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On the Design of a Compliant Mechanism with Non-Uniform Thermal Effects using Evolutionary Structural Optimization

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Abstract

In this paper, an evolutionary optimization procedure is proposed for the design of compliant thermal microactuators subjected to non-uniform temperature fields. During recent decades topology optimization techniques have been shown to be efficient tools to conceive these kinds of distributed compliant mechanisms. The procedure applied in this paper is based in the evolutionary structural optimization (ESO) method, which has been successfully applied to several optimum material distribution problems but not for non-uniformly heated compliant mechanisms including conduction and convection effects. The validity of this technique is demonstrated by an example and the design obtained is compared favourably with the analytical solutions.

Keywords: optimization, compliant, mechanisms, microactuators, topology, evolutionary, thermal.

1 Introduction

Thermal compliant microactuators provide displacements of different points in the device through non-uniform temperature fields by virtue of their specially designed topology and shape. They obtain force and motion transmission capabilities through elastic deformation and from the flexibility of its components and, as a result, give large forces or displacements transmission and rise to compact integrated monolithic systems, since they can be built using fewer parts comparing to conventional rigid-body mechanisms. Traditional compliant mechanisms work under the application of a force at an input port and generate the desired force or deflection at the output port. When the input load is applied, flexible links will deform and therefore flexural joints will bend, transferring the work through these deflections. Thermal micro actuators are those compliant mechanisms onto which thermal loading is applied as input instead of force. These systems function based on the thermal expansion of the

compliant mechanism material while being heated, and convert efficiently very large forces, associated with thermal auction, into deflection.
One of the first classic methods of choice for converting thermal auction to motion or displacement are the bimorph devices, composed of two materials with different thermal expansion coefficients. Using a single material to achieve the same effect in the plane by virtue of the shape was a newer idea introduced more recently. In this case the compliant mechanism is not heated uniformly as in classical bimorph systems. Modern micro actuators are based on electro-thermal actuation in which the heating is accomplished by Joule heating and deformation is achieved by the non-uniform temperature field that deforms the compliant mechanism when electric current is applied. The compliant mechanisms deforms differently depending on the particular topology and shape of the system. The Guckel actuator is one of the best examples of an electro-thermal compliant (ETC) microactuator. Its simple design consists of two arms of different thickness, where due to the different resistive nature of them one gets more heated and elongates more than the other, causing the device to deform and bend laterally [1]. The v-shaped thermal actuator, commonly referred to as a "chevron", is another well known thermal actuator, used in applications requiring high force and reliability [2]. This actuator is based on the constrained thermal expansion of four angled beams through material heating and results in motion of the center shuttle. In all cases the topology of the compliant mechanism critically affects system performance, as well as the temperature distribution, convection, thermal expansion parameters, etc. Accordingly, optimization methods adapted for these tasks are needed and this paper is concerned with the topology optimization of thermal compliant mechanisms and the development of an evolutionary procedure for systematic design of non-uniformly heated microactuators. With the use of finite element analysis in combination with the optimization strategy adopted in this investigation it is possible to address and solve the design problem discussed above.
The last few decades have seen dramatic improvements in the engineering design and topology optimization processes. Solutions obtained by standard sizing and shape optimization methods always maintain the same initial topology and many competing topologies are not explored. For this reason topology optimization algorithms have become increasingly important as potential tools in engineering design.
The goal of structural topology optimization is to determine the optimal distribution of material for a given design domain that minimizes a given cost function and satisfies a series of constraints. The design goals for structures and compliant mechanisms are quite similar, and the same topology optimization methods may therefore be adapted to design both types of elements. In the case of compliant mechanisms, designs must incorporate flexibility as a preferred effect, in contrast to the stiffness. Additionally, a compliant mechanism also needs to be stiff enough to be able to sustain applied loads.
In the field of compliant mechanisms design, we can distinguish two approaches, a kinematic synthesis approach and a continuum synthesis approach. The first method is based on traditional rigid-body kinematicks, where the basic configuration is obtained by knowledge from this field and is converted to partially compliant

mechanism with flexural segments [3]. The second mayor approach is a continuum synthesis approach for design of actuators based on a fully compliant mechanism with lumped compliance. The first applications based on this strategy appeared in Ananthasuresh et al. [4]. A later approach by Sigmund [5] modelled the output load by a spring which captures the nature of the work piece held at the output port of the compliant mechanism and allows control of the input-output behaviour using the mechanical advantage as objective function. An equivalent but different approach is based on the maximization of the ratio of two mutual energies, where two different finite element problems are considered [6]. Frecker et al presented also the synthesis of compliant topologies with multiple input and output ports, using as objective function a combination of the mechanical and geometrical advantage of the mechanism [7]. Path generating mechanisms have been also treated in the work by Saxena and Ananthasuresh [8], as well as compliant thermal microactuators topology optimization. Concerning the problem of compliant thermal microactuators topology optimization under a uniform temperature field was solved and tested with micro scale prototypes by Jonsmann et. al [9], and several systematic procedures for topology optimization of electro-thermally actuated compliant mechanics can be found in the works by Sigmund [10] and Yin and Ananthasuresh [11]. It was demonstrated that electro-thermal mechanisms behaviour could be significantly different with and without modelling convection [12]. A comprehensive thermal modelling for these devices was presented in [13]. Recently, topology optimization of thermally actuated compliant mechanisms considering time-transient effect has been also analyzed by Li et. al [14].

Most of these works use a SIMP interpolation scheme for the design domain parameterization, based on a penalized variable density approach [15], with subtle modifications and several adjustable tuning parameters for efficiency of the method. This class of parameterization generally is coupled with optimality criteria [16] or moving asymptotes algorithms [17]. The pioneering technique based on homogenized materials for finding the optimal topology of a structure by Kikuchi and Bendsoe [18] has been also employed for the solution of the topology optimization problem of compliant mechanisms [19]. During the last years more suitable methods have appeared to compliment the traditional methods, like the recently developed level-set method, successfully used in this field of optimization [20]. Finally, different heuristic or intuition based methods have been proposed to minimize compliance or other objective functions, like genetic algorithms [21], or the evolutionary method, also known as Evolutionary Structural Optimization (ESO) [22]. The ESO method has been successfully applied to several structural optimization problems so far, like stiffness [23], frequency [24] or buckling [25], and extended to design dependent loads [26] or complex cases like optimal design of absorption structures [27]. This work group has successfully applied the ESO method for planar and 3D compliant mechanism design [28], and showed that it seems to be promising for the case of thermally actuated devices as well, when it was used to solve the simple case of compliant mechanisms subjected to uniform temperature fields [29].

This work generalizes the evolutionary structural optimization method for thermally actuated compliant mechanism design where the loads arise due to a non-uniform change in the temperature, considering also convection effects. It will be done by means of an additive version of the ESO method. To suppress the formation of checkerboard patterns a sensitivity filter scheme is introduced, which is used to effectively overcome the mesh-dependency problem as well. This scheme is complemented by a sensitivity averaging scheme that helps to stabilize the process occasional chaotic behaviours which make the objective function and topology difficult to converge [30]. The procedure has been implemented as part of a general optimization computer program called Odessy [31] and tested in several numerical applications and benchmark examples to validate the approach.

2 Evolutionary structural optimization method

ESO stands for Evolutionary Structural Optimization, which is a design method based on the simple concept of gradually removing inefficient material from structure. The method was first proposed by Xie and Steven [32] and has since been continuously developed to solve a wide range of topology optimization problems. The initial concept of the method leads to a rejection criterion based on the local stress level, where the low stressed material is assumed to be under-utilized and is therefore removed progressively. Later the commonly used mean compliance was used to apply the evolutionary procedure for stiffness optimization problems. To enable more specific design objectives, one of the crucial issues associated with evolutionary topology optimization is to evaluate the topological sensitivities. Therefore, a sensitivity number for the mean compliance is defined and indicates de increase of the objective function as a result of the removal of an element. Since topology optimization problems may frequently consider objective function other than stiffness and constraints other than structural volume, different authors dived into various extended topology optimization problems computing the corresponding expressions for the sensitivity analysis of several objective functions. A comprehensive compilation of these investigations can be found in the monograph book by H. Huang and Y. M. Xie [33].

A natural consequence of the basic rejection ESO process was the exploration of the evolutionary growth of a structure starting from a minimum initial kernel, exploring the concept of achieving an optimum structure by adding material. As the name suggests, in the additive evolutionary structural optimization (AESO) method, elements were introduced in the areas of the ground structure where they are needed, and examples demonstrated that this method was capable of producing correct optimal shapes of structures [34]. In the case of structural topology optimization, this technique leaded to regions which were lightly stressed and material should be removed. Since AESO cannot remove elements, a new method was conceived, in order to combine the removing and addition attributes of both methods, called bi-directional evolutionary structural optimization (BESO), which has been applied to several structural topology optimization problems.

As shown by this work group for the case of compliant mechanisms optimum design under directly applied forces [35], is necessary and sufficient to use the additive version of the method to obtain the optimal solutions for thermally actuated problems.

3 Problem formulation

As shown in Figure 1a, we will consider an arbitrary design domain with several types of mechanical and thermal boundary conditions. Optimization will be posed as a material distribution problem where a limited amount is to be distributed in a larger specified design domain to fulfil certain objectives. In this case, the objective of thermal compliant mechanisms design is to obtain the topology that maximizes the displacement u_{out} of the output port when the design domain is subjected to a non-uniform temperature field Φ (x, y). Most actuator applications require the mechanism to resist an output force when interacting with its surroundings, which, in general, may not be known a priori. For such cases, a spring model is usually proposed to approximate this force with a constant K at the output port.

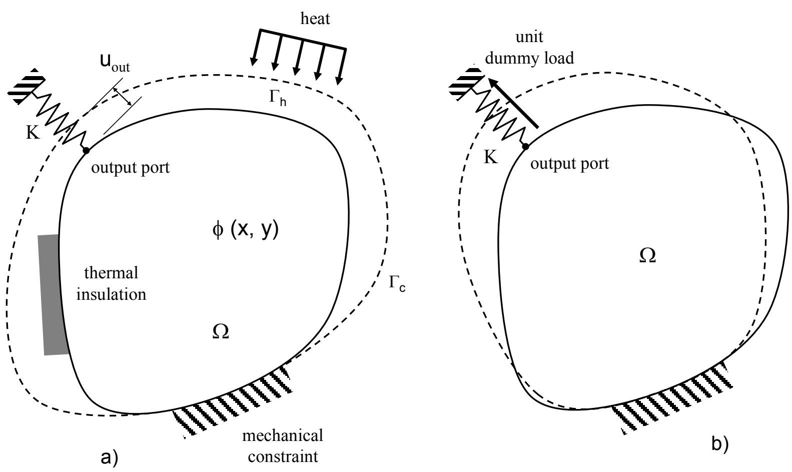


Figure 1: Design domain subjected to thermal load and unit dummy case.

The flexibility requirement can be captured by using the concept of mutual mean compliance, based on the reciprocal theorem for linear elasticity. For this purpose it is usual to adopt a second artificial load case, where a unit dummy load is applied at the output port in the direction of the desired displacement, as shown in Figure 1b. The finite element equilibrium equations that need to be solved for the real thermal load case are:

$$\boldsymbol{K_T \Phi} = \boldsymbol{F_T} \qquad\qquad \boldsymbol{K U_1} = \boldsymbol{F_1}(\boldsymbol{\Phi}) \tag{1}$$

where $\mathbf{K_T}$ is the thermal stiffness matrix, $\mathbf{\Phi}$ denotes de nodal variable temperature field vector and $\mathbf{F_T}$ represents the nodal heat source vector. The stiffness matrix $\mathbf{K}$ corresponds to the thermal-stress analysis, and obviously represents the regular stiffness matrix of the structure for elastic analysis, where the stiffness of the spring is also included. Finally $\mathbf{F_1}$ is the thermal load nodal vector and $\mathbf{U_1}$ is the nodal displacement vector. The equivalent thermal vector $\mathbf{F_1}$ is calculated from the temperature distribution, $\mathbf{\Phi}$, obtained in the heat flow analysis, in the following way:

$$F_1 = \int_{\Omega} \boldsymbol{B}^T \boldsymbol{E} t \alpha \Phi [1,1,0]^T dA = \boldsymbol{T\Phi} \tag{2}$$

where **B** is the strain-displacement matrix, **E** is the elastic coefficients constitutive matrix and α is the thermal expansion coefficient. This expression can be rewritten using the transformation matrix **T** between nodal temperature and nodal equivalent thermal force. This will be useful for derivation in the sensitivity analysis. It should be remembered that the thermal stiffness matrix $\mathbf{K_T}$ contains two terms since convection may occur on a part of the boundary, denoted by Γ_c, where it appears that a modification of the stiffness matrix occurs. The first term corresponds to the conductivity constitutive matrix, and the second one contains the convection coefficient on the boundary [36].

Concerning the second load case where only a unit dummy load is applied in the output port, we have the well know elastostatic finite element equilibrium equation:

$$\boldsymbol{KU_2} = \boldsymbol{F_2} \tag{3}$$

where **K** is the stiffness matrix. Likewise, this matrix contains the stiffness of the finite elements in the design model as well as the output spring stiffness, and $\mathbf{F_2}$ is a vector with the value 1 at the degree of freedom corresponding to the output point and with zeros at all other places. We call $\mathbf{U_2}$ the nodal displacement vector for this auxiliary load case.

Relying on both load cases we can use the following equation to express the displacement at the output port as:

$$u_{out} = \boldsymbol{U_2^T K U_1} \tag{4}$$

that is, the objective function we want to maximize for compliant mechanism design.

Summarizing, the finite element formulation of the optimization problem can be written as:

$$\begin{aligned} &\textit{Minimize} && -\boldsymbol{U_2^T K U_1} \\ &\textit{Subject to} && \boldsymbol{K_T \Phi} = \boldsymbol{F_T} \\ & && \boldsymbol{KU_1} = \boldsymbol{F_1}(\boldsymbol{\Phi}) \\ & && \sum_{i=1}^{n} v_i \le V^* \qquad v_i \in \{0, v_i^e\} \end{aligned} \tag{5}$$

where v_i is the actual element volume, v_i^e represent the total volume of the element and V* refers to the prescribed total volume in the design. Here we will take v_i as discrete design variables that state the absence (0) or presence (v_i^e) of an element. The overall objective of the formulated problem is to gradually add elements of volume v_i^e which results in the maximum increase of output displacement until the constrained total volume reaches its given limit.

4 Sensitivity analysis

This chapter shows the computation of the α_i sensitivity numbers for all the finite elements in the design model. This numbers will describe the effect of element addition on the output displacement and the material volume (see Figure 2). The volume change calculation is straightforward because the addition of an element will be traduced in a change of volume Δv_i equal to the total volume of the selected element, v_i^e. Obviously, if all the elements in the mesh have the same size, all of them will be increased by equal volume portions and Δv_i will be the same for all the elements in the mesh. The change in the output displacement can be found to be given by equilibrium conditions before and after the change.

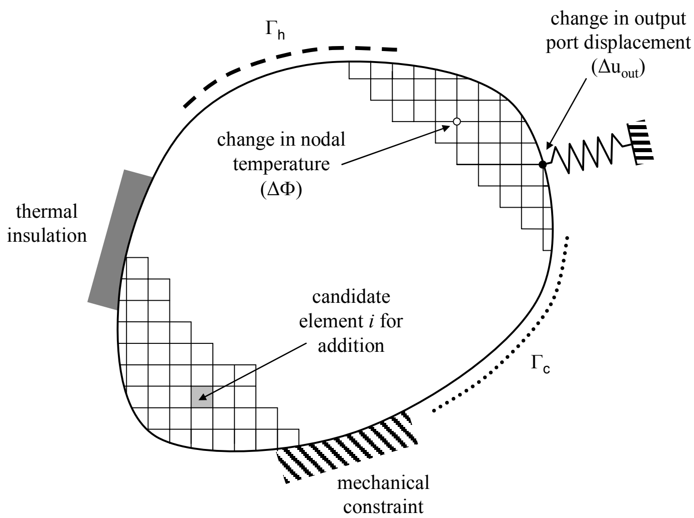


Figure 2: Element addition and sensitivity analysis.

From Equations (1) and (3) we get

$$\begin{aligned}(\boldsymbol{K} + \Delta\boldsymbol{K})(\boldsymbol{U_1} + \Delta\boldsymbol{U_1}) &= (\boldsymbol{F_1} + \Delta\boldsymbol{F_1}) \\ (\boldsymbol{K} + \Delta\boldsymbol{K})(\boldsymbol{U_2} + \Delta\boldsymbol{U_2}) &= \boldsymbol{F_2}\end{aligned} \tag{6}$$

where it can be noticed that for the first load case of thermo elastic analysis, the applied load vector, $\mathbf{F_1}$, depends on the design variables. One of the differences between this design problem and the directly applied forces case is that sensitivities have to take the nodal load vector change into account. The nodal force vector in the second load case is assumed to be zero, since the unit dummy load does not depend on the design variables and it is unaffected by the material distribution over the design domain. By subtracting Equation (6) from Equations (1) and (3) and neglecting higher order elements, we get:

$$\Delta \boldsymbol{K} \boldsymbol{U}_{1} + \boldsymbol{K} \Delta \boldsymbol{U}_{1} = \Delta \boldsymbol{F}_{1}$$
$$\Delta \boldsymbol{K} \boldsymbol{U}_{2} + \boldsymbol{K} \Delta \boldsymbol{U}_{2} = \boldsymbol{0} \tag{7}$$

Similarly, the variation in output displacement can be written from Equation (4) in the following way

$$\Delta u_{out} = \Delta \boldsymbol{U}_{2}^{T} \boldsymbol{K} \boldsymbol{U}_{1} + \boldsymbol{U}_{2}^{T} \Delta \boldsymbol{K} \boldsymbol{U}_{1} + \boldsymbol{U}_{2}^{T} \boldsymbol{K} \Delta \boldsymbol{U}_{1} \tag{8}$$

After substituting the Equations (7) in the first and the last term of Equation (8)

$$\Delta u_{out} = -\boldsymbol{U}_{2}^{T} \Delta \boldsymbol{K} \boldsymbol{U}_{1} + \boldsymbol{U}_{2}^{T} \Delta \boldsymbol{K} \boldsymbol{U}_{1} + \boldsymbol{U}_{2}^{T} \left(\Delta \boldsymbol{F}_{1} - \Delta \boldsymbol{K} \boldsymbol{U}_{1} \right) \tag{9}$$

and simplifying, we obtain

$$\Delta u_{out} = \boldsymbol{U}_{2}^{T} \Delta \boldsymbol{F}_{1} - \boldsymbol{U}_{2}^{T} \Delta \boldsymbol{K} \boldsymbol{U}_{1} \tag{10}$$

This expression gives the change Δu_{out} in the specific displacement component at the output port due to altering the i-th element. According to definition of the finite element stiffness matrix, when i-th element is introduced to the domain, only the stiffness corresponding to the added element is affected. Therefore, the variation of the stiffness matrix shown in Equation (10), $\Delta\mathbf{K}$, can be obtained in a simple manner:

$$\Delta \boldsymbol{K} = \boldsymbol{K}' - \boldsymbol{K} = \boldsymbol{K}_{i} \tag{11}$$

where $\mathbf{K}'$ is the stiffness matrix of the resulting structure after the i-th element is added and $\mathbf{K_i}$ is the stiffness matrix of the added element. In contrast, the computation of the load vector variation, $\Delta\mathbf{F_1}$, is a bit cumbersome, because it denotes the variable temperature field in the design domain and must be obtained based on the heat flow equilibrium equation. The global load vector given by Equation (2) is obtained by integration over the entire region Ω, and as it is well known it is calculated as a summation of integrations over each element, where Φ contains the necessary temperatures at the nodal points of each element. Taking differences in the thermal load vector of Equation (2), we get:

$$\Delta \boldsymbol{F}_{1} = \int_{\Omega} \boldsymbol{B}^{T} \Delta \boldsymbol{E} t \alpha \Phi [1,1,0]^{T} dA + \int_{\Omega} \boldsymbol{B}^{T} \boldsymbol{E} t \alpha \Delta \Phi [1,1,0]^{T} dA = \Delta \boldsymbol{T} \boldsymbol{\Phi} + \boldsymbol{T} \Delta \boldsymbol{\Phi} \tag{12}$$

in terms of the transformation matrix $\mathbf{T}$. Now the first term of Equation (10) will read

$$\boldsymbol{U}_{2}^{T} \Delta \boldsymbol{F}_{1} = \boldsymbol{U}_{2}^{T} \Delta \boldsymbol{T} \boldsymbol{\Phi} + \boldsymbol{U}_{2}^{T} \boldsymbol{T} \Delta \boldsymbol{\Phi} \tag{13}$$

Again, the variation of the elastic coefficient matrix, $\Delta\mathbf{E}$, is obtained directly because corresponds to the added element's elastic matrix with the full value of young modulus:

$$\Delta \boldsymbol{E} = \boldsymbol{E}' - \boldsymbol{E} = \boldsymbol{E}_i \tag{14}$$

since before the addition the soft kill method had assigned a very low elastic modulus to void elements. As previously stated, the alteration, addition in our case, of a conductive element will lead the temperature field Φ to changing by a value that must be computed using the heat equilibrium equation, and affects the second term of the integral in Equation (13). When obtaining the variation of the output displacement written in Equation (10), it was not necessary to calculate the variation of the displacement vectors explicitly. However, in this case the second term of Equation (13) contains the variation of the heat flow problem unknown field, Φ, which in turn must be obtained by taking the derivative of the equilibrium Equation $\mathbf{K}_T\boldsymbol{\Phi} = \mathbf{F}_T$. In our topology design problem we work with a low number of constraints comparing the number of design variables. Thus, the most effective method for calculating the variations of the nodal temperatures is to use the adjoint method. Differentiating the equilibrium equation we have

$$\Delta \boldsymbol{K}_T \boldsymbol{\Phi} + \boldsymbol{K}_T \Delta \boldsymbol{\Phi} = \Delta \boldsymbol{F}_T \tag{15}$$

and assuming that the variation of an element has no effect on the heat load vector, that is, it is not design dependent,

$$\boldsymbol{K}_T \Delta \boldsymbol{\Phi} = -\Delta \boldsymbol{K}_T \boldsymbol{\Phi} \tag{16}$$

Now an arbitrary but fixed adjoint vector Λ can be introduced as

$$\boldsymbol{\Lambda}^T \boldsymbol{K}_T \Delta \boldsymbol{\Phi} = -\boldsymbol{\Lambda}^T \Delta \boldsymbol{K}_T \boldsymbol{\Phi} \tag{17}$$

and it can be easily proven that one can compute the second term of Equation (12) multiplied by $\mathbf{U}_2^T$ in the following way

$$\boldsymbol{U}_2^T \boldsymbol{T} \Delta \boldsymbol{\Phi} = -\boldsymbol{\Lambda}^T \Delta \boldsymbol{K}_T \boldsymbol{\Phi} \tag{18}$$

if the adjoint vector it satisfies the adjoint equation

$$\boldsymbol{K}_T \boldsymbol{\Lambda} = \boldsymbol{T} \boldsymbol{U}_2 \tag{19}$$

Therefore the sensitivity analysis requires only on additional thermal analysis to be solved at each iteration. Moreover, previous factorization of the thermal stiffness matrix can be used and only forward and backward substitutions are needed to solve Equation (19). Combining the obtained relations it can be found easily that:

$$\Delta u_{out} = \left(\boldsymbol{U}_2^T \Delta \boldsymbol{T} - \boldsymbol{\Lambda}^T \Delta \boldsymbol{K}_T \right) \boldsymbol{\Phi} - \boldsymbol{U}_2^T \Delta \boldsymbol{K} \boldsymbol{U}_1 \tag{20}$$

Recalling that when an element is introduced to the domain, only the properties corresponding to the added element are affected, the necessary vector and matrix changes can be simply calculated at one element level. Therefore, from Equation (10), when i-th element is added to the design domain, the displacement change would be

$$\Delta u_{out} = \boldsymbol{U}_2^{T^i} \Delta \boldsymbol{F}_1{}^i - \boldsymbol{U}_2^{T^i} \boldsymbol{K}^i \boldsymbol{U}_1{}^i \tag{21}$$

where $\mathbf{K^i}$ is the element stiffness matrix, as it was stated in Equation (11), and the displacement vectors, $\mathbf{U_1}^\mathbf{i}$ and $\mathbf{U_2}^\mathbf{i}$, contain the nodal displacement of the candidate element . The first term where the nodal vector displacement of the unit dummy load case is multiplied by the change in the thermal vector can be computed again only in terms of the i-th element which sensitivity we want to evaluate after being added to the domain

$$\boldsymbol{U}_2^{T^i} \Delta \boldsymbol{F}_1{}^i = \boldsymbol{U}_2^{T^i} \Delta \boldsymbol{T}^i \boldsymbol{\Phi}^i + \boldsymbol{U}_2^{T^i} \boldsymbol{T}^i \Delta \boldsymbol{\Phi}^i = \boldsymbol{U}_2^{T^i} \boldsymbol{T}^i \boldsymbol{\Phi}^i - \boldsymbol{\Lambda}^{T^i} \boldsymbol{K}_T{}^i \boldsymbol{\Phi}^i = \left(\boldsymbol{U}_2^{T^i} \boldsymbol{T}^i - \boldsymbol{\Lambda}^{T^i} \boldsymbol{K}_T{}^i \right) \boldsymbol{\Phi}^i \tag{22}$$

In Equation (22) $\mathbf{K_T}^\mathbf{i}$ is the element thermal stiffness matrix and $\mathbf{\Phi^i}$ denotes the nodal temperatures of the element. Here $\mathbf{T^i}$ and $\mathbf{\Lambda^{Ti}}$ correspond to the transformation matrix and adjoint vector of the candidate element, respectively. Summarizing, the output displacement variation would be obtained by

$$\Delta u_{out} = \left(\boldsymbol{U}_2^{T^i} \boldsymbol{T}^i - \boldsymbol{\Lambda}^{T^i} \boldsymbol{K}_T{}^i \right) \boldsymbol{\Phi}^i - \boldsymbol{U}_2^{T^i} \boldsymbol{K}^i \boldsymbol{U}_1{}^i \tag{23}$$

All the matrix and vectors over each element can be easily calculated using the results available from the finite element analysis of the thermoelastic and static problems for both the thermal load and the virtual unit load, respectively, defined in the two load cases of Equations (1) and (3). We have seen that it is also necessary and additional adjoint equation to be solved (Equation (19)) at each iteration, but the changes in the required stiffness matrixes and in the nodal vectors are not difficult to calculate since they correspond directly to the added element matrixes and vectors.

5 Optimization algorithm

The element addition strategy proposed in this work is based on the Evolutionary Structural Optimization method. The traditional principle of ESO is that the structure evolves towards an optimum by eliminating inefficient elements of the finite elements mesh inside the design domain. First a design domain with a given boundary and load conditions is defined. Then the necessary finite element analysis are performed, where the required element matrixes and vectors are computed to determine displacements for the load cases considered, The next step is the sensitivity analysis, where a sensitivity number α_i is calculated for all elements. Finally volume can be reduced gradually by eliminating under-utilized portions from the structure removing elements of the smallest or highest sensitivity number, depending on the optimization problem. In this case we will adopt the additive version of the method, where elements with largest α_i will be added to the design domain. This process is repeated until the structure reaches the prescribed volume. The optimal design of the mechanism is obtained by repeating the cycle of finite elements analysis and element additions until the volume reaches the prescribed value, producing the largest increase of u_{out} for the given volume. It is easy to understand that depending on the volume limit, we may get an unconnected structure for low values or a solution that does not fulfil the flexibility requirement if too much material is present in the design domain. If we do not want to end up with an unconnected structure we should not to use too small values for the final volume. For large predefined volume limits it is also recommended to check if a convergence criterion (defined in terms of the relative change in the objective function in several successive iterations) is less than a given tolerance, to verify if the maximum value of u_{out} has been reached and stop before two much elements are added to the design domain:

$$\frac{\sum_{k=1}^{m} u_{out}^{i-k+1} - \sum_{k=1}^{m} u_{out}^{i-m-k+1}}{\sum_{k=1}^{m} u_{out}^{i-k+1}} \leq \varepsilon \tag{24}$$

where i is the current iteration number, ε is the convergence tolerance and m is an integer number that denotes the number of iterations over which the change in the objective functions is calculated. It would be also interesting to select the best solution by comparison out of solutions generated for different final volume fractions. The addition of elements from the mesh is obtained directly by a soft kill method, where optimization starts with a discretized design domain full of elements with a low elastic modulus and when an element is added, they are assigned the real isotropic elastic modulus. It is mandatory to define an element addition ratio to control the inclusion of elements and ensure that not too many elements are added in a single iteration. From numerical experience it is determined that the element addition or rejection ratio should not be larger than 1 % in the structural evolutionary method [37]. In this paper we will adopt the smallest possible ratio in order to improve the accuracy of the solution and ensure a smooth change in the output displacement, so the total element amount will be increased only by one in each iteration. The sensitivity numbers α_i obtained in the previous section could become zero order discontinuous across element boundaries when the continuum structure is discretized using low order elements, which may lead to checkerboard problems [38]. The presence of checkerboard patterns causes difficulty in interpreting and manufacturing the solution obtained. Another problem related to topology optimization is the so-called mesh dependency problem, that refers to the problem of obtaining different topologies when different finite element meshes are used. This situation usually is not desired, since ideally mesh refinement should result in a better modeling of the same optimal design and better description of boundaries, but not in a more detailed or qualitatively different structure [39].
To suppress the formation of these patterns, here we will apply the sensitivity filter scheme to the proposed element addition strategy [30].

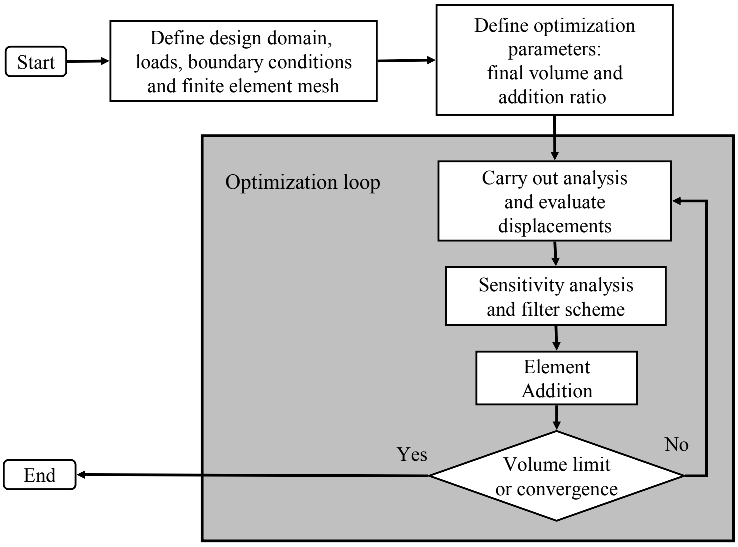


Figure 3: Flowchart of the optimization procedure.

The previous smoothing technique will be complemented with an averaging scheme in order to stabilize the optimization process. It has been often observed that oscillations may happen in the evolution history due to discrete nature of the optimization method, which makes the objective function and topology difficult to converge. Huang and Xie has found that averaging the sensitivity number with its historical information is an effective way to solve this problem.
The flow chart in Figure 3 summarizes the additive version procedure that will be used in this work.

6 Example and discussion

This example is presented to illustrate that thermally actuated compliant mechanisms optimization problems can be dealt with the described additive topology optimization algorithm. Moreover, after a visual inspection of the results and due to the simplicity of the problem, the output displacement can be approximated analytically to check and justify the optimum topologies obtained for different temperatures fields.
The mechanical material properties adopted for the example are chosen to those of a common silicon material, Young's modulus E = 200 GPa, Poison's ratio 0.3 an thermal expansion coefficient $\alpha = 20.10^{-5}$ ºC^{-1}. This example represent a rectangular domain of 10 × 20 mm where the left edge is clamped (see Figure 4). The thickness of the plate is 1 mm and it has been discretized using 3200 four node elements mesh. To illustrate that the proposed method is able to predict different optimal topologies when the temperature distribution changes, first we will use a prescribed and fixed non-uniform temperature field. Figure 4 shows the temperature distribution that will be assumed in this test example. Temperature variation is linear in *y* direction and constant in *x*, where we recall that $\Phi_1 = T_1 - T_\infty$ and $\Phi_2 = T_2 - T_\infty$. Taking advantage of the symmetry of the problem, it is taken to be the same in the top and bottom

sides of the design domain. The example was solved for three different temperature fields: $\Phi_1 = \Phi_0 = 100$ ºC, $\Phi_1 = 2\Phi_0 = 200$ ºC and $\Phi_1 = 300$ ºC. Obviously the first case is a particular situation where the temperature field is uniformly distributed over the design domain. In all cases the final volumes of the designs are 10% of the initial volume.

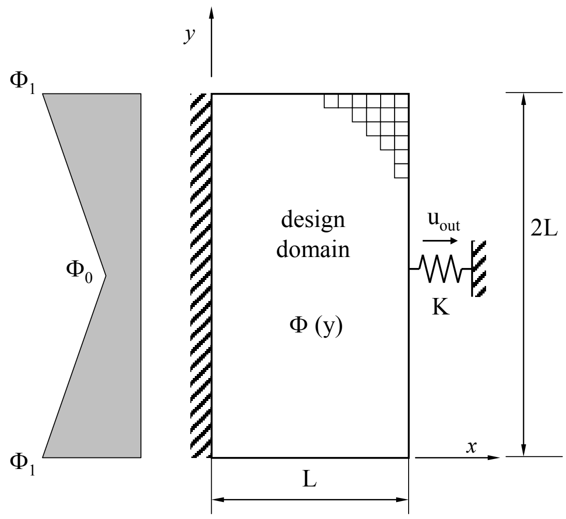


Figure 4: Design domain and boundary conditions with linear temperature variation.

Top row in Figure 5 contains the optimized topologies for different temperature distributions, where a two bar truss-like structure is obtained. The optimization procedure redistributes material in order to form a hinge-like region where two bars are connected, so that when temperature is raised, the elongation of members results in a horizontal displacement of the output port. The first case is a singular case, where both bars are merged in a unique horizontal bar. Deformation and temperature distribution patterns of the optimized actuators are included in the bottom row of Figure 5. As expected, horizontal displacement at the output port is higher for larger variations of the temperature field in the compliant mechanism. It is interesting to note that the inclination of the vertical members is different depending on the temperature variation.

This result can be obtained analytically in an approximate way comparing obtained topologies with a truss structure consisting of two bars hinged at the ends (Figure 6a). The reaction force of the spring can be derived in a straight forward manner from the equilibrium equations and the compatibility relation in node C, using strength of materials' classic formulas:

$$R = \alpha \left[\frac{(\Phi_1 - \Phi_0) L sen\theta}{2\cos^3\theta} + \frac{\Phi_0 L}{\cos^2\theta} \right] \Bigg/ \left[\frac{1}{K} + \frac{L}{2EA\cos^3\theta} \right] \qquad (25)$$

where E and A denote the elastic modulus and the section area, respectively.

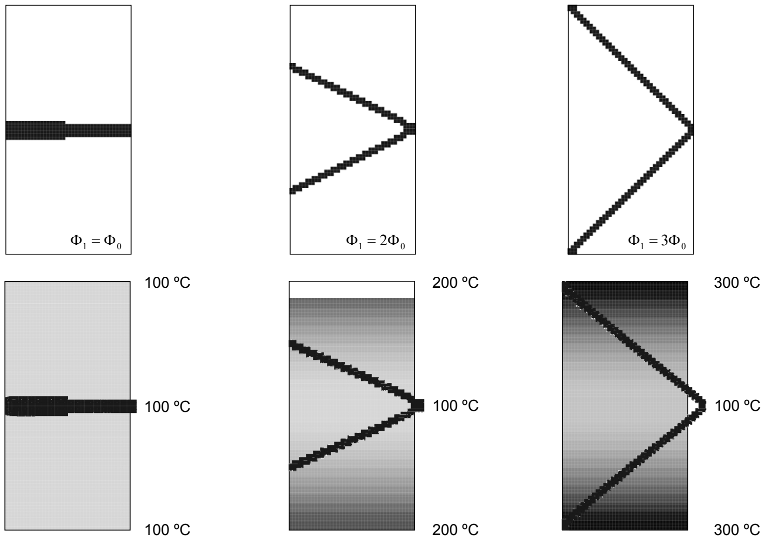


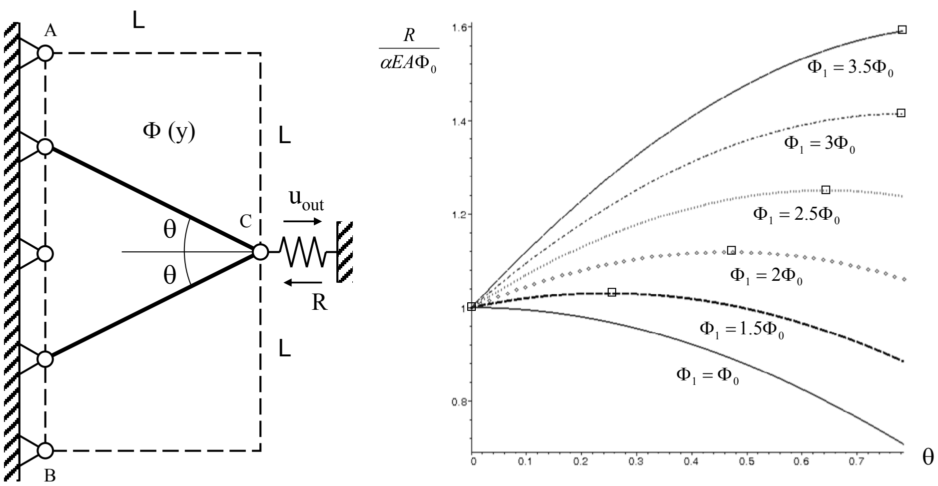
Figure 5: Temperature fields and optimum topologies.

For this example we have chosen deliberately a high stiffness spring, so that for simplicity we may assume that the spring constant K is infinite, and Equation (25) can be approximated by

$$R = \alpha EA\left[\left(\Phi_1 - \Phi_0\right) sen\theta + 2\Phi_0 \cos\theta\right] \tag{26}$$

Taking the first derivative, we can easily calculate the optimum angle for a range of different values of Φ_1 / Φ_0 and plot the dimensionless reaction force variation in terms of θ (see Figure 6b):

$$\frac{\partial R}{\partial \theta} = \alpha EA\left[\left(\Phi_1 - \Phi_0\right)\cos\theta - 2\Phi_0 sen\theta\right] = 0 \Rightarrow tg\theta = \frac{\Phi_1 - \Phi_0}{2\Phi_0} = \frac{1}{2}\left(\frac{\Phi_1}{\Phi_0} - 1\right) \tag{27}$$



a) b)

Figure 6: Curves from the analytical solution.

This inclination angle will maximize the output reaction force and, for a fixed high value of the spring constant, the output displacement will be the largest possible. When temperature distribution is almost uniform, we get the maximum reaction force and horizontal displacement at output port for small values of angle θ.
Actually, if $\Phi_1 / \Phi_0 = 1$, we get a single horizontal bar. On the contrary, when temperature is higher at the upper side of the design domain, dilatation in members is higher for larger values of θ and it is more effective to increase the inclination of the bars for maximum displacement in node C. It can be noted that the optimum angle θ is increasing as the slope of the linear temperature distribution gets higher, showing that obtained topologies agree well with this analytical results. For the case of high Φ_1 / Φ_0 ratios the optimum value of angle θ is always the maximum allowable value, $\pi/4$ in this case. It can be seen that the resulting topologies are very close to that of the analytically obtained optimum designs but, obviously, there must exist a discrepancy between both numerical results, accredited to the perfect hinges and straight bars assumed in the analytical model where the bending of the continuum solutions are not included, and affect the overall performance of the compliant mechanism.

7 Conclusions

It has been demonstrated the ability of the evolutionary structural optimization method for thermal compliant actuators topology design subjected to non-uniform temperature fields. An additive version of this method has been adopted in order to achieve the optimum design, since the traditional ESO method's element removal technique is not efficient in this case. The most efficient discrete element addition is achieved adding elements in regions with elements with positive sensitivity number during the optimization process, which gives the largest increase of the output displacement for the prescribed volume. The example prove that the evolutionary optimization method is a promising tool for design of electro-thermal actuators, since the solutions obtained converge to the topologies expected if compared to approximated analytical calculations. Since the final solution depends on the pre-assigned volume, if we do not want to end up with a highly non optimal solution, a tolerance should be defined for the convergence check, and stop the process when the maximum output port is reached since it is clear that further addition of material will not improve the solution. Mesh dependency problems and convergence histories of the objective function are greatly improved by introducing a filtering scheme and by averaging the sensitivity numbers. Further work will include the expansion of this topology optimization algorithm to incorporate complicated thermally actuated compliant mechanisms specifications, like electro-thermal actuators subjected to non temperature fields derived from Joule heating, since supplying constant heat flux might not be efficient in practice. Future investigation should explore the application of the bi-directional algorithm, which would help to yield a more stable convergence and a general behaviour closer to that of other optimization methods.

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