shell four-node elements SHELL. The girder symmetry and that of loading were made use of. The loading test was simulated by the incrementation of a loading step in the Euler method. The load-carrying capacity was determined as the loading rate at which the matrix of tangential toughness determinant Kt of structure approached zero with accurateness of 0.1 %. The incrementation run was decremented automatically. For steel grade S235, bilinear kinematic material hardening was supposed. Further on, it was assumed that the onset of plastification occurred when the Mises stress exceeded the yield stress.

At this junction it should be mentioned that the theoretically predicted ultimate loads very well correlated (the difference being only a few p.c.) with their experimental counterparts. Thereby the reliability of the theoretical results was confirmed.

The main conclusion of this study (see [2]) is that it is not indispensable to abide by the currently used and rather stringent web tolerances, and that plate girders can be used without their webs being straightened.

Of course, this conclusion holds only for the products of well equipped and highly accredited steel fabricators, having a staff with high expertise and experience, for which it can be expected that the standard of workmanship achieved will be sufficiently high and resulting imperfections reasonably small. From our study it follows that the amplitude of web initial curvature should not be larger than web depth/100.

3 Breathing

3.1 The “breathing” phenomenon in thin-walled construction and the fatigue behaviour generated by it

The fully developed kind of post-buckling behaviour described hereabove in section 2 occurs in building construction, i.e. in structural systems under the action of quasi-constant loading. The situation is however different in the case of steel bridgework, crane supporting girders and similar systems, which are exposed to many times repeated loading. Then their plate elements, for example their webs, if they are slender, repeatedly buckle out of their plane. This phenomenon, being now usually termed web “breathing”, induces significant cumulative damage process in the “breathing” webs and we can ask the obvious question of whether the “breathing” phenomenon leads to a significant “erosion” of the post-critical reserve of strength described above.

And to research on this problem the authors turned their attention several years ago.

Given the complex character of the cumulative damage process in “breathing” webs, it was crystal clear that a very important role should be played by experiments. The tests, their number already exceeding two hundred (220 tests on webs subjected to predominant shear and 54 ones on webs under patch loading), were conducted in three laboratories, viz. at (i) the Institute of Theoretical and Applied Mechanics of the Czech Academy of Sciences, (ii) Klokner Institute of the Czech Technical University, and (iii) the Research Institute of Materials.

The large number of tests proved to be indispensable in view of the large scatter which is characteristic of all “breathing” experiments.

But at this juncture it is useful to say a few words about the character of the test girders used.

Like most girders tested now by the writers at the Institute of Theoretical and Applied Mechanics in Prague, they are fairly large, most having a web 1000mm deep, so that their character is not far from that of ordinary girders.

All the girders were fabricated in the steel fabricator of Division 7 of the Company METROSTAV plc., using the same technological procedures as are applied there in the fabrication of ordinary steel bridges. It is important to note that, in the fabrication of the test girders, no attempt was made to diminish (by heat treatment) the initial curvature of the web generated during the process of girder fabrication.

3.2 The initiation and propagation of fatigue cracks in webs subjected to repeated (i) predominantly shear and (ii) partial edge loading and their effect on failure mechanism

The main impact of the cumulative damage process generated by the “breathing” phenomenon is the initiation and propagation of fatigue cracks. They initiate in the crack-prone areas at the toes of the fillet welds connecting the “breathing” web sheet with the girder flanges and transverse stiffeners.

In the case of slender webs exposed to repeated predominant shear, the fatigue cracks initiate near those web panel corners from which the diagonal buckled web pattern, and consequently also the diagonal tension (stress) band, emanate, see Figure 2. Then they propagate so that in the end they cut off this corner from the rest of the web, in the final stage the fatigue crack frequently growing perpendicularly to the tension diagonal. The girder then fails like one having a large opening in the web see Figure 3a*,* Figure 3b shows, for the sake of comparison, the collapse mechanism of a twin-girder, but under the effect of constant (unrepeated) shear.

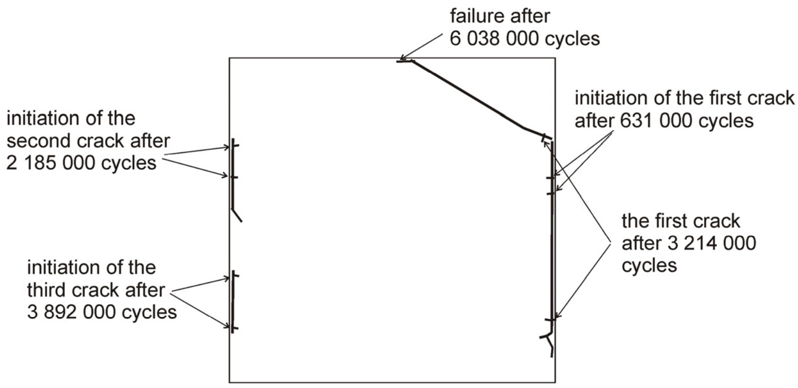
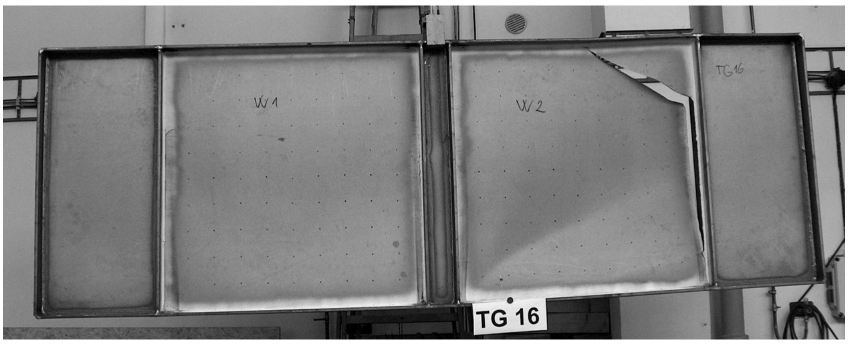


Figure 2: The initiation and propagation of fatigue cracks in webs subjected to repeated predominantly shear.

(a)



(b)

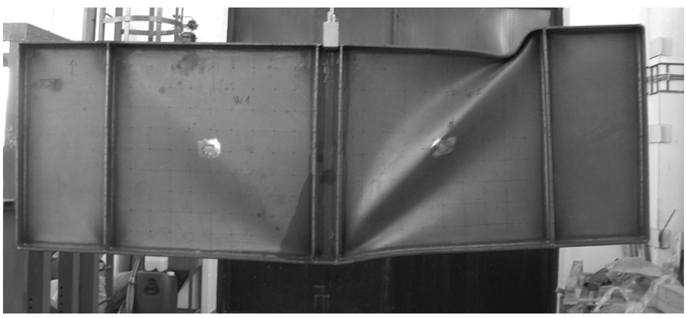


Figure 3: The failure mechanism of the whole girder: (a)repeated predominantly shear, (b)constant predominantly shear.

In the case of slender webs subjected to repeated patch (partial edge) loading, the fatigue cracks initiate in the zones where the segmental buckled pattern in the web sheet is “anchored” into the loaded flange (Figure 4). Then they propagate along the loaded flange towards the point of application of the patch load, so that in the end they cut off the supporting web sheet from the loaded flange, which then, under this load, buckles inwardly and the girder collapses, see Figure 5a. For the sake of comparison, Figure 5b shows the failure mechanism of the same girder, but subjected to a constant patch load.

Some details of the failure mechanism of the girder subjected to a repeated partial edge load can be seen in Figure 6.

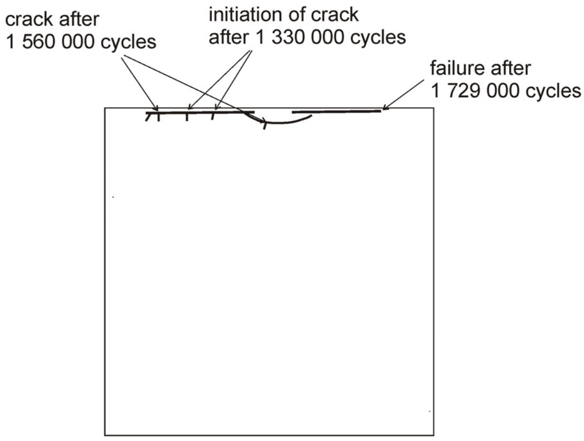
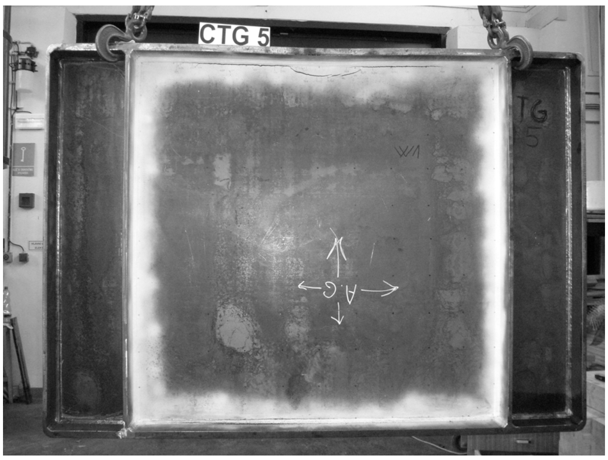


Figure 4: The initiation and propagation of fatigue cracks in webs subjected to repeated partial edge loading.

(a) **(b)**



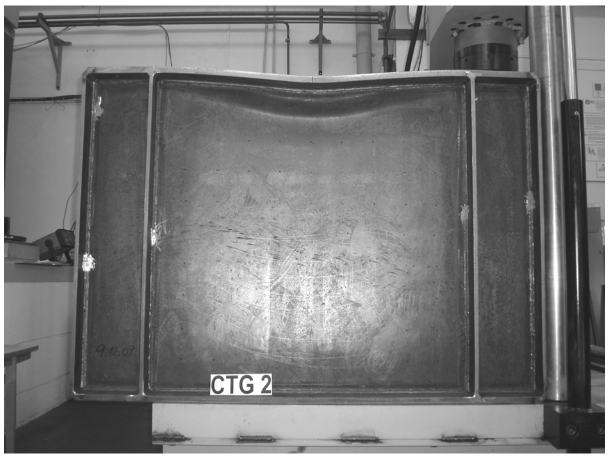


Figure 5: The failure mechanism of the whole girder: (a) repeated partial edge loading, (b)constant partial edge loading.

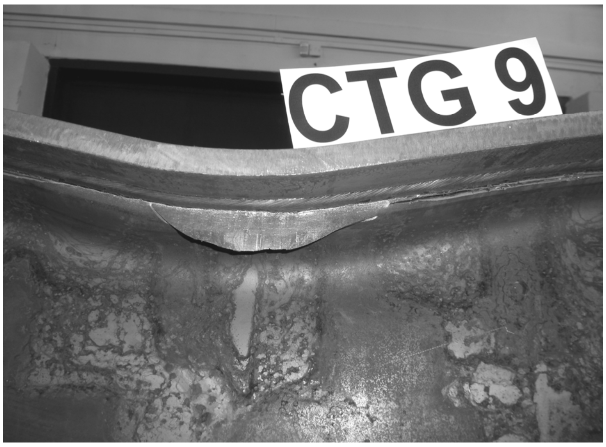


Figure 6: Details of the failure mechanism of the whole girder subjected to partial edge loading.

3.3 Partial "erosion" of the post-buckled behaviour in webs subjected to repeated (i) predominantly shear and (ii) partial edge loading

For the purpose of practical design it is important to know what portion of the post-buckled strength, i.e. of the maximum load that a girder is able to sustain, is "eroded" when the girder is exposed to many times repeated loads.

The authors were able to shed some light on this interesting question because for each series of their fatigue tests, they also carried out a few static experiments on girders having the same dimensions.

In the first part of the Prague test girders, the webs were subjected to combined shear and bending, with the effect of shear predominating. Let us therefore measure the loading of each web panel by shear force V, its intensity being determined by ratio V/V_{cr}, when V_{cr} is the critical shear force (given by the linear buckling theory), calculated for a web panel clamped into the flanges and hinged on the transverse stiffeners.

In the other part, the webs were subjected to a partial edge load F, and the intensity of loading can therefore be measured by ratio F/F_{cr}, where F_{cr} is the corresponding linear-buckling-theory critical load calculated in two ways, viz. (i) by the formula established by Rockey and Bagchi [4], who assumed that the patch load F is in equilibrium with shear flows acting along the two vertical edges of the web, F_{cr}^{RB}, (ii) by the formula derived by White and Cottingham [5] on the assumption that the patch load F is in equilibrium with the vertical reactions at the two supports of the web placed under the lower flange of the girder, F_{cr}^{WC}.

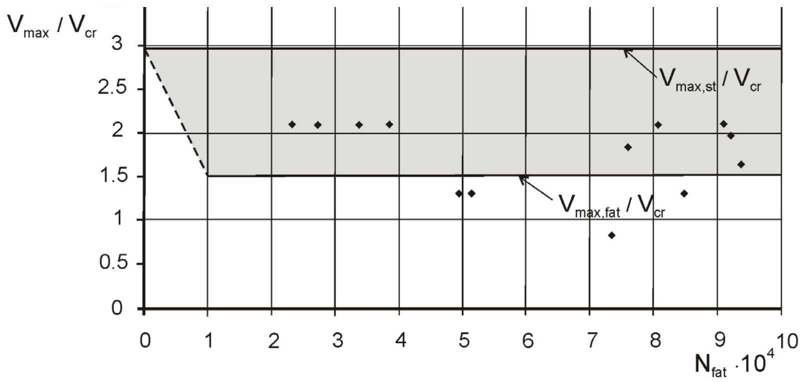
The ratios V/V_{cr} and F/F_{cr} then also determine the remaining post-buckled reserve of strength.

The results of the writers′ experiments are shown in Figures 7 and 8. The corresponding ratios V_{max}/V_{cr} and F/F_{cr} are plotted on the vertical axis, the related numbers N of loading cycles on the horizontal axis.

A very significant impact of the cumulative damage process on the fatigue tests can easily be seen in Figures 7 and 8. As the Prague breathing experiments exhibited, like most fatigue tests of all kinds, a large scatter, an average line of the breathing tests is also given in these figures. The area above it gives an average value of the cumulative-damage-induced “erosion” of the maximum sustainable load.

An analysis of Figures 7 and 8 shows that in the case of many times repeated loads, the web - so to speak - does not have enough time to develop the aforementioned classical mechanism, since its behaviour is over-shadowed by the initiation and propagation of fatigue cracks, and the failure mechanism of the girder is more complex. And the cumulative damage process is not here parallel to, but instead of classical buckling failure, which it entirely replaces and therefore determines the maximum sustainable load. Figure 7 is divided into two parts: that denoted by (a) concerns $0 < N \leq 10^5$ (i.e. the interval of N where the gradient of the test results is greatest and therefore deserves a separate figure), the part (b) then holds for $N > 10^5$. Figure 8 is also in two parts; (a) for webs with side ratio $\alpha = 1$, (b) for those with $\alpha = 2$.

(a)



(b)

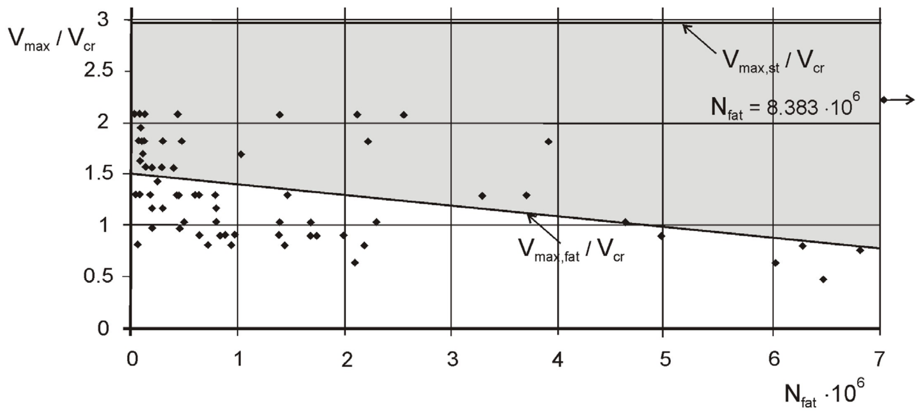
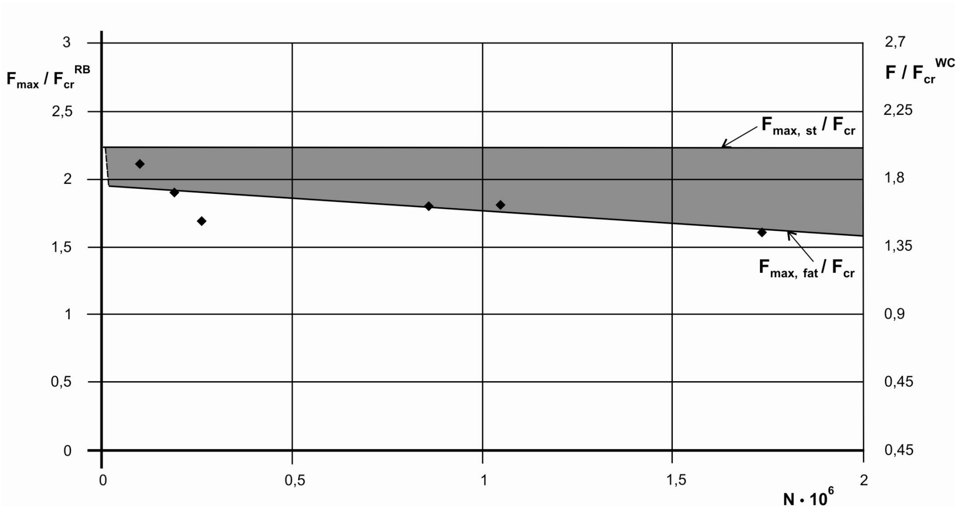


Figure 7: The cumulative-damage-induced “erosion” of the maximum sustainable load – repeated predominant shear.

(a)



(b)

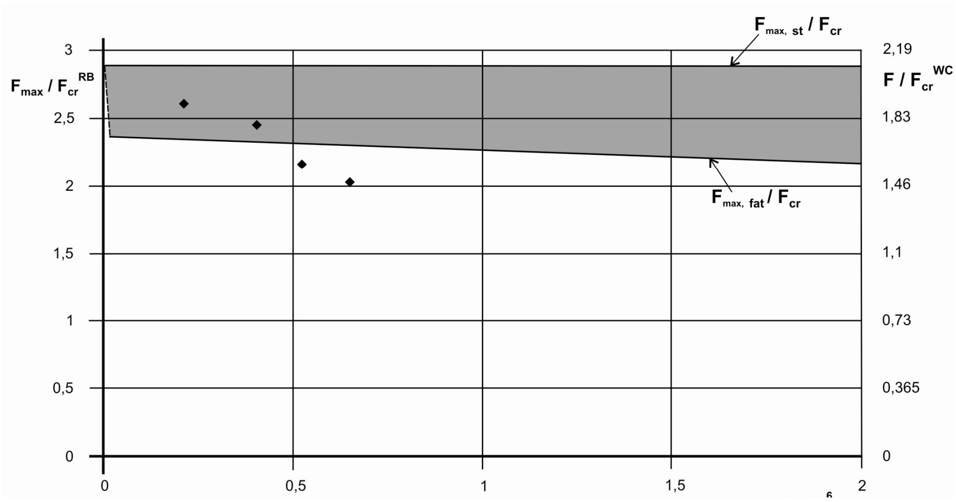


Figure 8: The cumulative-damage-induced "erosion" of the maximum sustainable load – repeated partial edge loading.

4 Design

It follows from the above analysis that the problem of web "breathing" can play a very important role and therefore cannot be disregarded; on the contrary, it can significantly affect the design of steel bridges, crane-supporting girders and other structural systems under the action of many times repeated loads.

And to establish a reliable method for the analysis of the "breathing" webs of thin-walled girders is the main objective of the authors´ research.

In so doing, the authors follow the general features of the design philosophy proposed by Maquoi and Škaloud [3], according to which two limit states are introduced in the analysis, viz. (i) the fatigue limit state, (ii) that of serviceability.

While the fatigue limit state can be related to the failure of the girder (i.e. to unrepairable damage – which is acceptable in view of the fact that the fatigue limit state can never be attained during the planned life of the girder), the limit state of serviceability should be related to a much more limited, easily repairable degree of damage. In the case of steel girders with "breathing" webs, this means that, in the course of the useful fatigue life of the girder, either no or very small fatigue cracks can develop, such as to be easily kept under control, or easily retrofitted in case of need. That is why this limit state can successfully be used for governing inspections of the structure concerned for the occurrence of breathing-induced fatigue cracks.

The corresponding S-N curves for combined shear and bending, with the effect of shear predominating, were presented by the authors in [6, 9] and are being continuously improved in the light of new experimental data. Those for repeated patch loading are going to be established by the authors once they have enough experimental evidence at their disposal, i.e. after they have completed their new currently conducted test series.

In this contribution, the authors are going to establish a simpler, more user-friendly method such as to make it possible entirely to disregard, as long as loading is kept under a certain limit, the impact of web breathing.

In so doing, the writers consider just the results of the experiments carried out on their large-size girders, so that the test girders used are similar, in both the fabrication procedures (see section 3.1) and the size of girders, to the characteristics of ordinary steel girders, and consequently the formulae obtained can reliably be applied in design.

The authors´ test results, related to the initiation of the first fatigue crack, are plotted in Figures 9 and 10, namely in Figure 9 for webs subjected to repeated predominantly shear and in Figure 10 for webs under the effect of repeated patch loading.

For our objective, however, i. e. to derive a formula limiting the maximum load so that the effect of web breathing can be neglected for any numbers N of loading cycles, it is the threshold value of the test results (determined in the optics that only 5% of the results can be under this value) which is more important.

4.1 Predominantly shear

In Figure 9 it can be seen that for webs subjected to combined shear and bending, this requirement is fulfilled when

$$\frac{\Delta V}{V_{cr}} \leq 0.6$$

where $\Delta V = V_{max} - V_{min}$ is the range of the shear force in the girder web, V_{cr} the critical shear force of the web determined by linear buckling theory and calculated with regard to the actual boundary condition of the web. If rewriting relationship in shear stresses τ (shear force V/web area A), we get

$$\frac{\Delta \tau}{\tau_{cr}} \leq 0.6 \qquad (2)$$

Hence, taking account of $\Delta\tau = \tau_{max} - \tau_{min}$, as long as

$$\tau_{max} \leq \tau_{min} + 0.6\tau_{cr} \tag{3}$$

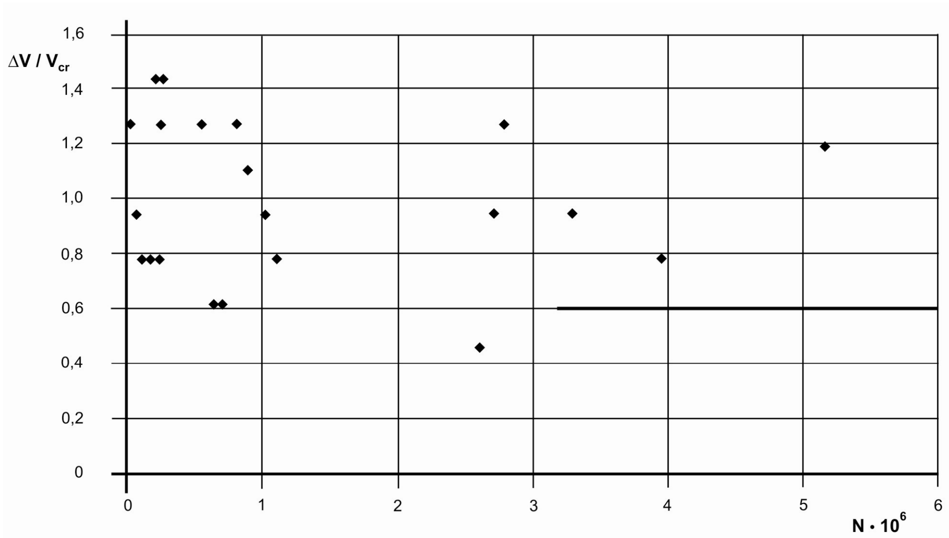


Figure 9: $\Delta V/V_{cr}$-values related to the initiation of the first fatigue crack in webs subjected to predominantly shear.

the impact of web breathing can be completely disregarded for any number N of loading cycles, since there is no danger of the initiation of any fatigue cracks generated by web breathing and, of course, even no danger of this kind of fatigue failure.

It remains to qualify what we mean by predominantly shear. Analysing again the authors´ experiments, the last formula can be applied to webs loaded by repeated combined shear τ and bending σ when $\tau/\sigma \geq 1$.

4.2 Partial edge loading

From Figures 10a and b it follows that for webs under the action of a partial edge load F the requirement hereabove is satisfied when

(i)

$$\frac{\Delta F}{F_{cr}^{RB}} \leq 1.4 \tag{4a}$$

and consequently (taking account of $\Delta F = F_{max} - F_{min}$)

$$F_{max} \leq F_{min} + 1.4F_{cr}^{RB} \tag{4b}$$

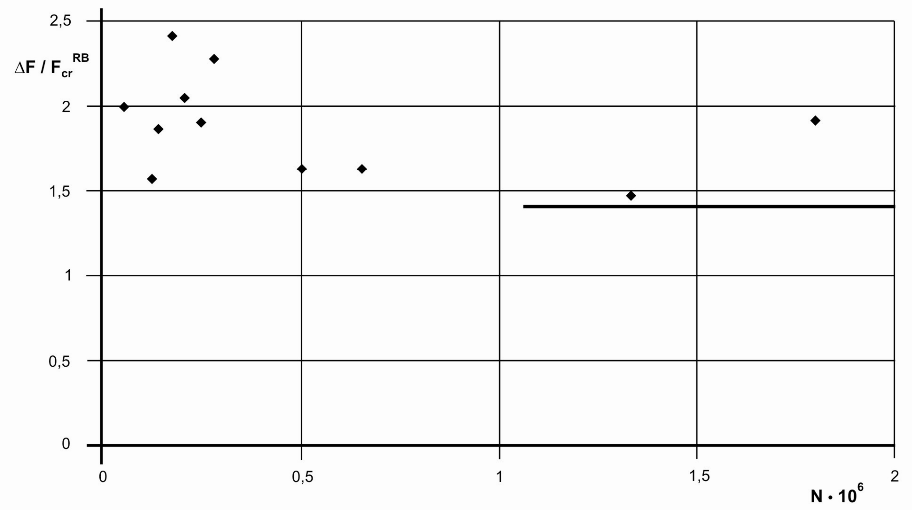
(ii)

$$\frac{\Delta F}{F_{cr}^{WC}} \leq 1.3 \tag{5a}$$

and therefore

$$F_{\max} \leq F_{\min} + 1.3\ F_{cr}^{WC} \tag{5b}$$

(a)



(b)

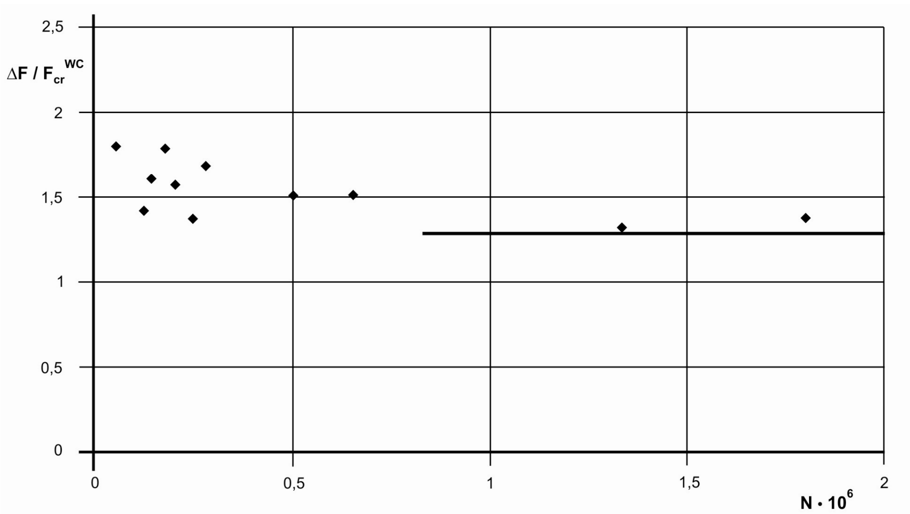


Figure 10: $\Delta F/F_{cr}$-values related to the initiation of the first fatigue crack in web subjected to a partial edge load F
(a) critical load F_{cr}^{RB} calculated by Rockey and Bagchi´s approach
(b) critical load F_{cr}^{WC} calculated by White and Cottingham´s approach

5 Conclusions

Using numerous experiments, the authors studied the breathing and fatigue behaviour of the slender webs of thin-walled steel girders subjected to many times repeated (i) predominantly shear and (ii) partial edge loading. They concluded that the cumulative damage process generated by web breathing can lead to a significant reduction of the beneficial post-buckled behaviour of the webs. However, they also established simple user-friendly formulae such as to make it possible to disregard the detrimental effect of web breathing if the maximum values of the loads acting on the girder are kept under certain limits.

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