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Extended Isogeometric Analysis of Plates with Curved Cracks

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Abstract

Fracture analysis of an isotropic two-dimensional body with a curved crack using the extended isogeometric analysis (XIGA) is investigated. XIGA is a newly developed numerical approach which benefits from the advantages of the isogeometric analysis method and the extended finite element method [1, 2]. It has been successfully applied for analysis of cracked bodies where the straight crack remains straight in the parametric space. In this paper, XIGA is extended for fracture analysis of curved cracks by adopting specific mapping techniques. Sub-traingles and almost polar techniques alongside applying blending function method are employed for integration over elements intersected by the crack. The accuracy of the proposed approach is investigated for a plate with an arc-shaped central crack with different arc angles.

Keywords: extended isogeometric analysis, curved crack, enrichment functions, linear elastic fracture mechanics, mapping, stress intensity factor.

1 Introduction

Fracture analysis of an isotropic two-dimensional body with a curved crack using the extended isogeometric analysis (XIGA) is discussed in this article.

Crack propagation simulation in structural analysis using novel numerical methods has been an active research topic for the past two decades due to the existence of singularity at the crack tip and remeshing necessity to accomodate the evolving geometry. Therefore, a new generation of numerical methods have been developed, including meshfree methods [3-12] and the extended FEM (XFEM) [13-18] which belongs to the class of partition of unity methods (PUM). One of the principal features of these methods is their capability in the analysis of moving discontinuous problems such as crack propagation, without remeshing or rearranging of the nodal points. In the XFEM, a priori knowledge of the solution is locally added to the approximation space. This enrichment allows for capturing particular features such as discontinuities and singularities which are present in the solution exactly.

On the other hand, isogeometric analysis (IGA) is another robust computational approach recently proposed by Hughes *et al.* [19, 20]. This method offers the possibility of integrating the finite element analysis (FEA) into conventional NURBS-based CAD design tools. In general, it is necessary to convert data between CAD and FEA packages to analyze new designs during the development. IGA employs complex NURBS geometry (the basis of most CAD packages) in the FEA software directly. This allows for models to be designed, tested and adjusted in one go, using a common data set.

The isogeometric formulation using NURBS basis functions has been recently enriched via XFEM to solve linear fracture mechanics problems with incompatible meshes while obtaining solutions with higher order convergence rates and high levels of accuracy [1, 2]. This approach, entitled the extended isogeometric analysis (XIGA) [2], benefits from the advantages of its origins: XFEM and IGA; while it is capable of analyzing crack growth problems without any remeshing requirement, complex geometries can be modeled with few elements and higher order inter-element continuities are satisfied.

XIGA has been successfully employed for analysis of bodies with straight cracks in the parametric space and parent element. For improving the accuracy of integration by the Gauss quadrature rule, the "sub-triangles approach" and the "almost polar technique" have been utilized for split and crack tip elements, respectively [2]. The principal difficulty with curved cracks is the special treatment required for the integration of sub-triangles which one of its sides is not straight in the parametric space.

In this contribution, specific mappings based on the blending function method are used for integration over the elements cut by a non-straight part of the crack, and is extended to analysis of structures having curved cracks in the parametric space. Consequently, no limitation remains for considering arbitrary non-smooth and smooth strong discontinuities in the XIGA.

In order to impose Dirichlet boundary conditions, the Lagrange multiplier method is used. Mixed-mode stress intensity factors (SIFs) are evaluated by means of the interaction integral to determine the fracture properties of domain.

Finally, a plate with an arc-shaped central crack with different arc angles is analyzed by the proposed method. Comparison of the numerical results with analytical solutions available in the literature shows the efficiency and validity of the present approach.

2 Extended isogeometric analysis (XIGA)

isogeometric analysis (IGA) [19, 20] has recently been enriched using superior concepts of the extended finite element method (XFEM) [13, 14] for analysis of isotropic cracked problems [1, 2]. In this new extended isogeometric analysis (XIGA) [2], classical IGA space is extrinsically enriched by some additional functions. These functions are obtained from the product of global enrichment functions and some classical NURBS functions. In this case, discontinuous problems can be efficiently analyzed so that the remeshing necessity is vanished in moving discontinuous problems, such as crack propagation simulations. In XIGA, some basis functions are selected to be enriched by the Heaviside function for modeling crack edges, and by crack tip enrichments for improving the accuracy of solution field near the crack tip. In XIGA, the solution field is approximated in the form of

$$\begin{aligned}\mathbf{u}^h\left(\xi^1,\xi^2\right) = \textstyle\sum_{i=1}^{n_{en}} R_i^{p,q}\left(\xi^1,\xi^2\right)\mathbf{u}_i + \sum_{j=1}^{n_H} R_j^{p,q}\left(\xi^1,\xi^2\right) H\mathbf{a}_j \\ + \textstyle\sum_{k=1}^{n_Q} R_k^{p,q}\left(\xi^1,\xi^2\right)\sum_{\alpha=1}^{4} Q_\alpha \mathbf{b}_k^\alpha\end{aligned} \tag{1}$$

where $\{R_i^{p,q}\left(\xi^1,\xi^2\right)\}$ are the NURBS basis functions of orders p and q in ξ^1 and ξ^2 directions, respectively, at the point (ξ^1,ξ^2) in the parametric space $[0,1]\times[0,1]$. $\{\mathbf{a}_j\}$ are the vectors of additional degrees of freedom which are related to the modeling of crack faces, $\{\mathbf{b}_k^\alpha\}$ are the vectors of additional degrees of freedom for modeling the crack tip, n_{en} is the number of nonzero basis functions for a given knot span, n_H is the number of basis functions that have crack face (but not crack tip) in their support domain and n_Q is the number of basis functions associated with the crack tip in their influence domain. H is the generalized Heaviside function [21],

$$H\left(\mathbf{X}\right) = \begin{cases} +1 & \text{if } \left(\mathbf{X}-\mathbf{X}^*\right).\mathbf{e}_n > 0 \\ -1 & \text{otherwise} \end{cases} \tag{2}$$

where $\mathbf{e}_n$ is the unit normal vector of crack alignment in point $\mathbf{X}^*$on the crack surface which is the nearest point to $\mathbf{X}\left(\xi^{\mathbf{1}},\xi^{\mathbf{2}}\right)$.

In eq. (1), $Q_\alpha\,\{\alpha=1,2,3,4\}$ are the crack tip enrichment functions whose roles are reproducing the singular filed around crack tips,

$$\{Q_\alpha\}_{\alpha=1}^4 = \left\{\sqrt{r}\sin\frac{\theta}{2},\sqrt{r}\cos\frac{\theta}{2},\sqrt{r}\sin\theta\sin\frac{\theta}{2},\sqrt{r}\sin\theta\cos\frac{\theta}{2}\right\} \tag{3}$$

where (r,θ) are the local crack tip polar coordinates with respect to the tangent to the crack tip in the physical space.

Readers are referred to [2] for more information about XIGA formulation and implementation.

3 Numerical integration

The Gauss quadrature rule is utilized for numerical integration. For integrating over elements which are not cut by part of a crack, the usual procedure for isogeometric analysis [19, 20] is employed. Other elements require special techniques for increasing the accuracy of numerical integration because of existence of discontinuity within them. In this contribution, “sub-triangles” and “almost polar” techniques are applied for split and crack tip elements, respectively.

Firstly, it is needed to map the crack curve from physical space to the parametric space. A simple way is to consider sufficient points on the crack and map them to the parametric space using a conventional technique for solving nonlinear algebraic equations such as the Newton-Raphson method. Then, by applying the B-Spline/NURBS curve fitting, one can obtain the related knot vector and control points of the crack curve in the parametric space (see [22]).

Elements cut by the curved crack are split into sub-triangles where some of them have a curved edge. It is noted that subdivision is only applied to perform the numerical integration and no new degrees of freedom are introduced in such elements. In addition, an element may contain breakpoints (the points that correspond to the knots used for crack definition in the parametric space) inside its curved edge. These points represent continuity reduction points and should be taken into account in the subdivision process. Figure 1 schematically shows such a subdivision procedure in the parametric space.

The transformation required for mapping between spaces are shown for some sub-triangles in Figure 1. Transformation T_1 maps the parametric space into the physical space. T_2 transforms the standard triangle parent element to a straight-sided triangle in the parametric space while $T_2^{'}$ transform the standard triangle parent element into a triangle with a curved edge in the parametric space, as illustrated in Figure 2. By using the isoparametric mapping and the blending function method, transformations T_2 and $T_2^{'}$ are defined as [23, 24],

$$T_2 : \boldsymbol{\xi} = \left(1 - \overline{\xi^1} - \overline{\xi^2}\right) \boldsymbol{\xi}_1 + \overline{\xi^1}\boldsymbol{\xi}_2 + \overline{\xi^2}\boldsymbol{\xi}_3 \tag{4}$$

$$T_2^{'} : \boldsymbol{\xi} = \frac{1 - \overline{\xi^1} - \overline{\xi^2}}{1 - \overline{\xi^1}} \mathbf{C}\left(\lambda\left(\overline{\xi^1}\right)\right) + \frac{\overline{\xi^1 \xi^2}}{1 - \overline{\xi^1}}\boldsymbol{\xi}_2 + \overline{\xi^2}\boldsymbol{\xi}_3 \tag{5}$$

where

$$\lambda\left(\overline{\xi^1}\right) = \lambda_1 + (\lambda_2 - \lambda_1)\,\overline{\xi^1} \tag{6}$$

λ_1 and λ_2 are the parametric values of the curve defined in the parametric space, corresponding to the points $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$, respectively.

In order to adopt the “almost polar technique”, an additional transformation T_3 needs to be applied for the sub-triangles, where one of their vertices coincides with the crack tip,

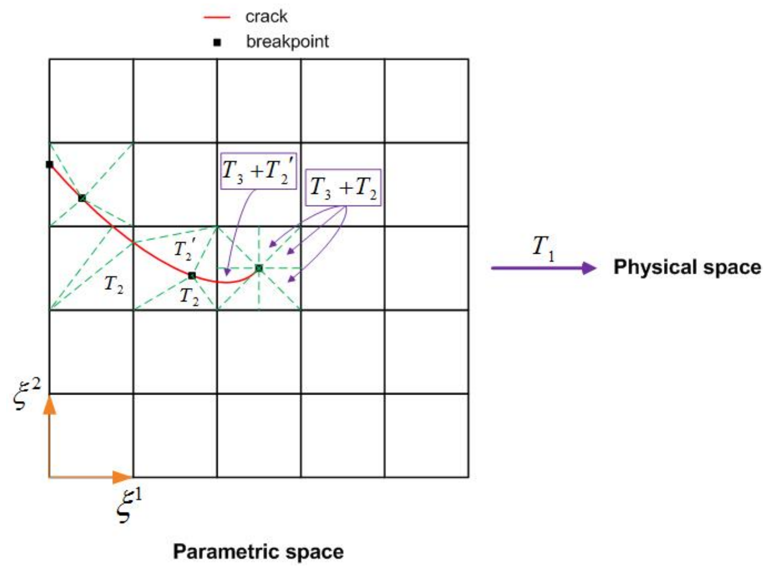


Figure 1: Sub-triangulation in the parametric space and transformation required for mapping between spaces.

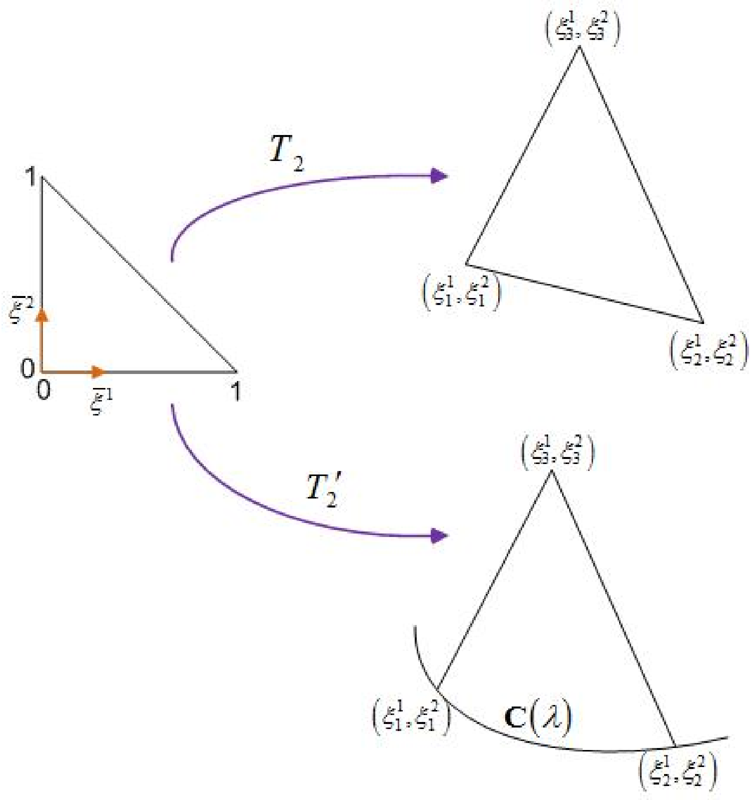


Figure 2: Transformations from the standard triangular parent element into straight-sided and one curved edge triangles in the parametric space.

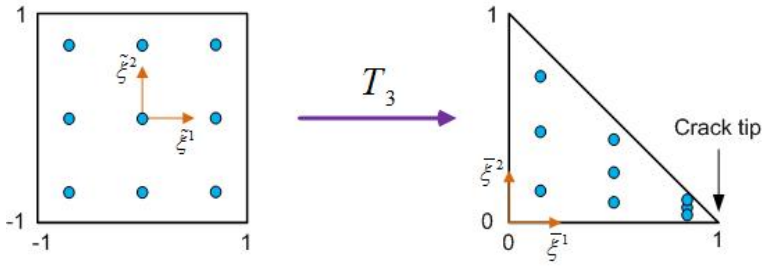


Figure 3: Transformation T_3 from a square into a triangle with the crack tip on its vertex.

$$T_3 : \begin{array}{c} \overline{\xi^1} = \frac{1}{2}\left(1 + \tilde{\xi^1}\right) \\ \overline{\xi^2} = \frac{1}{4}\left(1 - \tilde{\xi^1} + \tilde{\xi^2} - \tilde{\xi^1}\tilde{\xi^2}\right) \end{array} \tag{7}$$

4 Numerical simulation

An arc-shaped crack in an infinite plate under uniaxial tension is considered. A finite plate model, sufficientlty large with respect to the crack length is simulated, as shown in Figure 4. The analytical stress intensity factors, as given in [25], are:

$$\begin{array}{c} K_I = \frac{\sigma_o}{2}\sqrt{\pi R \sin\beta}\left[\frac{\left[1-\sin^2(\beta/2)\cos^2(\beta/2)\right]\cos(\beta/2)}{1+\sin^2(\beta/2)} + \cos\left(3\beta/2\right)\right] \\ K_{II} = \frac{\sigma_o}{2}\sqrt{\pi R \sin\beta}\left[\frac{\left[1-\sin^2(\beta/2)\cos^2(\beta/2)\right]\sin(\beta/2)}{1+\sin^2(\beta/2)} + \sin\left(3\beta/2\right)\right] \end{array} \tag{8}$$

where R is the radius of the circular arc and 2β is the subtended angle of the arc. The computations are performed for $R = 0.25$ and different values ofβ (30°, 45°, 60°).

NURBS basis functions of cubic order are used for analysis of the problem. 4×4 Gauss quadrature, sub-triangles technique with 13 Gauss points in each sub-triangle and almost polar technique with 4×4 in each sub-triangle are used for integration. The Lagrange mutiplier method is utilized for imposition of essential boundary conditions (see [2]). For selection of crack tip enrichment basis functions, the topological enrichment scheme is applied (see [2]).

A structured mesh containing 2209 elements in a narrow band near the crack is used, as illustrated in Figure 5. The mesh size in the vicinity of the crack is $h_e = 10/\left(9 \times 39\right)$, and away from the crack is $h_e = 10/9$. The total number of control points is 2500. The adopted discretization is clearly not the best choice, but in order to obtain an optimal one, error estimation and adaptive procedures are required, which are under development by the authors.

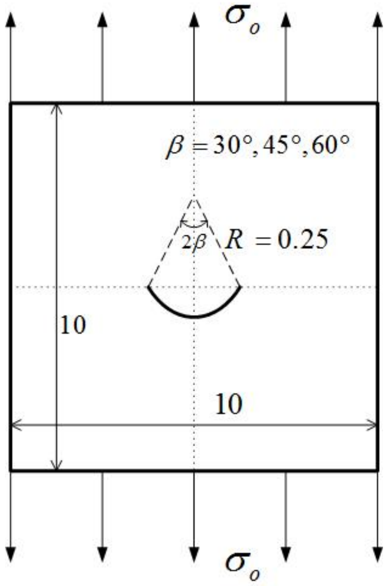
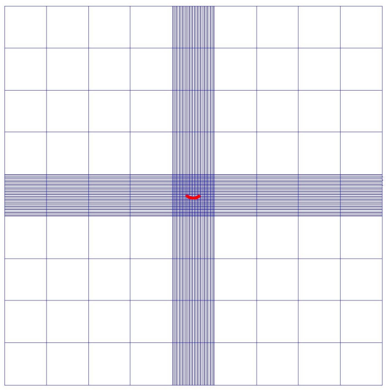
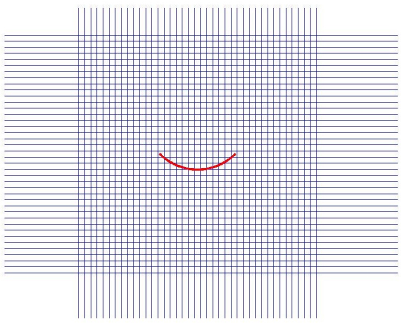


Figure 4: Arc-shaped crack under far-field uniaxial tension.



(a)



(b)

Figure 5: Discretization of the plate: (a) whole view; and (b) near the crack tip.

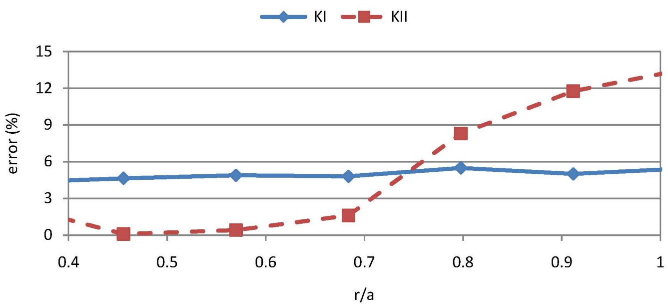


Figure 6: Errors (%) of mixed-mode SIFs for different ratios of interaction integral radius r to $a = R\sin\beta$ when $\beta = 30°$.

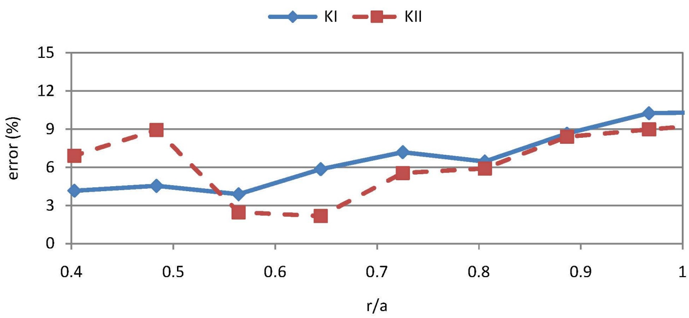


Figure 7: Errors (%) of mixed-mode SIFs for different ratios of interaction integral radius r to $a = R\sin\beta$ when $\beta = 45°$.

The mixed-mode stress intensity factors are computed for different choices of the radii in the domain integral computations and the errors (%) are shown in Figures 6, 7 and 8 for β = 30°, 45° and 60°, respectively. The XIGA results are in good agreement with the reference solution. The use of appropriate path-independent integrals for curved (circular arc- shaped) cracks [26] is required to further improve the SIF computations.

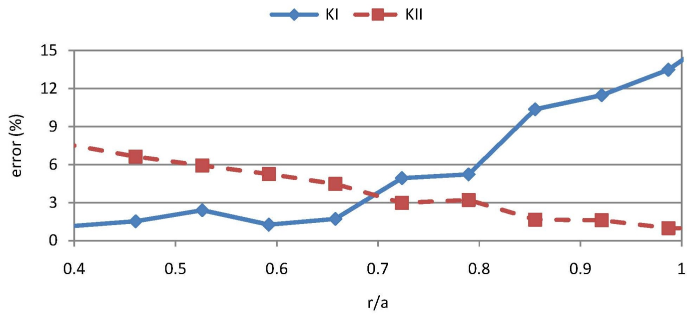


Figure 8: Errors (%) of mixed-mode SIFs for different ratios of interaction integral radius r to $a = R \sin \beta$ when $\beta = 60°$.

5 Conclusion

XIGA has been further extended for fracture analysis of curved-shape cracked bodies. Specific mapping techniques based on the blending function method are applied for more accurete integration over split and crack tip elements where the crack is not straight in the parametric space. As a result, no limitation remains for considering arbitrary non-smooth and smooth strong discontinuities in the XIGA. Results of fracture analysis of the plate containing arc-shaped central crack shows the validity and accuracy of the proposed approach.

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