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A Subsoil Model based on Numerical Integration of a Nonlinear Halfspace

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Abstract

The stress strain analysis of plate foundation structures in interaction with the subsoil is presented. The contact finite element method is based on a non-linear modified model of a three-dimensional elastic half-space in accordance with the European and Czech standards. The arbitrary shape and general course of the loaded area for the nodal points is enabled by the use of four- and eight-node isoparametric elements, numerical integration and Jacobean transformation. The results of numerical examples are compared with other solutions.

Keywords: foundation structure, soil-structure interaction, linear and nonlinear half-space, Gauss numerical integration, finite element method.

1 Introduction

The soil-structure interaction influences not only the settlement and the stress of a foundation, but also the other bearing elements of building structures. The objective of this paper is also results improving and developing the numerical methods of soil-structure analysis based on the CSN EN standard [1], [2], [3] and finite element method (FEM). The numerical integration and solving of large non-linear equations by means of iteration methods do not create any problem with the sharp growth of computer techniques [21], [23]. The basis of the proposed subsoil model is numerical integration of an elastic half-space, which is loaded by an arbitrary shape of the loaded area.

This is possible by use of the Jacobean matrix of transformation and isoparametric contact plate elements. This proposal's original solution was successfully used for various tasks in structure practice [10], [11], [13]. The state of stress in an elastic half-space has been dealt with so far for certain simple shapes and for certain loading behaviours only because in practice mathematical difficulties are faced even in the simplest cases. The author of this paper has managed to solve the integral only for some vertical lines that cross the centre, edge or corner points of a rectangular surface which is subject to a constant or, at least, linear load [7], [8], [9].

The author believes that according to information which is in available literature [1], [7], [8], [9], [12], [14], [15] there is not any explicit expression of stress components for general loading defined by four different intensities in corners of a general quadrangle. For that purpose, the analytical solution has been replaced with a numerical one. Considering accuracy of integration, Gauss' quadrature formulae have been used [6]. The spatial integral can be calculated by the numerical integration without any major difficulties only if the loading surface is a rectangle. In case of other shapes (including a triangle which is often used in FEM), the calculation is somewhat complicated because integration limits are variable.

The solution that eliminates such drawbacks is the use of transformation relations by means of the transformation Jacobian matrix, which utilized shape functions of isoparametric elements [4], [13]. Any loading surface can be expressed by means of four-node or eight-node isoparametric elements with the general load defined in node points [13].

2 Loading the half-space with the general load

In accordance with [13] relations for all six elements of the stress tensor in the elastic half-space have been derived in [6] for a single load.

$$\{\sigma\} = \{\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy}\} \tag{1}$$

The stress in the point can be theoretically described by integration of effects of an elementary force, dP, acting on the surface, dA. Let us express the differential of the stress component, dσ, by means of the vertical force differential, dP. Having integrated along the loading area, A, rather general relations can be derived for the individual components of the state of stress [10], [11], [13], [20]:

$$\sigma_x = \int_A \frac{dP_z}{2\cdot\pi}\cdot\left\{\frac{3x^2\cdot z}{r^5} - (1-2\nu)\cdot\left[\frac{r^2 + r\cdot z - z^2}{r^3\cdot(r-z)} - \frac{x^2\cdot(2r-z)}{r^3\cdot(r-z)^2}\right]\right\} \tag{2a}$$

$$\sigma_y = \int_A \frac{dP_z}{2\cdot\pi}\cdot\left\{\frac{3y^2\cdot z}{r^5} - (1-2\nu)\cdot\left[\frac{r^2 + r\cdot z - z^2}{r^3\cdot(r-z)} - \frac{y^2\cdot(2r-z)}{r^3\cdot(r-z)^2}\right]\right\} \tag{2b}$$

$$\sigma_z = \int_A \frac{dP_z}{2\cdot\pi}\cdot\frac{3\cdot z^3}{r^5} \tag{2c}$$

$$\tau_{yz} = \int_A \frac{dP_z}{2\cdot\pi}\cdot\frac{3\cdot y\cdot z^2}{r^5} \tag{2d}$$

$$\tau_{zx} = \int_A \frac{dP_z}{2\cdot\pi}\cdot\frac{3\cdot x\cdot z^2}{r^5} \tag{2e}$$

$$\tau_{xy} = \int_A \frac{dP_z}{2\cdot\pi}\cdot\left[\frac{3\cdot x\cdot y\cdot z}{r^5} - (1-2\nu)\cdot\frac{x\cdot y\cdot(z-2r)}{r^3\cdot(r-z)^2}\right] \tag{2f}$$

Having substituted the elementary force differential

$$\mathrm{dP_z} = \mathrm{p_z(x,y)\cdot dA} \tag{3}$$

where p_z (x,y) is the function describing the progress of load, p_z, along the surface, A, the final formula can be derived for the magnitude of all 6 components of the state-of-stress tensor, $\{\sigma\}$, resulting from effects of the general continuous load, p_z (x,y), [10], [11], [13], [20].

$$\sigma_x = \int_A \frac{p_z}{2\cdot\pi}\cdot\left\{\frac{3x^2\cdot z}{r^5} - (1-2\nu)\cdot\left[\frac{r^2+r\cdot z-z^2}{r^3\cdot(r-z)} - \frac{x^2\cdot(2r-z)}{r^3\cdot(r-z)^2}\right]\right\}\cdot dA \tag{4a}$$

$$\sigma_y = \int_A \frac{p_z}{2\cdot\pi}\cdot\left\{\frac{3y^2\cdot z}{r^5} - (1-2\nu)\cdot\left[\frac{r^2+r\cdot z-z^2}{r^3\cdot(r-z)} - \frac{y^2\cdot(2r-z)}{r^3\cdot(r-z)^2}\right]\right\}\cdot dA \tag{4b}$$

$$\sigma_z = \int_A \frac{p_z}{2\cdot\pi}\cdot\frac{3\cdot z^3}{r^5}\cdot dA \tag{4c}$$

$$\tau_{yz} = \int_A \frac{p_z}{2\cdot\pi}\cdot\frac{3\cdot y\cdot z^2}{r^5}\cdot dA \tag{4d}$$

$$\tau_{zx} = \int_A \frac{p_z}{2\cdot\pi}\cdot\frac{3\cdot x\cdot z^2}{r^5}\cdot dA \tag{4e}$$

$$\tau_{xy} = \int_A \frac{p_z}{2\cdot\pi}\cdot\left[\frac{3\cdot x\cdot y\cdot z}{r^5} - (1-2\nu)\cdot\frac{x\cdot y\cdot(z-2r)}{r^3\cdot(r-z)^2}\right]\cdot dA \tag{4f}$$

In practice, mathematical difficulties are faced in case of integration even in the simplest cases [7], [8], [9]. The authors of this paper have managed to solve the integral only for some vertical lines (that cross the centre, edge or corner points of a rectangular surface which is subject to constant or, at least, linear load). For the points located in general positions at the vertical lines, the stress can be determined for rectangular areas by superposition of simple loading patterns (both positive and negative ones).

For example *Steinbrenner (1934)* described the final relations for 3 components of the state of stress caused by the even continuous load, p, under a corner of the rectangular loaded area L x B. More general expressions for the calculation of the stress components for any x, y, z coordinated were derived by *Korotkin (1938)* and are mentioned by *Florin (1959)* in his work [7], see [13]. Those expressions apply to any evenly loaded rectangular surface with the length of sides being 2.a and 2.b. The integration is performed in the intervals from –a to +a and from –b to +b.

3 Isoparametric elements

In addition to the Cartesian system of x,y coordinates, let us introduce dimensionless coordinates ξ, η. It is required that in the nodal points, the values should be the required unit values [4], [11], [13]. Relations between the Cartesian system and the dimensionless coordinate system can be described as follows:

$$x = x(\xi,\eta) = \sum_{i=1}^{r} N_i(\xi,\eta).x_i \qquad y = y(\xi,\eta) = \sum_{i=1}^{r} N_i(\xi,\eta).y_i \qquad (5a, 5b)$$

where the r upper limit in the sum is given by the number of the nodal points in the component. For instance, the shape functions of a four-node element can be described as follows:

$$N_1(\xi,\eta) = 0{,}25 \cdot (1 - \xi + \eta + \xi\eta) \qquad (6a)$$

$$N_2(\xi,\eta) = 0{,}25 \cdot (1 + \xi - \eta - \xi\eta) \qquad (6b)$$

$$N_3(\xi,\eta) = 0{,}25 \cdot (1 + \xi + \eta + \xi\eta) \qquad (6c)$$

$$N_4(\xi,\eta) = 0{,}25 \cdot (1 - \xi + \eta - \xi\eta) \qquad (6d)$$

4-node element

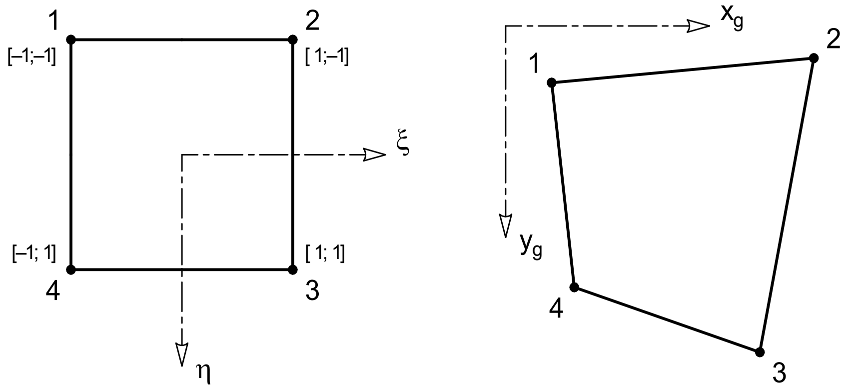


Figure 1: Transformation of the unit element into a curvilinear 4-node element

Similarly, the shape functions for an 8-node element can be described as follows:

- for the corner nodes

$$N_1(\xi,\eta) = 0{,}25.(-1 + \xi\eta + \xi^2 + \eta^2 - \xi^2\eta - \xi\eta^2) \qquad (7a)$$

$$N_2(\xi,\eta) = 0{,}25.(-1 - \xi\eta + \xi^2 + \eta^2 - \xi^2\eta + \xi\eta^2) \qquad (7b)$$

$$N_3(\xi,\eta) = 0{,}25.(-1 + \xi\eta + \xi^2 + \eta^2 + \xi^2\eta + \xi\eta^2) \qquad (7c)$$

$$N_4(\xi,\eta) = 0{,}25.(-1 - \xi\eta + \xi^2 + \eta^2 + \xi^2\eta - \xi\eta^2) \qquad (7d)$$

- for the intermediate nodes in centres of the sides

$$N_5(\xi,\eta) = 0{,}5.(1-\eta-\xi^2+\xi^2\eta) \quad (8a)$$

$$N_5(\xi,\eta) = 0{,}5.(1+\xi-\eta^2-\xi\eta^2) \quad (8b)$$

$$N_5(\xi,\eta) = 0{,}5.(1-\eta-\xi^2+\xi^2\eta) \quad (8c)$$

$$N_5(\xi,\eta) = 0{,}5.(1+\xi-\eta^2-\xi\eta^2) \quad (8d)$$

The advantage of the isoparametric elements is their ability to transform a unit square element to any shape, including a triangle, rectangle or a circle approximation [13], [20].

8-node element

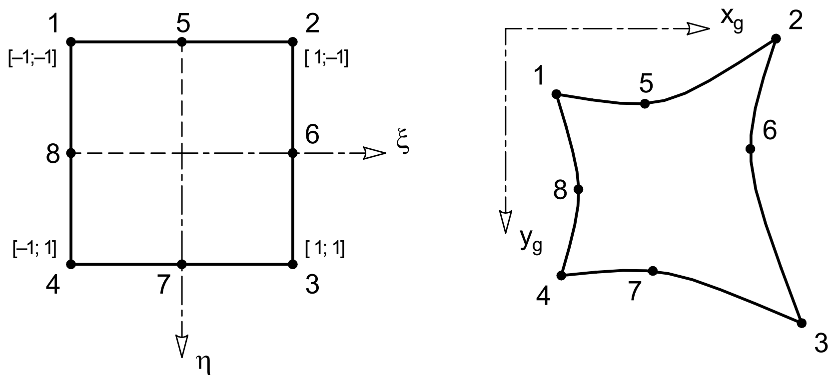


Figure 2: Transformation of the unit element into a curvilinear 8-node element

4 Calculating the stress by the numerical integration

In order to avoid a demanding search for analytical functions that would meet requirements applicable to general integrals that need to be solved in order to calculate the stress components, numerical integration is used. Because rather numerous integration points and high accuracy are needed, the best solution seems to be the *Gauss's quadrature formulae* [4], [6], [13]. In order to calculate the load p_z (x,y) on the surface of the half-space caused by the contact stress, σ_c (x,y), under a slab element, it would be recommended again to use the shape functions, N_i, however, with the x,y variables. If the magnitude of the contact stress in the nodal points is known, the development of the contact stress inside the element can be approximated.

$$p_z(x,y) \equiv \sigma_c(x,y) = \sum_{i=1}^{r} N_i(x,y)\cdot\sigma_{ci} \quad (9a)$$

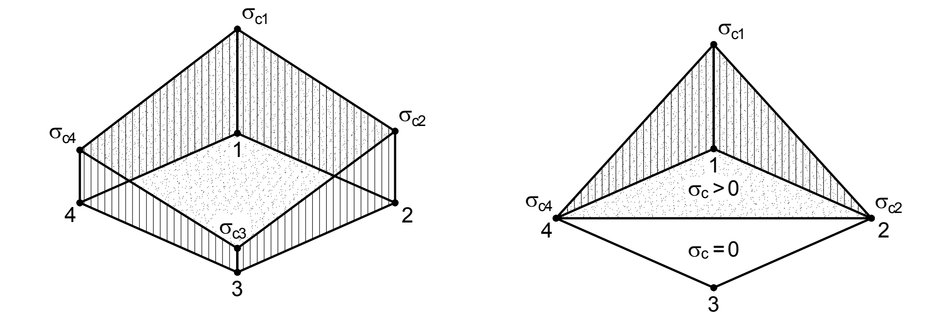
where σ_{ci} is the contact stress in the i^{th} node of the element,
r is the number of nodes in the element.

The horizontal load can be solved by the same way [13], [17], [24].

$$p_x(x,y) \equiv \tau_{cx}(x,y) = \sum_{i=1}^{r} N_i(x,y) \cdot \tau_{cxi} \tag{9b}$$

$$p_y(x,y) \equiv \tau_{cy}(x,y) = \sum_{i=1}^{r} N_i(x,y) \cdot \tau_{cyi} \tag{9c}$$

4-node element – contact pressure



8-node element – contact pressure

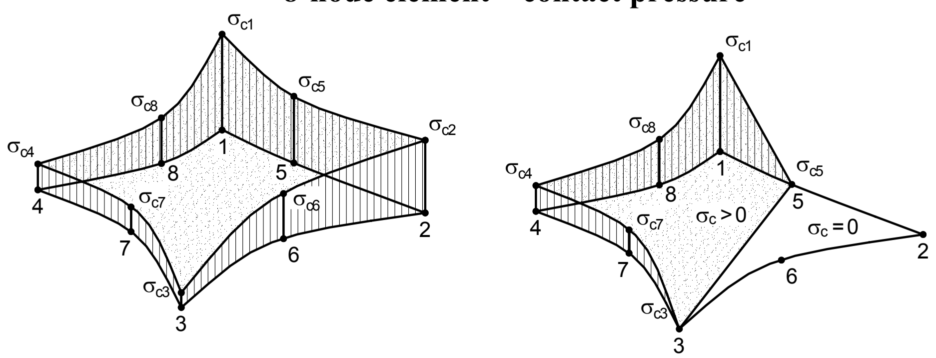


Figure 3: Development of the contact stress in the element

The spatial integral can be calculated by the numerical integration without any major difficulties only if the loading surface is a rectangle. In case of other shapes (including a triangle which is often used in FEM), the calculation is somewhat complicated because integration limits are variable. The solution which eliminates such drawbacks is the use of transformation relations by means of the *transformation Jacobian.* Using this method, any component of the state of stress can be calculated in a homogeneous elastic half-space under any load surface. This solution eliminates difficulties faced so far during attempts to apply the norm model [1], [2], [3] of the subsoil in the FEM interaction tasks. The use of isoparametric elements is becoming now more and more important.

If the rule on transformation of the coordinates is used, the integral can be transformed into variables ξ, η. The differential of the area can be expressed as follows:

$$dA = dx \cdot dy = \det[J] \cdot d\xi \cdot d\eta \tag{10}$$

where [J] is the *Jacobian functional matrix*

$$[J]=\begin{bmatrix}\dfrac{\partial x}{\partial\xi} & \dfrac{\partial y}{\partial\xi}\\ \dfrac{\partial x}{\partial\eta} & \dfrac{\partial y}{\partial\eta}\end{bmatrix} \tag{11}$$

and its determinant is the *Jacobian of transformation*

$$\det[J]=\left|\partial(x,y)/\partial(\xi,\eta)\right| \tag{12}$$

Having substituted the variables and having introduced new integration limits, the integral can be transformed to [13]:

(13a)

$$\sigma_x=\int_{-1}^{1}\int_{-1}^{1}\frac{p_z(\xi,\eta)}{2\cdot\pi}\cdot\left\{\frac{3\cdot\xi^2\cdot z}{r^5}-(1-2\nu)\cdot\left[\frac{r^2+r\cdot z-z^2}{r^3\cdot(r-z)}-\frac{\xi^2\cdot(2r-z)}{r^3\cdot(r-z)^2}\right]\right\}\cdot\det[J]\cdot d\xi\cdot d\eta$$

(13b)

$$\sigma_y=\int_{-1}^{1}\int_{-1}^{1}\frac{p_z(\xi,\eta)}{2\cdot\pi}\cdot\left\{\frac{3\cdot\eta^2\cdot z}{r^5}-(1-2\nu)\cdot\left[\frac{r^2+r\cdot z-z^2}{r^3\cdot(r-z)}-\frac{\eta^2\cdot(2r-z)}{r^3\cdot(r-z)^2}\right]\right\}\cdot\det[J]\cdot d\xi\cdot d\eta$$

$$\sigma_z=\int_{-1}^{1}\int_{-1}^{1}\frac{p_z(\xi,\eta)}{2\cdot\pi}\cdot\frac{3\cdot z^3}{r^5}\cdot\det[J]\cdot d\xi\cdot d\eta \tag{13c}$$

$$\tau_{yz}=\int_{-1}^{1}\int_{-1}^{1}\frac{p_z(\xi,\eta)}{2\cdot\pi}\cdot\frac{3\cdot\eta\cdot z^2}{r^5}\cdot\det[J]\cdot d\xi\cdot d\eta \tag{13d}$$

$$\tau_{zx}=\int_{-1}^{1}\int_{-1}^{1}\frac{p_z(\xi,\eta)}{2\cdot\pi}\cdot\frac{3\cdot\xi\cdot z^2}{r^5}\cdot\det[J]\cdot d\xi\cdot d\eta \tag{13e}$$

$$\tau_{xy}=\int_{-1}^{1}\int_{-1}^{1}\frac{p_z(\xi,\eta)}{2\cdot\pi}\cdot\left[\frac{3\cdot\xi\cdot\eta\cdot z}{r^5}-(1-2\nu)\cdot\frac{\xi\cdot\eta\cdot(z-2r)}{r^3\cdot(r-z)^2}\right]\cdot\det[J]\cdot d\xi\cdot d\eta \tag{13f}$$

where

$$r^2=\xi^2+\eta^2+z^2 \tag{14}$$

A good solution is to use the *Gauss' quadrature formulae* when calculating numerically the integral to the *"unit"* square areas. When calculating the stress components, the integral is transformed to a double summation where ξ, η are replaced with the respective integration points ξ_p, η_q

$$\sigma_x=\sum_{p=1}^{n}\sum_{q=1}^{n}\alpha_p\cdot\alpha_q\,\frac{p_z(\xi_p,\eta_q)}{2\cdot\pi}\cdot\left\{\frac{3\cdot\xi_p{}^2\cdot z}{r^5}-(1-2\nu)\cdot\right. \tag{15a}$$

$$\left.\cdot\left[\frac{r^2+r\cdot z-z^2}{r^3\cdot(r-z)}-\frac{\xi_p^2\cdot(2r-z)}{r^3\cdot(r-z)^2}\right]\right\}\cdot\det\left[J(\xi_p,\eta_q)\right]$$

$$\sigma_y = \sum_{p=1}^{n}\sum_{q=1}^{n}\alpha_p \cdot \alpha_q \frac{p_z(\xi_p, \eta_q)}{2 \cdot \pi} \cdot \left\{ \frac{3 \cdot \eta_q{}^2 \cdot z}{r^5} - (1-2\nu) \cdot \right. \tag{15b}$$

$$\left. \cdot \left[\frac{r^2 + r \cdot z - z^2}{r^3 \cdot (r-z)} - \frac{\eta_q^2 \cdot (2r-z)}{r^3 \cdot (r-z)^2} \right] \right\} \cdot \det\left[J(\xi_p, \eta_q)\right]$$

$$\sigma_z = \sum_{p=1}^{n}\sum_{q=1}^{n}\alpha_p \cdot \alpha_q \cdot \frac{p_z(\xi_p, \eta_q)}{2 \cdot \pi} \cdot \frac{3 \cdot z^3}{r^5} \cdot \det\left[J(\xi_p, \eta_q)\right] \tag{15c}$$

$$\tau_{yz} = \sum_{p=1}^{n}\sum_{q=1}^{n}\alpha_p \cdot \alpha_q \cdot \frac{p_z(\xi_p, \eta_q)}{2 \cdot \pi} \cdot \frac{3 \cdot \eta_q \cdot z^2}{r^5} \cdot \det\left[J(\xi_p, \eta_q)\right] \tag{15d}$$

$$\tau_{zx} = \sum_{p=1}^{n}\sum_{q=1}^{n}\alpha_p \cdot \alpha_q \cdot \frac{p_z(\xi_p, \eta_q)}{2 \cdot \pi} \cdot \frac{3 \cdot \xi_p \cdot z^2}{r^5} \cdot \det\left[J(\xi_p, \eta_q)\right] \tag{15e}$$

$$\tau_{xy} = \sum_{p=1}^{n}\sum_{q=1}^{n}\alpha_p \cdot \alpha_q \cdot \frac{p_z(\xi_p, \eta_q)}{2 \cdot \pi} \cdot \left[\frac{3 \cdot \xi_p \cdot \eta_q \cdot z}{r^5} - (1-2\nu) \cdot \frac{\xi_p \cdot \eta_q \cdot (z-2r)}{r^3 \cdot (r-z)^2} \right] \cdot \det\left[J(\xi_p, \eta_q)\right] \tag{15f}$$

Another advantage is also that it is simpler to calculate now the development of the contact stress for the "unit" square only, without any transformation to the real loading surface. For this reason, the shape functions $N(\xi_p, \eta_q)$ can be used directly in the calculation [10], [11], [13] .

5 Example and Comparison of Calculating Stress

In order to check whether it would be a good solution to use the numerical integration for the practical calculation of the stress components caused by the general load of the half-space surface, certain examples have been calculated and the results have been confronted with the known analytical solution. The accuracy of the calculated stress, σ_Z, has been tested for different numbers of the integration points and for different depths of the circular areas or square/triangle areas of the elastic half-spaces subject to an even load in corners. Correctness of the *Jacobian transformation* has been verified as well [10], [11], [13], [20].

The results were, for example, also tested for the circular area loaded by the unit load of $p_Z = 1.0$ MPa. The circular area is approximated by an 8-node isoparametric element with the diameter of r = 1.0 where the circle coordinates are met for each nodal point, Figure 4. The results in [13] show the solutions for 2, 4, 6 and 8 Gauss' integration points. For purposes of comparison, the calculation has been made for 96 integration points and the exact solution [13].

The accuracy is higher if there are more integration points and if the depth is higher. But in this case, the results do not match exactly with those calculated in the numerical integration, even for 96 integration points. The reason is that the circular border is replaced with the 2nd order curve in line with approximation of the side of the 8-node isoparametric element [13], [20].

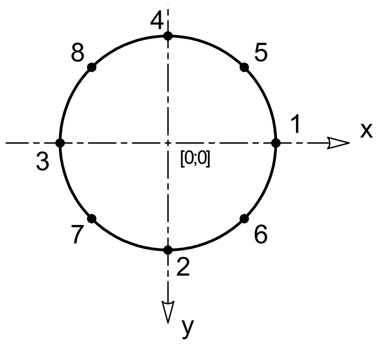


Figure 4: Coordinates of the approximated circular area

The exact magnitude of the circular area is

$$A_{teor} = \pi \cdot 1{,}0^2 = 3{,}141593 \tag{16a}$$

while the integrated area of the circle approximated by the 8-node element is

$$A_{aprox} = 3{,}104569 \tag{16b}$$

The loading area is smaller with the numerical integration. It results also in a little lower values of the calculated stress, if compared with the accurate solution, see Figure 5.

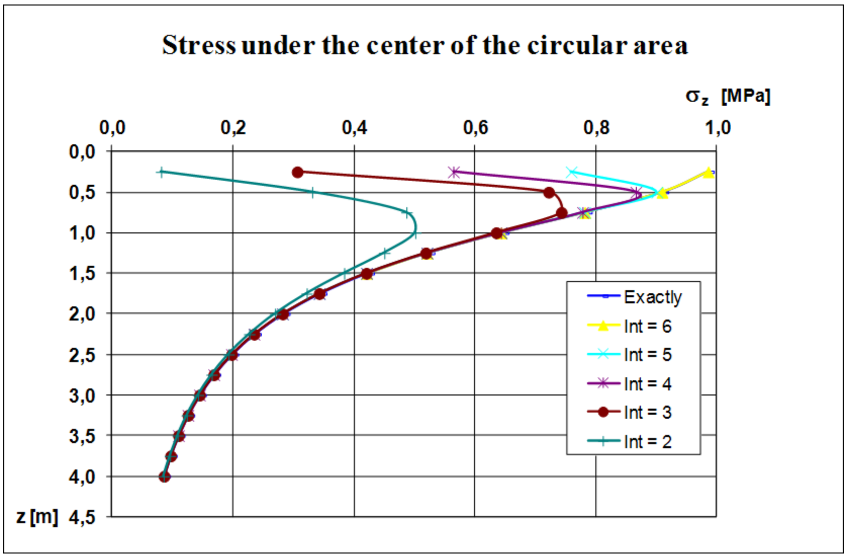


Figure 5: Results of the numerical integration in the centre of the circular area

6 Subsoil model and contact element

When calculating settlement with respect to structure strength of earth pursuant to the ČSN 73 1001 [1] or ČSN EN 1997-1 [3], a definite integral in the <0,z_z> interval can be used as follows [13]:

$$s_z(x,y) = \int_0^{z_z} \frac{\sigma_z(x,y,z)}{E_{oed}(x,y,z)} \cdot dz - \int_0^{z_z} \frac{m(x,y,z)\cdot\sigma_{or}(x,y,z)}{E_{oed}(x,y,z)} \cdot dz = s_{z,ol} - s_{z,or} \quad (17)$$

$z_z(x,y)$ deformation zone depths need to be defined considering the condition that the $\sigma_z(x,y,z_z)$ resultant vertical stress on the lower edge of the deformation zone is zero. This means

$$\sigma_z(x,y,z_z) = \sigma_{ol}(x,y,z_z) - m(x,y)\cdot\sigma_{or}(x,y,z_z) = 0 \quad (18)$$

The non-linear equation above is to be solved numerically. The interval halving method seems to be appropriate in this case.

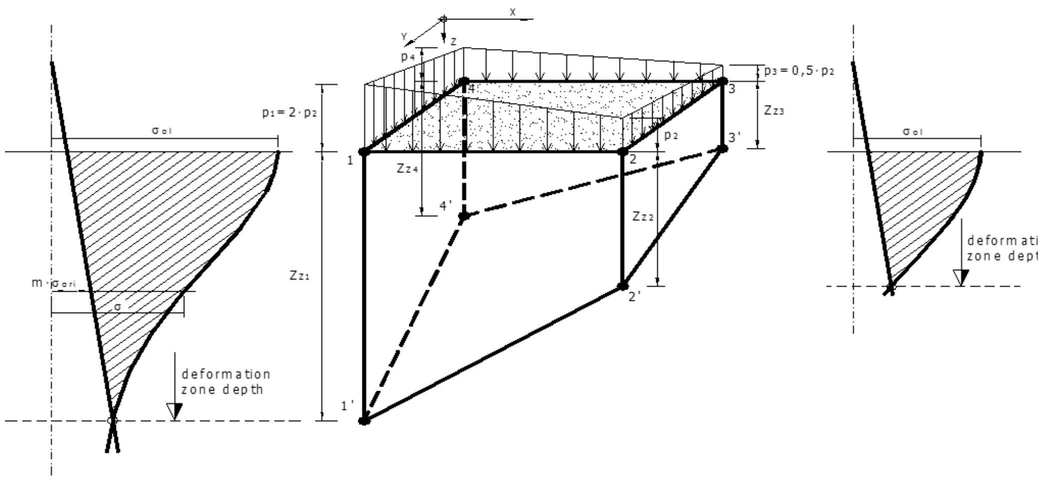


Figure 6: Isoparametric contact element with 4 nodes and active zone determination

The isoparametric element soil stiffness matrix has been derived by using an identical approach as in the case of the element stiffness matrix [4], [9], [11], [13].

$$[K_{ep}] = \sum_{p=1}^{n}\sum_{q=1}^{n} \alpha_p \cdot \alpha_q \cdot [N(\xi_p,\eta_q)]^T \cdot [C(\xi_p,\eta_q)] \cdot [N(\xi_p,\eta_q)] \cdot \det[J(\xi_p,\eta_q)] \quad (19)$$

For the plane element, the matrix of contact functions [C] can be expressed pursuant to [13] as:

$$[C] = \begin{bmatrix} C_{1x} & 0 & 0 \\ 0 & C_{1y} & 0 \\ 0 & 0 & C_{1z} \end{bmatrix} \quad (20)$$

Where

$$C_{1x} = \sum_{i=1}^{r} N_i(\xi,\eta).C_{1x,i}\,,\; C_{1y} = \sum_{i=1}^{r} N_i(\xi,\eta).C_{1y,i}\,,\; C_{1z} = \sum_{i=1}^{r} N_i(\xi,\eta).C_{1z,i} \quad \text{(21a, 21b, 21c)}$$

The friction parameters C_{1x}, C_{1y}, and stress parameter C_{1z}, represent the contact functions that can be, similarly as the contact stress and shear stresses in the iso-parametric plane element, approximated using the shape functions N_i and functional values $C_{1z,i}$, $C_{1x,i}$ or $C_{1y,i}$ in individual nodes [10], [11], [13].

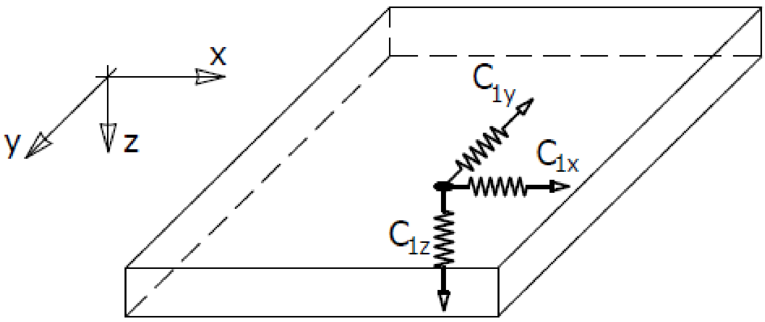


Figure 7: Scheme of nodal parameters C_x, C_y and C_z

The values of the contact functions can be determined for the individual node elements in FEM as the stress or share in the footing bottom and deformation of the node [13], [16], [17], [18], [19].

$$C_{1x,i} = \frac{\tau_{xz,i}}{s_{xi}}\,, \quad C_{1y,i} = \frac{\tau_{yz,i}}{s_{yi}}\,, \quad C_{1z,i} = \frac{\sigma_{z,i}}{s_{zi}} \qquad \text{(22a, 22b, 22c)}$$

When calculating an interaction task, it is essential to define such course of the contact stress that could result in the same deformation of the soil and the plate [11], [18]. Since the relation between the modified elastic half-space load and modified elastic half-space settlement is that of non-linearity, FEM non-linear methods need to be employed to solve the mentioned task. The iteration method seems to be the best choice in this case, since it converges towards a technically accurate solution as early as after approximately 7-8 iteration steps [10], [11], [13]. An advantage of the suggested iteration solution used to solve the non-linear task consists in a possibility of checking the plate structure stiffness with respect to the cracking limit [22]. The method also enables the limit contact stress to be checked in the footing bottom, eliminating thus tensile stress between the plate and soil (so-called one-sided bonds).

7 Example of Soil – Circular Plate Interaction

An example of circular plate based on *Mindlin theory* [5], [10], [11], [13] has been used as the one of a number of comparison examples [13], [20]. A circular plate having a radius of 1 m and parameter of thickness h = 0.1 m, concrete modulus of elasticity $E_c = 22.95.10^3$ MPa, $\nu_c = 0.2$ is placed on soil, $E_p = 5$ MPa, $\nu_p = 0.4$, $\gamma = 19$

kN.m^{-3}. The surcharge coefficient, m = 0.2, has been determined pursuant to the ČSN 73 1001 [1] or ČSN EN 1997-1 [3] standard. The whole plate is subject to an even, continuous load, p_z = 100 kPa. Figures 8 to 9 give contact stress values depending on a number of iteration steps and integration points.

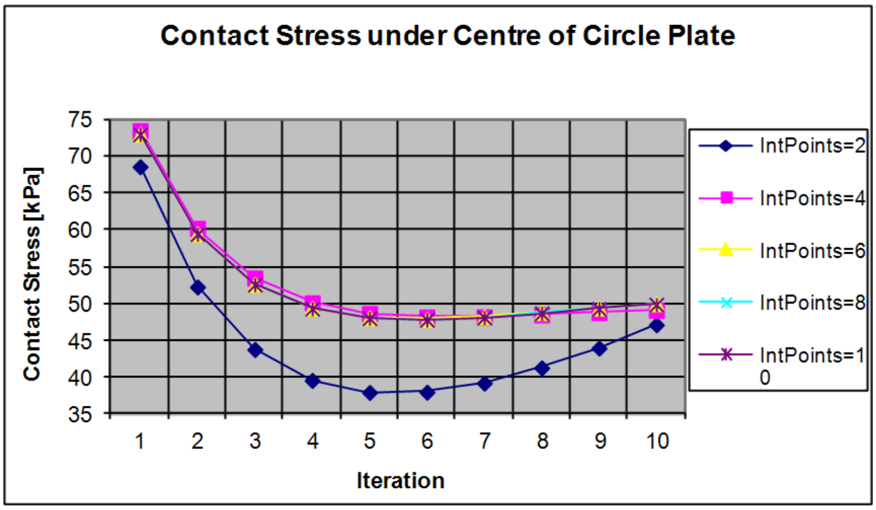


Figure 8: Contact stress dependency on integration points and number of iteration

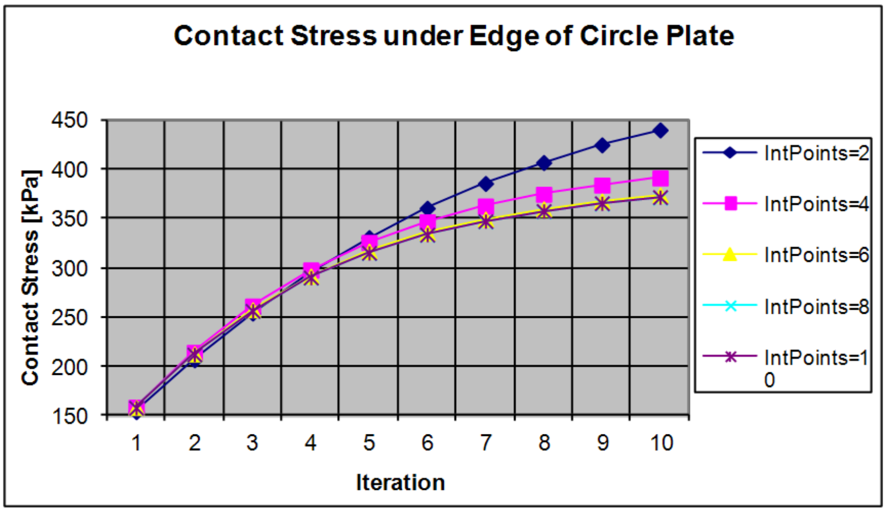


Figure 9: Contact stress dependency on integration points and number of iteration

Should contact stress under foundation corners exceed the plastic area creation limit in the soil, stress in the footing bottom shall redistribute, deformation in the foundation shall change and internal forces in the foundation structure shall also redistribute. The resulting settlement and deformation of the plate for 0th and 12th iteration step is on Figure 10. New possibilities of FEM solution together with simulation based reliability assessment and decreasing the time for solvers and integration procedures are offered by the methods of parallel programming [21], [23].

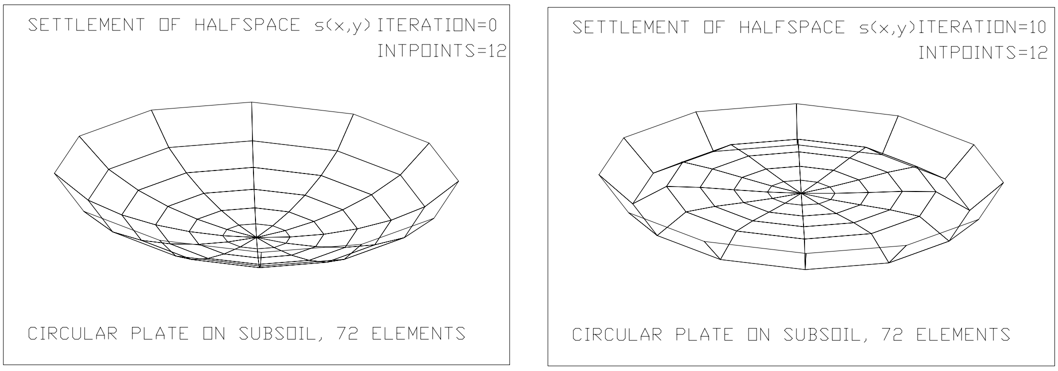


Figure 10: Settlement course s(x,y) of half-space under circular plate with 72 elements, 0th and 12th iteration with 12 integration points

8 Conclusion

This solution eliminates the difficulties faced so far during attempts to apply the reference model of the subsoil in the finite element interaction tasks [9], [14], [15], [25]. The suitability of the numerical integration for the practical calculation of the stress components resulting from the general load of the half-space surface was verified with particular examples and the results were compared with the known solution [10], [11], [13], [20].

The accuracy of the calculated stress has been tested for different numbers of the integration points and for different depths of the circular, square or triangular areas of the elastic half-spaces subject to an even load in the corners. The correctness of the Jacobian transformation has also been verified.

This approach can be used to calculate each of six components of the state-of-stress tensor in the elastic half-space at any point loaded by a general load and for any shape of the load surface, which is defined by the network of four-node or eight-node isoparametric elements. The final effects of the vertical load by P_z (e.g., effects of pressure by vehicle wheels or by vertical contact stress in the footing bottom) [10], [11], [13] and horizontal loads by P_x , P_y (e.g. the breaking and start-off force or friction in the footing bottom) [15], [16], [17], [19], [24] can be superposed or linearly combined. In general, this approach can be used to describe the effects of any P force which acts at an angle, since it can be always resolved into the components P_x, P_y and P_z. This approach has been successfully applied for interactions between the foundation and subsoil [10], [11], [13], [20]. It can be, however, used for other geotechnical tasks [12], [15], [25], [26] and is suitable model for slide joints [24].

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